

Solution to question 12.13.3.33

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Question: Suppose that 6% of the people with blood group O are left handed and 10% of those with other blood groups are left handed 30% of the people have blood group O. If a left handed person is selected at random, what is the probability that he/she will have blood group O?

Solution: Let us consider two random variables A and B ,

Random Variable	Event
A	Blood Group O
A'	Any other blood group
B	Left-Handed person

We need to find the value of $\Pr(A|B)$. We are given that,

$$\Pr(A) = 0.3 \quad (1)$$

$$\therefore \Pr(A') = 1 - \Pr(A) \quad (2)$$

$$= 1 - 0.3 \quad (3)$$

$$= 0.7 \quad (4)$$

$$\Pr(B|A) = 0.06 \quad (5)$$

$$\Rightarrow \frac{\Pr(BA)}{\Pr(A)} = 0.06 \quad (6)$$

$$\Rightarrow \Pr(BA) = 0.06 \Pr(A) \quad (7)$$

$$= 0.018 \quad (8)$$

$$\Pr(B|A') = 0.1 \quad (9)$$

$$\Rightarrow \frac{\Pr(BA')}{\Pr(A')} = 0.1 \quad (10)$$

$$\Rightarrow \Pr(BA') = 0.1 \Pr(A') \quad (11)$$

$$= 0.07 \quad (12)$$

We know that,

$$A + A' = 1 \quad (13)$$

$$AA' = 0 \quad (14)$$

We can write $\Pr(B)$ as:

$$\Pr(B) = \Pr(B(A + A')) \quad (15)$$

$$= \Pr(BA + BA') \quad (16)$$

By inclusion-exclusion principle,

$$\Pr(B) = \Pr(BA) + \Pr(BA') + \Pr((BA)(BA')) \quad (17)$$

$$= \Pr(BA) + \Pr(BA') + \Pr((BB)(AA')) \quad (18)$$

$$= \Pr(BA) + \Pr(BA') \quad (19)$$

By substituting values from equation (8) and (12),

$$\Pr(B) = 0.018 + 0.07 \quad (20)$$

$$= 0.088 \quad (21)$$

So, $\Pr(A|B)$ can be written as,

$$\Pr(A|B) = \frac{\Pr(BA)}{\Pr(B)} \quad (22)$$

$$= \frac{0.018}{0.088} \quad (23)$$

$$= \frac{9}{44} \quad (24)$$

Hence, if a left handed is selected at random, the probability of the person having blood group O is $\frac{9}{44}$.