Homework 1

1.

(a) 225

(b) After gamma correction, the intensity of inner square should be 2 times of the outer square, which means

$$\frac{\left(\frac{225}{255}\right)^{\gamma} \cdot I_{max}}{\left(\frac{128}{255}\right)^{\gamma} \cdot I_{max}} = 2 \quad \Rightarrow \gamma = 1.22$$

2.

(a) let the pixel value be x, then

$$\frac{\left(\frac{x+1}{255}\right)^2}{\left(\frac{x}{255}\right)^2} \geq 1.02 \quad \Rightarrow \frac{x+1}{x} \geq 1.00995 \Rightarrow x \leq 100.5$$

$\therefore$ when $x = 1, 2 \cdots 100$, the step will be visible.

(b) let the pixel ~~value~~ value be x

$$\frac{\left(\frac{x+1}{255}\right)^2 I_{max} + 0.01 I_{max}}{\left(\frac{x}{255}\right)^2 I_{max} + 0.01 I_{max}} \geq 1.02 \quad \Rightarrow 2x + 1 \geq 0.02 x^2 + 13.005$$

$$\Rightarrow \quad 6.4 \leq x \leq 93.5$$

$\therefore$ when $x = 7, 8, 9, \cdots 93$, the step will be visible.

(c) for (a) if $\gamma = 1$

$$\frac{\frac{x+1}{255}}{\frac{x}{255}} \geq 1.02 \quad \Rightarrow x \leq 50 \qquad \therefore x = 1, 2, \cdots 50$$

for (b),

$$\frac{\left(\frac{x+1}{255}\right) I_{max} + 0.01 I_{max}}{\left(\frac{x}{255}\right) I_{max} + 0.01 I_{max}} \geq 1.02 \quad \Rightarrow x \leq 47.45$$

$\therefore x = 1, 2 \cdots 47$

3.

(a) let $\frac{\left(\frac{x+1}{255}\right)^{2.2}}{\left(\frac{x}{255}\right)^{2.2}} = f(x)$ then the precision will be.

the max value of $(f(x)-1)$ $(x \geq 2.55)$, which is $\left(\frac{1}{3}\right)^{2.2} = 8.9\%$

(b) let the number of bits be b, then.

$$\max\left(\frac{\frac{x+1}{2^b-1}}{\frac{x}{2^b-1}}\right) - 1 \leq 8.9\% \qquad \left(x \geq \frac{2^b-1}{100}\right)$$

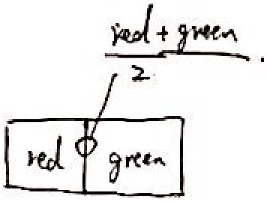
$$\Rightarrow x \geq 11.24 \qquad \rightarrow \frac{2^b-1}{100} \geq 11.24 \quad \Rightarrow b \geq 11$$

$\therefore$ at least 11 bits are needed.

4.

(a) $\frac{1}{\sqrt{2.2}}$, $\sqrt{2.2}$ (b) $\sqrt{2}$, $\frac{1}{\sqrt{2}}$

(c) Because if not, ~~把~~ some pixels may be distorted when doing some kind of modification.



For example, if we want to blur 2 colors, if we use the linear image, then the color in the middle will be $\frac{red+green}{2}$.

However, if not, the color in the middle will be (assume $\gamma = \frac{1}{2}$)

$\frac{\sqrt{red}+\sqrt{green}}{2}$, the after gamma decode, the color displayed will be

$$\left(\frac{\sqrt{red}+\sqrt{green}}{2}\right)^2 > \left(\sqrt{\frac{red+green}{2}}\right)^2.$$ so the color will be unreal.