Solution to question 12.13.3.33

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Question: Suppose that 6% of the people with blood group O are left handed and 10% of those with other blood groups are left handed 30% of the people have blood group O. If a left handed person is selected at random, what is the probability that he/she will have blood group O?

Solution: Let us consider two random variables A and B,

Random Variable	Value	Event
A	1	Blood Group O
A	0	Any other blood group
B	1	Left-Handed person
B	0	Right-Handed person

We need to find the value of $\Pr(A = 1|B = 1)$. We are given that,

$$\Pr(A = 1) = 0.3 \tag{1}$$
$$\therefore \Pr(A = 0) = 1 - \Pr(A = 1) \tag{2}$$
$$= 1 - 0.3 \tag{3}$$
$$= 0.7 \tag{4}$$
$$\Pr(B = 1|A = 1) = 0.06 \tag{5}$$
$$\implies \frac{\Pr((B = 1)(A = 1))}{\Pr(A = 1)} = 0.06 \tag{6}$$
$$\implies \Pr((B = 1)(A = 1)) = 0.06 \Pr(A = 1) \tag{7}$$
$$= 0.018 \tag{8}$$
$$\Pr(B = 1|A = 0) = 0.1 \tag{9}$$
$$\implies \frac{\Pr((B = 1)(A = 0))}{\Pr(A = 0)} = 0.1 \tag{10}$$
$$\implies \Pr((B = 1)(A = 0)) = 0.1 \Pr(A = 0) \tag{11}$$
$$= 0.07 \tag{12}$$

We know that,

$$(A = 1) + (A = 0) = 1 \tag{13}$$
$$(A = 1)(A = 0) = 0 \tag{14}$$

We can write $\Pr(B = 1)$ as:

$$\Pr(B = 1) = \Pr((B = 1)((A = 1) + (A = 0))) \tag{15}$$
$$= \Pr((B = 1)(A = 1) + (B = 1)(A = 0)) \tag{16}$$

By inclusion-exclusion principle,

$$\Pr(B = 1) = \Pr((B = 1)(A = 1)) + \Pr((B = 1)(A = 0)) \tag{17}$$

By substituting values from equation (8) and (12),

$$\Pr(B) = 0.018 + 0.07 \tag{18}$$
$$= 0.088 \tag{19}$$

So, $\Pr(A = 1|B = 1)$ can be written as,

$$\Pr(A|B) = \frac{\Pr((B = 1)(A = 1))}{\Pr((B = 1))} \tag{20}$$
$$= \frac{0.018}{0.088} \tag{21}$$
$$= \frac{9}{44} \tag{22}$$

Hence, if a left handed is selected at random, the probability of the person having blood group O is $\frac{9}{44}$.