1 Term_structure_evolution_HJM.py — Heath–Jarrow–Morton (HJM) framework

Purpose

Simulates forward rate curve evolution under the HJM model with different volatility structures.

Learning focus

- **HJM key idea**: Models the entire forward rate curve directly rather than the short rate.
- The drift term is not arbitrary — it is determined by the chosen volatility structure to ensure no-arbitrage.
- Different volatility functions change the behaviour of the curve:
  - Constant volatility: smooth, uniform shifts.
  - Humped volatility: more movement in mid-term maturities.
  - Two-factor volatility: more realistic, allowing different behaviours across short and long ends.

Experiment

Run compare_volatility_models() to see how the forward curve shape at the horizon changes with each volatility specification — you'll notice steepening or flattening depending on the term structure of vol.

2 LMM_forward_rate_vol_surface.py — Libor Market Model (LMM) with volatility surface calibration

Purpose

Calibrates a simplified LMM to synthetic market implied vol data, fitting both term structure and moneyness effects.

Learning focus

- **LMM** models discrete forward rates and is widely used for pricing interest rate derivatives like caps, floors, and swaptions.
- Volatility surface calibration ensures the model matches observed market implied volatilities across strikes and maturities.
- The calibration parameters (β, a, b, c, d) capture:
  - β: term structure decay
  - a, b: level and slope of maturity effect

  - c: curvature (smile)
  - d: skew (moneyness tilt)

Experiment

Modify the synthetic market parameters in create_synthetic_market_data() to create a steeper smile or stronger skew and see how calibration adjusts the parameters.

3 Short_rate_model_comparisons.py — Vasicek, CIR, and Hull–White short rate models

Purpose

Compares three classic short rate models using simulated paths, terminal distributions, and volatility patterns.

Learning focus

- **Vasicek model**: Mean-reverting Gaussian process with constant volatility; can produce negative rates.
- **Cox–Ingersoll–Ross (CIR) model**: Mean-reverting square-root process; keeps rates non-negative.
- **Hull–White model**: Vasicek with time-dependent drift to fit the initial yield curve exactly.

Key comparisons

- Distribution: CIR has skewness and no negative rates; Vasicek can go negative; Hull–White adapts drift over time.
- Volatility: CIR volatility is rate-dependent; Vasicek and Hull–White have constant volatility, but Hull–White's drift term changes rate dynamics.

Experiment

Increase sigma and see which model's terminal distribution spreads the most. Also, check rolling volatility plots to see how each model responds over time.