Solution to question 12.13.3.67

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Question: Let $\Pr(A) = \frac{7}{13}$, $\Pr(B) = \frac{9}{13}$, $\Pr(AB) = \frac{4}{13}$. Then $\Pr(A'|B)$ is equal to

(a) $\frac{6}{13}$
(b) $\frac{4}{13}$
(c) $\frac{4}{9}$
(d) $\frac{5}{9}$

Solution: We are given that:

$$\Pr(A) = \frac{7}{13} \tag{1}$$

$$\Pr(B) = \frac{9}{13} \tag{2}$$

$$\Pr(AB) = \frac{4}{13} \tag{3}$$

We know that:

$$A + A' = 1 \tag{4}$$

$$AA' = 0 \tag{5}$$

Hence, we can say that:

$$\Pr(B) = \Pr(B(A + A')) \tag{6}$$

$$= \Pr(AB + A'B) \tag{7}$$

By inclusion-exclusion principle,

$$\Pr(B) = \Pr(AB) + \Pr(A'B) + \Pr((AB)(A'B)) \tag{8}$$

$$= \Pr(AB) + \Pr(A'B) + \Pr((BB)(AA')) \tag{9}$$

$$= \Pr(AB) + \Pr(A'B) \tag{10}$$

We get that,

$$\Pr(A'B) = \Pr(B) - \Pr(AB) \tag{11}$$

Substituting values from (2) and (3),

$$\Pr(A'B) = \frac{9}{13} - \frac{4}{13} \tag{12}$$

$$= \frac{5}{13} \tag{13}$$

Then, $\Pr(A'|B)$ is:

$$\Pr(A'|B) = \frac{\Pr(A'B)}{\Pr(B)} \tag{14}$$

$$= \frac{\frac{5}{13}}{\frac{9}{13}} \tag{15}$$

$$= \frac{5}{9} \tag{16}$$

Hence, option (d) is correct.