Solution to question 12.13.3.33

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Question: Suppose that 6% of the people with blood group O are left handed and 10% of those with other blood groups are left handed 30% of the people have blood group O. If a left handed person is selected at random, what is the probability that he/she will have blood group O?

Solution: Let us consider two random variables A_i and B_i,

Random Variable	i	Description
A_i	0	Any other Blood Group
	1	Blood Group O
B_i	0	Right Handed Person
	1	Left Handed Person

We need to find the value of $\Pr(A_1|B_1)$. We are given that,

$$\Pr(A_1) = 0.3 \tag{1}$$
$$\Pr(B_1|A_1) = 0.06 \tag{2}$$
$$\Pr(B_1|A_0) = 0.1 \tag{3}$$

The above equations can be written as:

$$\therefore \Pr(A_0) = 1 - \Pr(A_1) \tag{4}$$
$$= 1 - 0.3 \tag{5}$$
$$= 0.7 \tag{6}$$
$$\Pr(B_1|A_1) = 0.06 \tag{7}$$
$$\implies \frac{\Pr(B_1A_1)}{\Pr(A_1)} = 0.06 \tag{8}$$
$$\implies \Pr(B_1A_1) = 0.06\Pr(A_1) \tag{9}$$
$$= 0.018 \tag{10}$$
$$\Pr(B_1|A_0) = 0.1 \tag{11}$$
$$\implies \frac{\Pr(B_1A_0)}{\Pr(A_0)} = 0.1 \tag{12}$$
$$\implies \Pr(B_1A_0) = 0.1\Pr(A_0) \tag{13}$$
$$= 0.07 \tag{14}$$

Hence, we get,

$$\Pr(B_1A_1) = 0.018 \tag{15}$$
$$\Pr(B_1A_0) = 0.07 \tag{16}$$

We know that,

$$A_1 + A_0 = 1 \tag{17}$$
$$A_1A_0 = 0 \tag{18}$$

We can write $\Pr(B_1)$ as:

$$\Pr(B_1) = \Pr(B_1(A_1 + A_0)) \tag{19}$$
$$= \Pr(B_1A_0 + B_1A_1) \tag{20}$$

By inclusion-exclusion principle,

$$\Pr(B_1) = \Pr(B_1A_1) + \Pr(B_1A_0) + \Pr((B_1A_1)(B_1A_0)) \tag{21}$$
$$= \Pr(B_1A_1) + \Pr(B_1A_0) + \Pr((B_1B_1)(A_1A_0)) \tag{22}$$
$$= \Pr(B_1A_1) + \Pr(B_1A_0) \tag{23}$$

By substituting values from equation (15) and (16),

$$\Pr(B_1) = 0.018 + 0.07 \tag{24}$$
$$= 0.088 \tag{25}$$

So, $\Pr(A_1|B_1)$ can be written as,

$$\Pr(A_1|B_1) = \frac{\Pr(B_1A_1)}{\Pr(B_1)} \tag{26}$$
$$= \frac{0.018}{0.088} \tag{27}$$
$$= \frac{9}{44} \tag{28}$$

Hence, if a left handed is selected at random, the probability of the person having blood group O is $\frac{9}{44}$.