

1 *Payoff_digital_vanilla.py*

Purpose

Compares the net payoff at expiry for:

- A **vanilla European call** — payout increases linearly above the strike.
- A **digital (binary) call** — fixed payout if the underlying finishes above strike, zero otherwise.

Learning focus

- The vanilla call payoff = $\max(S_T - K, 0) - \text{premium}$. Gains grow as spot rises.
- The digital call payoff = fixed_amount if $S_T > K$ else $0 - \text{premium}$. Gains do not increase with spot — the only question is whether you finish above the strike.
- Premiums differ because the digital call offers a capped payoff profile.

Key intuition

- Vanilla calls reward *how far* you finish above strike.
- Digital calls reward *whether* you finish above strike.
- Digital options are cheaper, but all-or-nothing — they behave more like a probabilistic bet than a participation in upside.

Experiment

Increase $\text{digital}_{\text{payoff}}$ and observe how the break-even point shifts closer to the strike. This simulates a structure with a larger fixed payout and higher odds of profitability.

2 *Delta_vs_spot_digital.py*

Purpose

Explores the **Delta** of a digital option as spot price moves and time passes — plus a hedging simulation.

Learning focus

- **Digital Delta shape:**
 - Peaks sharply around the strike, especially close to expiry.
 - Flattens quickly away from strike.
- This spike means the option's sensitivity to small spot moves is **extremely concentrated near strike** — making hedging tricky.

- The simulation shows how spot, Delta, and price evolve over time for specific random paths.

Key intuition

- Digital options are **path-independent in payoff**, but **path-sensitive in hedging** because their Delta can swing dramatically near strike.
- As maturity approaches, a small change in spot price near strike can cause huge changes in Delta, requiring rapid hedge adjustments — this is known as **Gamma risk**.

Experiment

Reduce T_{days} to 5 and see how the Delta spike becomes even sharper — it will almost resemble a vertical wall at the strike, illustrating why market makers demand higher spreads for short-dated digitals.