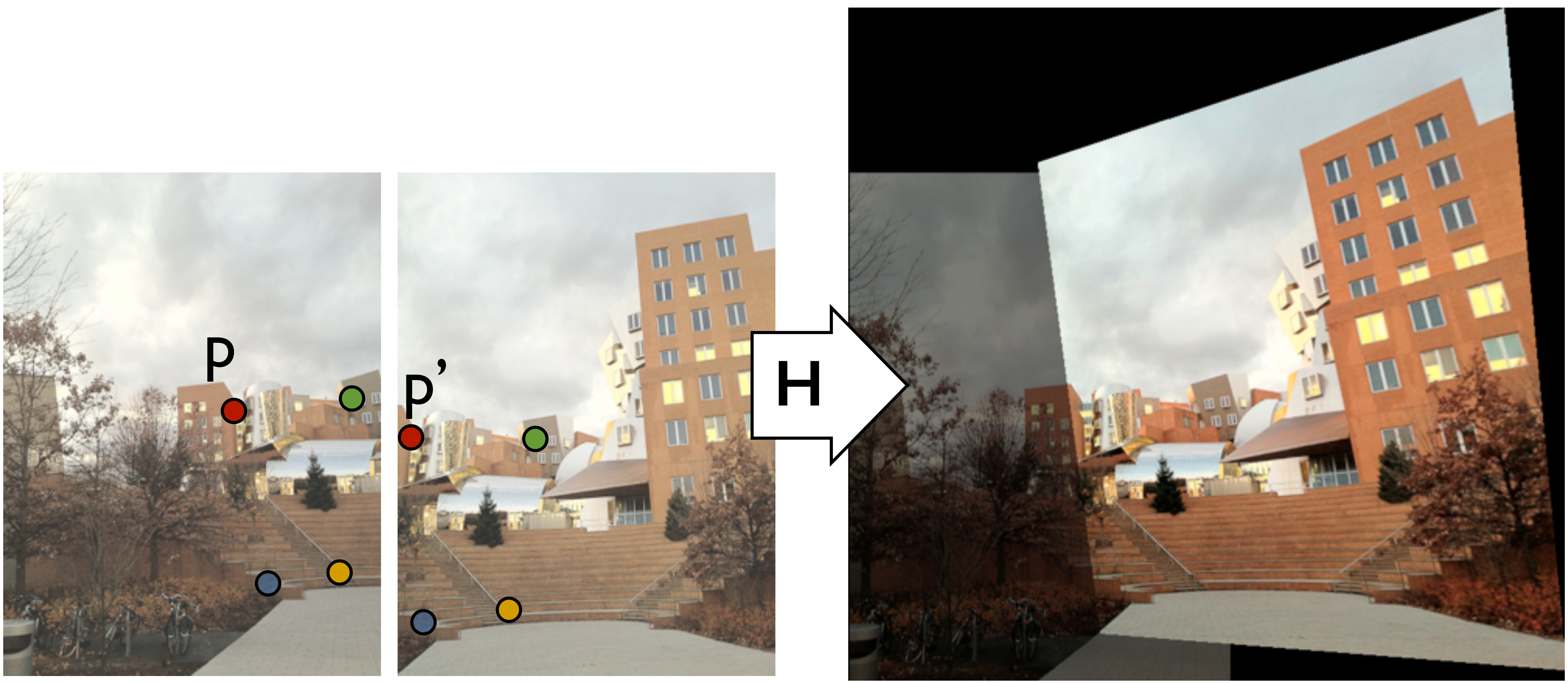
Solving for homographies

Taken from Fredo Durand

Goal

- Given correspondences
- Find homography matrix H that maps the p_i to p_i'



Warning

- In what follows I use the order y, x
  - At least I will try
- This will make life easier for pset 6

Homography equation

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} y \\ x \\ 1 \end{pmatrix} = \begin{pmatrix} y'w' \\ x'w' \\ w' \end{pmatrix}$$

- We are given pairs of corresponding points
  - x, y, x’, y’ are known
- Unknowns: matrix coefficients and w’

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- **We are given pairs of corresponding points**
  - x, y, x’, y’ are known
- **Unknowns: matrix coefficients and w’**
  - but w’ is easy to get:

$$w' = gy + hx + i$$

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$$w' = gy + hx + i$$

- **For a pair of points (x, y)->(x’,y’) we have**

$$ay + bx + c = y'(gy + hx + i)$$

$$dy + ex + f = x'(gy + hx + i)$$

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- Unknowns: a, b, c, d, e, f, g, h, i
  - Linear!

How many pairs?

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} y \\ x \\ 1 \end{pmatrix} = \begin{pmatrix} y'w' \\ x'w' \\ w' \end{pmatrix}$$

- **Each correspondence pair gives us two equations**

$$ay + bx + c = y'(gy + hx + i)$$

$$dy + ex + f = x'(gy + hx + i)$$

- **How many unknowns?**
  - 9
  - but H is defined up to scale. Four pairs are

Forming the linear system

- We have 4x2 linear equations in our 8 unknowns
- Represent as a matrix systemAx=B:

$$\Bigg(\quad A \quad \Bigg) \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \end{pmatrix} = B$$

- Now we need to fill matrix A and vector B

Forming the matrix

$$ay + bx + c = y'(gy + hx + i)$$

$$dy + ex + f = x'(gy + hx + i)$$

$$\begin{pmatrix} a & b & c & d & e & f & g & h & i \\ & & & & & & & & \\ & & & & & & & & \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \end{pmatrix} = \begin{matrix} \cdot \\ \cdot \\ \cdot \\ \\ \\ \cdot \\ \cdot \\ \cdot \end{matrix}$$

Forming the matrix

$$ay + bx + c = y'(gy + hx + i)$$

$$dy + ex + f = x'(gy + hx + i)$$

$$\begin{array}{c} \begin{matrix} a & b & c & d & e & f & g & h & i \end{matrix} \\ \begin{pmatrix} & & & & \cdots\cdot & & & & \\ y & x & 1 & 0 & 0 & 0 & -yy' & -xy' & -y' \\ & & & & \cdots\cdot & & & & \\ & & & & \cdots\cdot & & & & \end{pmatrix} \end{array} \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \end{pmatrix} = \begin{matrix} . \\ . \\ . \\ 0 \\ . \\ . \\ . \end{matrix}$$

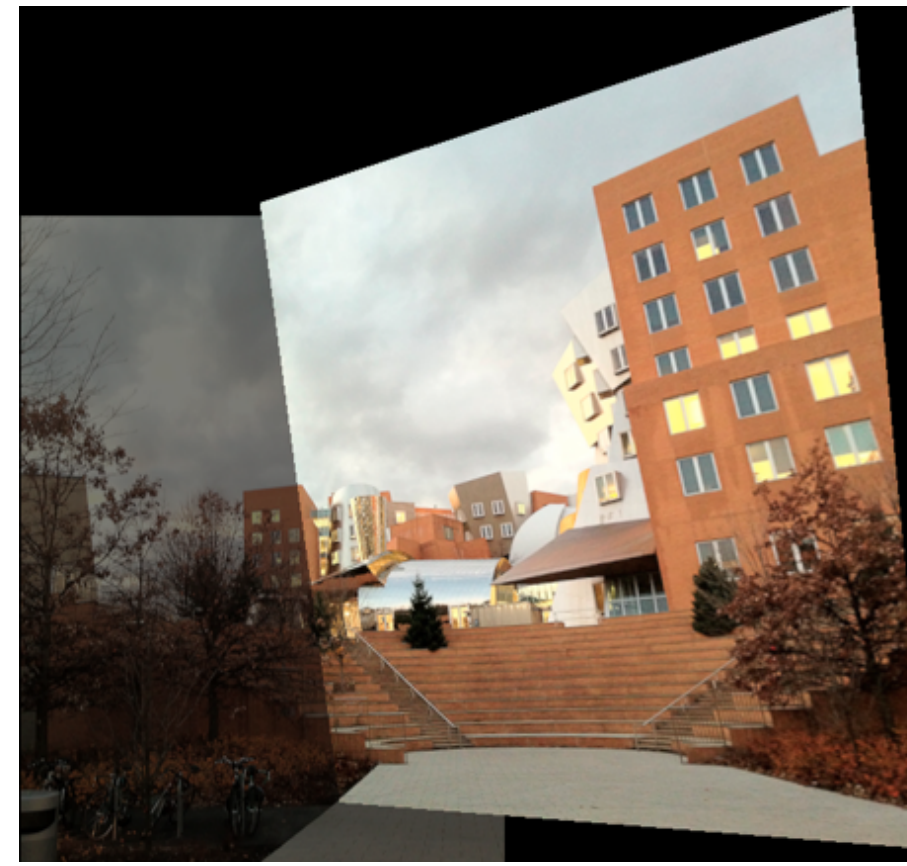
I'll let you do the x case for pset 6

Recap

- We have four pairs of points

- Looking for homography H

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} y \\ x \\ 1 \end{pmatrix} = \begin{pmatrix} y'w' \\ x'w' \\ w' \end{pmatrix}$$



- Formed a big 8x9 linear system Ax=0
  - where x is the 9 homography coefficients

$$\begin{pmatrix} & & & & \cdots\cdots & & & & \\ y & x & 1 & 0 & 0 & 0 & -yy' & -xy' & -y' \\ & & & & \cdots\cdots & & & & \\ & & & & \cdots\cdots & & & & \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \\ f \\ g \\ h \\ i \end{pmatrix} = \begin{matrix} . \\ . \\ . \\ 0 \\ . \\ . \\ . \end{matrix}$$

Solve for scale invariance

- We know that there exists a full family of solutions kH, for any non-zero k
- i.e., if (a, b, c, d, e, f, g, h, i) is a solution, so is (ka, kb, kc, kd, ke, kf, kg, kh, ki)

- Adding more correspondences won't help

Dirty solution

- Hope that *i* is not 0
- Set it arbitrarily to 1
  - either create an 8x8 matrix
  - or add a last row that says i should be 1

- In practice i is rarely zero
  - But it’s still dirty
- You can use this solution for pset 6

Cleaner solution

- Use SVD
- The singular vector with singular value 0 is a solution

- See your favorite linear algebra textbook

Dirty-clean: Three versions

- Dirty with i=1
  - 9x9:
    - RHS is zero almost everywhere, except last row
    - last row just says i=1
  - 8x8:
    - substitute i=1 from beginning
    - RHS is usually non-zero
- Clean with SVD: 8x9
  - Ax=0