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Solution to question 12.13.3.33

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Question: Suppose that 6% of the people with blood group O are left handed and 10% of those with other blood groups are left handed 30% of the people have blood group O. If a left handed person is selected at random, what is the probability that he/she will have blood group O?

Solution: Let us consider two random variables A and B. We are given that,

RV	Value	Description	Representation
A	0	Any other Blood Group	A'
	1	Blood Group O	A
В	0	Right Handed Person	<i>B'</i>
	1	Left Handed Person	В

$$Pr(A) = 0.3 \tag{1}$$

$$Pr(B|A) = 0.06$$
 (2)

$$\Pr(B|A') = 0.1\tag{3}$$

So, we can write that,

$$\Pr(A) = 0.3 \tag{4}$$

$$\therefore \Pr(A') = 1 - \Pr(A) \tag{5}$$

$$= 1 - 0.3$$
 (6)

$$=0.7\tag{7}$$

$$Pr(B|A) = 0.06$$
 (8)

$$\implies \frac{\Pr(BA)}{\Pr(A)} = 0.06 \tag{9}$$

$$\implies \Pr(BA) = 0.06 \Pr(A)$$
 (10)

$$= 0.018$$
 (11)

$$\Pr(B|A') = 0.1$$
 (12)

$$\implies \frac{\Pr(BA')}{\Pr(A')} = 0.1 \tag{13}$$

$$\implies \Pr(BA') = 0.1 \Pr(A') \tag{14}$$

$$= 0.07$$
 (15)

Hence,

$$Pr(BA) = 0.018$$
 (16)

$$Pr(BA') = 0.07$$
 (17)

We know that,

$$A + A' = 1 \tag{18}$$

$$AA' = 0 \tag{19}$$

We can write Pr(B) as:

$$Pr(B) = Pr(B(A + A'))$$
 (20)

$$= \Pr(BA + BA') \tag{21}$$

By inclusion-exclusion principle,

$$Pr(B) = Pr(BA) + Pr(BA') + Pr((BA)(BA'))$$
(22)

$$= \Pr(BA) + \Pr(BA') + \Pr((BB)(AA'))$$
 (23)

$$= \Pr(BA) + \Pr(BA') \tag{24}$$

By substituting values from equation (16) and (17),

$$Pr(B) = 0.018 + 0.07 \tag{25}$$

$$= 0.088$$
 (26)

So, Pr(A|B) can be written as,

$$Pr(A|B) = \frac{Pr(BA)}{Pr(B)}$$
 (27)

$$=\frac{0.018}{0.088}\tag{28}$$

$$=\frac{9}{44}\tag{29}$$

Hence, if a left handed is selected at random, the probability of the person having blood group O is $\frac{9}{44}$.