

Solution of question 1.2.2

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Question : We are given a triangle and the mid-points D, E, F of the sides AB, BC, AC respectively. We have to find the normal form of equation of lines AD, BE, CF.

Solution :

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (1)$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad (2)$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (3)$$

The mid points D, E, F of sides AB, BC, AC are :-

$$\mathbf{D} = \begin{pmatrix} -\frac{7}{2} \\ \frac{1}{2} \end{pmatrix} \quad (4)$$

$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} \quad (5)$$

$$\mathbf{F} = \begin{pmatrix} -\frac{3}{2} \\ \frac{5}{2} \end{pmatrix} \quad (6)$$

Now, the slope vector of line DA(\mathbf{m}) is :-

$$\mathbf{m} = \mathbf{D} - \mathbf{A} \quad (7)$$

$$\Rightarrow \mathbf{m} = \begin{pmatrix} -\frac{9}{2} \\ \frac{3}{2} \end{pmatrix} \quad (8)$$

\mathbf{m} can be taken as $\frac{\mathbf{D}-\mathbf{A}}{2}$, $\frac{\mathbf{D}-\mathbf{A}}{3}$, $\frac{\mathbf{D}-\mathbf{A}}{4}$ etc. because it will be eliminated while calculating the normal form of line. Now, we have to find \mathbf{n}^\top such that

$$\mathbf{n}^\top \mathbf{m} = 0. \quad (9)$$

\mathbf{n} can be written in the form :-

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \quad (10)$$

$$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -\frac{9}{2} \\ \frac{3}{2} \end{pmatrix} \quad (11)$$

$$= \begin{pmatrix} \frac{3}{2} \\ \frac{9}{2} \end{pmatrix} \quad (12)$$

So the transpose of \mathbf{n} ,

$$\mathbf{n}^\top = \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \quad (13)$$

Normal form of line AD is :

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{A}) = 0 \quad (14)$$

$$\Rightarrow \mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{A} \quad (15)$$

$$\Rightarrow \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (16)$$

$$\Rightarrow \frac{1}{2} \begin{pmatrix} 3 & 9 \end{pmatrix} \mathbf{x} = \frac{1}{2} \begin{pmatrix} 3 & 9 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (17)$$

$$\Rightarrow \begin{pmatrix} 3 & 9 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 & 9 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (18)$$

$$\Rightarrow \begin{pmatrix} 3 & 9 \end{pmatrix} \mathbf{x} = -6 \quad (19)$$

