Solution of Question 1.3.5

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Question: We are given a triangle with vertices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and the point of intersection of the altitudes drawn from the vertices $\mathbf{B}$ and $\mathbf{C}$ is $\mathbf{H}$. We have to prove that

$$(\mathbf{A}-\mathbf{H})^{\top}(\mathbf{B}-\mathbf{C}) = 0 \tag{1}$$

Solution:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{2}$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{3}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{4}$$

The point of intersection of altitutes BE_1 and CF_1 is:

$$\mathbf{H} = \begin{pmatrix} \frac{17}{6} \\ \frac{-5}{6} \end{pmatrix} \tag{5}$$

Now,

$$\mathbf{A}-\mathbf{H} = \begin{pmatrix} \frac{-11}{6} \\ \frac{-1}{6} \end{pmatrix} \tag{6}$$

$$(\mathbf{A}-\mathbf{H})^{\top} = \begin{pmatrix} \frac{-11}{6} & \frac{-1}{6} \end{pmatrix} \tag{7}$$

$$\mathbf{B}-\mathbf{C} = \begin{pmatrix} -1 \\ 11 \end{pmatrix} \tag{8}$$

$$(\mathbf{A}-\mathbf{H})^{\top}(\mathbf{B}-\mathbf{C}) = \begin{pmatrix} \frac{-11}{6} & \frac{-1}{6} \end{pmatrix}\begin{pmatrix} -1 \\ 11 \end{pmatrix} \tag{9}$$

$$= \frac{11}{6} - \frac{11}{6} \tag{10}$$

$$= 0 \tag{11}$$

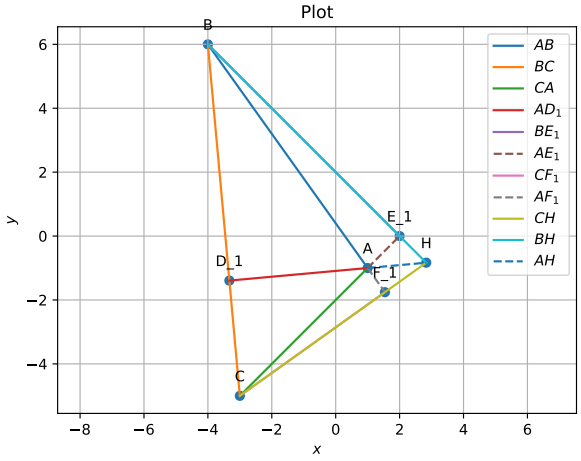

Fig. 0. Plot