

Solution to question 11.16.3.34

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Question: Prove if the given statement is true or false - The probability of intersection of two events A and B is always less than or equal to those favourable to the event A.

Solution: We have to prove that:

$$\Pr(AB) \leq \Pr(A) \quad (1)$$

We know that,

$$BB' = 0 \quad (2)$$

$$B + B' = 1 \quad (3)$$

$$\implies \Pr(A) = \Pr(A(B + B')) \quad (4)$$

$$= \Pr(AB + AB') \quad (5)$$

By using inclusion-exclusion principle,

$$\Pr(A) = \Pr(AB) + \Pr(AB') - \Pr((AB)(AB')) \quad (6)$$

$$= \Pr(AB) + \Pr(AB') - \Pr((AA)(BB')) \quad (7)$$

$$= \Pr(AB) + \Pr(AB') \quad (8)$$

We know that the value of probability ranges from 0 to 1.

$$0 \leq \Pr(AB') \leq 1 \quad (9)$$

Adding $\Pr(AB)$ both sides

$$\Pr(AB) \leq \Pr(AB) + \Pr(AB') \quad (10)$$

Substituting value from equation (8)

$$\implies \Pr(AB) \leq \Pr(A) \quad (11)$$

Hence, the given statement is true.