Practical Work - Water Waves and Sea State Models

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1 Introduction

The main objective of this computer work is to present practical applications with respect to the lectures *Water Waves and Sea State Models*. We intend to introduce the most important environmental source of structure loading: sea waves. The two main parts of the course are addressed:

- The deterministic approach, with the study of different mathematical models used to represent the free surface gravity waves and the associated underlying flow.
- The statistical approach, with the practical use of the concept of the wave spectrum.

The study of those wave fields will be complemented with the analysis of their interaction with a specific structure. Matlab software will be used for the solution of the problem. A Matlab application is provided to you for the study of the deterministic waves and you will develop your own code for the treatment of irregular wave field and wave-structure interactions.

Another objective of this practical work is to provide you with valuable tools for the experiments in the hydrodynamic facilities of Centrale Nantes you will do in December. The wave tank you will have access to exhibits the following characteristics:

- Ocean wave basin (see Fig. 1)
  - Dimensions: $50m \times 30m \times 5m$
  - Segmented wavemaker with 48 independent flaps (hinge rotation axis at $d = 2.147m$ from the bottom)
  - Wave periods: $T \in [0.5s; 4s]$

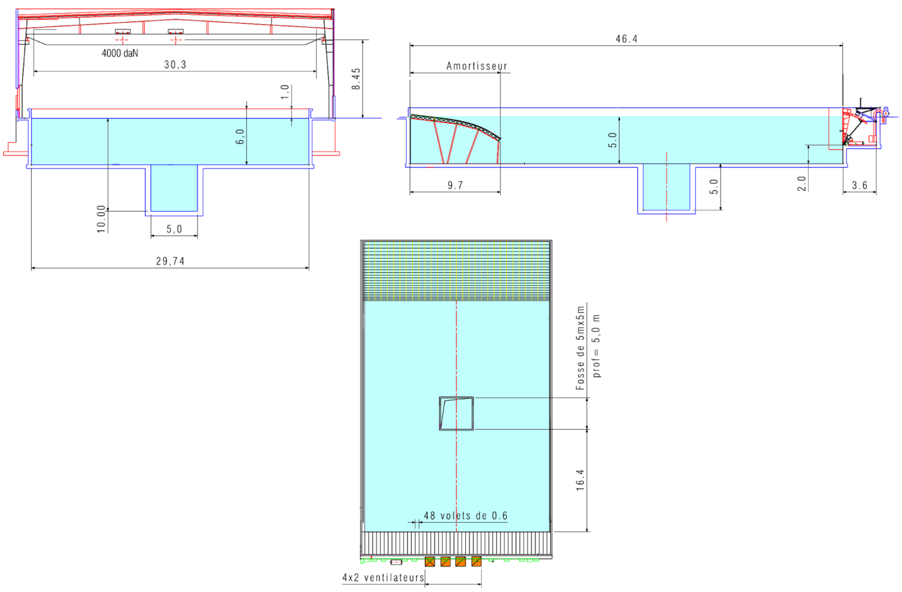

Figure 1: Scheme of the ocean wave basin

Instructions

- In all the following studies, the parameters characterizing the studied wave field have to be explicitly **given** and **justified**.
- It is of particular importance to **check that the assumptions of the chosen model are fulfilled**.
- The influence of the parameters associated with the wave field is usually achieved using its **non-dimensional characteristics**.

2 Weakly non-linear regular dispersive waves

We assume that waves are periodic in space and time and that the following parameters describe the studied waves

- λ: wavelength
- T: wave period (in time)
- A: amplitude of the wave
- d: water depth (assumed constant)
- Related parameters
  - $k = \frac{2\pi}{\lambda}$ the wave number
  - $H = 2A$ the crest-to-trough wave height
  - $\omega = \frac{2\pi}{T}$ the angular frequency

Furthermore, one assumes that we are dealing with *dispersive waves*. This means:

- Dispersive parameter: $\mu = kd \geq O(1)$ (or eq. μ is not small)

Finally, the *non-linearity* of the considered waves is assumed to be *small*:

- Steepness: $\epsilon = kA \ll 1$

2.1 Objectives and Theory

When studying weakly non-linear dispersive waves, the Stokes theory is the classical model for representing regular waves. The initial part of this practical work intends to make you use this model in order to discuss: i) the validity of the model and ii) the physics of the resulting wave field.

For the first stage of this work, you will study the Stokes solution, that is described in the reference [2]. This paper describes the solution up to 5^{th} order. It is reminded that the order refers to the non-linearity characterized by the steepness $\epsilon = ka$.

In order to quantify the accuracy of the results produced, a reference solution is used in the tool we provide. This numerical solution of the fully non-linear set of equations is described in [1] and is available freely in Mathworks repository.

We refer to the Water Waves and Sea States Modelling course for the description of the first, second and third-order Stokes models. In addition, you can refer to the original work of [2].

2.2 Matlab Application

A Matlab application has been developed in order to help you handle the different theories and study their respective features through the use of a graphical user interface that should allow you to change easily the parameters and generate the figures necessary for your report.

2.2.1 Installation & use

For the installation, you need:

- Download from Hippocampus the necessary files (included in the archive provided to you at the beginning of the practical work).
- Extract the archive `Student_pack.zip`.
- Extract the archive `Application_v10.zip`.
- Install the application in Matlab: tab 'Apps', click on 'Install App', select the file `ECN_Stokes5th.mlappinstall` located in the previously extracted directory.

Once the installation is performed, you should have access in a menu 'My Apps' to this application that you can launch by simply clicking on the corresponding icon. A capture of the graphical interface is depicted in figure 2.

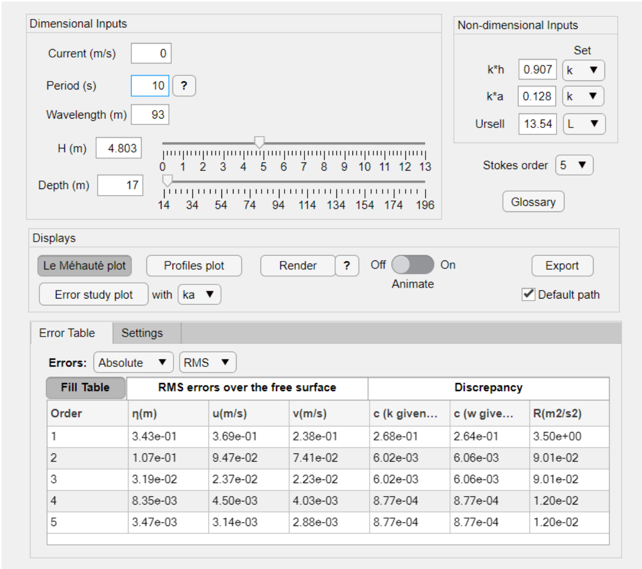

Figure 2: Graphical User Interface of the Matlab App

You can explore freely the capabilities of the tool as a first exercise. The important points to indicate:

- 'Le Méhauté plot' displays the current choice of wave parameters in a graph similar to the one proposed by Le Méhauté [3], see figure 3.
- You can select the wave you are interested in by clicking directly in the previously generated figure of Le Méhauté.
- The errors of the Stokes solution are computed with respect to a non-linear reference numerical solution [1]. They are computed for different wave quantities and are provided as absolute or relative:
  - 'Fill Table' displays for the wave parameters selected the errors of the different orders of the Stokes solution.
  - 'Error study plot' runs a study on the errors of the Stokes solutions when varying the non-dimensional parameters ϵ or μ.
- 'Export' allows you to export the figures displayed on the screen in a directory that can be chosen.

2.2.2 Questions

Q.1. Le Méhauté plot [3] provides information about the adequate model to use to address a wave problem from an engineer's perspective. Following for instance the proposed boundary between Stokes 1^{st} and 2^{nd} order, discuss the observed errors and conclude on the use of this plot.

Q.2. Observing the wave profile, discuss the behaviour of the Stokes solution at the limits identified in Le Méhauté plot, namely $H_0/\lambda_0 \simeq 0.14$ and $H\lambda^2/h^3 = 26$.

Q.3. Run the complete 'error study' with respect to ka and kh (standing for kd in this App). Report the results of this convergence study and analyze them in detail. You can focus on the study of the relative errors on the free surface elevation η.

Q.4. Using Froude scaling, compare the non-dimensional parameters ka and kd corresponding to waves at model scale and full scale.

Choose an intermediate water depth configuration for which Le Méhauté plot suggests the use of Stokes 3^{rd} order theory.

Q.5. Report and analyze the free surface elevation as well as the vertical profiles of velocities and pressure.

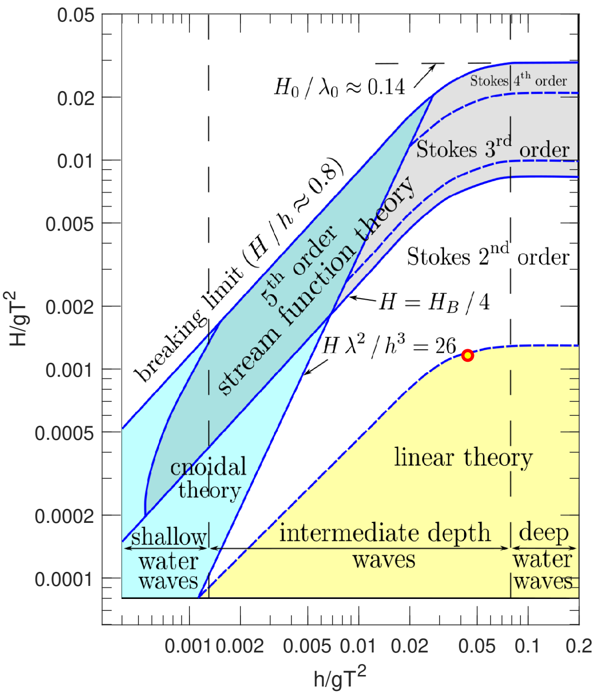

Figure 3: Le Méhauté plot [3] (h standing for water depth)

Q.6. Using the volume view ('Render'), animate the wave field and stop. Add the current and discuss of the influence of this phenomena on the wave parameters. Start again the animation and discuss of what you observe.

Modify the wave parameters that you identify as relevant in order to answer the following question. You will limit your study to waves propagating without current.

Q.7. Report the influence of both those parameters on the wave field. The different physical features associated with the waves should be detailed.

2.3 Matlab functions

The rest of the practical work uses a set of Matlab functions that are provided to you. They are used in the main program `WaterWaves_LabWork.m` that is given to you and that you will modify in order to solve the questions to come.

The most useful functions are :

- `kfromw.m` that allows the solution of the dispersion relation $\omega^2 = gk\tanh(kd)$.
- `elevation_linear.m` that constructs the free surface elevation from the linear model (Stokes 1^{st} order).
- `velocity_pressure_linear.m` that constructs the velocity and presure fields from the linear model (Stokes 1^{st} order).
- `Le_Mehaute.m` that allows you to simply set the position of the waves chosen in Le Méhauté graph.

The command `help name_of_the_function` will give you details with respect to its use. Then, in order to help you understand how to handle those functions, you will answer the following questions.

Q.8. Sketch the evolution of the phase and group velocities as a function of the dispersion parameter. It is useful to make those velocities dimensionless with respect to the phase velocity in deep water and/or shallow water for a more in-depth analysis.

Application to experiments

The dispersion relation is essential for the post-processing of experimental data obtained in the context of a wave basin experiment. The finite length of the wave basin induces the possible presence of reflected waves on the measured signal.

Q.9. For a given location in the wave tank, evaluate what is the measurement time window free of reflections for different choices of wave frequencies. It relies on the definition of when the data processing can start t_{start} and when it has to be stopped t_{stop}. This study has to be done for the ECN wave basin described in the Introduction.

Complementary, the dispersion relation is used to choose the wave parameters for a given set of experiments.

Q.10. For the whole range of wave frequencies available in the ECN wave tank, evaluate the amplitudes of the generated waves for a range of tests performed at *constant steepness*.

The linear Stokes theory provides the most simple way to represent weakly non-linear dispersive waves.

Q.11. Perform a temporal animation (on the screen, no need for a video file) of the free surface elevation together with the velocity/pressure fields for a realistic configuration tested in the ocean wave tank of ECN that can be modelled thanks to this linear Stokes theory.

3 Irregular waves

When dealing with irregular waves, it is possible (assuming linear theory) to describe waves with the variance spectral density (or equivalently the energy spectral density), known as the **wave spectrum** $S_\eta(f)$. Remind that we assume linear superposition

$$\eta(x,t) = \sum_i \eta_i(x,t) = \sum_i a_i cos(k_i x - \omega_i t + \varphi_i) \tag{1}$$

Relation between variance and corresponding spectrum is given for each frequency component by

$$\frac{1}{2}(a_i)^2 = S_\eta(f_i)\Delta f_i \tag{2}$$

3.1 Wave spectrum

Real sea states are approached with "classical" spectra. The main parameters are

- Significant wave height $H_s = H_{m_0}$
- Peak period T_p

We remind here of two widely used wave spectra.

- Pierson-Moskovitz: valid for fully developed sea

$$S(f) = \frac{5}{16 f^5}\frac{H_s^2}{T_p^4}\exp\left(-\frac{5}{4T_p^4 f^4}\right)$$

- JONSWAP: valid for wind sea to fully-developed sea

$$S(f) = \alpha\frac{5}{16 f^5}\frac{H_s^2}{T_p^4}\exp\left(-\frac{5}{4T_p^4 f^4}\right)\gamma^{\exp\left(\frac{-(f-\frac{1}{T_p})^2 T_p^2}{2\sigma^2}\right)}$$

$$\int_0^\infty S(f)df = \frac{H_s^2}{16} \quad \Rightarrow \quad \alpha \quad \text{and} \quad \begin{cases} \sigma = 0.07 & \text{for} \quad f < f_p = \frac{1}{T_p} \\ \sigma = 0.09 & \text{for} \quad f \geq f_p = \frac{1}{T_p} \end{cases}$$

In this study, the JONSWAP spectrum is studied. It is a function of H_s, T_p and γ. Fix a value to H_s and T_p (reasonable!) that you will report.

Q.12. Look at the influence of γ parameter on the shape of the spectrum.

Q.13. Give the wave profile at a given location as a function of time for different values of γ (make sure to use the same random phases for each choice of γ). Explain the differences observed, possibly making a link with the shape of the spectrum (previous question).

Q.14. Once the surface elevation is known (choose one value for γ), it is also possible to have a look at the corresponding velocity and pressure field. Describe the observed field.

In real conditions, the sea states are often complex and made of components that have different physical origins (typical swell and wind sea).

Q.15. Evaluate the wave spectrum of a mixed sea state composed of one swell and one wind sea. Give explicitly the characteristics chosen for each part. Compute the corresponding free surface elevation.

3.2 Directional seas - Optional

Directionality may also play a key role in wave propagation (and its resulting wave loadings). The free surface elevation becomes

$$\eta(x, y, t) = \sum_i \sum_j \eta_{ij}(x, y, t) = \sum_i \sum_j a_{ij} cos(\overrightarrow{k_{ij}}.\vec{x} - \omega_{ij} t + \phi_{ij}) \tag{3}$$

We remind that a directional spectrum is usually written under the form:

$$S_\eta(f, \theta) = S_\eta(f).D(\theta) \tag{4}$$

The spreading function $D(\theta)$ may be of several forms. The most classical is a cos^{2s} spreading under the following form (for $-\pi \leq \theta - \theta_m \leq \pi$)

$$D(\theta) = \frac{2^{2s-1}}{\pi} \frac{[\Gamma(s+1)]^2}{\Gamma(2s+1)} cos^{2s}\left(\frac{\theta - \theta_m}{2}\right) \tag{5}$$

with the Gamma function defined by (if n is a positive integer $\Gamma(n) = (n-1)!$)

Q. optional: Represent the 3D free surface elevation at a given time. Look at the influence of the s parameter on the free surface elevation, as well as θ_m.

4 Wave-structure interactions

In this part, you will choose which case you want to study. **Only one of the following case has to be treated** among the two:

- Small body
- Large body

4.1 Small bodies: application to wave loading

We consider a tubular element of a fixed jacket structure (*cf.* Fig. 4). The geometry of this element is cylindrical and assumed to be vertical. We intend to evaluate the loading induced by waves on this element. We assume we will be in the theory of small bodies (*i.e.* large K_C).

Usually, in the framework of small bodies theory, the force exerted on an object in regular waves is characterized by the Morison formula. Considering a vertical cylinder, the horizontal projection of the latter allows us to express the force per unit length $f_x = \frac{F_x}{L}$ (perpendicular to the flow, *i.e.* wave direction of propagation)

$$f_x(z, t) = \rho C_M S \frac{\partial U(z, t)}{\partial t} + \frac{1}{2} \rho C_D D U(z, t) \left| U(z, t) \right|$$

with U the horizontal velocity induced by the flow which is considered constant, S the section of the cylinder and D its diameter. The total force exerted on the cylinder is simply obtained by integration along the cylinder:

$$F_x(t) = \int_{-d}^{\eta} f_x(z, t) dz$$

Figure 4: Jacket for offshore wind farm

The $C_M = 1 + C_m$ and C_D coefficients are determined by experiments and are function of Keulegan-Carpenter number $K_C = \frac{A\omega T}{D}$ as well as Reynolds number $Re = \frac{A\omega D}{\nu}$ (or $\beta = Re/K_C$). At low K_C, inertia is dominating while at large K_C, this is the drag which dominates. From a physical point of view, low K_C (< 2 or 4 depending on β), the flow is attached to the cylinder and potential theory is applicable.

Figures 5 present C_M and C_D coefficients respectively for a cylinder in an oscillating current. The Matlab function `Morison_coeff_Sarpkaya.m` returns the values of those coefficients for a given value of Re and K_C.

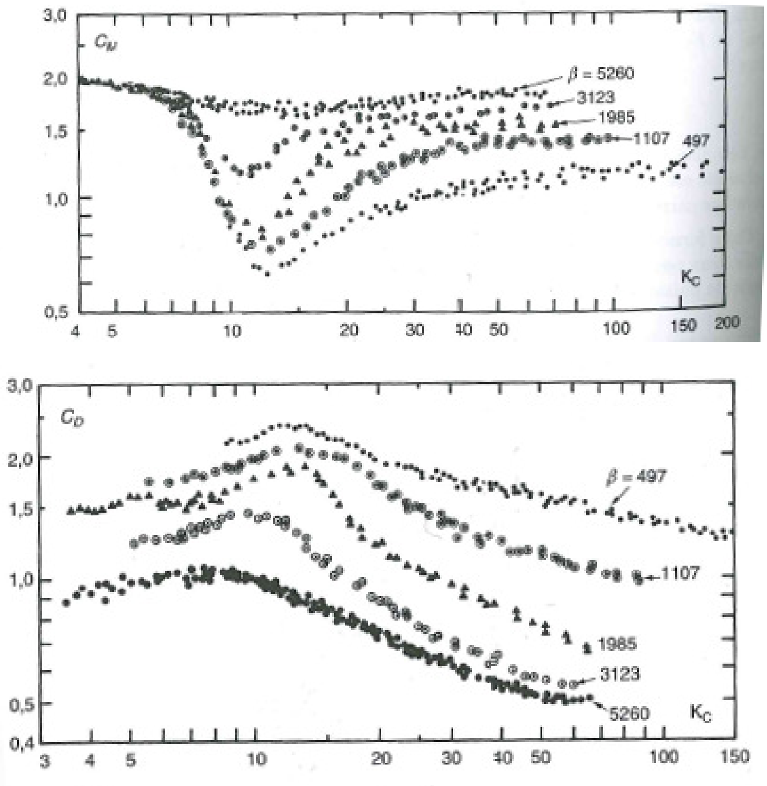

Figure 5: Coefficient $C_M = (1 + C_m)$ (top) and C_D (bottom) for a cylinder in an oscillating flow as function of K_C and β [4]

Q.16. Evaluate the horizontal force exerted by a regular wave on the chosen structure.

- **a.** Give the characteristics of the chosen structure as well as the wave considered.
- **b.** Describe in detail the procedure used to evaluate this force.
- **d.** In the analysis of the results, it is useful to compare the drag force component with the added mass one.

Q.17. Reminding the different assumptions made, how can you enhance the evaluation of wave loadings on the structure studied?

Q. optional: Evaluate the loading in the case of an irregular wave. You can refer to the lecture and to the DNV standards (for offshore wind structures) which are provided: `DNV.pdf` pages 78 to 80.

4.2 Large bodies: response spectrum

We consider a large dimension body in an irregular wave field. Viscous effects may be neglected as a first approximation, diffraction and radiation phenomena being preponderant. A seakeeping numerical code enables the computation of the transfer function (Response Amplitude Operator: RAO) of the considered system along its degrees of freedom.

We refer to the seakeeping course (and the corresponding practical applications). We study a cargo ship hull of type series 60 (see Fig. 6). RAO along the 6 degrees of freedom are provided. Be careful, they are given with respect to the period of the wave in the fixed reference frame. See following files: `RAO-0nds.dat` and `RAO-Froude0p22.dat` with and without forward speed ($Fr = 0.22$) respectively.

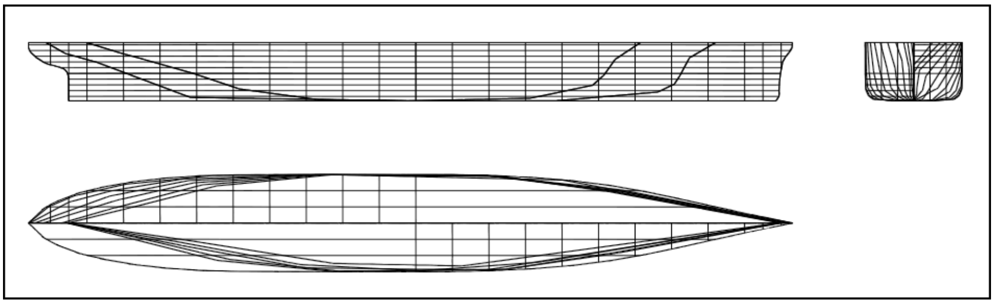

Figure 6: Series-60 ship hull shape

Q.16. After the choice of an adequate sea state, determine the response spectrum (in heave) of the ship of interest:

- **a.** Without forward speed
- **b.** With forward speed in a head sea

Q.17. From the response spectra, represent a temporal signal of the movement of interest. Evaluate the significant movements as well as the characteristics periods (that you may compare to the wave spectrum).

Q. optional: Evaluate the influence of the forward speed in the case of following seas.

References

[1] Didier Clamond and Denys Dutykh. Accurate fast computation of steady two-dimensional surface gravity waves in arbitrary depth. *Journal of Fluid Mechanics*, 844:491–518, 2018.

[2] John D Fenton. A fifth-order stokes theory for steady waves. *Journal of waterway, port, coastal, and ocean engineering*, 111(2):216–234, 1985.

[3] Bernard Méhauté. An introduction to hydrodynamics and water waves. 1976.

[4] Turgut Sarpkaya. Vortex shedding and resistance in harmonic flow about smooth and rough circular cylinders at high reynolds numbers. Technical report, NAVAL POSTGRADUATE SCHOOL MONTEREY CA, 1976.