Solution of question 1.2.2

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Question : We are given a triangle and the mid-points D, E, F of the sides AB, BC, AC respectively. We have to find the normal form of equation of lines AD, BE, CF. Answer :

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{2}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{3}$$

The mid points D, E, F of sides AB, BC, AC are :-

$$\mathbf{D} = \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix} \tag{4}$$

$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} \tag{5}$$

$$\mathbf{F} = \begin{pmatrix} \frac{-3}{2} \\ \frac{5}{2} \end{pmatrix} \tag{6}$$

Now, the slope vector of line DA($\mathbf{m}$) is :-

$$\mathbf{m} = \mathbf{D} - \mathbf{A} \tag{7}$$

$$\implies \mathbf{m} = \begin{pmatrix} \frac{-9}{2} \\ \frac{3}{2} \end{pmatrix} \tag{8}$$

$\mathbf{m}$ can be taken as $\frac{\mathbf{D}-\mathbf{A}}{2}, \frac{\mathbf{D}-\mathbf{A}}{3}, \frac{\mathbf{D}-\mathbf{A}}{4}$ etc. because it will be eliminated while calculating the normal form of line. Now, we have to find $\mathbf{n}^\top$ such that

$$\mathbf{n}^\top \mathbf{m} = 0. \tag{9}$$

$\mathbf{n}$ can be written in the form :-

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{10}$$

$$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{-9}{2} \\ \frac{3}{2} \end{pmatrix} \tag{11}$$

$$= \begin{pmatrix} \frac{3}{2} \\ \frac{9}{2} \end{pmatrix} \tag{12}$$

So the transpose of n,

$$\mathbf{n}^\top = \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \tag{13}$$

Normal form of line AD is :

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{A}) = 0 \tag{14}$$

$$\implies \mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{A} \tag{15}$$

$$\implies \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{16}$$

$$\implies \frac{1}{2} \begin{pmatrix} 3 & 9 \end{pmatrix} \mathbf{x} = \frac{1}{2} \begin{pmatrix} 3 & 9 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{17}$$

$$\implies \begin{pmatrix} 3 & 9 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 & 9 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{18}$$

$$\implies \begin{pmatrix} 3 & 9 \end{pmatrix} \mathbf{x} = -6 \tag{19}$$