Solution of question 1.2.2

Gagan Singla* – $EE22BTECH11021$

Question :-
We are given a triangle and the midpoints D, E, F of the sides AB, BC, AC respectively. We have to find the normal form of equation of lines AD, BE, CF.
Answer :-

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{2}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{3}$$

The mid points of sides AB, BC, AC can be found by the formula $\dfrac{\mathbf{A}+\mathbf{B}}{2}$, $\dfrac{\mathbf{B}+\mathbf{C}}{2}$, $\dfrac{\mathbf{A}+\mathbf{C}}{2}$ respectively for all the three sides.
Hence,

$$\mathbf{D} = \frac{\mathbf{B}+\mathbf{C}}{2} \tag{4}$$

$$= \frac{\begin{pmatrix} -4 \\ 6 \end{pmatrix} + \begin{pmatrix} -3 \\ -5 \end{pmatrix}}{2} \tag{5}$$

$$= \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix} \tag{6}$$

$$\mathbf{E} = \frac{\mathbf{A}+\mathbf{C}}{2} \tag{7}$$

$$= \frac{\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -3 \\ -5 \end{pmatrix}}{2} \tag{8}$$

$$= \begin{pmatrix} -1 \\ -3 \end{pmatrix} \tag{9}$$

$$\mathbf{F} = \frac{\mathbf{A}+\mathbf{B}}{2} \tag{10}$$

$$= \frac{\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -4 \\ 6 \end{pmatrix}}{2} \tag{11}$$

$$= \begin{pmatrix} \frac{-3}{2} \\ \frac{5}{2} \end{pmatrix} \tag{12}$$

The parametric equation of line AD is:

$$\mathbf{m} = \mathbf{D} - \mathbf{A} \tag{13}$$

$$\implies \mathbf{m} = \begin{pmatrix} \frac{-9}{2} \\ \frac{3}{2} \end{pmatrix} \tag{14}$$

Now,

$$\mathbf{x} = \mathbf{A} + k\mathbf{m} \tag{15}$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + k \begin{pmatrix} -9/2 \\ 3/2 \end{pmatrix} \tag{16}$$

$\mathbf{m}$ can be taken as $\frac{\mathbf{D-A}}{2}$, $\frac{\mathbf{D-A}}{3}$, $\frac{\mathbf{D-A}}{4}$ etc. because of the k(constant) present with $\mathbf{m}$ in the equation.

Now, we have to find $\mathbf{n^T}$ such that $\mathbf{n^T m} = 0$.

Here, $\mathbf{n^T} = \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix}$

Hence, normal form of line AD is :

$$\mathbf{n^T}(\mathbf{x} - \mathbf{A}) = 0 \tag{17}$$

$$\mathbf{n^T x} = \mathbf{n^T A} \tag{18}$$

$$\begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{19}$$

$$\frac{1}{2}\begin{pmatrix} 3 & 9 \end{pmatrix} \mathbf{x} = \frac{1}{2}\begin{pmatrix} 3 & 9 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{20}$$

$$\begin{pmatrix} 3 & 9 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 & 9 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{21}$$

$$\begin{pmatrix} 3 & 9 \end{pmatrix} \mathbf{x} = -6 \tag{22}$$