Labwork: lattice Boltzmann method (LBM)

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1 Introduction

In the present labwork session during 4 hours, we are going to implement a numerical scheme of lattice Boltzmann method (LBM) in a Matlab script in order to simulate a 2D Poiseuille flow driven by a body force. As shown in Figure 1, between two fixed solid walls (no-slip), a fluid flow is driven by a horizontal acceleration g (body force per unit mass). Periodic boundary condition is applied on the left and right boundaries of the computational domain. The analytical solution is given as $u(y) = gy(H - y)/(2\nu)$ with H and ν being the distance between two solid walls and the kinematic viscosity, respectively.

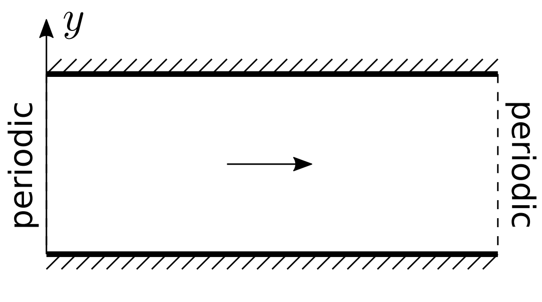

Figure 1: Periodic Poiseuille flow driven by a body force.

The objective of this labwork is to apply the LBM formulation in a practical exercise to better understand the major steps and features of the method. A Matlab script to be completed is available on the Hippocampus server.

2 Mathematical formulation

In the present labwork, we will adopt the single-relaxation-time (SRT) lattice Boltzmann equation (LBE) given as

$$\bar{f}_a(\boldsymbol{x}+\boldsymbol{\xi}_a\Delta t, t+\Delta t) = \bar{f}_a(\boldsymbol{x},t) - \frac{\Delta t}{\bar{\tau}}\left(\bar{f}_a(\boldsymbol{x},t) - f_a^{eq}(\boldsymbol{x},t)\right) + \Delta t\left(1-\frac{\Delta t}{2\bar{\tau}}\right)F_a(\boldsymbol{x},t), \tag{1}$$

where the equilibrium distribution function f_a^{eq} is given as

$$f_a^{eq} = \rho w_a\left(1+\frac{\boldsymbol{\xi}_a\cdot\boldsymbol{u}}{c_s^2}+\frac{(\boldsymbol{\xi}_a\cdot\boldsymbol{u})^2}{2c_s^4}-\frac{\boldsymbol{u}\cdot\boldsymbol{u}}{2c_s^2}\right), \tag{2}$$

and the force-related term F_a is calculated by

$$F_a = \rho w_a\left(\frac{\boldsymbol{\xi}_a-\boldsymbol{u}}{c_s^2}+\frac{\boldsymbol{\xi}_a\cdot\boldsymbol{u}}{c_s^4}\boldsymbol{\xi}_a\right)\cdot\boldsymbol{g}, \tag{3}$$

with $\boldsymbol{g} = g\boldsymbol{e}_x$. In addition, the macroscopic variables are computed as

$$\begin{cases} \rho = \sum_a \bar{f}_a, \\ \rho\boldsymbol{u} = \sum_a \boldsymbol{\xi}_a\bar{f}_a + \frac{\Delta t}{2}\rho\boldsymbol{g}. \end{cases} \tag{4}$$

3 Numerical implementation

3.1 Lattice and discretization

We are going to use the 'D2Q9' lattice, as shown in Figure 2, with which the weight coefficient w_a is given as $w_0 = 4/9$, $w_{1-4} = 1/9$ and $w_{5-8} = 1/36$. The speed of sound is $c_s = 1/\sqrt{3}\Delta x/\Delta t$ and the kinematic viscosity is $\nu = c_s^2(\bar{\tau} - 0.5\Delta t)$. Here, we take $\Delta x = 1$ and $\Delta t = 1$.

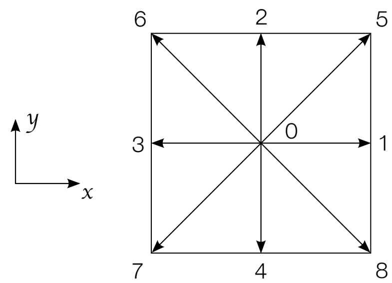

Figure 2: D2Q9 lattice.

The computational domain is a rectangle area, which will be discretized using $N_x \times N_y$ lattice nodes. Since the investigated problem is periodic in x-direction, the value of N_x could be small, e.g. $N_x = 3$.

3.2 Boundary condition

Two types of boundary condition will be used in this labwork, which are the no-slip boundary condition for the upper and lower solid walls and the periodic condition for the left and right boundaries, as shown in Figure 1. The no-slip boundary condition will be tackled with the (1) bounce-back rule and (2) the Zou-He's method.

3.3 Initial condition

Initially at $t = 0$, we impose that $\rho(\boldsymbol{x}, 0) = 1.0$ and $\boldsymbol{u}(\boldsymbol{x}, 0) = \mathbf{0}$. Given these macroscopic variables, we can compute the equilibrium distribution function $f_a^{eq}(\boldsymbol{x}, 0)$ and we will initialize the distribution function as $\bar{f}_a(\boldsymbol{x}, 0) = f_a^{eq}(\boldsymbol{x}, 0)$.

3.4 Time iteration

Within each time-step, we will carry out the LBM computation as follows:

(1) Compute the post-collision distribution function $\bar{f}_a^{pc}$ as

$$\bar{f}_a^{pc}(\boldsymbol{x}, t) = \bar{f}_a(\boldsymbol{x}, t) - \frac{\Delta t}{\bar{\tau}} \left(\bar{f}_a(\boldsymbol{x}, t) - f_a^{eq}(\boldsymbol{x}, t) \right) + \Delta t F_a(\boldsymbol{x}, t), \tag{5}$$

(2) Propagate the post-collision distribution function to the neighbouring nodes

$$\bar{f}_a(\boldsymbol{x} + \boldsymbol{\xi}_a \Delta t, t + \Delta t) = \bar{f}_a^{pc}(\boldsymbol{x}, t), \tag{6}$$

(3) Calculate the macroscopic variables ρ and $\boldsymbol{u}$ using Equation (4)

(4) Calculate the equilibrium distribution function f_a^{eq} with Equation (2)

(5) Calculate the force-related term F_a with Equation (3)

(6) Goto (1) for next time-step until the desired total iteration numbers

4 Post-processing

Works to be presented in the report:

- Numerical discretization parameters (lattice resolution, $\bar{\tau}$, etc.)
- Convergence history
- Comparison of the velocity profile between the numerical and analytical solutions

- Comparison between the two no-slip boundary conditions
- Mesh convergence study
- Conclusion and remarks

5 To go further (optional)

Increase the size of N_x and implement Zou-He's velocity inlet boundary condition and pressure outlet boundary condition. You can either impose a constant velocity profile or a paraboic one at the inlet. Show the velocity and pressure fields after convergence and then select three sections to plot the velocity profiles in y-direction.