Solution of question 1.2.2

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Question :-
We are given a triangle and the midpoints D, E, F of the sides AB, BC, AC respectively. We have to find the normal form of equation of lines AD, BE, CF.
Answer :-

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{2}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{3}$$

The mid points D, E, F of sides AB, BC, AC are :-

$$\mathbf{D} = \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix} \tag{4}$$

$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} \tag{5}$$

$$\mathbf{F} = \begin{pmatrix} \frac{-3}{2} \\ \frac{5}{2} \end{pmatrix} \tag{6}$$

Now, the slope vector of line DA($\mathbf{m}$) is :-

$$\mathbf{m} = \mathbf{D} - \mathbf{A} \tag{7}$$

$$\implies \mathbf{m} = \begin{pmatrix} \frac{-9}{2} \\ \frac{3}{2} \end{pmatrix} \tag{8}$$

$\mathbf{m}$ can be taken as $\frac{\mathbf{D-A}}{2}, \frac{\mathbf{D-A}}{3}, \frac{\mathbf{D-A}}{4}$ etc. because it will be eliminated while calculating the normal form of line.

Now, we have to find $\mathbf{n}^T$ such that $\mathbf{n}^T.\mathbf{m} = 0$. Let $\mathbf{n}^T$ be equal to $\begin{pmatrix} a & b \end{pmatrix}$.

This gives us,

$$\begin{pmatrix} \frac{-9}{2} \\ \frac{3}{2} \end{pmatrix} \begin{pmatrix} a & b \end{pmatrix} = 0 \tag{9}$$

$$\frac{-9}{2}a + \frac{3}{2}b = 0 \tag{10}$$

$$3a = b \tag{11}$$

Now, any choice of a and b can me made, but for easier understanding we are using $a = \frac{3}{2}$ and $b = \frac{9}{2}$ respectively.

So, $\mathbf{n}^T = \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix}$

Normal form of line AD is :

$$\mathbf{n}^T(\mathbf{x} - \mathbf{A}) = 0 \tag{12}$$

$$\mathbf{n}^T\mathbf{x} = \mathbf{n}^T\mathbf{A} \tag{13}$$

$$\begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix}\mathbf{x} = \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{14}$$

$$\frac{1}{2}\begin{pmatrix} 3 & 9 \end{pmatrix}\mathbf{x} = \frac{1}{2}\begin{pmatrix} 3 & 9 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{15}$$

$$\begin{pmatrix} 3 & 9 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 3 & 9 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{16}$$

$$\begin{pmatrix} 3 & 9 \end{pmatrix}\mathbf{x} = -6 \tag{17}$$