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Mathematical Models to Understand Sequential Choice Data Derived From the Balloon Analogue Risk Task (BART)

Author: Kilian Sennrich

Immatriculation number: 18-051-060

Correspondence email: kilian.sennrich@stud.unibas.ch

Examiner: Prof. Dr. Jörg Rieskamp & PD Dr. Thorsten Pachur

Supervisor: PD Dr. Thorsten Pachur

Institution: Economic Psychology, Faculty of Psychology, University of Basel

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Declaration of Scientific Integrity

The author hereby declares that she/he has read and fully adhered the Code for Good Practice in Research of the University of Basel.

Abstract

Risky decisions are rarely a one-time event but are often encountered in a repetitive manner. To investigate human behaviour within decisions under risk, the Balloon Analogue Risk Task (BART) has been used in the past, where subjects can earn a reward by inflating a virtual balloon without exploding it. Further, several mathematical models have been proposed that allow to study the underlying cognitive processes of dynamic risk-taking. These models have been discussed and extended in recent years and have also been employed to compare different populations in respect to dynamic risk-taking. This thesis holds a systematic literature search to answer the following question: What cognitive processes have been suggested regarding dynamic risk-taking and how do these processes affect human behaviour? The results show that dynamic risk-taking is essentially based on four cognitive processes: learning, representation of the risk structure of the situation, option evaluation, and response selection. Furthermore, it is shown that especially the sensitivity to payoff gains and the consistency with which a subject follows its own evaluation can be claimed as predictors for negatively associated behaviours.

Keywords

BART, balloon analogue risk task, sequential risk-taking, dynamic risk, cognitive model, computational modelling

Introduction

Risk accompanies humans in every aspect of life. In almost every situation, people make decisions that involve risk. It usually starts in the morning: Should we get up after turning off the alarm clock? Should we close our eyes for another 5 minutes? And continues throughout the day: Shall we eat junk food for lunch? Or shall we stick to a salad? Should we have an after-work beer to reward ourselves for the day? Or should we drink water instead? Further, decisions under risk are not limited to everyday situations. For example, when one decides to invest money on the stock market: How can we increase one's savings through clever investment? What are the possible negative consequences? Or when one decides to participate in unprotected sex: What could be the positive or negative consequences? Even though these situations may be very different in a way, they may rely on rather similar cognitive processes. These common cognitive processes and how they affect human behaviour are what this thesis is about.

Risk plays an essential role in many scientific disciplines. The definition varies greatly depending on the field of application (Schonberg et al., 2011). In cognitive science, the definition of risk as behaviour that can inflict potential harm to oneself or to others has gained acceptance (Mata et al., 2018; Steinberg, 2008). The focus on negative consequences incorporates that potential harm often manifests itself in an externally visible characteristic such as drug addiction or imprisonment. Risk is always related to the decision to obtain potential gains or losses with a certain (or uncertain) probability. Individuals differ in their willingness to accept risk, and these individual differences may also influence the affiliation with certain groups (e.g., drug users, prisoners).

Closely related to a person's willingness to accept risk is the concept of risk preference. Risk preference refers to an individual's tendency to engage in behaviours and activities that lead to personal rewards but may also have serious negative consequences in the long term (Steinberg, 2008). Excessively eating junk food for example, can be perceived as an immediate reward, but may potentially lead to cardiovascular disease in the future.

Risk preference is a dynamic construct that is moderated by situational factors. A person's risk preference can vary greatly in different life domains (Hanoch et al., 2006). Furthermore, situational motivation (Scholer et al., 2010) and emotions (Lerner & Keltner, 2001), as well as the context and the way the situation presents itself (e.g., framing: Tversky & Kahneman, 1986), can influence risk preference. Such factors must be controlled for when comparing the risk preference of different individuals.

Humans often encounter similar risks repeatedly. People who regularly engage in the lottery, for example, are repeatedly faced with the same decision to either invest some money to potentially win millions or to save the money instead. Other examples are buying and selling stocks, diagnosing in medical settings, forecasting the weather, stealing, and using drugs. Decisions under risk are thus rarely a one-time event, but rather a dynamic process in which one is repeatedly confronted with similar decisions. This phenomenon is described here by the term "dynamic risk". The outcome of previous decisions can be used to refine the decision strategy in upcoming decisions and therefore maximize the outcome. For example, when a person invests in the stock market for the first time, sooner or later he or she is likely to lose some money. Since the person wants to maximize gains and minimize losses, this loss will probably lead to an adjustment of the investment strategy. Yet, another cognitive process of decisions under risk becomes obvious, dynamic risk-taking relies to a large extent on learning processes. Knowledge about the outcome in previous decisions flows into upcoming decisions.

Even though a lot of research has been done on different aspects of learning, there are not yet many theories that deal with learning in dynamic risk-taking environments (Wallsten et al., 2005). An approach that gained popularity in recent years is to adapt Bayesian updating/learning in this regard (e.g. Wagenmakers et al., 2018; Wallsten et al., 2005). Bayesian learning goes back to Bayes' theorem, which determines the probability for an event when information about that event is already known (conditional probability). In Bayesian learning theory, assumptions about an upcoming event are modelled with the prior distribution. After the event, the knowledge gained about the event is combined with the assumptions of the prior distribution, which produces the posterior distribution. The principle of Bayesian learning now states that the posterior distribution can be taken as the prior distribution for the second event. A detailed description of Bayesian Learning can be found in the theory section (Bayesian Learning).

Dynamic risk-taking seems to involve more than one cognitive process. It therefore appears inadequate attempting to measure it as an overarching construct. Thus, the question arises, how to examine these underlying cognitive processes separately. One answer to the question relies on computational modelling. Computational modelling is a technique to understand behavioural data using mathematics, physics and computer science (National Institute of Biomedical Imaging and Bioengineering, 2020; Wilson & Collins, 2019). With the aid of mathematical equations, complex phenomena (such as dynamic risk) can consequently be broken down to their underlying processes. Computational models are developed by first making assumptions about the process to be modelled. These assumptions are then incorporated into a model using mathematical expressions. In fact, there are no mathematical procedures that can causally test or verify assumptions. Hence, computational modelling relies on model comparisons for their validation (Cavagnaro et al., 2013; Lewandowsky & Oberauer, 2018). In other words, models with different assumptions are benchmarked against each other for their explanatory power on behavioural data. The assumptions of the model that best fits the data, i.e., that has prevailed over the other models, are then accepted as the most probable (further details on mathematical modelling and model comparison are described in the theory section).

Mathematical models alone do not provide insight into cognitive processes. They must first be fitted to behavioural data. Behavioural data can take the form of decisions, reaction times and other observable behaviours and are usually obtained from experiments or tasks (Wilson & Collins, 2019). These tasks/experiments must simulate a situation in which the cognitive processes to be described by the models are elicited. Many experimental designs have been proposed to simulate dynamic risk scenarios (Bechara et al., 1994; Grant et al., 2000; Rogers et al., 1999), but most of them could only show a low correlation (convergent validity) with self-assessments in risky situations (e.g. Mitchell, 1999; Petry, 2001). However, this is not the case with the Balloon Analogue Risk Task (BART) developed by Lejuez et al. (2002). In the BART, the subject is confronted with a virtual balloon that needs to be inflated. With each decision to inflate the balloon further, the risk for an explosion increases. After every pump, the subject receives a small amount of money credited to a temporary bank, which is transferred to a permanent account when the subject decides to stop pumping. However, if the balloon bursts, the amount in the temporary bank is deleted and the subject goes away empty-handed (a detailed elaboration of the BART can be found in the theory section). Subjects learn to dynamically optimize their risk strategy from balloon to balloon.

Several models have been proposed to describe the cognitive processes underlying dynamic risk-taking in the BART (Rolison et al., 2012; van Ravenzwaaij et al., 2011; Wallsten et al., 2005). These models have been extensively reviewed and discussed in recent years, providing new insights into the cognitive processes underlying dynamic risk-taking (Guan, 2019; Pleskac, 2008; Pleskac & Wershbale, 2014; Schürmann et al., 2019; Wershbale & Pleskac, 2010). Further, the models have also been used to reveal differences in the cognitive processes concerning dynamic risk-taking of different populations (Bexkens et al., 2019; Khodadadi et al., 2010; Kim et al., 2020; Prause & Lawyer, 2014; Rolison et al., 2012; Seaman, 2015; Wichary et al., 2015). In order to create a simple terminus, research that proposed model elements and tested them is referred to here by the term "principal research" and research that deals with population differences regarding dynamic risk-taking is referred to by the term "population research". This distinction however does not come without limitations since it suggests a clear dividing between the two research areas. But this does not reflect the reality. At least to some extent, most of the publications addressed in this thesis make contributions to both areas.

After almost 20 years of research with the BART, it is time to summarize the findings of both areas. To accomplish this and to identify all relevant publications, this thesis includes a systematic literature search. The results of this search will be instrumentalized to answer the following question: *Which cognitive processes have been suggested to underlie dynamic risk-taking and how do these processes affect human behaviour?*

What becomes apparent when analysing the results of the systematic literature search is that publications dealing mainly with population differences do not always take into account the latest findings from principal research. Currently, a trend in population research is to instrumentalize a cognitive model (BSR; further details can be found in the theory section), which has already been further developed by principal research. This is in fact an issue, because it means that population research cannot realise its full potential. A third part of this thesis will elaborate on this matter.

The Present Thesis

To answer the research question at hand, a systematic literature search is carried out. This consists of a forward citation search based on Wallsten et al. (2005) and a keyword search. In advance to listing the results of the searches, the experimental paradigms to measure dynamic risk-taking, some details on computational modelling and Bayesian learning theory, as well as a description of the models attempting to describe the cognitive processes underlying dynamic risk-taking will be introduced. The results section is divided into two parts. The first part describes research contributions aimed at better understanding the cognitive processes underlying dynamic risk-taking. The second part presents research results that examine population differences in the cognitive processes underlying dynamic risk-taking. It is discussed that decisions under risk depend on four main cognitive processes (learning processes, representation of the probability of burst, option evaluation, and response selection) and that elevated scores in these processes are also linked to real-world dynamic risk-taking behaviours. Further, a contribution on the issue of the current trend to instrumentalize early models for outlining population differences concerning dynamic risk-taking will be made.

Theory

This section introduces key concepts and paradigms of dynamic risk-taking. First, a detailed description of the BART and its extension called Angling Risk Tasks will be given. Second, important concepts of mathematical modelling as well as Bayesian learning are explained. Finally, models specifically designed for the BART are presented.

The Balloon Analogue Risk Task

The Balloon Analogue Risk Task (BART), first described by Lejuez et al. (2002), is a computerized experimental design to measure risk-taking in controlled conditions. During the BART procedure, a subject is rewarded for higher risk-taking, up until a certain point, where taking more risk results in poorer reward. In the experiment, subjects are confronted with a virtual balloon that contains a monetary reward. On each trial, subjects were faced with the decision to either (1) pump the balloon and therefore increase the monetary amount, or (2) secure the balloon and receive the reward. With increasing size of the balloon, the nonstationary probability of burst P_{burst} increases such that

$$P_{burst} = \frac{1}{\alpha - n_{pumps}} \tag{1}$$

, where α indicates the maximal possible pumps and n_{pumps} indicates the times a subject decides for pumping (1) in advance. In the original experiment, Lejuez et al. (2002) set α to be 8, 32 and 128. Subjects were not informed about the true value of α. One’s earnings can be maximized by deciding for (2) when

$$n_{pumps} = \frac{\alpha}{2}. \tag{2}$$

In the original study by Lejuez et al. (2002), each subject processed a total of 90 balloons (trials). To assess risk-taking of a single subject, the mean value of all trials in which the balloon did not burst, can be calculated. In order to make learning effects visible, the mean values of all balloons, except the exploded ones, can be compared over 30 trials each. The graphical user interface of the BART is shown in figure 1 on the left.

In contrast to its counterparts, the BART provides good reliability (Lejuez et al., 2002; White et al., 2008) and validity (Aklin et al., 2005; Crowley et al., 2006; Fernie et al., 2010; MacPherson, Magidson, et al., 2010; MacPherson, Reynolds, et al., 2010). However, the BART may not cover the whole range of real-world dynamic risk scenarios. For example, for someone facing the decision to use a drug, the associated risk may be well known in advance. Since the risk structure in the BART is not revealed to the subject, it cannot account for this scenario. For this reason, Pleskac (2008) presented an extension to the BART called The Angling Risk Tasks.

The Angling Risk Tasks

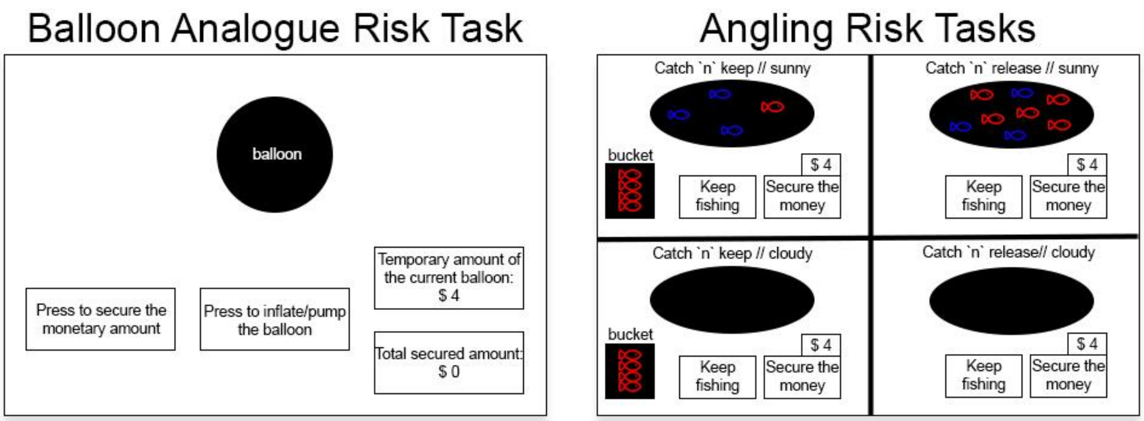
The Angling Risk Tasks (ART) are a modular class of sequential risk-taking tasks. Their main goal is to examine learning and decision making as distinct processes. In the ART, subjects are confronted with a fishing scenario. In a total of 30 rounds, they must catch as many fish from a pond as they can. The fish have different colours. If a red fish is caught, the subject is credited with a monetary amount, if a blue fish is caught, the round ends and the monetary amount is lost. Subjects have the choice of either casting the rod and possibly catching another red fish or securing the monetary amount. As in the BART, the goal in the ART is to maximize one's earnings. The graphical user interface of the ART is shown in figure 1 on the right.

The strength of the ART lies in its modularity, i.e., the possibility to manipulate (1) learning and (2) decision making individually as independent variables. Learning (1) can be manipulated with different weather conditions. In good weather, the pond, illuminated by the sun, shimmers transparently, revealing the fish and their colour. In this case, subjects do not have to learn anything about the distribution of the fish in the pond, which is why this scenario is referred to as well-defined. In bad weather, however, the sun cannot shine through the pond, and therefore the fish are not visible. As a result, subjects are forced to first learn about the number of fish in the pond. In fact, this ill-defined scenario is identical to the BART. Decision making (2) depends on the subject's mental representation of the stochastic environment underlying the task. This environment can eighter be stationary or nonstationary. The latter is present in the BART. In the ART, the stochastic environment can be manipulated by the release law of the pond. The stationary model states that for each decision, the probability of fishing a blue fish (not desirable) remains the same. This is signalled to the subject by releasing each fish caught back into the pond (catch 'n' release; sampling-with-replacement). In contrast, the probability of catching a blue fish increases continuously in the nonstationary model. This is signalled to the subject by placing the caught fish in a bucket so that they can no longer return to the pond (catch 'n' keep; sampling-without-replacement).

The ART turn out to be a generalization of the BART and offer the advantage of being applicable to a wider range of real-world scenarios. The BART and ART provide a good foundation of data to study the processes underlying dynamic risk-taking. As previously outlined, these data can then be further analysed using cognitive models.

Figure 1

The Balloon Analogue Risk Task & The Angling Risk Tasks



Note. Both images show a possible scenario of the task after the 4th decision.

Mathematical Modelling

Mathematical modelling is a tool for breaking down complex mechanisms into more fundamental sub-processes. There are different types of mathematical models. The models attempting to explain the cognitive processes underlying dynamic risk-taking (BART-models) are generally referred to as axiomatic models (Cavagnaro et al., 2013). Axiomatic models attempt to produce an outcome (behavioural response) by mathematically defining the relationships between the axioms (propositions) and the model parameters. One advantage of axiomatic models is that they are easily justifiable, since they merely translate mathematically the assumptions previously made in a verbal model. A distinction must be made between predictive axiomatic models and descriptive axiomatic models (Wilson & Collins, 2019). The former tries to predict the outcome with best possible accuracy, the latter tries to explain as much variance of the outcome as reasonably possible. BART-models are all descriptive in nature.

Model parameters are numerical latent variables that are representing subject-specific properties. They can either be fixed, i.e., predefined, or free. Free parameters must be estimated from the experimental data for each subject individually. There are various methods for parameter inference (Thomopoulos, 2017), but the method widely used in BART-models is called Maximum Likelihood Estimate (MLE). Roughly speaking, the Maximum Likelihood method selects the value as the parameter estimate, whose distribution makes the observed data seem most plausible (Wilson & Collins, 2019):

$$L(x_1, x_2 \dots, x_n; \theta) = \prod_{i=1}^{n} P(x_i; \theta) = P(x_1; \theta) * P(x_2; \theta) * \dots * \; P(x_n; \theta)\,. \tag{3}$$

Hereby x_1 - x_n are input values from the data and θ (theta) represents the parameter to be estimated. There are three main limitations with the maximum-likelihood-estimate. First, the distribution of the data must be known, otherwise the validity of the estimate is not guaranteed. Second, the estimates can include systematic errors when the sample size is too small and third the method only guarantees to find local minima, which is not guaranteed to be the global minimum. The latter may also lead to wrongful estimates but can be controlled for by conducting multiple maximum-likelihood-estimates and choosing the highest scoring one or by conducting a grid search in advance.

To validate the axiomatic assumptions regarding the mathematical model, model comparisons are being performed. To do this, one typically starts by implementing models with different assumptions. The logic is thereby the same as in the classical experiment: Different levels (assumptions) of the independent variable are being compared with respect to a criterion, i.e., the dependent variable. In general, it will not be sufficient to accept the assumptions of the model that explains the largest proportion of variance of the dependent variable. Otherwise, the model that had the most free parameters would always be accepted, since it would always demonstrate the best goodness-of-fit (this phenomenon is generally described by the term overfitting). However, the basic cognitive processes would not be plausibly represented and the model itself would not be robust. For this reason, one makes use of so-called information criteria. Information criteria calculate a score that balances goodness-of-fit and model simplicity. Model simplicity is measured by the number of model parameters, where fewer parameters mean more simplicity. Information criteria therefore penalize models that have too many parameters. There are several different information criteria, the best known being the Akaike information criterion (AIC; Akaike, 1974) and the Bayesian information criterion (BIC; Schwarz, 1978):

$$AIC = 2k - 2\,ln\big(\hat{L}\big)\,, \tag{4}$$

$$BIC = k \cdot ln(n) - 2\,ln\big(\hat{L}\big)\,. \tag{5}$$

Hereby, *k* is the number of estimated parameters, *n* represents the number of values (in the dataset) and $\hat{L}$ is the estimated maximum value of the maximum-likelihood function (equation 3). There are in dept comparisons on the two criteria (Burnham & Anderson, 2002, 2004), but no further explanation is given here, as this does not add any value to the thesis. It should nevertheless be mentioned that the BIC is considered to be the more conservative measure.

Any discussion of information criteria must inevitably end with perhaps the best-known aphorism in statistics: *“All models are wrong”* (First mentioned by Box, 1976). The statement aims at the fact that even the model that performs best in comparisons may not be based on the true axioms. Rather, it is likely that the "true model" that represents the actual reality is not among the candidates in the first place. The resulting relativism however does not mean that models are not useful. In many cases, they can still provide insights into the state of affairs (e.g., dynamic risk) that they are trying to describe.

Bayesian Learning

As described in the introduction, a mayor issue of modelling dynamic risk-taking is the mathematical implementation of learning. Many BART-models have adopted a Bayesian approach for this purpose. This approach goes back to the so-called Bayes theorem, which describes the probability of an event if some details about this event are already available in advance:

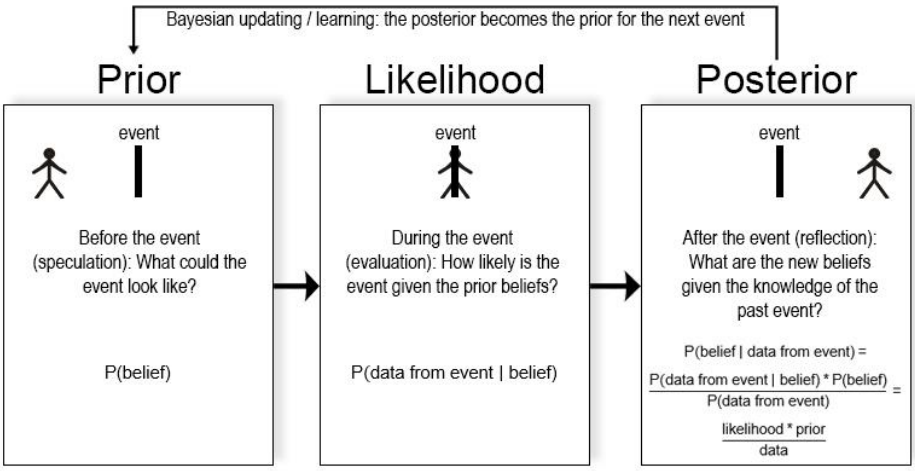
$$P(event|prior\ knowledge) = \frac{P(event\ \&\ prior\ knowledge)}{P(prior\ knowledge)}$$
$$= \frac{P(prior\ knowledge|event) * P(event)}{P(prior\ knowledge)} \tag{6}$$

Intuitively, Bayes theorem seems to be very useful because it works in a similar way to how humans think: For example, in the BART, when one is faced with the decision to continue pumping or to stop, one might consider the probability that the balloon will burst within the next pump (event), given the experience from previous balloons and the knowledge of how many pumps were already performed on the current balloon (prior knowledge). Nevertheless, Bayes' theorem did not find resonance for a long time. This is due to the fact that in the continuous case, sooner or later, not rudimentary integrable functions would have to be solved, which was not possible for a long time. Only computational approximation methods like the Markov Chain Monte Carlo Simulation have opened up new possibilities for the Bayes Theorem (Koch, 2007).

In Bayesian theory, subjects' beliefs about an event can be represented by a prior distribution (due to technical reasons often a beta, gamma, or normal distribution). This distribution includes all knowledge that was known before the first event occurred. The mean of the prior distribution reveals the subject's belief on the probability that the event will not happen, and the variance can be interpreted as the subject's uncertainty about this belief (Wallsten et al., 2005). After the event took place, the gained experience and knowledge can be combined with the prior distribution and a posterior distribution can be calculated. The posterior distribution represents the subjects updated beliefs after the first event took place. It can then be interpreted as the prior distribution for the second event (Koch, 2007). When the second event took place, the further gained experience and knowledge can again be combined with the prior beliefs and a new posterior distribution can be calculated. This potentially endless process is called Bayesian updating / learning and is illustrated in figure 2. In the BART, the prior distribution can be construed as a subject's belief about the outcome of the forthcoming decision to pump or to stop. After the pump has been executed, the subject has certainty about the outcome and can adjust its beliefs with this knowledge. Subsequently, the updated beliefs again act as the new prior for the next decision. This principle can be used to simulate learning processes in a naturalistic manner.

Figure 2

The Bayesian learning process.



Note. Bayes' theorem works similarly to human learning. The prior becomes a posterior via the likelihood.

Models to Explain the Mechanisms Underlying Risk and Learning in a Dynamic Task

The first attempt to model behavioural data from the BART was undertaken by Wallsten et al. (2005). Their work has suggested three classes of models, which mainly differ in the number of cognitive processes they account for. All models have in common that they are probabilistic in nature.

The most primitive model neither assumes subjects to learn from past balloons, nor to adjust their strategies in upcoming ones. In particular, the role of this model is to provide a baseline for more complex models to beat. This derives the name "Baseline Model".

Baseline Model

The Baseline Model does not assume any learning from past trials. Therefore, a subject's probability $\hat{r}$ to pump at balloon *h* on any given opportunity *i* can be estimated (MLE) as a constant free parameter such that

$$\hat{r} = \frac{\sum_{h=1}^{f} a_h}{\sum_{h=1}^{f} a_h + \sum_{h=1}^{f} d_h} \tag{7}$$

for $\{ \hat{r} \mid 0 \leq \hat{r} \leq 1\}$, where a_h denotes the number of pumps the subject took on balloon *h* and d_h is a dummy variable indicating whether the balloon exploded on the last pump (Yes: $d_h = 0$; No: $d_h = 1$).

The Baseline Model consists of only one parameter. Nevertheless, it plays an important role because it provides a baseline that must be beaten by more complex models. One of these more complex models proposed by Wallsten et al. (2005) is the so-called "Target Model".

Target Model

The Target Model aims to model learning processes that occur from balloon to balloon. It is built around the assumption that subjects target a specific number of pumps for the first balloon t_1^*. This number is set in advance and will not be updated until either the targeted number of pumps is reached or the balloon bursts. After each balloon, the target number will be updated. If the previous balloon burst, the target number for the next balloon is adjusted downwards, and if the subject secured the monetary amount of the previous balloon, the target number is adjusted upwards (formula 9). The adjustment is expected to become more marginal as the number of balloons processed increases. As the subject comes closer to its targeted number, the probability of pumping $r_{h,i}$ on opportunity *i* of balloon *h* decreases in a logistic manner such that

$$r_{h,i} = \frac{1}{1 + e^{\beta \delta_{h,i}}} \tag{8}$$

where $\delta_{h,i}$ = $i - t_h^*$ (t_h^*: targeted number for balloon h) and $\{\beta \mid 0 < \beta\}$. The targeted number for the next pump t^*_{h+1} can be calculated according to the function

$$t^*_{h+1} = \begin{cases} round\left(t_h^* + a_1 e^{-ah}\right), & if\ subject\ stops \\ round\left(t_h^*[1 - a_2 e^{-ah}]\right), & if\ balloon\ explodes \end{cases} \tag{9}$$

for $\{a_1, a_2, \alpha \mid 0 < a_1, a_2, \alpha\}$. a_1 and a_2 are constants indicating the strength of the adjustment after each balloon, parameter α modulates the speed with which the knowledge acquired in previous balloons is converted and the parameter β modulates the decrease of the probability to pump, the closer subjects get to the target number. All free parameters can be estimated with MLE.

The Target Model assumes that subjects manage risk decisions depending on past risk choices and their outcome. However, it does not consider the subjective evaluation of the gains and losses between pumping decisions. To account for such processes, Wallsten et al. (2005) developed another class of models that can generally be described as Bayesian Learning and Evaluation Models.

Bayesian Learning and Evaluation Models

The plural in the title suggests that this class consists of multiple models. In fact, eight models were suggested with slightly different assumptions on the subject's cognitive processing of the BART. All models assume that subjects' beliefs about the explosion probability of the first balloon can be modelled by a prior distribution and make use of Bayesian updating/learning to understand how subjects adjust their decisions as they gain experience. However, they differ in three key assumptions about the subject's mental representation of the BART. That is, (1) first, how the subjects mentally represent the probability of burst, (2) second, how often and when subjects evaluate their pumping strategies (option evaluation), (3) and third, how subjects adjust their learning with increasing experience (response selection).

(1) The BART does not stipulate that subjects know the probability of burst of the balloons. However, in order to maximize gains, subjects must have as good a representation of this probability as possible. Wallsten et. al. (2005) suggest two possibilities of how this representation could look like. Either (1a) subjects could falsely believe that the probability of burst is stationary and therefore the same for each pump, or (1b) they could correctly believe that the probability of burst increases with each pump performed (nonstationary).

(1a) When stationary, the implementation remains relatively simple. Since the subjects' decision (pumping or stopping) satisfies the requirements of a binomial distribution, it is appropriate to use a beta distribution with free parameters a_0 and m_0 as a prior, where greater values in a_0 can be interpreted as higher belief that balloon h will not explode within the next pump (formally defined as $q_h = 1 - p_h$, where p_h is the subjective probability that the balloon will explode) and greater values in m_0 can be interpreted as higher certainty about q_h. The prior provides information on the subject's assumptions about the probability of burst. Specifically,

$$E(q_1) = \frac{a_0}{m_0} \tag{10}$$

denotes the subjects' belief about q_1 (belief for the first balloon; no prior knowledge available) and

$$var(q_1) = \frac{a_0(m_0 - a_0)}{m_0^2(m_0 + 1)} \tag{11}$$

denotes the subjects' certainty about q_1 (further details on the interpretation are described in the theory section). a_0 and m_0 are both estimated from the data using MLE.

(1b) When it is assumed that the probability of burst increases with each pump, the implementation is more complicated since there is no naturally plausible distribution for this scenario. Wallsten et al. (2005) overcome this issue by modelling the prior with a gamma distribution equipped with two free parameters μ_0 and σ_0. In both variants, the prior can be updated with experience.

(2) After each decision, subjects evaluate the outcome that resulted from the prior chosen option. This evaluation process follows Prospect Theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992). Prospect Theory asserts that gains and losses are evaluated against a reference point, the status quo, with losses weighted more heavily than gains. This can be formalized with the value function *v(x)* such that

$$v(x) = \begin{cases} x^{\gamma^+}, & for\ gains\ relative\ to\ the\ status\ quo, (x > 0) \\ -\theta * |x|^{\gamma^-}, & for\ losses\ relative\ to\ the\ status\ quo, (x < 0) \end{cases} \quad (12)$$

for $\{\gamma^+, \gamma^-, \theta \mid 0 < \gamma^+, \gamma^-, \theta\}$. x describes the status quo (often as a monetary amount), γ^+ and γ^- describe subjects' sensitivity to gains and losses and θ indicates the decline of the value of an option over time. Wallsten et al. (2005) hypothesize that subjects either (a) sequentially evaluate their decision-making behaviour after each pump or (b) update it after each completed balloon.

The former (2a) implies that the amount of money deposited into the temporary bank from the last pump $(i - 1)x$ becomes the subjects' new status quo, which indicates that potential gains and losses are considered when evaluating the current decision. The choice to stop has no influence on the status quo. Thus, expected values for pumping or stopping on a given opportunity i can be calculated according to

$$E_i(pump) = (1 - p_i)x^{\gamma^+} - p_i * \theta * \left((i-1)x\right)^{\gamma^-}, \quad (13)$$

$$E_i(stop) = 0 \quad (14)$$

with p_i denoting the probability of burst given $(i - 1)$ successful pumps.

The latter (2b) implies that the subject specifies an optimal number of pumps prior to each balloon. The status quo is fixed on the value in the temporary bank at the beginning of each balloon (i.e., 0). Every increase of the monetary amount is therefore evaluated as a gain (γ^+). Thus, the expected value for pump i can be calculated such that

$$E_h(i\ pumps) = \pi_{h,i}(ix)^{\gamma^+} \quad (15)$$

where $\pi_{h,i}$ denotes the probability of successfully pumping balloon *h* *i*-times without exploding it.

(3) At each pumping opportunity, subjects are required to decide whether to pump again or to stop and secure the monetary amount. The current decision is influenced by the option evaluation process detailed above (2). When modelling this process, a case distinction must be made. If it is assumed that the decision-making behaviour is evaluated sequentially (2a), the probability to pump $r_{h,i}$ decreases as a logistic function such that

$$r_{h,i} = \frac{e^{\beta\left((1-p_i)x^{\gamma^+} - p_i * \theta * \left((i-1)x\right)^{\gamma^-}\right)}}{1 + e^{\beta\left((1-p_i)x^{\gamma^+} - p_i * \theta * \left((i-1)x\right)^{\gamma^-}\right)}} \tag{16}$$

for $\{\beta \mid 0 < \beta\}$, with β being a free parameter reflecting the consistency with which subjects' follow their own targeted evaluation.

If it is assumed that the decision-making behaviour is evaluated after each completed balloon (2b), the probability to stop strictly increases the closer the subject gets to the prior defined targeted number. Again, this can be implemented with a logistic expression

$$r_{h,i} = \frac{1}{1 + e^{\beta(i-g_h)}} \tag{17}$$

with g_h denoting the number of pumps, that optimizes the expected value of equation 15. The probabilistic nature of response selection implies that subjects are not completely sure of their own evaluation and understanding of the task structure. This uncertainty is reflected in the way subjects translate their targeted evaluation into concrete decisions. Wallsten et al. (2005) suggested that this translation process either (3a) remains constant with experience or (3b) or changes. If it remains constant (3a), this means that over the span of the BART, subjects do not change their behaviour of following their own evaluation and therefore β remains constant. If it changes (3b), this means that with oncoming balloons subjects develop more (or less) confidence in their evaluation and therefore adhere more or less strongly to their targeted evaluation. Mathematically, this can be formulated as

$$\beta = h^c, \tag{18}$$

where h is the current balloon and c is a free parameter indicating the increase/decrease rate of the sensitivity over time.

Wallsten et al. (2005) presented several hypotheses about the cognitive processes underlying the BART. They combined the above-described components into complete models (called Models 1-8) and tested their performance on a behavioural data set. It is said at the outset that Model 3 provided the best fit to the data and was therefore taken up by most of the work that followed later. Therefore, it is briefly described here.

Model 3 is generally known as the Bayesian Sequential Risk-Taking Model (BSR; Pleskac & Wershbale, 2014; Schürmann et al., 2019; Wershbale & Pleskac, 2010; Wichary et al., 2015). The model assumes that subjects have a stationary representation of the explosion probability (1a), which means that the prior is modelled with a beta distribution (a_0 & m_0). Furthermore, the option evaluation takes place prior to each balloon (2b; γ^+) and the response sensitivity remains constant with experience (3a; β). The model is thus based on a total of four free parameters.

Van Ravenzwaaij et al. (2011) found that the free parameters concerning Model 3 may be further condensed because of their similarity.

Van Ravenzwaaij 2-Parameter Model

In an analysis of the Wallsten et al. (2005) Model 3, van Ravenzwaaij et al. (2011) pointed out, that remarkable similarity can be found between the a_0 and m_0 parameter and the γ^+ parameter. They assumed that subjects who are more willing to seek risk at the beginning (a_0 and m_0) may be also more sensitive to gains (γ^+) while pumping. Based on their considerations, van Ravenzwaaij et al. (2011) presented two 3-Parameter Models and a 2-Parameter Model. Only the 2-Parameter Model could withstand tests with simulated data. Under the condition that subjects know the actual probability of burst, the parameters a_0 and m_0 can be omitted in the 2-Parameter Model. This then implies that subjects no longer learn based on gains and losses. However, the response consistency and the sensitivity to changes in payoffs are still included in the model with the parameters β and γ^+.

The model of van Ravenzwaaij et al. (2011) further differs from the Wallsten et al. (2005) models in the way it estimates parameters. Other than using MLE for parameter estimation, van Ravenzwaaji et al. (2011) used a Bayesian method called Bayesian Hierarchical Modelling. In the hierarchical approach, the individual parameters are derived from an overarching group distribution (usually a normal distribution), with its own parameters (hyperparameters). If the group distribution of a free parameter has a large variance, it can be concluded that the individual parameters in the population are widely heterogeneous. One advantage that Bayesian Hierarchical Modelling offers is that individual parameters can be estimated relatively well even with little data.

Expectancy Valence Model

The Expectancy Valence Model by Busemeyer and Stout (2002) was not originally created for the BART but rather for another experimental design called Iowa Gambling Task (IGT; also known as the Bechara Gambling Task). However, the BART and the IGT share several common properties, making it easy to adapt the Expectancy Valence Model for the BART.

The Expectancy Valence Model assumes that subjects experience an affective reaction (valence) through gains and losses. The valence v for balloon h is derived from the importance subjects ascribe to wins w and losses l and the actual amount won Win_{h-1} or lost $Loss_{h-1}$ in the previous balloon:

$$v_h = w * Win_{h-1} - l * Loss_{h-1}. \tag{19}$$

Together with the subject's individual learning rate a, where $\{a \mid -1 \leq a \leq 1\}$, and the number of pumps in the previous decision n_{h-1} the valence influences the target number of pumps in the upcoming choice t_h^* in the following manner:

$$t_h^* = v_h + (1 + a) * n_{h-1}. \tag{20}$$

The subject’s individual learning rate a can be interpreted as memory from previous decisions or adaptability of the personal decision strategy based on experience from previous choices (Rolison et al., 2012). The targeted number of pumps is probabilistic in nature, meaning that the closer the subject gets to the targeted number, the more likely she/he is to stop. In the Expectancy Valence Model this is implemented using a logistic equation such that

$$r_{h,i} = \frac{1}{1 + e^{\beta \cdot t_h^*}}, \tag{21}$$

where $r_{h,i}$ is the subjects' probability to pump on balloon h and β is a consistency parameter (similar to the one of Wallsten et al. (2005)). Thus, the Expectancy Valence Model consists of a total of 5 free parameters that can be estimated using MLE.

Method

To extract relevant studies for this thesis, a forward citation search and a keyword search were carried out using the online database "ISI Web of Science”. All searches were conducted on October 20, 2020.

The forward citation search was based on Wallsten et al. (2005) and included only citations in the English language. This particular article was chosen since it was the first to describe cognitive mechanisms underlying the BART. "ISI Web of Science" listed 115 citations. 13 of which met the inclusion criteria described above.

To search for articles in the extended frame, a keyword search was conducted using the following search term: *(“Balloon Analogue Risk Task” OR BART) AND (“Cognitive Model*” OR “Cognitive Process*” OR “Cognitive Mechanism*”)*. Searches were restricted to publications from Psychology and Neuroscience published between 2005 and October 2020. The keyword search resulted in the listing of 50 articles. No additional article could be identified to meet the inclusion criteria. However, all 13 articles identified with the forward citation search could also be identified with the keyword search, which can be interpreted as an indication of the functioning of both search procedures.

Abstracts and titles of all articles were examined according to the following criteria: The study must either (1a) examine the cognitive processes underlying the BART and/or (1b) examine population differences regarding dynamic risk-taking with the BSR and the BART. (1a) The cognitive processes must be implemented in a model and compared with other models. (1b) For population differences, the study must investigate real-world dynamic risk-taking either by direct estimation of model parameters from two populations or with a correlation analysis (e.g., correlation between self-reports and model parameters).

Results

Research with cognitive models underlying the BART can be broadly divided into two subfields. The first subfield aims to understand the cognitive processes underlying dynamic risk-taking. Research in this subfield mainly focusses on contributions and adjustments of these processes. The second subfield aims at understanding real-world dynamic risk-taking phenomena. By estimating and comparing the free parameters of the BSR from two or more populations (e.g., control group vs. pathological group), differences in underlying cognitive processes become apparent. Alternatively, estimated free parameters of a representative sample can be correlated with a self-reported attribute. If the correlation is significant, a relationship between the free parameter and the attribute is assumed. It should be noted that many publications contain contributions to both subfields. For the sake of comprehension, however, they are described separately here.

Which Cognitive Processes Have Been Suggested to Underlie Dynamic Risk-Taking?

There is a consensus that four main processes underlie dynamic risk-taking: learning processes, subjects' representation of the probability of burst, option evaluation and response selection. Various publications have proposed, extended, discussed, and tested models or model elements that aim to better understand these four processes. An overview can be found in table 1.

Table 1

Articles proposing, discussing, extending, and testing models or model elements that aim to capture the cognitive processes underlying dynamic risk-taking.

Learning	Option evaluation	Representation of the probability of burst	Response selection
Wallsten et al., 2005	Wallsten et al., 2005	Wallsten et al., 2005	Wallsten et al., 2005
Rolison et al., 2012	Wershbale & Pleskac, 2010	Pleskac, 2008	van Ravenzwaaij et al., 2011
	van Ravenzwaaij et al., 2011	Schürmann et al., 2019	
	Pleskac & Wershbale, 2014		

Note. Articles were sorted in descending order by their publication date.

The comparisons of the ten models (Baseline Model, Target Model, Model 1-8) from Wallsten et al. (2005) suggest that learning is a crucial cognitive process when making dynamic decisions under risk. The only proposed model that did not include learning was the Baseline Model, which had by far the worst AIC score (192.79). Further, it can be assumed that learning processes operate in a Bayesian fashion. The Target Model (AIC: 164.39), which does not employ Bayes rule for the modelling of learning processes, underperformed the Bayesian models. However, this comparison is not necessarily appropriate since the Target Model is not as complex as the Bayesian models. A better comparison can be carried out with the Expectancy Valence Model, which is very similar to the Bayesian models in terms of assumptions and complexity but does not implement learning processes by means of Bayes Rule. Still, the Expectancy Valence Model has been found inferior to the Bayesian models in model comparisons (Rolison et al., 2012; Wichary et al., 2015). This leads to the conclusion that learning in dynamic risk scenarios suits a Bayesian updating process.

As mentioned before, the best fitting model of Wallsten et al. (2005) is Model 3 (BSR; AIC: 154.14). It goes head-to-head with Model 1(AIC: 154.96) and Model 2 (AIC: 156.46). The three models differ in the way they implement the option evaluation (Models 1 & 2: sequentially for each pump; Model 3: prior to each balloon) and response sensitivity with experience (Models 1 & 3: constant; Model 2: increasing) but have in common that they all implement the mental representation of the explosion probability as a stationary process. This seems surprising when considering that the actual probability of burst in the BART takes a nonstationary form. In oppose to this finding, Pleskac's (2008) findings suggest that learning processes and the subjects' mental representation of the explosion probability vary depending on the exact structure of the task. By fitting two Wallsten et al. (2005) models to data from the ART, one with a stationary, one with a nonstationary implementation of the subjects' mental representation of the probability of burst, Pleskac (2008) found that in either case the cognitive model that best fit the data was the one that correctly embodied the task structure: In the catch 'n' keep task, the model with the non-stationary implementation fit better, and in the catch 'n' release task, the stationary representation fit better. This suggests that subjects adapt their mental representations to the stochastic environment of the task. The contributions of both Wallsten et al. (2005) and Pleskac (2008) were again picked up in a study by Schürmann et al. (2019), who measured subjects' self-reported risk perception on the probability of burst and found drastic differences in the subjects' assumptions and the actual objective explosion probability: After experiencing the first balloon, subjects tended to substantially overestimate the probability of burst. Even after all the balloons were completed, these false assumptions persisted. The finding of systematic overestimation of the explosion probability complements the discussion of Wallsten et al. (2005) and Pleskac (2008) by providing further evidence that subjects may have difficulty in accurately representing the risk structure of the task. Schürmann et al. (2019) further point out the crucial role of risk perception in predicting individual choice behaviour in the BART.

The BSR of Wallsten et al. (2005) suggests that for each decision, subjects equally evaluate their options of either continuing to pump or stopping and securing the amount of money in the temporary bank. By measuring subjects response times, Wershbale and Pleskac (2010) provided evidence that this assumption conflicts with studies stating that working memory becomes progressively slower as more information is being processed (e.g. Atkinson et al., 1969). Subjects in the BART tended to have shorter assessment times when they are far, and longer assessment times when they are close to their targeted number. Wershbale and Pleskacs' (2010) study further shows that assessment times become shorter as subjects gain experience with the BART and that subjects rather perform response evaluation cyclically after several "automatically" executed pumps, than at every pump opportunity. The latter led to a major redesign of the original BSR: In a following study, Pleskac and Wershbale (2014) suggested that the option evaluation process is divided into two distinct pathways that subjects might follow on each opportunity. With a certain probability p, subjects will initiate an assessment process at a given decision and match their completed number of pumps for the current balloon with their targeted number. The actual decision to continue pumping or to stop then follows from a drift diffusion process (decision field theory; Busemeyer and Townsend, 1993), in which the two options are substantiated with a multitude of accumulated evidence (a substantial part of this evidence is the outcome of the distance-to-the-target assessment). With the counter probability *1-p*, subjects will "automatically" repeat their last decision (i.e., without re-evaluation). The closer subjects' get to their previously defined targeted number of pumps, the more often they re-evaluate their decision. Further, subjects perform more assessments in early balloons than in late balloons (presumably due to greater confidence, experience, or fatigue). With subjects choosing between two different response pathways, Pleskac and Wershbale (2014) named their adapted version dualresponseBSR. It performed very well in comparisons.

The response selection process suggested by Wallsten et al. (2005) has not been further discussed in the literature yet. This is possibly due to the fact that the assumption of uncertainty about one's own option evaluation, expressed in the consistency parameter, is quite plausible. Support for this assumption is provided by van Ravenzwaaij et al. (2011) who implemented this process in their 2-Parameter Model. Even after the BSR was reduced to only two processes, one of which being the response selection process, it was able to prove its quality in a parameter recovery.

In sum, the results of the scientific literature indicate that individuals have difficulties in accurately representing the risk structure of the situation (i.e., the BART). In particular, this is indicated by the results of Schürmann et al. (2019), which show that subjects substantially overestimate the probability of burst. There is consensus that learning processes in dynamic risk-taking operate in a Bayesian manner. Among the results of the literature search performed for this thesis, not a single scientific publication was identified that reached a contrary conclusion. The option evaluation process follows Prospect Theory, with Wallsten et al. (2005) proposing that subjects define an optimal targeted number at the beginning of each balloon. Thus, any increase in the monetary amount is interpreted as a gain. Pleskac and Wershbale (2014) suggested that the evaluation process is not performed at each opportunity to pump (or to stop), but periodically, after several "unevaluated" decisions. The response selection process ultimately takes place under uncertainty about one's own evaluation.

How Does Dynamic Risk-Taking Affect Human Behaviour?

People differ in the way they deal with dynamic risk. The cognitive processes detailed above may vary among different populations. Studies that investigate such population comparisons are presented in table 2.

A first subdomain of dynamic risk-taking in which people of different populations differ, is their prior belief about the probability of burst (a_0 and m_0). People who carry a gun have higher scores in the a_0 parameter than people who don't, suggesting that people carry a gun because they do not believe in negative consequences that might occur because of it (Prause & Lawyer, 2014). Further, it appears that older people tend to have a more negative initial risk perception ($E(q_1)$; formula 10) than their younger counterparts (Rolison et al., 2012). There are also differences within the cohorts. For instance, older people who had a rather negative risk perception in the BART are more likely to live in the retirement community (Seaman, 2015).

A second subdomain of sequential risk taking in which people of different populations differ, is their sensitivity to changes in payoff gains (γ^+). In contrast to the healthy average person, drug users (Heroin: Khodadadi et al., 2010; Various: Pleskac, 2008; Wallsten et al., 2005), thiefs (Wallsten et al., 2005), prisoners (Wichary et al., 2015), persons participating in unprotected sex (Prause & Lawyer, 2014; Wallsten et al., 2005) and (mild to) borderline intellectual disability & behaviour disorder patients (Bexkens et al., 2019) seem be more sensitive to changes in payoff gains. In addition, evidence was found for differences between older and younger adults when it came to high risk conditions, suggesting that older people tend to be more open to reward-related risk taking (Cavanagh et al., 2012). However, these findings could not be replicated (Rolison et al., 2012).

A third subdomain of sequential risk taking in which people of different populations differ is the consistency with which they follow their evaluation (β). In contrast to the healthy average person, drug users (Heroin: Khodadadi et al., 2010; Various: Wallsten et al., 2005), prisoners (Wichary et al., 2015) and (mild to) borderline intellectual disability & behaviour disorder patients (Bexkens et al., 2019) seem to be less consistent in following their targeted evaluation.

In sum, differences between an average person and a person who exhibits negatively associated externalizing behaviours seem to manifest in the sensitivity to payoff gains and in the consistency of approaching one's own targeted evaluation. Altered scores in the prior belief about the probability of burst, on the other hand, are hardly evident in individuals associated to negative externalizing behaviour. Such changes are rather associated with age.

Table 2

Findings from research on population differences from studies that either correlated parameter values with self-reports or estimated parameters directly from samples of different populations.

Study	Attribute under investigation	Finding of significant differences in the free parameter			comment
		a_0 & m_0	γ^+	β	
Wallsten et al., 2005	Drug use	*No*	*Yes*	*Yes*	Cwq
	Unprotected sex	*No*	*Yes*	*No*	Cwq
	Seatbelt use	*No*	*No*	*No*	Cwq
	Reckless driving	*No*	*No*	*No*	Cwq
	Stealing	*No*	*No*	*Yes*	Cwq
Pleskac, 2008	Drug use	*NA*	*Yes*	*No*	Cwq
	Social attitude	*NA*	*No*	*No*	Cwq
	Health attitude	*NA*	*Yes*	*Yes*	Cwq
Khodadadi et al., 2010	Heroin abuse	*No*	*Yes*	*Yes*	Peds
van Ravenzwaaij et al., 2011	Alcohol use	*NA*	*No*	*No*	Peds; 2-parameter version of the BSR
Cavanagh et al., 2012	Age	*No*	*Yes*	*No*	Peds
Rolison et al., 2012	Age	*Yes*	*No*	*No*	Peds; only initial risk perceptions were significant, not the

					parameters themselfs.
Prause & Lawyer, 2014	Financial risk-taking	*No*	*No*	*No*	Cwq
	Sexual risk-taking	*No*	*Yes*	*No*	Cwq
	Substance use	*No*	*No*	*No*	Cwq
	Antisocial risk-taking	*Yes*	*No*	*No*	Cwq; Weapon Carry
Seaman, 2015	Gender	*No*	*No*	*Yes*	Cwq
	Age	*Yes*	*No*	*No*	Cwq
	Education	*No*	*No*	*No*	Cwq
	Vocabulary	*No*	*No*	*No*	Cwq
	Ability to read	*No*	*No*	*Yes*	Cwq
	Living in a retirement area	*Yes*	*No*	*No*	Peds; Sample with elderly people living in a retirement area vs. control group
Wichary et al., 2015	Prisoner women	*No*	*Yes*	*Yes*	Peds
	Prisoner men	*No*	*No*	*No*	Peds
Bexkens et al., 2019	Mild to Borderline Intellectual Disabilities	*No*	*Yes*	*Yes*	Peds
	Behavioural Disorder	*No*	*No*	*No*	Peds

Note. Cwq = Correlation with questionnaire; Peds = Parameter estimation from a representative sample; Due to reasons of comparability, only studies that could provide information on at least one of the free parameters were included.

Discussion

The present thesis investigated which cognitive processes have been suggested to underlie dynamic risk-taking and how these processes affect human behaviour. Each of the two sub-questions is answered here in a separate paragraph.

Which Cognitive Processes Have Been Suggested to Underlie Dynamic Risk-Taking?

Many new insights into the cognitive processes underlying dynamic risk-taking have been gained thanks to the development of the BART. It is time to summarize the achievements of the last few years. Overall, four processes underlying dynamic risk-taking have been suggested: Learning processes, representation of the risk structure of the scenario, option evaluation and response selection. Learning processes are essential for understanding dynamic risk-taking. The only model not incorporating learning processes is the Baseline Model, which was always inferior in comparisons with other models. Further, it can be assumed that learning processes underlying dynamic risk operate in a Bayesian fashion. Models that do not implement learning with Bayes rule, namely the Target Model and the Expectancy Valence Model, underperform in comparisons with the Bayesian models.

A second cognitive process underlying dynamic risk-taking is the acquisition of a mental representation of the probability of burst. Research on this process has not yet concluded, but it appears that subjects struggle with understanding the actual risk structure of the task. Although it is still unclear whether subjects assume a false risk scenario, it can be shown that subjects systematically overestimate the probability of burst.

A third cognitive process that occurs during dynamic risk taking is option evaluation. The consensus is that this process follows the value function of Prospect Theory. There is evidence that a targeted number of pumps is determined before the start of a new balloon and that an assessment on the discrepancy between the performed number of pumps and the targeted number is carried out for every decision to pump or to stop. Research with reaction times has revealed that this evaluation process is not carried out for every single decision, but rather cyclically after a multitude of decisions. The closer subjects get to their targeted number, the more frequently the evaluation process is carried out.

The last cognitive process involved in dynamic risk-taking is response selection. It has been suggested that subjects are unsure of their own option evaluation and their understanding of the risk structure of the BART, and that they therefore pump more or less consistent to their targeted number of pumps.

How do These Processes Affect Human Behaviour?

Risk preference describes the individual tendency to engage in risky situations if personal positive consequences can be obtained. By means of research on parameter comparisons of different populations, it is possible to investigate which cognitive processes are involved in risk preference. The BSR of Wallsten et al. (2005) found support for such parameter comparisons. A first insight from the literature is that only a few populations differ with respect to prior beliefs. Prior beliefs express how likely subjects believe a negative outcome of a decision is. Only gun carriers and older people show significantly different values to a control group in the a_0 and m_0 parameters. Taking into account that the attribute "gun carrier" arguably applies to a widely heterogeneous group, the meaning for a conclusion is in doubt. Consequently, altered values in these parameters can only be assumed to be predictors of increased age.

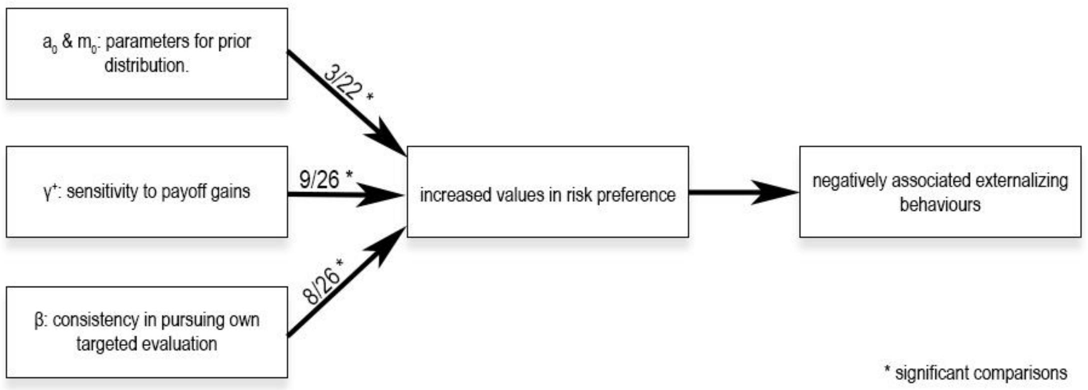
The most substantial differences in negatively associated externalizing behaviour can be attributed to the process of option evaluation. This is expressed primarily in the sensitivity to payoff gains. Compared to the normal population, the γ^+ parameter is elevated in various individuals with behavioural problems (e.g., prisoners, drug users, delinquents). It appears that high expectations of the positive consequences of a decision and the urge to receive these consequences lead to negative externalizing behaviours. The large number of publications coming to this conclusion prompts that an elevated score in the γ^+- parameter is a valid predictor of negatively associated behavioural problems.

Further, the consistency with which subjects follow their evaluation (expressed in the *β*-parameter) is decreased in individuals who exhibit negative externalizing behaviours. Individuals with decreased values in the *β*-parameter might be less confident about their evaluation or might not consider their previously made evaluation when selecting a response, which can often lead to the confrontation with negative consequences. Thus, the *β*-parameter is also a valid predictor of negatively associated behaviours.

In a nutshell, the existing research suggests that the sensitivity to payoff gains and the consistency with which individuals follow their own evaluation contribute to the level of a person's risk preference. High risk preference then leads to negatively associated behaviours like drug abuse and delinquency. This principle is illustrated in figure 3.

Figure 3

Associations between risk parameters and behaviour



Note. The a priori probabilities should be treated with caution. The circumstances of the studies are widely heterogeneous.

What is remarkable when analysing the publications from population research is that almost all of them instrumentalize the BSR model (see table 2). While this is good for the comparability of different studies, it is still somewhat surprising considering that the assumptions of the BSR have been discussed and revised many times in principal research. In the original publication by Wallsten et al. (2005), the BSR with an AIC of 154.14 had narrowly edged out Model 3 (AIC: 154.96). Model 2 was also within reach with an AIC of

156.46. Despite only marginal differences, the BSR prevailed for population comparisons. In addition, the assumptions made by the BSR have not all been replicated in subsequent publications of principal research. For example, Pleskac (2008) has provided evidence that subjects in the BART do not have a misrepresentation of the probability of burst as suggested by the BSR. Further, Pleskac and Wershbale (2014) and Wershbale and Pleskac (2010) have provided evidence against the BSR's assumption that subjects in the BART perform option evaluation for every decision. Thus, one can certainly argue that population comparisons using the BSR disregard important findings from principal research. Of course, this does not mean that the findings of population research are invalid or useless, but with the use of more recent models, one could potentially maximize the evidence gain. On the other hand, it should be emphasized that many publications contribute to both areas, i.e., principal research and population research, and that the authors often apply their findings of the cognitive processes directly to their population comparisons. Still, the move away from BSR on to newer models (such as the dualresponseBSR) would have a positive effect for both principal research and population research.

One of the more recent models proposed in the literature is the dualresponseBSR (Pleskac and Wershbale, 2014; as described in the results section). In contrary to the BSR, the dualresponseBSR makes use of response times to gain further insights into the cognitive processes underlying dynamic risk-taking in the BART. Pleskac and Wershbales' (2014) findings suggest that the average healthy subject will increase the assessment rate when coming closer to the targeted number. When comparing different populations concerning their dynamic risk-taking, this cognitive process could be of interest. It might be an important hypothesis, for example, that drug users will not intensify their assessment rate in contrast to a control group, which would further explain their increased risk preference. However, this hypothesis cannot be examined with the original BSR. Moreover, the free parameters of the original BSR are also estimated in the dualresponseBSR. Thus, the original BSR has no advantages over the dualresponseBSR (except that it is less time-consuming, since no response times must be collected, but this is a strong trade-off against the evidence gain).

In conjunction with the dualresponseBSR, it might also be appropriate to study dynamic risk-taking using the ART rather than the BART. The ART are a good generalization of the BART, allowing a better assessment of the subjects' mental representation of the risk structure of the task. Where in the traditional approach with the BSR and the BART population differences in the mental representation of the risk structure of the task cannot be further investigated, this is possible with the ART. It would arguably be an interesting hypothesis that smokers have more trouble accurately representing the risk scenario of the task than a control group. Such study would for instance be an important contribution to the Viscusi-Slovic tobacco debate (Slovic, 2001; Viscusi, 1992; Weinstein et al., 2005, 2004).

Several publications have provided model comparisons (e.g. Guan, 2019; Rolison et al., 2012; Wichary et al., 2015). Most have compared the BSR (Wallsten et al., 2005), the Expectancy Valence Model (Busemeyer and Stout, 2002), and the 2-Parameter-Model [1] (van Ravenzwaaij et al., 2011). Such comparisons are of great value because they provide further evidence that the BSR is superior to the other models, no matter the context. However, even here, the insights could be further maximized if reaction times were captured additionally. For example, Wichary et al. (2015) provide evidence that female prisoners differ from average females particularly in the γ^{+} - and β - parameters. This finding could have been extended with the measurement of reaction times: It would have been interesting to see whether the two populations also differed in terms of frequency of assessments (objectifiable with the dualresponseBSR) and in terms of the representation of the probability of burst (objectifiable with the ART). Such insight cannot be derived using the BSR, the Expectancy Valence Model and the 2-Parameter-Model alone. Thus, it can be argued that maximum insight cannot be obtained by using these models.

[1] It was pointed out by my supervisor that there has recently been an increasing number of publications only instrumentalizing the 2-Parameter-Model instead of the BSR (e.g., Guan, 2019). This development is problematic, because the subjects' belief about the probability of burst for the next pump is fixed in the 2-Parameter Model, which means that subjects do not learn. This assumption is a strong limitation in comparison to the BSR and leads to fewer insights into the cognitive processes of dynamic risk-taking. The 2-Parameter Model should therefore always be compared with another model, that does not hold this assumption.

Overall, it can be said that future publications on population comparisons should also incorporate more recent research results from principal research. The measurement of reaction times, self-reported risk perception, and the transition to the ART would contribute to gaining more sophisticated insights into the cognitive processes underlying dynamic risk-taking.

Limitations

There are several limitations that come with this thesis. With the BART, an experimental design was introduced that correlates with real-world dynamic risk situations. This is a necessary condition to better understand the phenomenon of dynamic risk-taking. However, this thesis may be limiting itself too much by focusing only on the BART. Indeed, findings from other experimental designs are not taken into account. Second, the BART cannot control for fear and anxiety, which can have a moderating effect on risk preference. Third, it should also be noted that the assumptions of the BART models do not extend to a physiological level. Thus, they cannot suggest how dynamic risk is processed at a neural scale. Fourth, the distinction between principal research and population research may not be appropriate, because it suggests a clear dividing between the two sub-fields. However, this is by no means the case. Finally, it is important to recall Box's aphorism (1976), which makes clear that even the model that performs best in model comparisons is probably not entirely correct. Information criteria can never prove assumptions but can only make statements about better and worse. Out of three wrong models, the model with the best AIC/BIC is also wrong.

Conclusion

This thesis examined the question of which cognitive processes underlie dynamic risk in the BART and how these processes affect externalizing behaviour. Four processes could be identified: Learning processes, representation of the risk structure of the scenario, option evaluation and response selection. It was further shown that the way individuals evaluate gains and the consistency with which they follow their evaluation, but not how they assess the probability of a negative consequence, can be taken as predictors of negatively associated externalizing behaviour. Many publications compare different populations with the BSR. Further insights into the differences between populations could be gained by considering reaction times and self-reported risk perception in future population comparisons, and by making more use of the ART.

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