Solution to question 12.13.3.33

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Question: Suppose that 6% of the people with blood group O are left handed and 10% of those with other blood groups are left handed 30% of the people have blood group O. If a left handed person is selected at random, what is the probability that he/she will have blood group O?

Solution: Let us consider two random variables A and B. We are given that,

RV	Value	Description	Representation
A	0	Any other Blood Group	A'
	1	Blood Group O	A
B	0	Right Handed Person	B'
	1	Left Handed Person	B

$$\Pr(A) = 0.3 \tag{1}$$
$$\Pr(B|A) = 0.06 \tag{2}$$
$$\Pr(B|A') = 0.1 \tag{3}$$

So, we can write that,

$$\Pr(A) = 0.3 \tag{4}$$
$$\therefore \Pr(A') = 1 - \Pr(A) \tag{5}$$
$$= 1 - 0.3 \tag{6}$$
$$= 0.7 \tag{7}$$
$$\Pr(B|A) = 0.06 \tag{8}$$
$$\implies \frac{\Pr(BA)}{\Pr(A)} = 0.06 \tag{9}$$
$$\implies \Pr(BA) = 0.06\Pr(A) \tag{10}$$
$$= 0.018 \tag{11}$$
$$\Pr(B|A') = 0.1 \tag{12}$$
$$\implies \frac{\Pr(BA')}{\Pr(A')} = 0.1 \tag{13}$$
$$\implies \Pr(BA') = 0.1\Pr(A') \tag{14}$$
$$= 0.07 \tag{15}$$

Hence,

$$\Pr(BA) = 0.018 \tag{16}$$
$$\Pr(BA') = 0.07 \tag{17}$$

We know that,

$$A + A' = 1 \tag{18}$$
$$AA' = 0 \tag{19}$$

We can write $\Pr(B)$ as:

$$\Pr(B) = \Pr(B(A + A')) \tag{20}$$
$$= \Pr(BA + BA') \tag{21}$$

By inclusion-exclusion principle,

$$\Pr(B) = \Pr(BA) + \Pr(BA') + \Pr((BA)(BA')) \tag{22}$$
$$= \Pr(BA) + \Pr(BA') + \Pr((BB)(AA')) \tag{23}$$
$$= \Pr(BA) + \Pr(BA') \tag{24}$$

By substituting values from equation (16) and (17),

$$\Pr(B) = 0.018 + 0.07 \tag{25}$$
$$= 0.088 \tag{26}$$

So, $\Pr(A|B)$ can be written as,

$$\Pr(A|B) = \frac{\Pr(BA)}{\Pr(B)} \tag{27}$$
$$= \frac{0.018}{0.088} \tag{28}$$
$$= \frac{9}{44} \tag{29}$$

Hence, if a left handed is selected at random, the probability of the person having blood group O is $\frac{9}{44}$.