Financial Market Uncovered – Article 13

Jump Diffusion: Pricing the Unhedgeable

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Summary

1 Introduction

The Black–Scholes–Merton model has long been a central tool in options pricing. It assumes that asset prices follow a continuous stochastic process, driven by Brownian motion with constant volatility. This framework produces closed-form solutions and facilitates dynamic hedging. However, it rests on assumptions that are systematically violated in practice.

In real markets, asset prices do not evolve smoothly. They exhibit sudden, discontinuous jumps triggered by macroeconomic announcements, earnings surprises, geopolitical shocks, or liquidity imbalances. These events lead to price movements that are far larger and more abrupt than what a normal distribution would predict.

Under the Black–Scholes framework, such events have near-zero probability. In contrast, historical data consistently show fat tails, skewed return distributions, and sharp volatility spikes, particularly around key events.

This discrepancy between theoretical assumptions and market behaviour has direct implications:

- Option prices deviate from model predictions, especially for out-of-the-money strikes.
- Implied volatility surfaces exhibit smiles and skews not explained by constant-volatility models.
- Risk metrics underestimate tail exposure, particularly in volatile regimes.

To address these shortcomings, jump diffusion models were developed. By combining standard diffusion with a jump process — often modelled as a compound Poisson process — they account for both continuous fluctuations and rare but significant shocks.

2 The Merton Jump Diffusion Model

In 1976, Robert C. Merton proposed an extension to the Black–Scholes model that incorporated discrete jumps into the dynamics of asset prices. The idea was straightforward: while prices generally evolve continuously, they occasionally experience abrupt changes due to major events. The standard geometric Brownian motion was therefore augmented by a jump component, producing what is now known as the Merton jump diffusion model.

This framework allows for a more realistic representation of asset return distributions by capturing two empirical facts:

- Fat tails: the probability of extreme price movements is higher than the normal distribution suggests.
- Return asymmetry: markets may exhibit upward or downward jump bias, depending on the underlying asset.

By combining a continuous diffusion term with a discrete jump process, the Merton model improves the modelling of risk, particularly in pricing out-of-the-money options and estimating tail-related metrics.

2.1 Mathematical Formulation and Key Intuition

The price dynamics of a risky asset under the Merton jump diffusion model are given by the following stochastic differential equation (SDE):

$$\frac{dS_t}{S_t} = (\mu - \lambda k)dt + \sigma dW_t + dJ_t$$

Where:

- S_t is the asset price at time t,
- μ is the expected return,
- σ is the volatility of the diffusion component,
- W_t is a standard Brownian motion (as in Black–Scholes),
- J_t is a jump process,
- λ the jump intensity, i.e. the average number of jumps per unit of time,
- k is the mean jump size, defined as $E[Y-1]$ where Y is the multiplicative jump,
- The term $(\mu - \lambda k)$ adjusts the drift to ensure no-arbitrage under the risk-neutral measure.

Drift term $(\mu - \lambda k)dt$): This is the average direction in which the price is expected to move over time. The adjustment by λk accounts for the fact that jumps — when they occur — also contribute to returns. Without subtracting this term, the model would overestimate expected growth.

Diffusion term (σdW_t): This is the random, continuous movement of prices — what we typically think of as "normal" market noise. It's the same component used in the Black–Scholes model, capturing day-to-day fluctuations due to news, trading flow, and general uncertainty.

Jump term (dJ_t): This is what makes the model different. It represents the possibility that the price suddenly jumps — up or down — due to major events. It doesn't happen all the time, but when it does, the effect is immediate and substantial. Think of it as a switch that flips on occasionally, causing a discrete shock to the asset price.

So in plain terms:

The price usually follows a continuous path shaped by volatility and drift, but from time to time, it experiences sharp jumps that can't be explained by ordinary market noise. The model blends both behaviours into a single equation.

The jump component J_t is typically modelled as a compound Poisson process:

$$J_t = \sum_{i=1}^{N_t} (Y_i - 1)$$

with:

- $N_t \sim Poisson(\lambda t)$ the number of jumps up to time t,
- Y_i i.i.d. random variables representing jump sizes (often assumed lognormal: $\ln(Y_i) \sim \mathcal{N}(\mu_j, \delta^2)$).

What sets this model apart is its ability to reproduce the discontinuities observed in real markets. While Brownian motion assumes that asset prices move smoothly — albeit unpredictably — jump diffusion assumes that prices can change abruptly in response to new information or structural shocks. This feature is not only theoretically appealing but also necessary when pricing options that are sensitive to tail events.

Imagine a company releasing quarterly results. Under Black–Scholes, the stock price might gradually price in market expectations. But in practice, we observe something different: a sudden revaluation the moment results are announced. Prices jump — not drift. Merton's model captures this behaviour by design.

By explicitly modelling these jumps, the framework provides better estimates of the probability of extreme price moves, leading to more accurate pricing for out-of-the-money options and better calibration of risk metrics such as Value at Risk or stress-test scenarios.

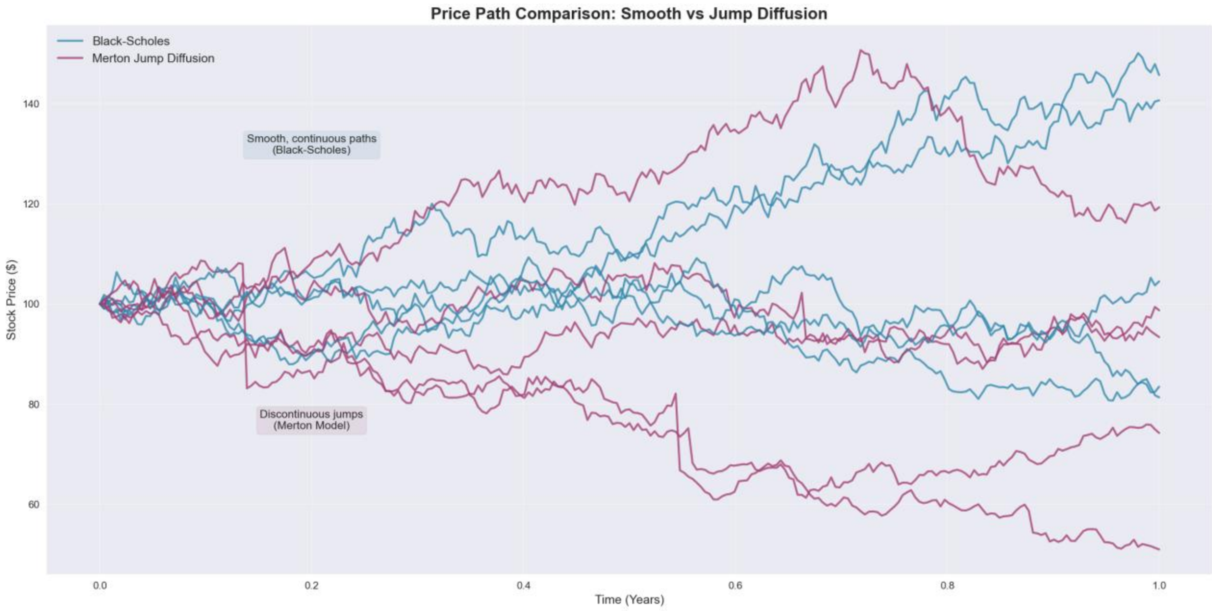

Figure 1: Simulated asset price paths under two models

The Merton jump diffusion model accounts for sudden, non-Gaussian events — such as earnings shocks, geopolitical news, or flash crashes — that are impossible to represent with pure diffusion. This distinction is central to understanding the need for jump-aware pricing and risk frameworks.

Traditional diffusion models assume markets move like a "random walk" with no sudden cliffs. But reality often features jumps — instant price shifts too large to be explained by volatility alone. This figure makes the contrast clear: smooth vs broken paths. The Merton model introduces these jumps explicitly, via a Poisson process that adds realism — and complexity — to how we model asset returns.

2.2 *Economic Meaning and Parameter Sensitivity*

Each parameter in the Merton jump diffusion model carries a distinct economic interpretation. While the model introduces mathematical complexity, the underlying concepts are intuitive — and particularly relevant for traders, risk managers, and anyone attempting to capture real-world market behaviour beyond the assumptions of pure diffusion.

The Jump Intensity λ

The parameter λ governs the **expected frequency of jumps** over a given time horizon. For example, if $\lambda = 2$, then on average two jumps are expected per year. A higher λ indicates a market environment where jumps occur more frequently — a setting that might correspond to unstable macroeconomic conditions, persistent political risk, or assets exposed to binary events such as regulatory approvals or earnings surprises.

In practice, λ is critical when modelling tail risk. An option portfolio exposed to sudden movements (for example, through short gamma positions) is particularly sensitive to this component. From a calibration perspective, λ tends to be inferred from the steepness of the volatility smile in short-dated options.

The Jump Size Distribution Y, Mean k

Jump sizes Y are modelled as multiplicative shocks to the asset price. Typically, their logarithms are assumed to follow a normal distribution, meaning that the actual jump sizes are lognormally distributed — a choice that preserves the positivity of prices.

The term $k = E[Y - 1]$ represents the **expected percentage change** in asset price per jump. It reflects both the average direction and the magnitude of the jumps. When $k > 0$, jumps are on average upward; when $k < 0$, they are downward. For equity markets, k is usually negative — capturing the empirical observation that crashes are faster and more severe than rallies.

This asymmetry in jump size is central to reproducing **skew** in implied volatility surfaces. For example, the sharp pricing of out-of-the-money puts compared to calls can be captured by introducing rare but significant negative jumps.

Volatility σ and Diffusion–Jump Interplay

Although the model introduces jumps, the traditional volatility parameter σ remains. It still describes the magnitude of day-to-day fluctuations — the continuous noise that dominates in stable markets.

However, the interplay between σ and the jump parameters is crucial. If σ is high and jumps are rare (λ small), the model behaves similarly to Black–Scholes. But when σ is low and jumps are frequent or large, jump risk dominates, and traditional hedging assumptions break down.

This dual structure allows the model to distinguish between *diffusive volatility* (reflecting regular trading and microstructure) and *jump risk* (reflecting discontinuities and informational shocks).

Drift Adjustment: $\mu - \lambda k$

The adjustment of the drift term by λk ensures that the model remains arbitrage-free under the risk-neutral measure. Economically, this reflects the fact that when jumps are incorporated, their expected effect on returns must be subtracted from the continuous drift to avoid double-counting.

From a practical perspective, this term rarely affects pricing directly but is important when simulating price paths or computing expected returns in a model-consistent way.

2.3 *Trading Insights*

The Merton jump diffusion model is not merely a theoretical refinement; it offers valuable insights for traders and market practitioners seeking to understand or exploit the pricing of discontinuities in asset returns. While not always used directly on trading desks, the intuition behind the model shapes how traders think about event risk, volatility surfaces, and hedging under uncertainty.

Out-of-the-Money Option Pricing and the Smile

One of the most tangible implications of introducing jumps is the impact on out-of-the-money (OTM) option prices. Under the Black–Scholes model, deep OTM options — especially puts — are often underpriced relative to what is observed in the market. This is because the model assumes an extremely low probability of large moves.

Jump diffusion models correct this by increasing the probability mass in the tails of the return distribution. In practical terms, this means:

- OTM puts (reflecting downside jump risk) become more expensive.
- The implied volatility curve takes on a smile or skew shape, particularly visible in short-dated maturities.

For volatility traders, this helps explain why certain options appear "expensive" despite low historical volatility — they are pricing in discrete risks not captured by standard diffusion.

Hedging and the Limits of Delta

In a continuous world, delta hedging works because small moves can be neutralised by rebalancing frequently. But in a world with jumps, no amount of delta hedging can protect against a discontinuous price move. The market may open at a level far beyond your hedge, rendering rebalancing strategies ineffective.

This insight has two major consequences:

- Traders holding short gamma positions face unhedgeable exposure to event risk.
- There is a jump risk premium embedded in option prices, especially near binary catalysts (e.g. earnings, central bank announcements, geopolitical events).

This is also why realised volatility and implied volatility can diverge meaningfully. A market may be quiet most of the time, but if one large jump is anticipated, implied volatility will remain elevated — reflecting the cost of insuring against that single event.

Scenario Analysis and Risk Management

For risk managers, the jump diffusion model provides a more robust framework for stress testing and scenario analysis. Rather than relying solely on variance-based measures like Value at Risk (VaR), the model makes it possible to simulate rare but impactful outcomes, such as:

- A 10% market gap following an earnings miss
- A policy shock that reprices an entire asset class overnight

While not all firms implement the full model in live systems, its conceptual role is well established: it challenges the assumption that risk can be fully managed through continuous rebalancing and Gaussian assumptions.

Trading Takeaway

The primary lesson for traders is that markets don't just exhibit more volatility — they exhibit *discontinuous* volatility. Jumps are not statistical noise; they are structural features of how information enters markets. Pricing them correctly, managing their impact, and recognising when the market is overstating or understating their likelihood are core elements of advanced options trading.

3 Parameter Intuition

While the Merton jump diffusion model introduces additional mathematical complexity compared to the Black–Scholes framework, its parameters have clear and intuitive interpretations for practitioners. Each one corresponds to a specific dimension of market behaviour: how often extreme moves occur, how large they are, and how they interact with everyday volatility.

Understanding how these elements combine is essential for translating the model from theoretical abstraction to actionable insight. Rather than treating the parameters as isolated inputs, experienced traders often think of them as coordinates on a regime map — a way to classify different market conditions and anticipate how they might affect pricing, volatility surfaces, and risk exposure.

3.1 The Jump Risk Regime Map

To bring the model's intuition into a trader's world, consider a two-dimensional regime map based on the key jump parameters:

- Jump frequency (λ) — how often do jumps occur?
- Jump size (k) — how large is each jump on average?

These two variables define a matrix of market environments:

Jump Frequency ($\boldsymbol{\lambda}$)	**Jump Size ($\boldsymbol{k}$)**	**Market Interpretation**
High	**High**	*Volatile and unstable*: Frequent, large jumps. Crisis regimes, distressed credit, EM FX.
High	**Low**	*Noisy but shallow*: Many small jumps. Political noise, high-frequency earnings names.
Low	**High**	*Complacent build-up, rare crashes*: Long periods of calm, then sudden breakdown. 2008.
Low	**Low**	*Diffusion-dominated*: Quiet regime, close to Black–Scholes. Most large-cap equity in normal times.

This matrix helps position assets or strategies in relation to their exposure to jump risk:

- A trader pricing short-dated options on a biotech stock pre-FDA decision might be in the low-frequency / high-jump regime — quiet most of the time, but explosive on binary news.
- A portfolio of frontier market sovereign bonds might sit in the high-frequency / high-jump corner — frequent repricings with large amplitude, often driven by external shocks.

- Liquid, large-cap equity indices in stable macro environments fall near the low / low zone — where jumps are negligible, and Black–Scholes holds reasonably well.

Importantly, the regime is not fixed. Markets move between these quadrants as conditions evolve. A calm environment with low volatility and rare jumps can quickly shift into an unstable, fat-tailed regime when macro or political risk escalates. This makes scenario mapping — not just point estimation — critical.

The matrix also explains why options with similar implied volatilities can behave very differently when a jump occurs. Two instruments may have the same ATM vol, but if one is exposed to rare, severe jumps and the other to regular mild ones, their tails — and risk profiles — will differ materially.

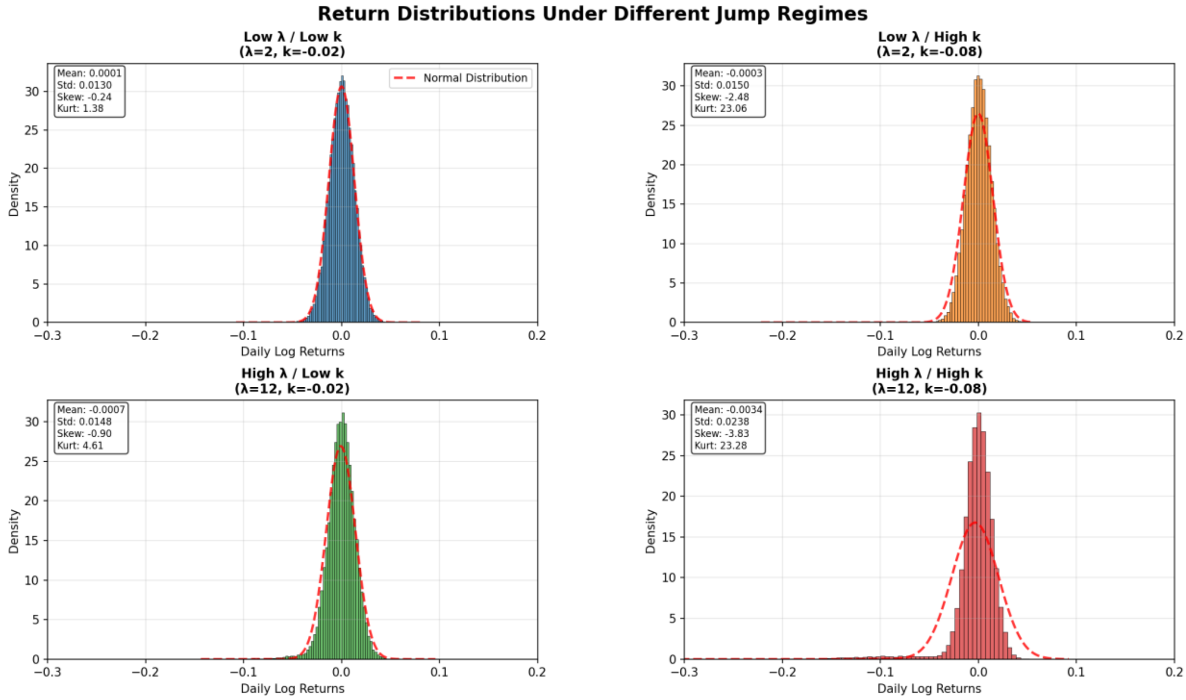

Figure 2: Simulated daily log return distributions under four jump diffusion regimes, varying frequency (

As we move from top-left (low jump risk) to bottom-right (high frequency + large jumps), the distributions become increasingly skewed and fat-tailed. Note how the Gaussian curve (red dashed line) systematically underestimates tail risk — particularly in the high-λ, high-k quadrant. This visual confirms the intuition behind the regime map: frequent large jumps do not just fatten tails — they shift the mean, distort skew, and collapse the assumption of "normal" returns entirely.

3.2 Practical Use: Scenario Planning and Stress Testing

The jump diffusion framework is not just a theoretical refinement. It offers a practical lens through which traders and risk managers can better anticipate market regimes and prepare for shocks. When used thoughtfully, the parameters of the Merton model become tools for forward-looking scenario design, not just backward-looking calibration.

A Different Approach to Tail Risk

Traditional volatility models often underestimate tail risk because they assume price changes are incremental and normally distributed. This leads to unrealistic confidence in hedging strategies and value-at-risk estimates — especially in markets where surprises are not rare, but routine.

By explicitly incorporating jump frequency (λ) and magnitude (k), the model provides a more realistic structure for building stress tests and imagining non-linear outcomes. Instead of asking, "What happens if the market moves 2%?", a jump-based approach asks:

"What if the next price print is 15% away from the last, and there was no opportunity to rebalance?"

This changes the nature of the risk conversation — especially for portfolios exposed to convexity, barrier structures, or illiquid hedging instruments.

Use Cases Across the Desk

- Trading: A short gamma book requires jump-aware risk buffers, particularly around macro releases or binary events. Using the regime map, a trader can size positions not just based on implied vol but also on jump asymmetry.
- Structuring: Designing a knock-out option or digital payoff without modelling jumps is dangerous. A single gap can wipe out the structure. The model allows for more conservative pricing and protection structuring.
- Risk Management: Scenario generation becomes richer. Instead of drawing returns from a normal distribution, jump-aware simulations can replicate observed crash dynamics — improving VaR, Expected Shortfall (CVaR), and reverse stress testing.

Realistic Stress Frameworks

Many banks and hedge funds already use Merton-style logic to simulate event-driven regimes. Rather than relying on historical windows, they impose artificial jump shocks calibrated to current macro risk. For example:

- 5% down-jump with 10% probability around central bank decisions.
- 20% drop as a stress test on emerging market FX pairs with historical precedents.

This style of thinking reframes risk as episodic and discontinuous — which is how it behaves in practice.

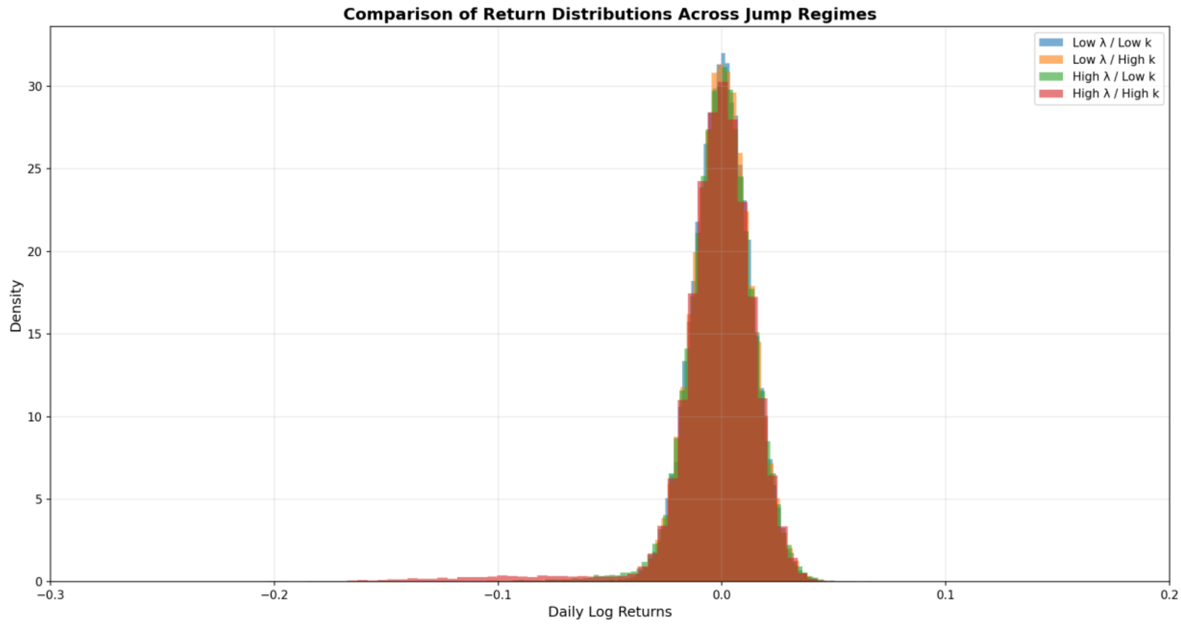

Figure 3: Overlay of return distributions under different jump regimes

While all are centred near zero, the tails and skewness vary significantly. Visually, this underscores the importance of modelling discontinuities explicitly — not just relying on historical volatility or Gaussian-based VaR. From a risk management standpoint, two return series might look "stable" based on their centre — but behave very differently in the tails. This is exactly where jump-aware stress testing adds value.

4 Option Pricing Under Jumps

Introducing jumps into asset price dynamics brings fundamental changes to how options are priced. Under the Black–Scholes framework, closed-form solutions exist for a wide range of vanilla derivatives, thanks to the model's reliance on continuous, normally distributed returns.

Once discontinuities are introduced, the mathematics becomes more complex — and so does the intuition.

In jump diffusion models, prices evolve via two distinct mechanisms: continuous Brownian motion and discrete jumps. These two components cannot be disentangled neatly, and as a result, option pricing under jumps no longer follows the same deterministic structure. There is no universal closed-form formula for European or American options under Merton's framework. Instead, pricing involves more advanced tools: partial integro-differential equations, transform methods, or simulation.

Yet the payoff is worth the complexity: jump diffusion models can generate volatility smiles, more accurately reflect market prices of tail risk, and allow for improved scenario analysis across strikes and maturities.

4.1 From PDEs to Pricing

In the Black–Scholes world, option pricing is derived from a partial differential equation (PDE) that reflects the dynamics of continuous price changes and the absence of arbitrage. The solution to this PDE gives us the classic Black–Scholes formula for European calls and puts.

With jumps, this PDE is transformed into a partial integro-differential equation (PIDE). Why? Because we must now account not only for the infinitesimal changes driven by Brownian motion, but also for the discrete jump events that may occur at any instant with a certain probability.

This extra complexity comes from a non-local term: the value of the option at a new post-jump price must be included in the pricing function — and since jumps can happen anywhere on the price axis, the model becomes *global* in its behaviour.

The PIDE for the value $V(S,t)$ of a European option takes the form:

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}+(r-q-\lambda k)S\frac{\partial V}{\partial S}-rV+\lambda\mathbb{E}[V(YS,t)-V(S,t)]=0$$

Each term reflects a piece of intuition:

- The second-order term represents the familiar diffusion (volatility) effect.
- The first-order term reflects the adjusted drift.
- The integral expectation term captures the impact of **jumps to a new price level** YS — which may be up or down, depending on the jump size distribution.

Because analytical solutions to this equation are generally not available, practitioners turn to one of three main numerical approaches:

1. **Fourier Transform Methods**

Especially for European options, models like Carr–Madan or COS method allow efficient computation using characteristic functions. These are well-suited for Merton's model and its extensions.

2. **Finite Difference Methods**

By discretising the PIDE on a grid, this approach enables approximate solutions. It is flexible but computationally intensive, especially when pricing early-exercise options.

3. **Monte Carlo Simulation**

Perhaps the most intuitive approach, Monte Carlo allows simulation of jump paths by superimposing Poisson-driven jumps onto standard diffusion. This method is highly adaptable and often used for exotic options, though it converges slowly for Greeks and early-exercise features.

Each method comes with trade-offs between accuracy, speed, and flexibility. For liquid European options, transform methods tend to dominate. For exotic payoffs or illiquid names, simulation is often preferred — especially when jump correlations or path dependency matter.

In the next section, we will explore how introducing jumps naturally produces volatility smiles and skews, and why this matters not just for pricing but for market interpretation.

4.2 Why Smiles and Skews Emerge Naturally

One of the most well-known limitations of the Black–Scholes model is its inability to explain the shape of observed implied volatility curves. In theory, if returns were normally distributed and volatility were constant, implied volatilities across strikes and maturities should be flat — a "volatility plateau." But in practice, they are anything but flat.

Markets consistently exhibit smiles, skews, and term structure effects in implied volatility. These patterns are not noise — they are structural features of how markets price risk. And they emerge naturally from models that include jumps.

Smiles: More Weight in the Tails

The presence of jumps introduces fat tails into the return distribution. In simple terms, it increases the probability of extreme outcomes — both positive and negative. For option pricing, this has immediate implications:

- Deep out-of-the-money (OTM) calls and puts become more valuable than the Black–Scholes model suggests.

- Implied volatility, as inferred from market prices, must rise at the wings to account for this added tail risk.

This results in the classic volatility smile: implied volatility is higher for low and high strike prices than for at-the-money (ATM) options. Under jump diffusion, this is not a calibration artifact — it is a direct consequence of the return process being non-Gaussian.

Skews: Asymmetric Jump Risk

In equity markets, downside jumps tend to be more frequent and more severe than upside ones. Negative jumps reflect panic, liquidations, or systemic news, while positive jumps are often capped by structural constraints (e.g. price limits, investor conservatism).

If jump sizes are skewed to the downside — as is often the case with equities or credit — then put options will embed more jump risk premium than calls. The result is a volatility skew: implied volatility declines with strike price. This is a well-documented feature in equity options markets and is naturally captured by jump diffusion models with asymmetric jump distributions (e.g. lognormal or double exponential).

Short-Dated Options Are the Most Sensitive

Jumps are discrete, low-probability events. Over long horizons, their effect blends into the diffusion process. But for short-dated options — especially those expiring around key events — jump risk dominates.

As a result, smiles are most pronounced for short maturities. This is something that standard stochastic volatility models struggle to reproduce alone, but jump diffusion models handle with ease. The market's anticipation of a possible discontinuity in the very near term is priced directly into short-term implied volatilities.

Interpretation for Traders

From a trading perspective, the model tells a clear story:

- If the market is assigning high implied vol to deep OTM puts, it's not just pricing in variance — it's pricing in jump risk.
- If short-dated options show an exaggerated smile, there may be a binary event ahead.
- If you're pricing exotic structures (e.g. barrier options), ignoring jumps leads to mispricing and mishedging.

The smile isn't a quirk — it's a signal. And jump diffusion gives you a language to interpret it.

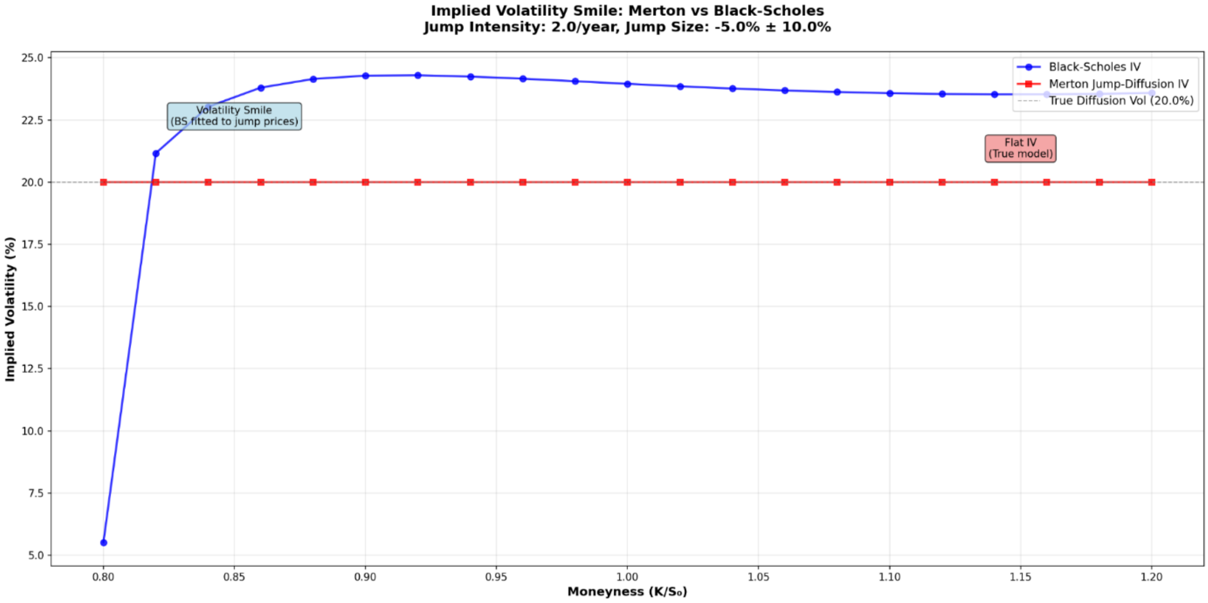

Figure 4: Implied volatility curves for European options under two models

The Merton model, despite having a flat "true" volatility, produces option prices that, when reinterpreted through Black–Scholes, generate a volatility smile — with elevated IV for OTM puts and calls. This illustrates how jumps naturally create fat tails, requiring higher implied volatilities to reconcile observed prices.

What's striking is that even though the "true" model has no volatility smile, the presence of jumps in price dynamics forces the market to embed one. Traders observing market prices through a Black–Scholes lens will interpret this as elevated risk at the tails — and rightly so.

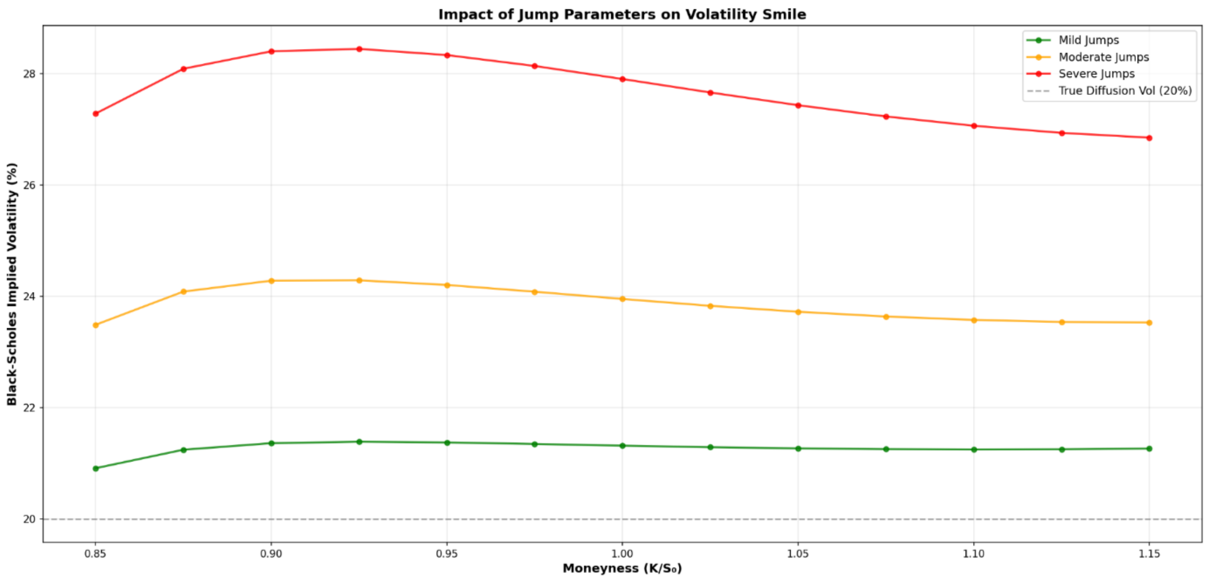

Figure 5: Effect of increasing jump severity on implied volatility smiles

This aligns with market intuition: the more pronounced the jump risk (in frequency and size), the more the market demands compensation in the wings — even if central volatility remains unchanged. As shown above, the structure of the smile is highly sensitive to jump characteristics. The more intense and asymmetric the jumps, the more convex the smile becomes — especially on the downside.

4.3 Hedging Implications

One of the foundational principles of the Black–Scholes framework is that an option can be perfectly hedged through continuous rebalancing. Small changes in the underlying asset's price can be offset by adjusting the delta — the sensitivity of the option's price to that of the underlying. This forms the basis for dynamic replication and risk-neutral pricing.

However, once jumps are introduced, this logic begins to break down.

Discontinuities Break Delta Hedging

Delta hedging assumes that the underlying moves smoothly and incrementally. In such a world, a trader can rebalance the hedge portfolio as frequently as needed to stay neutral to small changes.

But a jump, by definition, is a sudden and discontinuous price movement. The price may gap 5% or 10% between hedge adjustments — or worse, between market closes and opens. This results in slippage: the hedge is no longer effective, and the trader suffers a loss that cannot be recovered by rebalancing.

As a result, delta and gamma — the traditional tools of options risk management — become insufficient. The portfolio remains exposed to gap risk, especially around binary events, macro announcements, or illiquid periods.

Vega and the Cost of Uncertainty

Jumps also affect vega — the sensitivity of an option to changes in implied volatility. If markets anticipate a possible jump, implied volatility rises, and option prices become inflated — even if realised volatility remains low.

This disconnect between realised and implied volatility creates a jump risk premium. Traders who sell options in such environments are not simply short volatility — they are short unhedgeable uncertainty. The premium reflects not just variance, but convex exposure to events that standard hedges cannot neutralise.

Incomplete Markets, Incomplete Hedges

In a world without jumps, risk can be dynamically hedged, and replication arguments hold. Once jumps are introduced, the market becomes incomplete: not all risks can be hedged with traded instruments. This means:

- Option prices depend on investors' risk preferences, not just arbitrage conditions.
- Theoretical pricing becomes a blend of replication and utility-based valuation.

- Hedging strategies must now be probabilistic and scenario-driven, rather than purely deterministic.

In practical terms, this means accepting that hedging is no longer perfect — and that part of the option premium reflects insurance against the unknowable.

Practical Adjustments for Traders

- Wider hedging bands: During event risk, traders allow for larger tolerances in hedge ratios to avoid overtrading.
- Volatility buffers: Risk systems often overlay jump scenarios onto historical simulations, assigning extra capital to gap exposure.
- Structured protection: Traders may use calendar spreads, risk reversals, or digital overlays to partially neutralise jump risk.

Ultimately, jump diffusion reminds us that not all risk can be smoothed out with calculus. Some exposures must be priced, respected, and carried, not eliminated.

5 Practical Applications and Use Cases

Jump diffusion models are more than theoretical constructs — they offer direct, actionable value across pricing, trading, and risk functions. While not always implemented in full form on trading desks, their logic underpins how sophisticated market participants approach option valuation, hedge construction, and risk control in volatile or event-driven environments.

By acknowledging that asset prices do not move continuously, these models give practitioners a more realistic framework for managing uncertainty. They allow for the pricing of rare events, the sizing of tail exposure, and the design of structures that anticipate discontinuities.

5.1 In Pricing — From Vanilla to Tail-Sensitive Structures

Incorporating jump diffusion into pricing models fundamentally alters the valuation of instruments that are sensitive to tail risk, event probability, or barrier behaviour. This matters most in contexts where the payoff is highly non-linear or concentrated in the wings.

Deep Out-of-the-Money Options

Options that are far out-of-the-money — particularly puts — are often overpriced relative to Black–Scholes predictions. This is not a market inefficiency; it's a rational adjustment for the possibility of sudden, large price movements.

Jump diffusion explains this clearly:

- Rare but severe moves (captured by large k, low λ) increase the expected value of options that only pay in the tails.
- Even if the underlying is trading calmly, the market may embed a crash premium — particularly around macro events or earnings.

This helps explain why implied volatility is often elevated on the downside for equity indices, despite low realised volatility: the market is pricing the possibility of a discontinuity, not just noise.

Digital and Barrier Options

Exotic options with digital or barrier features are especially sensitive to jumps. In models without jumps, barrier crossing probabilities are derived assuming continuous paths. In practice:

- A jump can cause an immediate breach of the barrier — without warning.
- Standard pricing will underestimate the probability of knock-out or knock-in events.

Jump diffusion models adjust the transition density to reflect this. They increase the likelihood that a payoff condition is satisfied abruptly — especially near expiry. For traders structuring path-dependent products, ignoring jump risk leads to systematic underpricing of protection and misestimated sensitivities (e.g. incorrect delta at the barrier).

Event-Driven Option Pricing

Short-dated options around events (e.g. earnings releases, central bank decisions, geopolitical announcements) often show volatility spikes that cannot be explained by recent realised volatility alone. This is not irrational — it is the market pricing a binary jump scenario.

Jump diffusion models allow these scenarios to be embedded explicitly:

- With a specified jump probability and magnitude, one can derive the fair value of a straddle conditional on a gap occurring.
- Traders often use this logic to compare implied move vs expected event move, and position accordingly (e.g. long gamma ahead of NFP).

Even when not applied mechanically, the mental framework of jump-adjusted valuation is core to informed pricing.

5.2 In Risk — Measuring and Stressing the Unthinkable

Traditional risk frameworks — particularly those based on historical variance or Gaussian assumptions — often fail to capture the most important risks in financial markets: discontinuities, crashes, and regime shifts. These are precisely the risks that jump diffusion models are designed to incorporate.

By modelling rare, sudden events as a structural component of price dynamics, jump models provide a more robust foundation for both quantitative risk metrics and scenario-driven thinking.

Value at Risk and Conditional VaR (CVaR)

Standard VaR models assume that returns are driven by continuous processes, typically with normal or near-normal distributions. This leads to systematic underestimation of tail risk — particularly over short horizons where jumps matter most.

Jump diffusion allows for more realistic tail probabilities. In a Merton setting:

- The return distribution includes both frequent small moves and infrequent large jumps.
- This creates fatter tails — raising both VaR and CVaR estimates without needing to arbitrarily widen confidence intervals.

For risk managers, this is essential when:

- Assessing options books exposed to path-dependency or gap risk,
- Stress testing credit or emerging market portfolios, where discontinuities are frequent,
- Setting limits or margins in low-liquidity environments, where microstructure breaks can mimic jump-like events.

Scenario Generation and Reverse Stress Testing

Beyond statistical measures, jump models help define **narrative-based scenarios** that reflect actual market behaviour. Instead of tweaking volatility inputs, risk teams can build shocks directly into simulated paths:

- 5% downward jump on a single day, followed by volatility spike,
- Sector-specific gap moves (e.g. regulatory change for tech or energy),
- Overnight currency devaluation or fixed-income repricing.

These simulations better capture how portfolios behave when rebalancing is impossible — a key blind spot of diffusion-only models.

In reverse stress testing, the question becomes:

“What size of jump — and under what conditions — would break this portfolio?”

Merton-style logic provides the architecture to answer that question quantitatively, not just qualitatively.

Risk-Neutral Densities and Tail Awareness

Jump diffusion also sharpens the interpretation of risk-neutral distributions implied from options markets. When markets assign high probability to extreme moves (e.g. high put skew), jump models provide a structural explanation:

- Not just more variance, but discontinuous repricing is being anticipated.
- Traders pricing risk reversals, tail hedges, or convexity overlays can use jump-consistent models to match observed smile curvature more precisely.

This matters when designing hedges for tail exposure. Standard delta–vega hedges might neutralise small moves, but they leave a portfolio vulnerable to non-linear effects. Jump-aware thinking encourages the use of:

- Out-of-the-money options for asymmetric risk,
- Digitals and binaries to monetise tail exposure,
- Dynamic overlays to adapt as jump risk shifts over time.

5.3 In Trading — Navigating Convexity, Event Risk, and the Jump Premium

Incorporating jump diffusion into trading practice is not about applying every equation mechanically. It’s about recognising that certain types of risk — especially those linked to discontinuities — require different positioning, sizing, and interpretation.

The presence of jumps reshapes the payoff profile, hedging strategy, and risk premium embedded in a trade. It helps explain why some strategies lose money despite being delta-hedged, why others offer excess returns despite low volatility, and why certain options appear persistently expensive. These are not anomalies — they are the market pricing in the cost of unhedgeable risk.

Volatility Arbitrage Across Maturities

Many volatility arbitrage strategies rely on comparing implied volatility with expected realised volatility. But when jump risk is present, the realised variance may understate the true economic risk — especially for short maturities or tail-sensitive strikes.

Jump diffusion clarifies:

- Why short-dated implied vol remains elevated ahead of events, even when recent realised vol is low.
- Why vega-neutral positions may still lose money in jump-prone regimes: you're not short variance — you're short discontinuity.

Sophisticated vol traders adjust for this by:

- Using jump-aware realised variance estimators,
- Segmenting exposure into continuous and jump-attributed risk,
- Avoiding short-vol trades ahead of binary catalysts — or requiring larger risk premia to take them on.

Extracting the Jump Risk Premium

Jumps create risk premia that differ from traditional volatility premia. Because jump risk cannot be hedged away entirely, investors demand compensation for bearing it. This leads to systematic features in option markets:

- Puts tend to trade rich, especially far OTM — reflecting downside jump fears.
- Digital spreads and risk reversals reflect asymmetric pricing of positive and negative jump risk.

Traders can extract jump risk premium by:

- Selling OTM puts (with structural caution),
- Constructing reverse knock-outs or skew-neutral spreads that monetise steep implied skews,
- Positioning long convexity in names likely to surprise — not just in volatility, but in discrete price gaps.

Event-Driven Setups and Jump Positioning

Event risk — from earnings to central bank meetings — offers a natural testing ground for jump-aware thinking. Traditional trading around such events involves:

- Buying gamma via short-dated straddles,

- Estimating “implied move” vs “historical move”,
- Structuring delta-neutral trades with defined convex payoffs.

Jump diffusion provides the underlying justification. It models the market as a quiet diffusion process interrupted by occasional discontinuities — precisely the structure of event-driven trading.

Experienced traders use this framework to:

- Forecast skew steepening ahead of event dates,
- Infer market positioning from implied volatility shapes (e.g. front-loaded skew = fear),
- Time volatility compression after the jump window closes.

6 Calibration Challenges

While jump diffusion models offer compelling improvements in realism and flexibility, they come at a cost: they are difficult to calibrate, sensitive to assumptions, and fragile to noise. In theory, their parameters can be inferred from option prices or historical returns. In practice, calibration becomes a mix of art and approximation.

Unlike the Black–Scholes model, which requires only volatility (plus basic contract terms), jump models introduce at least three new parameters: the jump intensity λ, the average jump size k, and the jump size distribution itself (often characterised by its mean and standard deviation). Each of these is non-observable, and none are easily inferred with stability across time or assets.

As a result, most traders and risk managers use jump models for intuition and structure, but rarely for precision calibration — unless they're working on bespoke structures, exotics, or regulatory scenario frameworks.

6.1 Estimation Issues

The first problem in calibrating jump diffusion models is that jumps are rare events. Most historical return series — even over years — contain relatively few large discontinuities. This makes it hard to statistically separate a true jump process from fat-tailed diffusion or clustered volatility.

Historical Estimation: Limited Signal

When attempting to estimate jump parameters from returns data:

- Small samples and few tail events lead to unstable estimates.
- Volatility clustering (e.g. from GARCH effects) can be mistaken for jumps, and vice versa.
- Estimated jump frequencies vary widely depending on the sampling interval and filtering method.

In practice, any parameter estimated directly from returns is subject to considerable noise. This undermines the reliability of out-of-sample forecasting, and may lead to overfitting when applied mechanically.

Implied Estimation: Overfitting the Smile

The alternative approach is to calibrate jump models directly to market option prices. This seems promising — option surfaces contain rich information about risk-neutral distributions, and jumps influence the shape of the smile.

However, several issues arise:

- The same implied volatility surface can often be fit with multiple combinations of jump parameters, especially when stochastic volatility is also in the model.
- Short-dated options are highly sensitive to microstructure noise and bid–ask spreads.
- Long-dated options embed forward expectations of jump risk, which are not directly observable.

The result is a calibration landscape that is non-unique, non-smooth, and regime-dependent. Traders may find one set of parameters that fits well today, only to see it break down under new conditions tomorrow.

Jump Risk Is Not a Constant

Another difficulty is that jump behaviour is non-stationary. During calm periods, jump intensity appears low; in crisis regimes, it spikes. The market's perception of jump risk changes rapidly — far faster than any statistically inferred model can track.

For this reason, even the best-calibrated model is likely to lag current market dynamics unless supplemented by qualitative judgement or regime-switching logic.

6.2 Use in Practice

Despite the calibration challenges outlined above, jump diffusion models remain valuable — not because they produce perfect prices, but because they embed the right intuition about market behaviour. In real trading environments, these models are rarely used to mark books or automate pricing. Instead, they serve as a structural lens: a way to think clearly about where risk lives, how it behaves, and what can (or cannot) be hedged.

Traders: Intuition, Not Infrastructure

For most options traders, jump models are not directly plugged into the pricing engine. Instead, they are part of the mental framework:

- Understanding why OTM puts remain elevated even when realised volatility is low.
- Interpreting steep skews not just as a volatility artefact, but as a priced-in tail risk.
- Anticipating which products are most likely to suffer from jump exposure (e.g. barriers, digitals, short gamma books near catalysts).

Rather than estimate exact jump parameters, experienced traders infer the presence of jump risk from market shape — using the implied volatility surface as a diagnostic.

Structurers: Stress Awareness and Path Sensitivity

In exotic desks or structured product teams, jump diffusion models often appear behind the scenes — not for day-to-day pricing, but for backtesting, risk assessment, and stress design. They are especially useful in:

- Estimating early barrier breaches or digital triggers under jump-aware dynamics.
- Designing client protections against extreme moves (e.g. reverse knock-ins, tail hedges).
- Stress testing long-dated portfolios to non-diffusion risk — events that would be missed by pure stochastic volatility models.

Here, the goal is not exact calibration, but understanding whether certain payoffs are robust to the presence of large, sudden shocks.

Risk Desks: Scenario Engines, Not Forecast Tools

On the risk side, jump models are widely used as scenario engines. Their value lies not in precise replication of the market, but in expanding the space of possibilities:

- "What happens if the index gaps 7% overnight?"
- "What if three correlated assets jump simultaneously?"
- "How does this position behave under 2008-like discontinuities?"

Banks often build jump logic into stress testing platforms, reverse stress tools, or tail capital estimation engines. These are not real-time pricing systems — they are tools for identifying blind spots and capturing exposure to structural breaks.

As one practitioner put it: "You don't use jump models to trade the next 10bps. You use them to survive the next 10%."

Ultimately, jump diffusion models are not judged by how tightly they fit last week's smile. They are judged by how well they help practitioners think in discontinuities, and how effectively they embed the principle that risk is not always continuous, hedgeable, or Gaussian.

7 Variants and Hybrid Models

While the Merton model was a foundational breakthrough in asset pricing, its structure — combining constant volatility with Poisson-distributed jumps — still falls short in reproducing some of the richer dynamics observed in market data. Most notably, it struggles to explain:

- The volatility clustering seen in return series,
- The term structure of implied volatility skew,
- And the persistent asymmetry in how markets price upside versus downside tail risk.

To address these limitations, several extensions to the original model have emerged. These hybrid frameworks combine jumps with stochastic volatility, asymmetric jump distributions, or more flexible jump frequency dynamics, creating models that are both more realistic and better aligned with observed market prices.

While none of these variants are analytically simple, they are widely used in research, calibration, and risk systems, particularly for longer-dated, illiquid, or exotic structures.

7.1 Bates Model

The Bates model (1996) is perhaps the most natural evolution of Merton's framework. It combines the jump diffusion process with the Heston stochastic volatility model, creating a two-factor structure:

1. The asset price evolves with both a diffusion and jump component.
2. Volatility itself follows a mean-reverting square-root process (CIR-type), as in Heston.

Mathematically, the price process becomes:

$$\frac{dS_t}{dS} = (r - q - \lambda k)dt + \sqrt{v_t}dW_t^S + dJ_t$$

$$dv_t = \kappa(\theta - v_t)dt + \eta\sqrt{v_t}dW_t^v$$

Where:

- v_t is the instantaneous variance,
- κ, θ, and η govern the volatility mean-reversion,
- dJ_t is the same jump process from Merton (often lognormal),
- The Brownian motions W_t^S and W_t^v may be correlated.

Why It Matters

The Bates model is designed to capture both tail events and volatility dynamics:

- Jumps account for rare, large moves that generate fat tails and sudden repricings.

- Stochastic volatility captures volatility clustering, vol-of-vol, and persistent skew over time.

This dual structure significantly improves the model's ability to fit:

- Implied volatility smiles across strikes, and
- Term structures of volatility — especially useful for pricing long-dated options, where Merton alone would flatten out the smile.

Practical Application

In practice, the Bates model is used by:

- Volatility desks seeking to match smile curvature at multiple maturities,
- Exotic structurers pricing products like corridor variance swaps or long-dated callable exotics,
- Risk managers simulating market regimes with both jump and vol shocks.

Its flexibility comes at a computational cost — pricing typically relies on Fourier transform methods (e.g. Carr–Madan), Monte Carlo, or advanced finite difference schemes.

But for markets where tail risk and volatility dynamics matter equally — such as equity indices, FX, or commodities — the Bates model offers a far more complete picture than either Merton or Heston alone.

7.2 Kou Model

The Merton model assumes that jump sizes follow a lognormal distribution, which is symmetric in log-space and smooth in its tails. While this works reasonably well for modelling fat tails, it fails to capture a key empirical feature of markets: jump asymmetry.

Real-world returns are not just more leptokurtic than Gaussian — they are often skewed, especially during crises. Downward jumps tend to be larger, faster, and more frequent than upward ones. In equity markets, this is reflected in steep volatility skews and rich pricing of downside protection.

The Kou model (2002) addresses this directly by replacing the lognormal jump size distribution with a double exponential (DE) distribution. This allows for asymmetric jumps with fat tails — and better control over the steepness of the volatility smile.

Jump Size Specification

In the Kou model, the jump amplitude Y (expressed as a multiplicative factor) is such that:

$$ln(Y) \sim Double\ exponential$$

The double exponential (or Laplace) distribution has a density function:

$$f(y) = p\eta_1 e^{-\eta_1 y}\mathbf{1}_{y\geq 0} + (1-p)\eta_2 e^{\eta_2 y}\mathbf{1}_{y<0}$$

Where:

- $p \in [0,1]$: probability of an upward jump,
- η_1, η_2: exponential decay rates for up and down jumps.

This distribution has sharper peaks and heavier tails than the normal — and it can model asymmetric tail risk with high precision:

- A high η_2 implies that large negative jumps are more likely than large positive ones.
- The skew of the jump distribution becomes a controllable parameter, rather than an incidental feature.

Why It Matters

The Kou model allows traders and quants to fit steep volatility skews — especially those observed in short-dated options — without requiring excessive jump intensity or volatility. This is particularly useful when:

- The market prices in a high likelihood of a crash, but the diffusion volatility remains modest.
- You want to match observed pricing without exaggerating either σ or λ.

It also preserves analytical tractability:

- Kou derived semi-closed-form solutions for European call and put options using transform methods.
- This makes calibration and implementation more feasible than many other jump or stochastic models.

Use Cases in Trading

The Kou model is especially valuable in:

- Equity options, where downside jump risk dominates,
- Credit derivatives, where default is essentially a negative jump,
- Short-dated skew pricing, where market-implied distributions are highly skewed and leptokurtic.

For practitioners, it offers a cleaner separation between:

- Jump asymmetry (via η_1, η_2), and
- Jump frequency and size (via λ, k).

This separation provides more degrees of freedom when fitting implied volatility surfaces, especially when the smile is steep and asymmetric.

7.3 SVJD

The Stochastic Volatility with Jump Diffusion (SVJD) model brings together the two most important departures from the Black–Scholes world:

1. Volatility is stochastic — it evolves over time in a random, mean-reverting fashion (as in Heston),
2. Prices can jump — large, sudden changes are possible at any moment (as in Merton or Kou).

By combining these features, the SVJD framework is capable of reproducing nearly every major characteristic observed in option markets:

- Fat tails in return distributions,
- Volatility clustering over time,
- Implied volatility skews and smiles across strikes,
- And a term structure of skew across maturities.

This makes SVJD models particularly powerful for pricing and hedging exotic options, managing risk in volatile markets, and explaining the dynamics of implied volatility surfaces over time.

Core Model Structure

A generic SVJD model takes the form:

$$\frac{dS_t}{dS} = (r - q - \lambda k)dt + \sqrt{v_t}dW_t^S + dJ_t$$

$$dv_t = \kappa(\theta - v_t)dt + \eta\sqrt{v_t}dW_t^v$$

Here:

- v_t is the stochastic variance (as in Heston),
- dJ_t is a jump process (often Merton-type or double exponential),
- Correlation ρ between W_t^S and W_t^v generates skew in implied volatility.

Compared to the Bates model, SVJD is a broader label — encompassing any model that includes both stochastic variance and jump components. The specific jump structure (lognormal, double exponential, etc.) varies by implementation.

Why It Matters

SVJD models solve one of the persistent problems in option pricing: the smile flattens too quickly over time in pure jump models, and stochastic volatility alone can't explain short-dated skew. But when the two effects are combined:

- Short-dated skew is driven by jumps,

- Long-dated skew is driven by stochastic volatility (via mean-reverting variance and correlation structure),
- The full implied volatility surface becomes both realistic and dynamically responsive.

Use Cases in Practice

While SVJD models are computationally intensive, they are increasingly used in:

- Pricing long-dated exotics, where both forward vol dynamics and jump risk matter,
- Hedging structured products exposed to multiple Greeks over varying regimes,
- Risk-neutral density estimation, where fitting both tails and skew curvature is critical.

They are also used by:

- Hedge funds for tactical positioning around event risk,
- Banks for internal model validation and regulatory backtesting,
- Exotic desks for calibrating to multi-strike, multi-maturity surfaces with a single coherent framework.

Caution: Flexibility vs Overfitting

SVJD models offer enormous flexibility — but with it comes the risk of overfitting. With many parameters (vol of vol, mean reversion speed, jump intensity, jump size, correlation, etc.), a model can fit the data well without offering robust out-of-sample performance.

Experienced practitioners often fix or bound certain parameters, and rely more on structural fit than pure error minimisation.

7.4 Practical Takeaway

The landscape of jump diffusion models is rich and growing — but that doesn't mean more complexity is always better. Each variant serves a purpose, and the value lies in matching the model to the risk you're trying to understand or price.

Here's how to think about them:

Model	**Strength**	**Best For**
Merton	Simple, adds fat tails via symmetric jumps	Crash risk pricing, stress testing, intuition for tail-heavy markets
Bates	Jumps + stochastic volatility	Capturing term structure of skew, long-dated equity and FX options
Kou	Asymmetric jump distribution, fast decay	Fitting steep, short-dated skew in equity, credit, or binary event markets

SVJD	Flexible hybrid: stochastic vol + general jump process	Exotic structures, surface calibration, volatility clustering + tail modelling

Each step up the model ladder introduces:

- More parameters to estimate,
- More realism in capturing market dynamics,
- And greater risk of overfitting if applied blindly.

For Traders

- Don't overmodel if the trade doesn't demand it. For a short-dated vanilla, a basic jump model might suffice.
- Use these models to understand what the smile is telling you, not just to price.
- Recognise when you're taking on jump exposure: if your P&L breaks on a single print, you're long or short jump risk — model it accordingly.

For Structurers

- Exotic payoffs (barriers, digitals, callable range notes) demand models that can handle path sensitivity, tail risk, and volatility regimes.
- Matching both strike and maturity curvature often requires a combination of stochastic volatility and asymmetric jumps — no single model gets it all.

For Risk Managers

- Think in scenarios, not just in deltas and gammas.
- Jump models aren't about pricing the next 5bps — they're about surviving the next 10%.
- Use them to explore fragility, not to forecast means.

8 Conclusion

Financial markets are not smooth. They don’t move in quiet increments or respond predictably to every piece of information. They lurch, gap, and sometimes collapse — often when liquidity is thin, narratives are crowded, or uncertainty is suddenly repriced. These are not statistical outliers. They are structural features of how prices evolve in the real world.

Jump diffusion models begin with a simple but radical idea: that markets don't just vary — they break. And any model that ignores this risk is not just incomplete — it’s potentially dangerous.

By adding discontinuities to the modelling framework, we move closer to what traders actually experience:

- Option smiles and skews that emerge from asymmetry and tail risk.
- Hedging strategies that accept gap risk cannot be fully neutralised.
- Risk systems that stress beyond volatility to nonlinear price resets.

Over time, models have evolved from the elegant tractability of Black–Scholes to the richer, more robust formulations of Merton, Bates, Kou, and beyond. Each step reflects a shift in mindset: from continuous risk to discontinuous reality.

These models don’t just improve pricing. They improve judgment.

They give traders and risk managers a language to interpret extreme scenarios — to understand when the market is charging for fear, when it's ignoring fragility, or when it's preparing for an event that hasn’t yet happened.

In theory, there’s drift. In practice, there are jumps.

9 Python code

```
def simulate_merton_jump_diffusion(S0, mu, sigma, lambda_jump, mu_jump, sigma_jump, T, n_steps, n_paths):
    dt = T / n_steps
    dW = np.random.normal(0, np.sqrt(dt), (n_paths, n_steps))

    # Jump component
    jump_counts = np.random.poisson(lambda_jump * dt, (n_paths, n_steps))

    # Jump sizes (log-normal)
    jump_sizes = np.zeros((n_paths, n_steps))
    for i in range(n_paths):
        for j in range(n_steps):
            if jump_counts[i, j] > 0:
                individual_jumps = np.random.normal(mu_jump, sigma_jump, jump_counts[i, j])
                jump_sizes[i, j] = np.sum(np.exp(individual_jumps) - 1)

    S = np.zeros((n_paths, n_steps + 1))
    S[:, 0] = S0

    for i in range(n_steps):
        diffusion = (mu - 0.5 * sigma**2) * dt + sigma * dW[:, i]
        # Combined evolution
        S[:, i+1] = S[:, i] * np.exp(diffusion) * (1 + jump_sizes[:, i])

    return S

```

Figure 6: Merton Jump Diffusion Model python code

```python
def simulate_jump_diffusion_returns(n_sims=10000, dt=1/252, mu=0.08, sigma=0.20,
                                    jump_intensity=0, jump_mean=0, jump_std=0):
    diffusion_returns = np.random.normal(
        (mu - 0.5 * sigma**2) * dt,
        sigma * np.sqrt(dt),
        n_sims
    )

    if jump_intensity > 0:
        n_jumps = np.random.poisson(jump_intensity * dt, n_sims)
        jump_component = np.zeros(n_sims)
        for i in range(n_sims):
            if n_jumps[i] > 0:
                jumps = np.random.normal(jump_mean, jump_std, n_jumps[i])
                jump_component[i] = np.sum(jumps)
    else:
        jump_component = np.zeros(n_sims)

    total_returns = diffusion_returns + jump_component

    return total_returns

regimes = {
    'Low λ / Low k': {'lambda': 2, 'k': -0.02, 'k_std': 0.03},
    'Low λ / High k': {'lambda': 2, 'k': -0.08, 'k_std': 0.05},
    'High λ / Low k': {'lambda': 12, 'k': -0.02, 'k_std': 0.03},
    'High λ / High k': {'lambda': 12, 'k': -0.08, 'k_std': 0.05}
}

base_params = {
    'n_sims': 50000,
    'dt': 1/252,
    'mu': 0.08,
    'sigma': 0.20
}
```

Figure 7: Parameter Intuition python code to simulate log returns under different jump regimes

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