Ecole Centrale Nantes **Master 1 2021-2022**

Lab work : lifting profiles
Coding of the lifting line theory using MATLAB

Goals : (1) Code the prandt method (2) check with the results obtained with theory (3) compare the performances for 3 types of wings.

We consider 3 types of plane wings : semi-elliptic (straignt trailing edge), rectangular and triangular, that have a chord of *l(y)* and a span of *2L*. For a given calculation, **We will take the same span and surfaces for the 3 types of wings** in order to compare the performances. The fluid is air and have a density of ρ=1,2 kg.m^{-3} and The shape coefficient is $k_\infty = \pi$ (coefficient obtained for a plate with 2D model).
In your calculations, you will take aspect ratios higher that 4, angles of attack α lower than 15°, and an angle for the triangular wing lower than 30°.

1/ Coding of the prandtl equation

$$\Gamma(y) = k_\infty(y)\lambda(y)U_\infty\left(\alpha(y) + \frac{1}{4\pi U_\infty}\int_{-L}^{+L}\frac{d\Gamma(\eta)}{(\eta - y)}\right)$$

We take N points of discretization $y_i, i \in \{1,..N\}$ between –*L* and +*L*. The circulation Γ will be estimated on these points y_i and we will suppose that $\Gamma(y = -L) = \Gamma(y = +L) = 0$.
To avoid singular values for the induced velocity, we need to take control points $\eta_j, j \in \{1,..N-1\}$ located at the center of each elements, defined by your discretization.
Discretize the Prandtl equation ; note that you will need to write the discrete term of the induced velocity in order to explicitely express in the discretized equation the coefficients relative to the variables $\Gamma_j = \Gamma(\eta_j)$.

Once the linear system on the discrete circulation solved, it is possible to compute the lift and drag coefficients with the integral equations seen during the lectures.

2/ Results

- For the 3 types of wings, you have to check the convergence of the results on the circulation and the forces (drag and lift), according to your discretization (number of points along the span).

- In the case of the elliptic wing, check that the numerical results are in agreement with the theory.

(formulas $C_x = \dfrac{C_z^2}{\pi\lambda}$ and $\dfrac{dC_z}{d\alpha} = \dfrac{2k_\infty}{1 + \dfrac{2k_\infty}{\pi\lambda}}$ for different aspect ratio λ).

- Compare the performances for the 3 types of wing, for different angle of attack and aspect ratio.

3/ Write a report where you will give all your results and comments, and join the MATLAB programm