Assignment 1 – Migration Matrix

Your goal is to compute the migration / transition matrix using a discrete cohort method based on the data provided in the file transition_data.xlsx.

The entry in the data set comprises three components:

- Id: the id of the rated obligor
- Date: the date of the rating assessment
- Rating: the result of the assessment

The rating scheme used is based on 7 non-default states (1 – 7) and one default state (8).

Consider the following principles when calculating rating migrations based on the cohort method:

- Each obligor is rated one or several times in the period of 2016 to 2022
- Migration probabilities need to be calibrated on 1-year time horizons: end of year t until end of year t+1, end of year t+1 until end of year t+2, etc.
- In case of several rating assessments within a single year, only the latest assessment before year end is relevant for end of year rating
- An obligor stays in the assessed rating grade until a new assessment is observed

Example 1: The data set contains only one observation for obligor '36' on May 30th 2017 with rating grade 4. The relevant periods for obligor '36' are therefore:

Start of period	End of period	Start rating	End rating
31-12-2017	31-12-2018	4	4
31-12-2018	31-12-2019	4	4
31-12-2019	31-12-2020	4	4
31-12-2020	31-12-2021	4	4
31-12-2021	31-12-2022	4	4

Example 2: The data set contains two observations for obligor '39' on May 21st 2017 with rating grade 2 and May 23rd 2019 with rating grade 1. The relevant periods for obligor '39' are therefore:

Start of period	End of period	Start rating	End rating
31-12-2017	31-12-2018	2	2
31-12-2018	31-12-2019	2	1
31-12-2019	31-12-2020	1	1
31-12-2020	31-12-2021	1	1
31-12-2021	31-12-2022	1	1

Migration probability for a single period:

$N_{i,t}$ = number of obligors in grade i at the beginning of period t

$N_{ij,t}$ = number of obligors in grade i at t migrating to grade j until end of period t

$$\hat{p}_{ij,t} = \frac{N_{ij,t}}{N_{i,t}} \qquad (1)$$

Migration probability over several periods:

Compute the obligor-weighted average for averaging migration probabilities over several periods. The weight is chosen as number of obligors per cohort:

$$\hat{p}_{ij} = \frac{\sum_t N_{i,t}\hat{p}_{ij,t}}{\sum_t N_{i,t}} \qquad (2)$$

Inserting (1) into (2) leads to:

$$\hat{p}_{ij} = \frac{\sum_t N_{i,t}\left(N_{ij,t}/N_{i,t}\right)}{\sum_t N_{i,t}} = \frac{\sum_t N_{ij,t}}{\sum_t N_{i,t}} = \frac{N_{ij}}{N_i} \qquad (3)$$

The obligor-weighted average can be directly obtained by dividing the overall sum of migrations from i to j by the overall number of obligors that were in grade i at the beginning of the considered periods. The calculation of the final migration probability is thereby agnostic of the corresponding period.