Solution to question 11.16.3.34

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Question: Prove if the given statement is true or false - The probability of intersection of two events A and B is always less than or equal to those favourable to the event A.

Solution: We have to prove that:

$$\Pr(AB) \le \Pr(A) \tag{1}$$

We know that,

$$BB' = 0 (2)$$

$$B + B' = 1 \tag{3}$$

$$\implies \Pr(A) = \Pr(A(B+B'))$$
 (4)

$$= \Pr(AB + AB') \tag{5}$$

By using inclusion-exclusion principle,

$$Pr(A) = Pr(AB) + Pr(AB') - Pr((AB)(AB'))$$
 (6)

$$= \Pr(AB) + \Pr(AB') - \Pr((AA)(BB')) \quad (7)$$

$$= \Pr(AB) + \Pr(AB') \tag{8}$$

We know that the value of probability ranges from 0 to 1.

$$0 \le \Pr(AB') \le 1 \tag{9}$$

Adding Pr(AB) both sides

$$Pr(AB) \le Pr(AB) + Pr(AB')$$
 (10)

Substituting value from equation (8)

$$\implies \Pr(AB) \le \Pr(A)$$
 (11)

Hence, the given statement is true.

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