

Solution of question 1.2.2

Gagan Singla* – EE22BTECH11021

Question :-

We are given the three vertices of a triangle(A, B, C) and the midpoints D, E, F of the sides AB, BC, AC respectively. We have to find the equations of the sides AD, BE, CF.

Answer :-

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (1)$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad (2)$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (3)$$

The mid points of sides AB, BC, AC can be found by the formula $\frac{\mathbf{A} + \mathbf{B}}{2}$, $\frac{\mathbf{B} + \mathbf{C}}{2}$, $\frac{\mathbf{A} + \mathbf{C}}{2}$ respectively for all the three sides.

Hence,

$$\mathbf{D} = \frac{\mathbf{B} + \mathbf{C}}{2} \quad (4)$$

$$= \frac{\begin{pmatrix} -4 \\ 6 \end{pmatrix} + \begin{pmatrix} -3 \\ -5 \end{pmatrix}}{2} \quad (5)$$

$$= \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix} \quad (6)$$

$$\mathbf{E} = \frac{\mathbf{A} + \mathbf{C}}{2} \quad (7)$$

$$= \frac{\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -3 \\ -5 \end{pmatrix}}{2} \quad (8)$$

$$= \begin{pmatrix} -1 \\ -3 \end{pmatrix} \quad (9)$$

$$\mathbf{F} = \frac{\mathbf{A} + \mathbf{B}}{2} \quad (10)$$

$$= \frac{\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -4 \\ 6 \end{pmatrix}}{2} \quad (11)$$

$$= \begin{pmatrix} \frac{-3}{2} \\ \frac{5}{2} \end{pmatrix} \quad (12)$$

The parametric equation of line AD is:

$$\mathbf{m} = \mathbf{D} - \mathbf{A} \quad (13)$$

$$\Rightarrow \mathbf{m} = \begin{pmatrix} \frac{-9}{2} \\ \frac{3}{2} \end{pmatrix} \quad (14)$$

Now,

$$\mathbf{x} = \mathbf{A} + k\mathbf{m} \quad (15)$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + k \begin{pmatrix} -9/2 \\ 3/2 \end{pmatrix} \quad (16)$$

Now, we have to find \mathbf{n}^T such that $\mathbf{n}^T \mathbf{m} = 0$.

$$\text{Here, } \mathbf{n}^T = \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix}$$

Hence, normal form of line AD is :

$$\mathbf{n}^T(\mathbf{x} - \mathbf{A}) = 0 \quad (17)$$

$$\mathbf{n}^T \mathbf{x} = \mathbf{n}^T \mathbf{A} \quad (18)$$

$$\begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (19)$$

$$\begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \mathbf{x} = -3 \quad (20)$$