

Solution of question 10.15.1.23

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Question: A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

Solution: A coin toss can have only two outcomes which are:

- 1) Heads
- 2) Tails

Both of these outcomes are equally likely. Let us consider random variable X for the number of heads in the experiment.

Parameter	Value	Description
X	$\{0, 1, 2, 3\}$	Number of heads
n	3	Number of trails
p	0.5	Probability of success
q	0.5	Probability of Failure

The PMF of X is

$$\Pr(X = k) = {}^nC_k p^k q^{n-k} \quad (1)$$

$$= {}^3C_k \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{3-k} \quad \forall k = 0, 1, 2, 3 \quad (2)$$

$$= {}^3C_k \left(\frac{1}{2}\right)^3 \quad \forall k = 0, 1, 2, 3 \quad (3)$$

The Cumulative Distribution Function (CDF) of X is given by the probability that X is less than or equal to a given value k , for $k = 0, 1, 2, 3$. The CDF can be expressed as:

$$F_X(k) = \Pr(X \leq k) \quad (4)$$

$$= \sum_{i=0}^k {}^nC_i p^i q^{n-i} \quad (5)$$

$$= \sum_{i=0}^k {}^3C_i \left(\frac{1}{2}\right)^3 \quad (6)$$

Hanif will lose the game if the value of X is 1 or 2. Hence, we need to find $\Pr(X = 1) + \Pr(X = 2)$

$$\Pr(X = 1) + \Pr(X = 2) = {}^3C_1 \left(\frac{1}{2}\right)^3 + {}^3C_2 \left(\frac{1}{2}\right)^3 \quad (7)$$

$$= 3 \left(\frac{1}{8}\right) + 3 \left(\frac{1}{8}\right) \quad (8)$$

$$= 3 \left(\frac{1}{4}\right) \quad (9)$$

$$= 0.75 \quad (10)$$

Hence, the probability of hanif losing the game is 0.75