Solution to question 11.16.3.34

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Question: Prove if the given statement is true or false - The probability of intersection of two events A and B is always less than or equal to those favourable to the event A.

Solution: We have to prove that:

$$\Pr(A.B) \leq \Pr(A) \qquad (1)$$

We have to consider 3 cases:

1) If A and B are mutually exclusive

$$\Pr(A.B) = 0 \qquad (2)$$
$$\Pr(A) \geq 0 \qquad (3)$$
$$\implies \Pr(A.B) \leq \Pr(A) \qquad (4)$$

2) If A and B are independent

$$\Pr(A.B) = \Pr(A).\Pr(B) \qquad (5)$$
$$\Pr(B) \leq 1 \qquad (6)$$
$$\implies \Pr(A.B) \leq \Pr(A) \qquad (7)$$

3) If A and B are neither mutually exclusive nor independent
   By inclusion-exclusion principle,

$$\Pr(A.B) = \Pr(A) + \Pr(B) - \Pr(A + B) \qquad (8)$$

$$\text{We know that,} \qquad (9)$$
$$\Pr(A + B) \geq \Pr(B) \qquad (10)$$
$$\implies \Pr(A.B) \leq \Pr(A) \qquad (11)$$

All three cases justify the given statement. Hence, the given statement is true.