Universität Konstanz
Physikalisches Fortgeschrittenenpraktikum

22 January 2014

Lock-In Amplifier

Abstract:

A lock-in amplifier is an important instrument using phase sensitive detection to measure signals that are coded in noise. In the beginning of this report, the concept of 'noise' will be explained. After that, some electrical filters and their properties will be presented, as well as the basics of phase sensitive detection. The second part of this report will deal with the results of the experiment: after examining the behavior of the single parts the lock-in amplifier consists of, like preamp, band pass and phase sensitive detector, all those devices will be combined in order to measure a signal tainted by noise with the lock-in amplifier.

Authors:

Simon Hönl
René Sedlak

Tutor:

Qijun Liang

Contents

1. Noise

Any unwanted disturbance of a physical system caused by signals with unspecific frequency spectrum is called noise. Basically these disturbing signals separate into random noise, which can only be described stochastically and systematic noise, which is either caused by variations of the parameters like the gain or by electromagnetic interference of the measured signal with external sources. An important example for the last one is interference with the power grid. In this case the noise frequency equals the utility frequency and its harmonics. Thus in Europe the noise frequencies are 50 Hertz and 100 Hertz while in North America it is 60 Hertz and 120 Hertz. Systematic noise can be reduced by accurate grounding and careful shielding.

In the following, the main types of noise will be explained.

1.1 Thermal Noise

Thermal Noise, which is also called Johnson Noise, is caused in any electronic component by the Brownian Motion of the charge carriers. So thermal noise is random noise. Approximately thermal noise is so-called White Noise: its amplitude stays more or less constant over its complete bandwidth. The time-weighted average of the noise voltage is given by the following formula:

$$V(rms) = \sqrt{4kTR\Delta f}$$

Therefore the strength of the disturbing noise depends on the temperature T, the ohmic resistance R of the device and the measured frequency bandwidth Δf. k is the Boltzmann constant. Hence thermal noise can be reduced either by decreasing the temperature or by narrowing the measured frequency bandwidth.

1.2 1/f Noise

1/f noise is also known as flicker noise. According to its name its intensity grows as the reciprocal of the frequency. For low frequencies 1/f noise overshadows thermal noise, like it is shown in figure 1. The causes for 1/f noise are still mostly unknown so far. This kind of noise has already been observed on amplifiers, the base current of transistors and photo detectors.

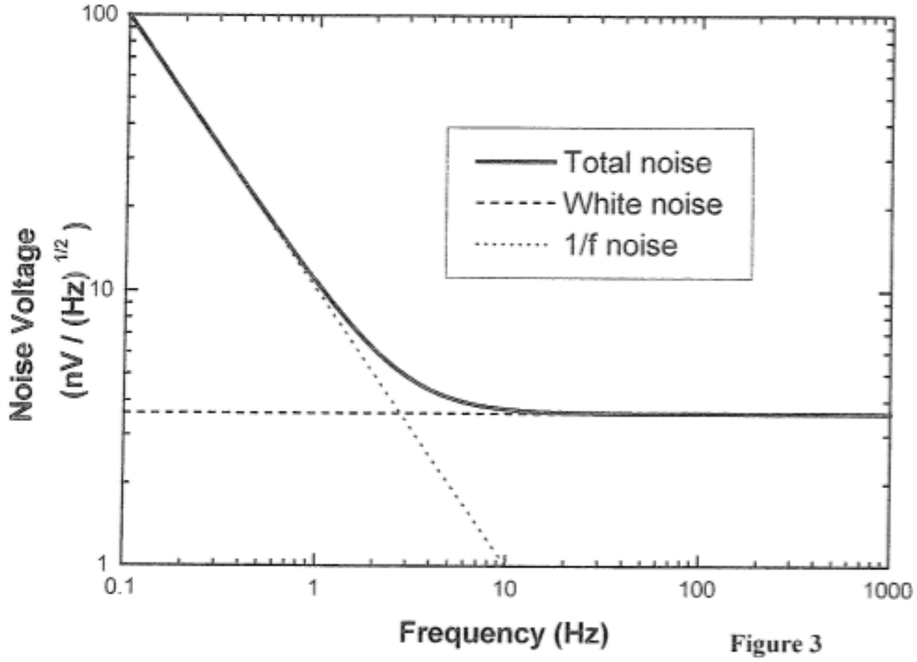

Figure 1: Noise voltage as a function of the frequency

1.3 Shot Noise

Like thermal noise shot noise is random noise. It arises from electrical current, which is represented by the flux of individual charge carriers (electrons). Like other particles they are subject to random fluctuations, which is reflected in density variations of the charge carrier flux and leads to a disturbing electrical current. For high currents fluctuations of several charge carriers are practically negligible. However shot noise is significant for low current. The averaged noise current is determined from the formula

$$I_{noise}(rms) = \sqrt{2eI\Delta f}$$

where e is the elementary charge, I is the electrical current and Δf is the measured frequency bandwidth.

2. Frequency Filters

Frequency filters are used to select or surpress specifically a certain frequency range out of a continuous electromagnetic signal. A distinction is made according to their utilization between low pass, high pass and band pass. It should be noted that generally every kind of filter reduces the signal-to-noise-ratio for random noise because noise of the respective frequencies is surpressed as well.

2.1 Low Pass

A low pass suppresses every frequency that exceeds a certain cutoff frequency f_c. Thus only lower frequencies can pass. The following figure shows the circuit diagram of the simplest version of a low pass, which just consists of one resistor and one capacitor.

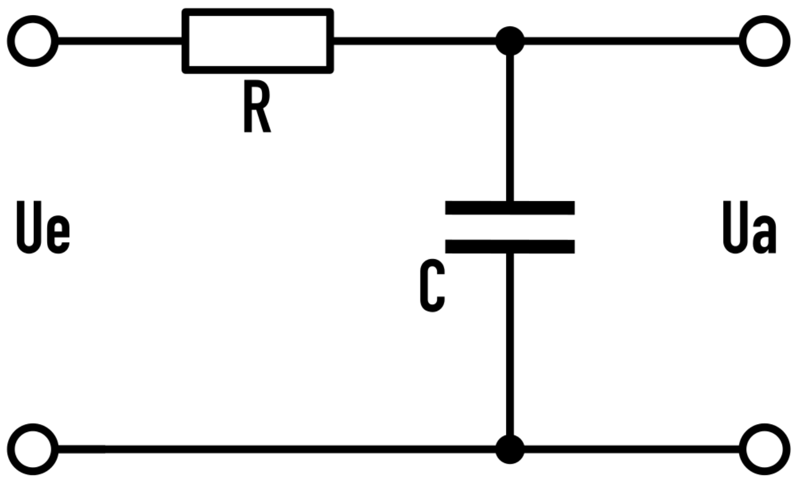

Figure 2: Circuit of a simple low pass consisting of resistor R and a capacitor C

The output voltage depends on the input voltage and the ratio of the AC resistances.

$$U_{out} = U_{in} \cdot \frac{|X_C|}{\sqrt{X_C^2 + R^2}} = U_{in} \cdot \frac{\frac{1}{\omega C}}{\sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}} = U_{in} \cdot \frac{1}{\sqrt{1 + (\omega CR)^2}}$$

with $\omega = 2\pi f$

The output voltage approaches zero at high frequencies f, whereas it converges to the input voltage at low frequencies. The cutoff frequency is defined to be the frequency at which the voltage is reduced by 3 dB, i.e. the output voltage is about half of the input voltage.

$$f_g = \frac{1}{2\pi RC}$$

The suppression of high frequencies leads to a phase shift φ, which converges to $-\frac{\pi}{2}$ for high frequencies.

It is given by

$$\tan\varphi = -\omega RC$$

Figure 3 shows the frequency-dependent behavior of the gain, which is the ratio of output voltage to input voltage, as well as the phase shifting.

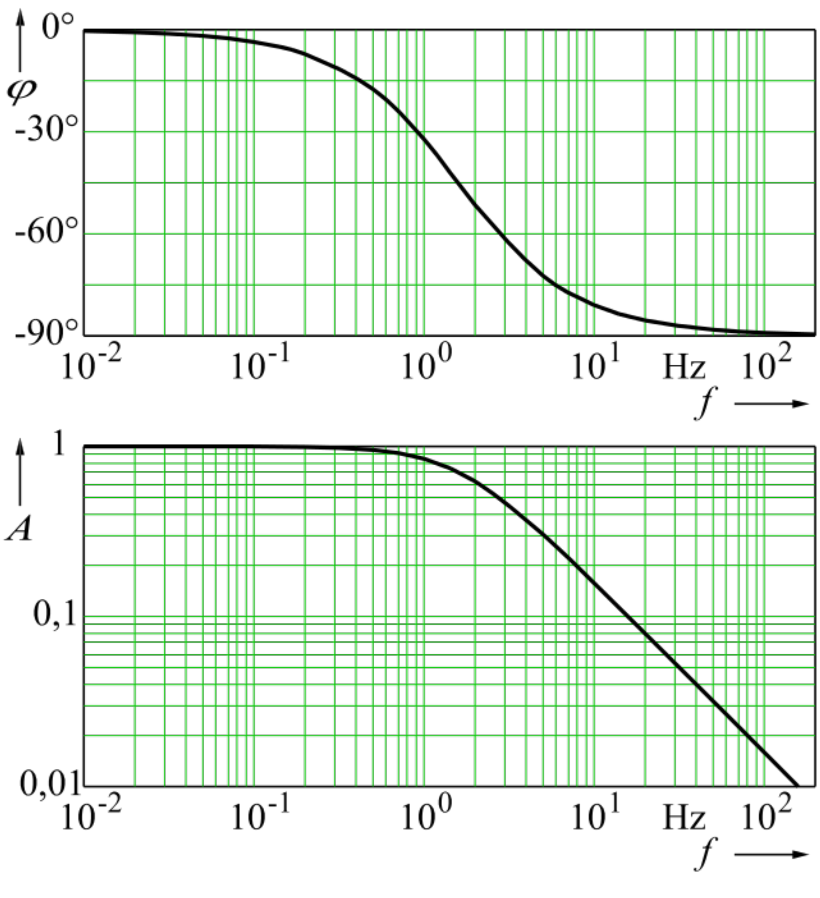

Figure 3: Bode diagrams of the amplitude A and phase shift of a low pass as functions of the Frequency f

2.2 High Pass

In contrast to the low pass allows frequencies that are higher than the cutoff frequency to pass, whereas lower frequencies are being suppressed. Its simplest version can be built by switching resistor and capacitor of the corresponding circuit of the low pass (see Figure 4).

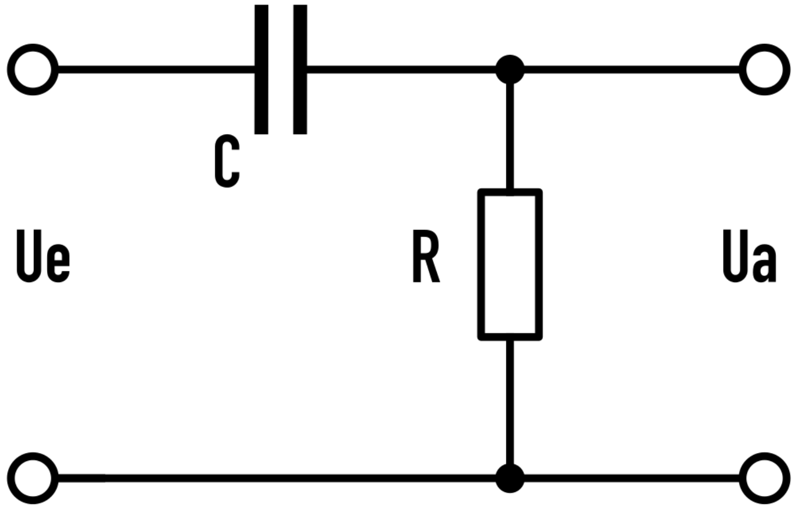

Figure 4: Circuit of a simple high pass consisting of resistor R and a capacitor C

Thus the output voltage is

$$U_{out} = U_{in} \cdot \frac{R}{\sqrt{X_C^2 + R^2}} = U_{in} \cdot \frac{R}{\sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}} = U_{in} \cdot \frac{\omega C}{\sqrt{1 + (\omega C)^2}}$$

At low frequencies the output voltage tends to zero, while it equals the input voltage at high frequencies. Once again the signal's phase φ gets shifted:

$$\tan \varphi = \frac{1}{\omega CR}$$

2.3 Band pass

The band pass selects a bandwidth Δf to be passed, while all frequencies below and above are being suppressed. The bandwidth Δf is limited by a lower cutoff frequency f_L and an upper cutoff frequency f_H.
Signals at frequencies within the bandwidth Δf are attenuated by less than 3 dB (see Figure 5). Band passes are characterized by the center frequency, which is defined by the geometrical mean of the upper and lower cutoff frequencies.

$$f_0 = \sqrt{f_L \cdot f_H}$$

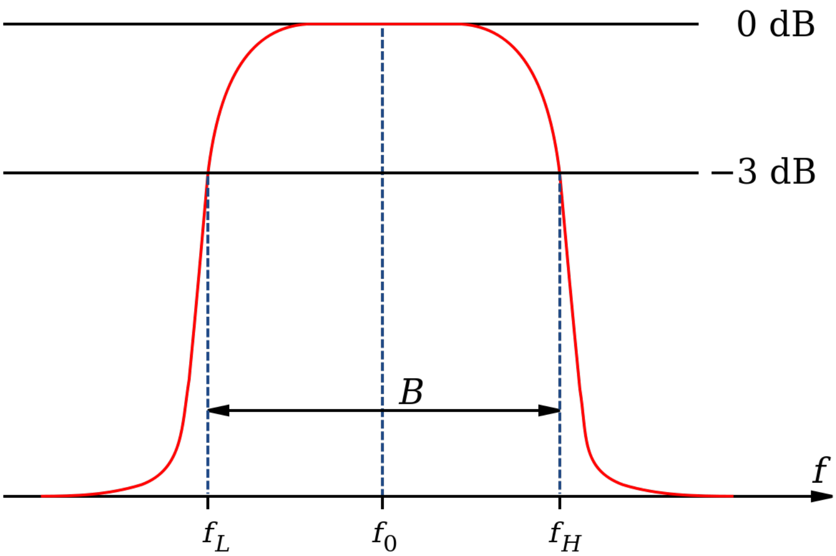

Figure 5: Gain of a band pass as a function of the frequency f

Being a combination of a low pass and a high pass a simple band pass consists of a combination of resistor, capacitor and inductor (Figure 6).

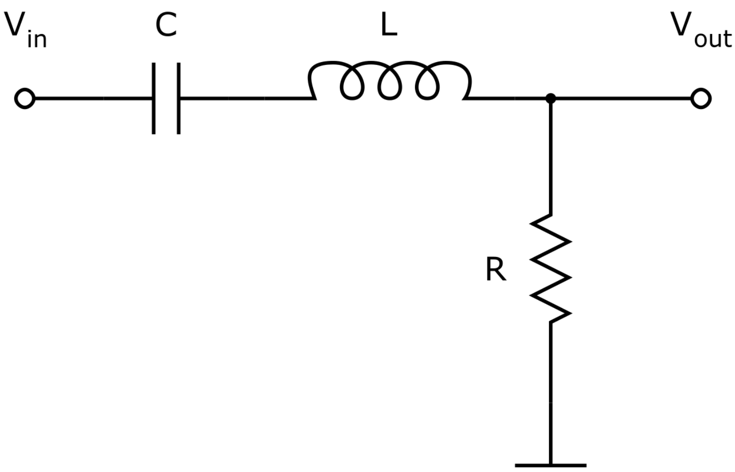

Figure 6: Circuit of a band pass consisting of resistor R, capacitor C and inductor L

Once again the output voltage can be calculated using the resistances:

$$U_{out} = U_{in} \cdot \frac{R}{\sqrt{X_C^2 - X_L^2 + R^2}} = U_{in} \cdot \frac{R}{\sqrt{\left(\frac{1}{\omega C}\right)^2 - (\omega L)^2 + R^2}}$$

$$= U_{in} \cdot \frac{1}{\sqrt{1 + \left(\frac{1}{\omega RC}\right)^2 - \left(\frac{\omega L}{R}\right)^2}}$$

The phase shift φ is given by

$$\tan\varphi = \frac{\frac{1}{\omega C} - \omega L}{R}$$

Tending towards the lower cutoff frequency the signal's phase shift converges to $-\frac{\pi}{2}$, at frequencies next to the upper cutoff frequency it approaches $\frac{\pi}{2}$. Figure 7 shows the Bode plots of the gain and the phase shift of the band pass.

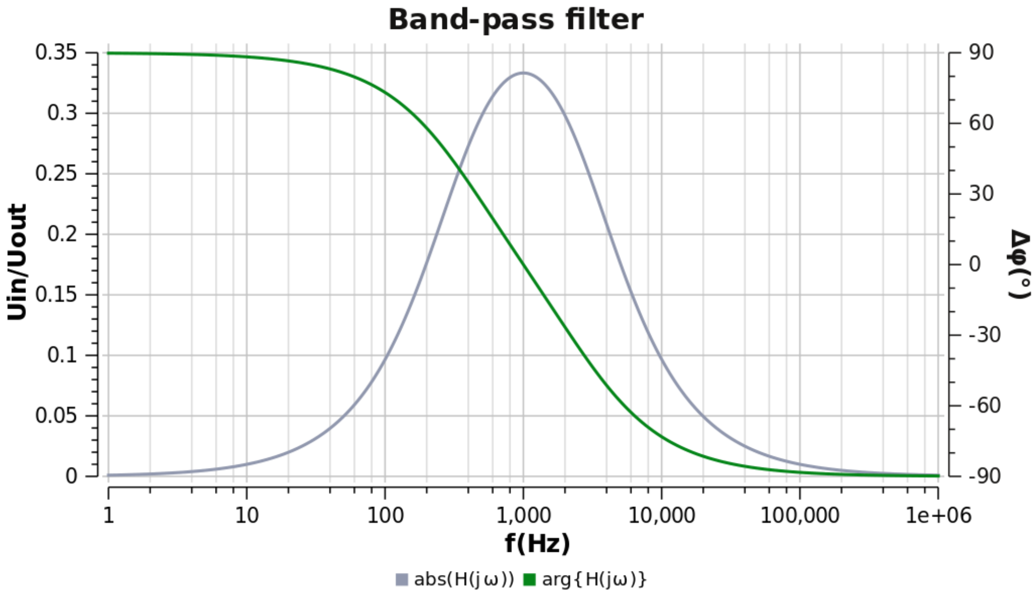

Figure 7: Bode plots of the gain (grey) and the phase shift (green) as functions of the frequency f

2.4 Types of Filters

Depending on their desired properties there are various types of filters which have individual characteristics concerning damping, phase response and group delay. Increasing the order of a filter means e.g. to connect two or more filters in series in order to strengthen the characteristics of the respective filter. In the following diagrams the amplitude and the group delay will be plotted as normalized functions of the frequency by the cutoff frequency $\omega_n = \frac{f}{f_0}$.

2.4.1 Bessel Filter

The most significant property of a Bessel filter is the constant group delay at the passing frequencies. Figure 9 shows the Bode plot of the group delay of a low pass Bessel filter. One can see that the group delay stays constant at low frequencies. That leads to a very good transmission behavior for square-wave signals, i.e. the response signal stays very close to a square-wave signal (Figure 10).

This is important for transmitting pulse signals. Since the group delay is constant the phase shift in the pass band is linearly dependent on the frequency.
One disadvantage of the Bessel filter is the slight side inclination of the amplitude so that the cutoff is not as sharp as it is for other types of filters (see Figure 8).

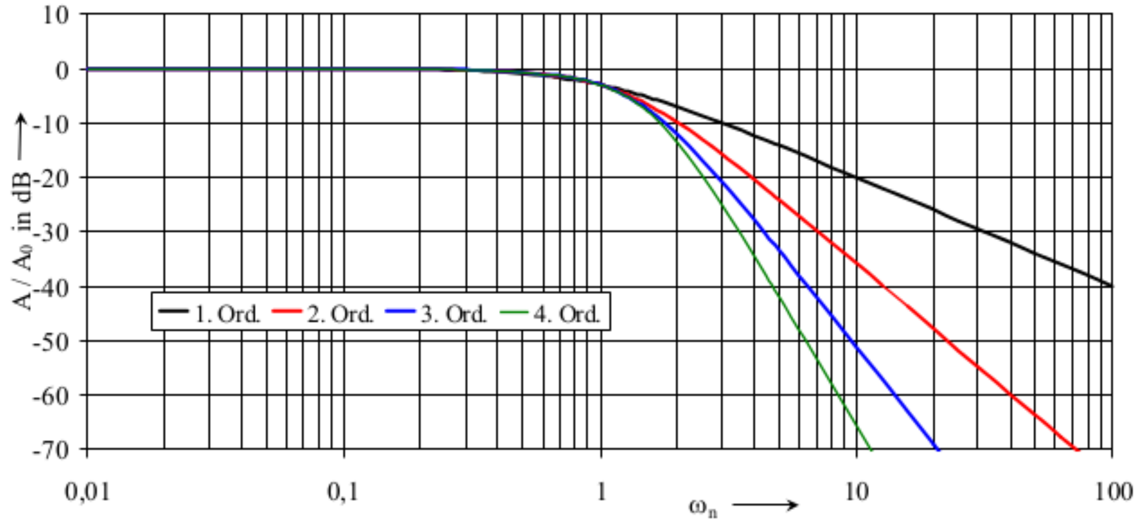

Figure 8: Bode plot showing the amplitude of a low pass Bessel filter

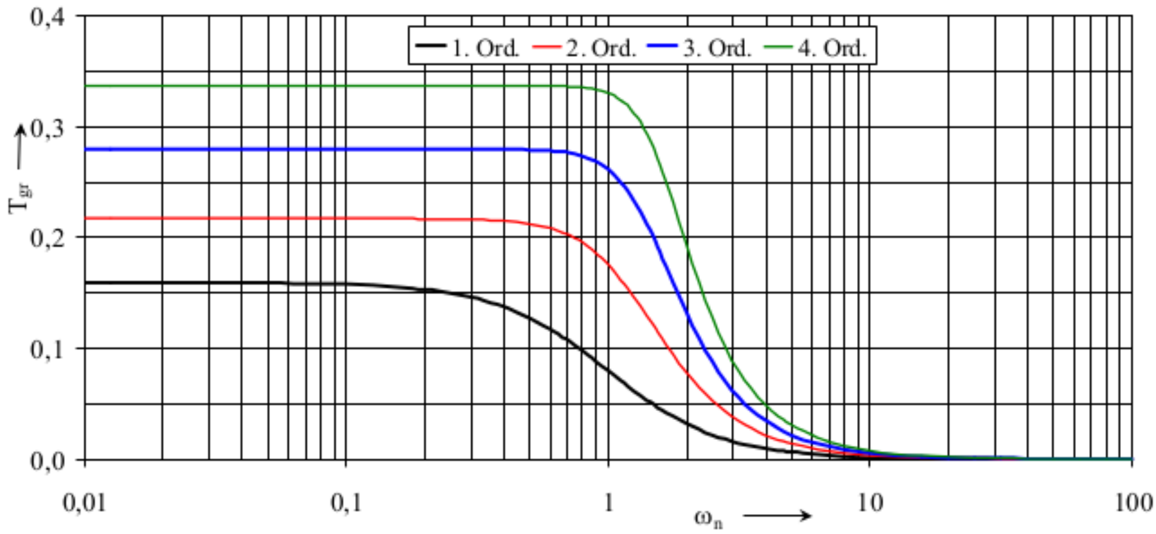

Figure 9: Bode plot showing the group delay of a low pass Bessel filter

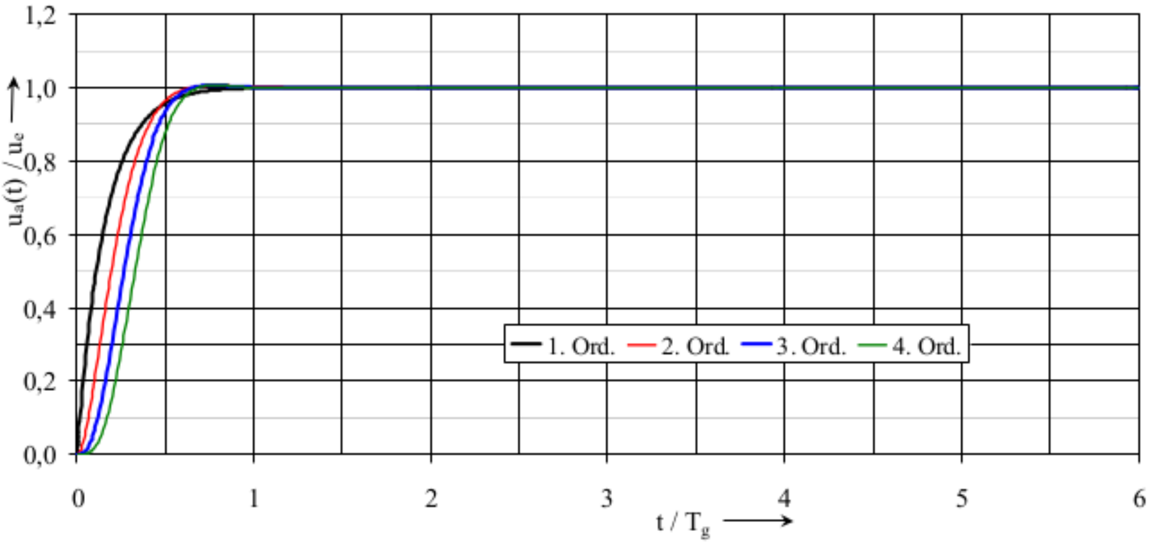

Figure 10: Step response of a Bessel filter

2.4.2 Butterworth Filter

The special feature of a Butterworth filter is the nearly constant gain in the passband. There is only a small overshoot close to the cutoff frequency (see figure 11). The phase shift is slightly non-linear and the group delay strongly depends on the frequency (see figure 12).
Figure 13 shows that the step response isn't as close to a square signal as it is for the Bessel filter.
The square of the absolute gain is given by

$$|A|^2 = \frac{A_0^2}{1 + \left(\frac{\omega}{\omega_0}\right)^{2n}}$$

A_0 is the DC gain, ω_0 the cutoff pulsatance and n the order of the filter.
Butterworth filters are often used to transfer narrow-band signals containing various frequencies whose amplitudes mustn't be damped.

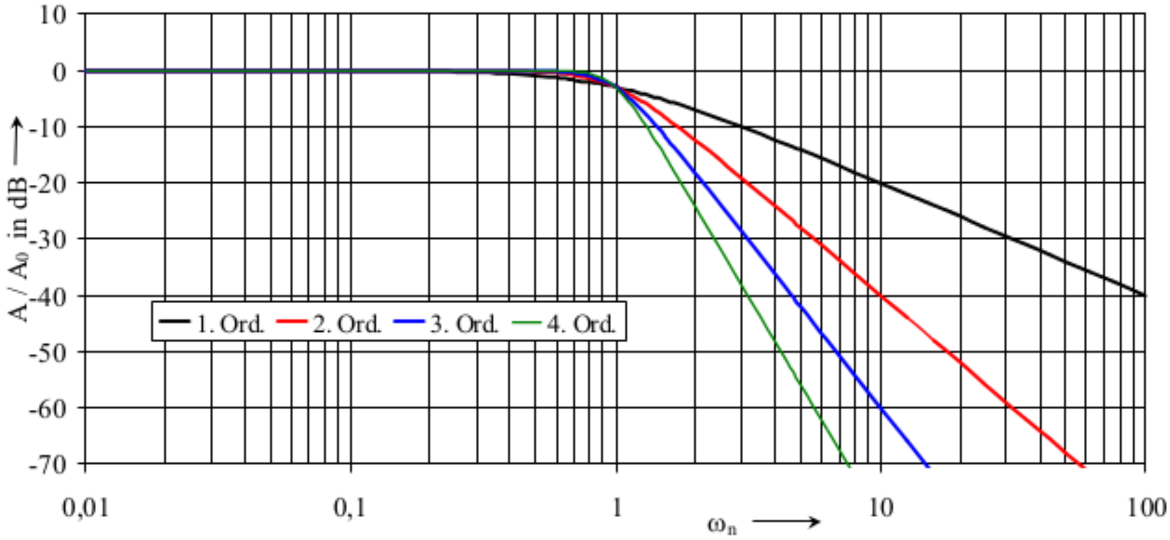

Figure 11: Bode plot showing the amplitude of a low pass Butterworth filter

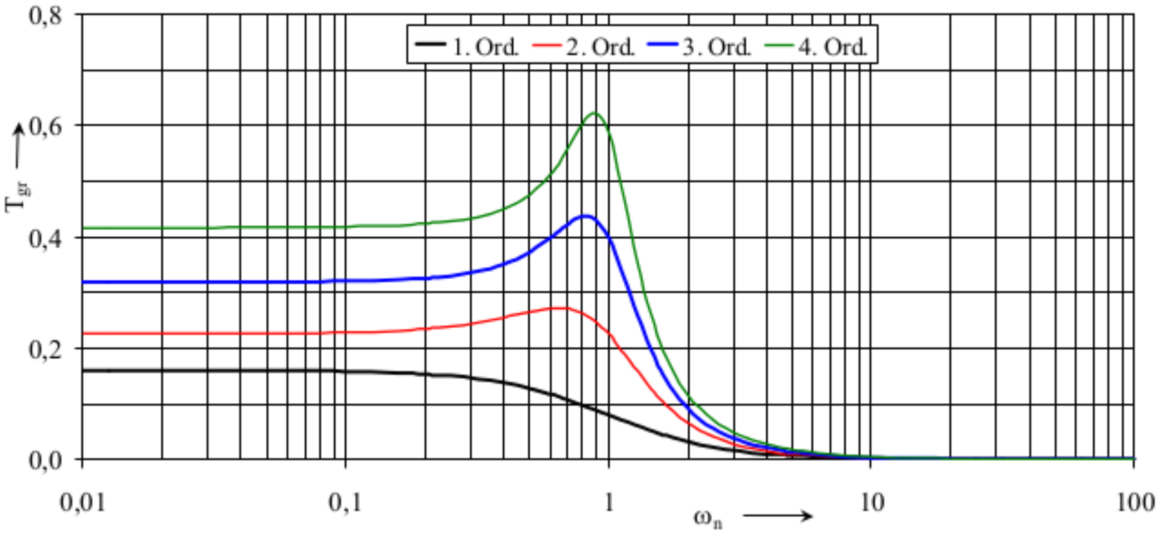

Figure 12: Bode plot showing the group delay of a low pass Butterworth filter

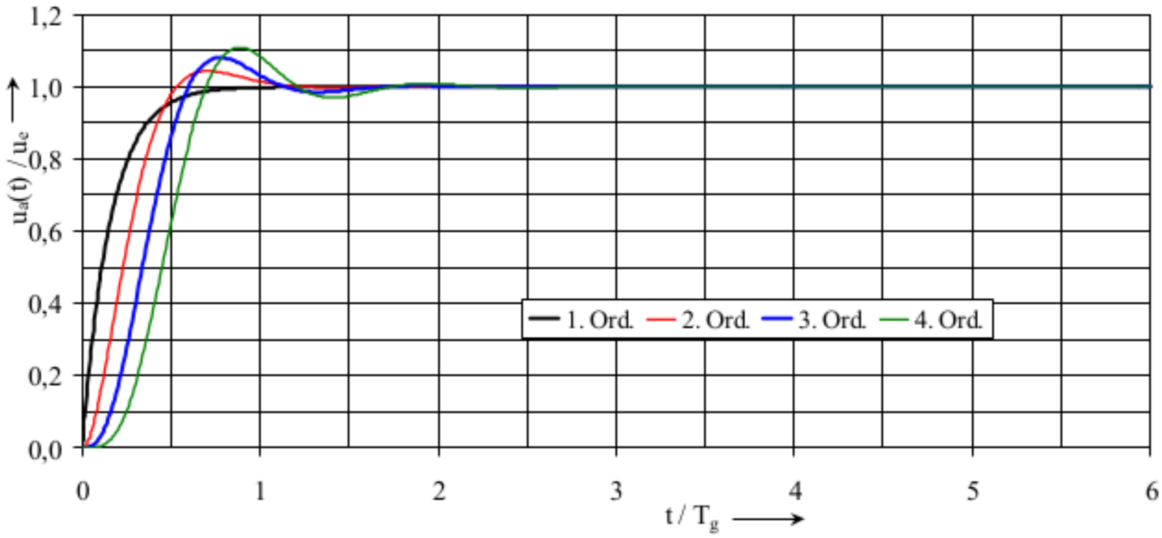

Figure 13: Step response of a Butterworth filter

2.4.3 Chebyshev Filter

The square of the absolute value of the voltage gain of a Chebyshev filter is

$$|A|^2 = \frac{k \cdot A_0^2}{1 + \varepsilon^2 \cdot T_n^2(x)}$$

where $T_n(x)$ are the Chebyshev polynomials, ε is the Ripple factor and k is a constant, which is selected so that $|A|^2 = A_0^2$ for $x = 0$.

The Chebyshev polynomials are given by

$$T_n(x) = \begin{cases} \cos\big(n \cdot \arccos(x)\big) \text{ for } 0 \leq x \leq 1 \\ \cosh\big(n \cdot \operatorname{arcosh}(x)\big) \text{ for x} > 1 \end{cases}$$

The Ripple factor describes the waviness of the output signal.

As it can be seen in figure 14, the Chebyshev filter has strong waviness in the passband, which of course leads to a very wavy step response (see figure 16). Additionally the group delay is strongly dependant on the frequency (see figure 15), so that a Chebyshev filter isn't really appropriate to transfer pulse or square wave signals properly.

The advantage of a Chebyshev filter is the sharp cutoff. Figure 14 shows that the gain decreases rapidly for frequencies higher than the cutoff frequency.

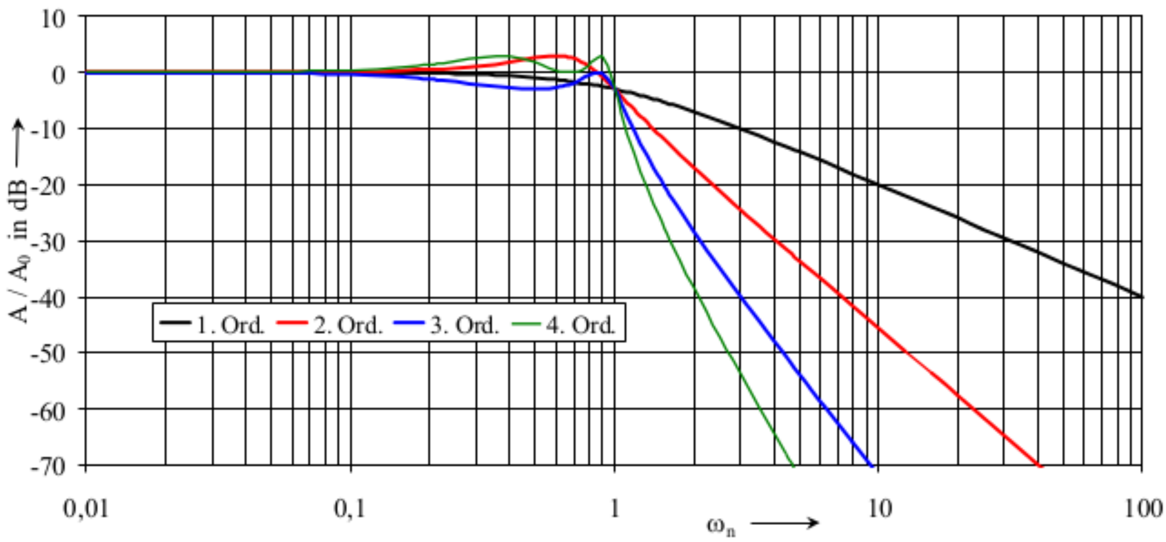

Figure 14: Bode plot showing the amplitude of a low pass Chebyshev filter

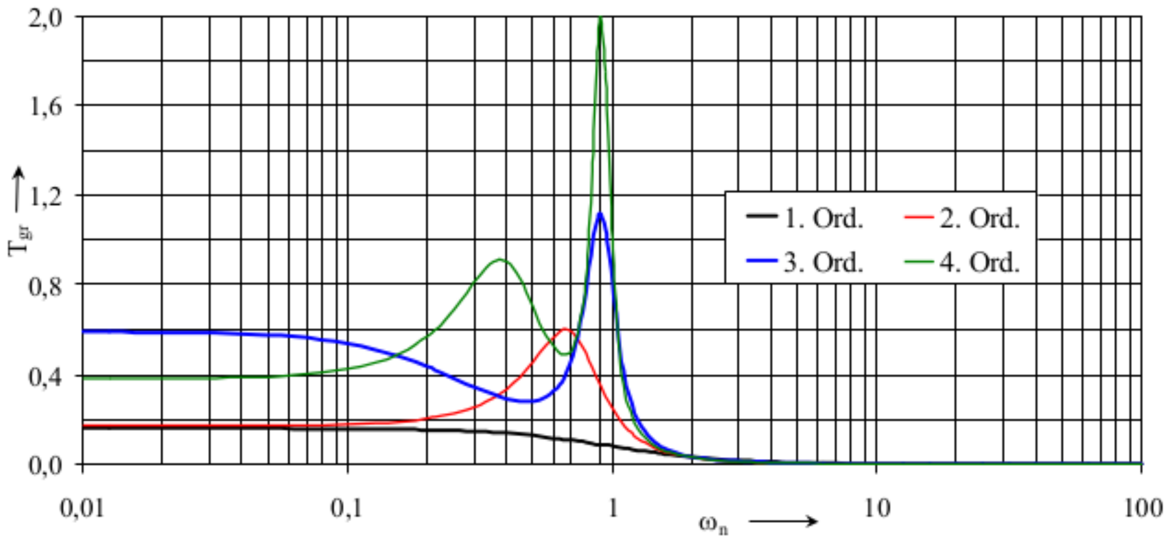

Figure 15: Bode plot showing the group delay of a low pass Chebyshev filter

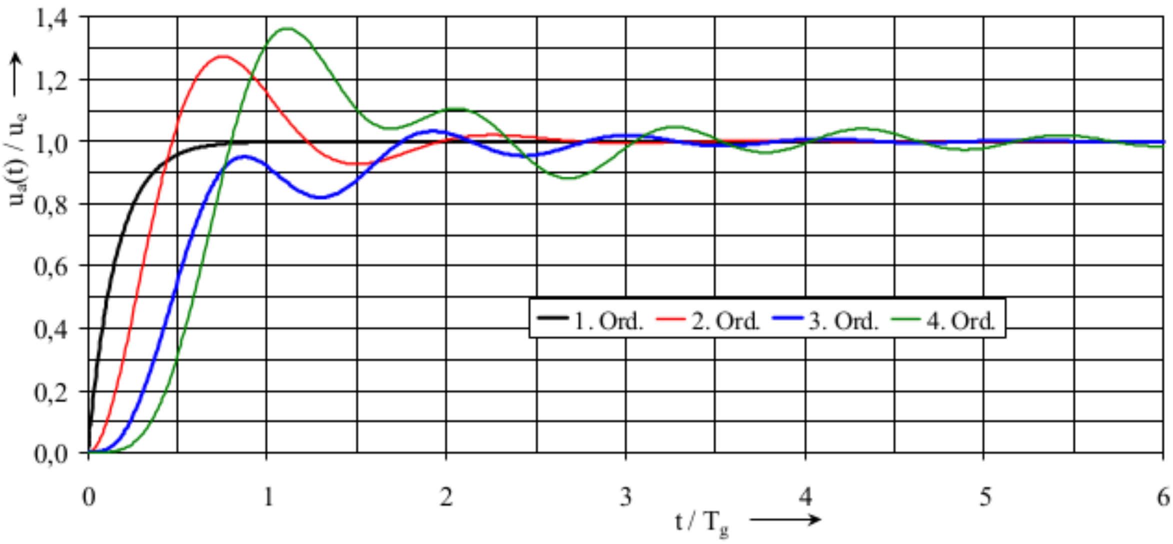

Figure 16: Step response of a Chebyshev filter

2.5 Multi Stage Filters

In practical use two or more filters are often combined and conducted in series in order to strengthen their properties (see figure 17). In that case the total roll-off is calculated by summing up the roll-offs of each single filter. If for example the roll-off of a filter is 6 dB/octave, it is 12 dB/octave for a combination of two of them, 18 dB/octave for combining three of them and so on.

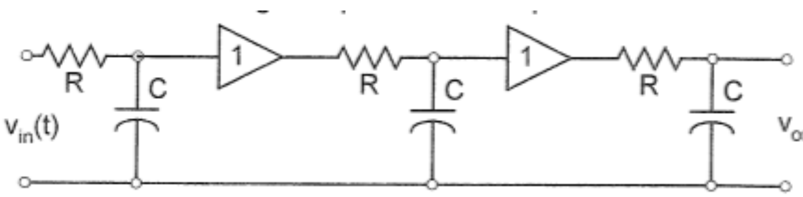

Figure 17: Circuit of a three-stage low pass filter

Figure 18 shows the plot of the voltage gain as a function of the frequency. Apparently the side inclination gets steeper for more combined filters. Unfortunately the ‘perfect’ filter that leads to a step function as an output signal cannot be built in real life.

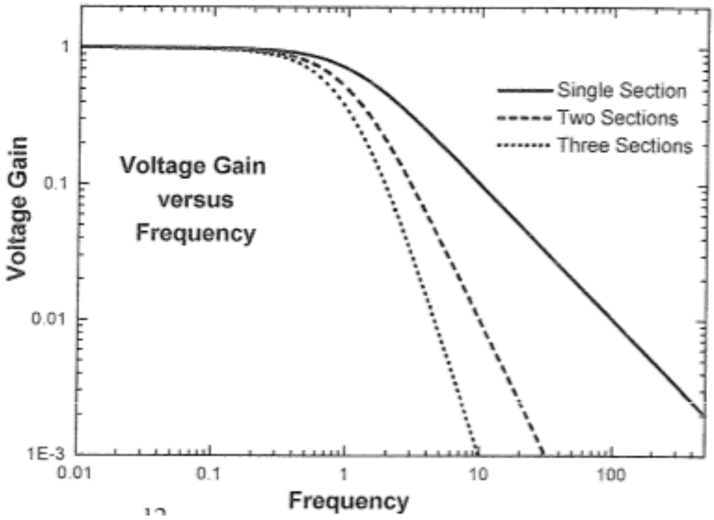

Figure 18: Voltage gain as a function of the frequency for a single-, two- and three-stage low pass

2.6 The Quality Factor Q

The quality factor Q describes the selectivity of the filter. It is definied as

$$Q = \frac{f_c}{\Delta f}$$

Increasing Q the full width at half power points Δf gets narrower while the maximum gain increases. At the same time the graph of the phase shift gets steeper (see figure 19).

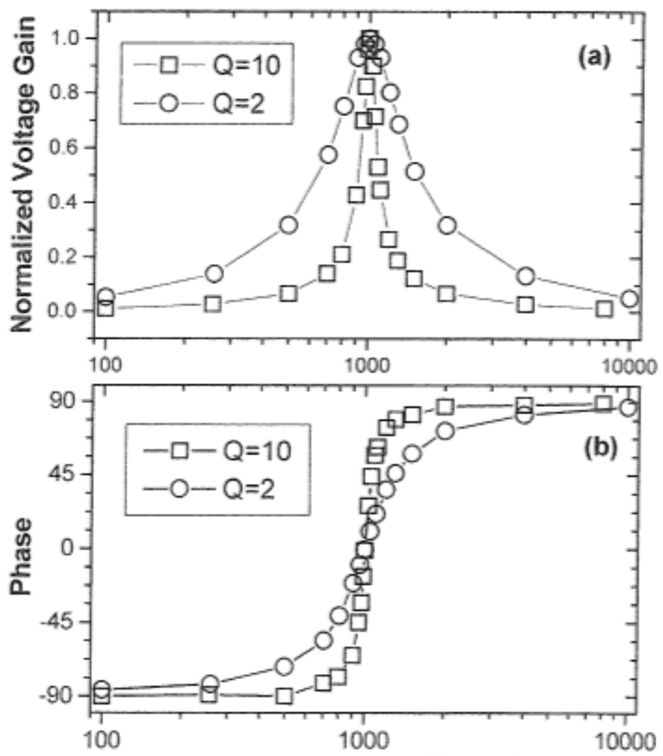

Figure 19: Bode plots showing the normalized voltage gain and the phase shift as functions of the frequency for different quality factors

In the experiment we will examine the effect of various quality factors on the output signal. Three special quality factors represent the Bessel, Butterworth and Chebyshev configuration of the filters.

Q	Configuration
0.577	Bessel
0.707	Butterworth
1	Chebyshev

3. The Lock-In Amplifier

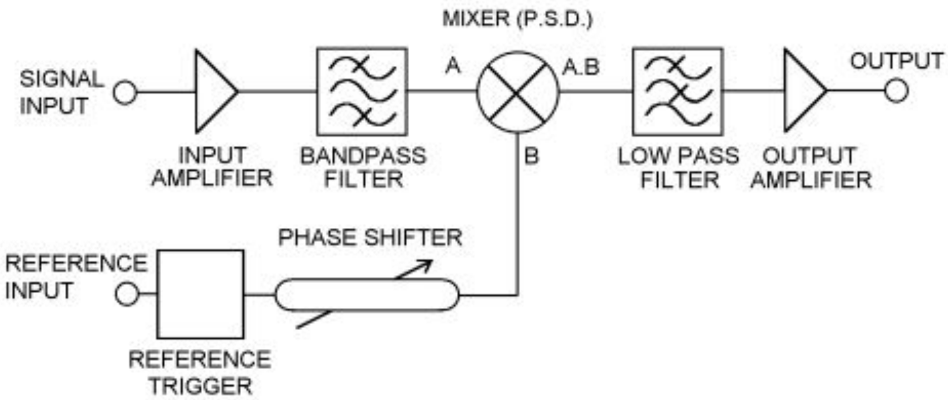

Figure 20: Block diagram of a lock-in amplifier

Figure 20 shows the block diagram of a lock-in amplifier. Our input signal consists of the signal we would like to detect and noise. The signal-to-noise-ratio can be very low. First of all the incoming signal passes a preamp, in the figure called input amplifier, so that both our desired signal und the noise are amplified. After that a band pass filters out the DC part and the high frequency noise of the input signal. That already increases the signal-to-noise-ratio. Additionally a reference signal is created which has the same frequency f_s as the signal we want to measure. The latter is also called coded signal since it is "coded" in noise.
The next part is the most important one, the phase sensitive detector (PSD). The input signal and the reference signal are multiplied in the mixer and then passed through a low pass filter whose time constant $\tau = RC$ is chosen $\tau \gg \frac{1}{2\pi f_s}$, so that we get a DC signal as output. With the phase shifter the phase shift $\Delta\varphi = \varphi_s - \varphi_r$ between the coded signal and the reference signal is adjusted to zero. This leads to a maximum DC level output. Now the great advantage of a phase sensitive detector becomes clear. Out of a vast amount of noise the PSD doesn't only select the signals with the wanted frequency, but also with the right phase. While the frequencies and phase angles of noise are statistically distributed, filtering the input signal by the two specific values of the coded signal increases the signal-to-noise-ratio remarkably.

Now we will have a short look on the multiplication of the two signals in the mixer. Assume that the input signal is given by $U_{in}(t) = U_{in}\cos(\omega_{in}t + \varphi_{in})$ and the reference signal is $U_r(t) = U_r\cos(\omega_r t + \varphi_r)$.
These two signals are multiplied in the PSD:

$$\begin{aligned} U_{PSD} &= U_{in} \cdot U_r \cdot \cos(\omega_{in}t + \varphi_{in}) \cdot \cos(\omega_r t + \varphi_r) \\ &= \frac{U_{in}U_r}{2}\{\cos[(\omega_{in} - \omega_r)t + (\varphi_{in} - \varphi_r)] - \cos[(\omega_{in} + \omega_r)t + (\varphi_{in} + \varphi_r)]\} \end{aligned}$$

As we choose $\omega_{in} = \omega_r$, the multiplier output signal becomes

$$U_{PSD} = \frac{U_{in}U_r}{2}\{\cos[(\varphi_{in} - \varphi_r)] - \cos[(2 \cdot \omega_{in})t + (\varphi_{in} + \varphi_r)]\}$$

and the following low pass subdues the AC signal, because its frequency is twice as high as the input frequency.
So the output signal of the low pass is a DC signal whose amplitude reaches its maximum value if the input phase and the reference phase are equal. However the signal is averaged to zero if the phase shift is 90° or 270°.

If the input signal is coded in noise all noise frequencies except those close to the coded frequency are filtered out. The passing noise modulates an AC output with the reference frequency which is added to the DC output signal. This is now the wanted coded signal liberated from disturbing noise.

4. Experiment – Setup and Analysis

4.1 Preamplifier

First of all we will have a look at the preamplifier, which is the first device to be passed by the coded signal. As a start we are interested in the frequency-dependent behavior of amplification and phase shift. For that we measured Bode plots for both the gain and phase shift of the amplified signal. We set the gain to 1.

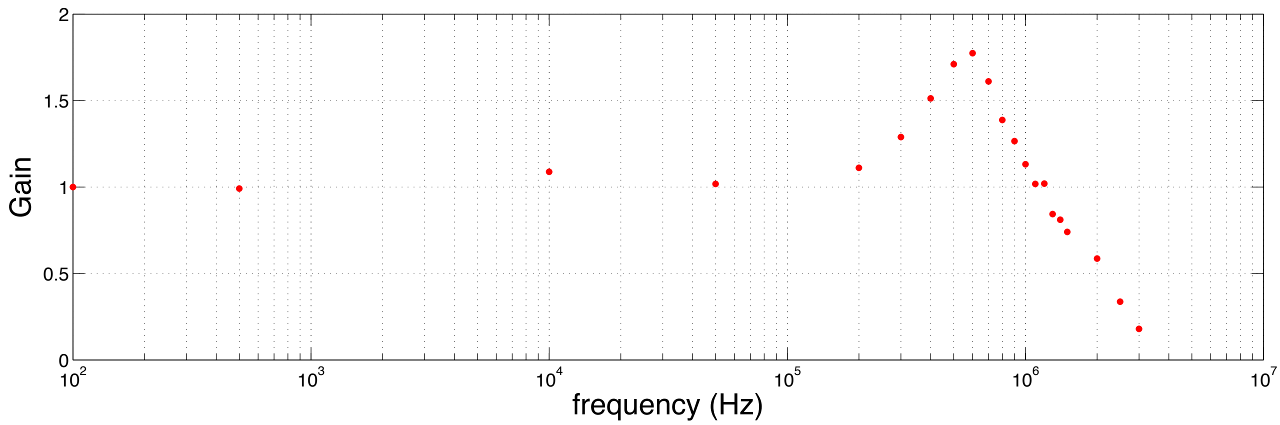

Figure 21: Bode plot of the gain

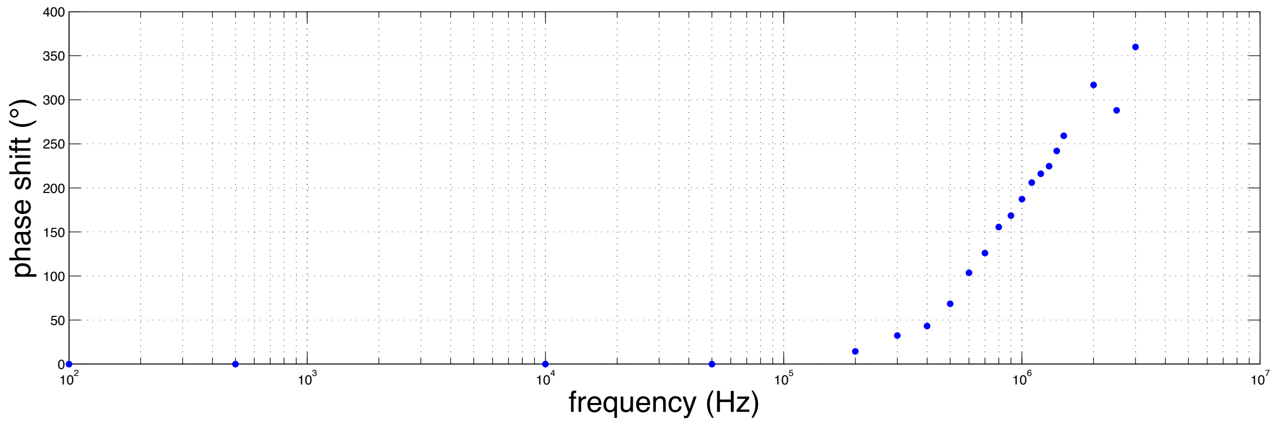

Figure 22: Bode plot of the phase shift

Figure 21 shows that at lower frequencies up to about 100-200 kHz the gain of the preamp remains at the chosen value. From 200 to 600 kHz it increases nearly to 1.8 but then decreases linearly to zero, which is reached at about 6 MHz. Figure 22 shows that there is no phase shift at frequencies lower than 100 kHz. At frequencies higher than 100 kHz the phase shift begins to grow more or less linearly until it approaches 360 ° at 2.5 MHz.

To sum it up, the two properties of the preamp, amplifying a signal by the chosen factor and leaving the phase unchanged, are not kept for too high frequencies.

Next the 3 dB frequency (i.e. the frequency at which the amplitude of the output signal is half of the amplitude of the DC output signal) is measured for different gain settings.

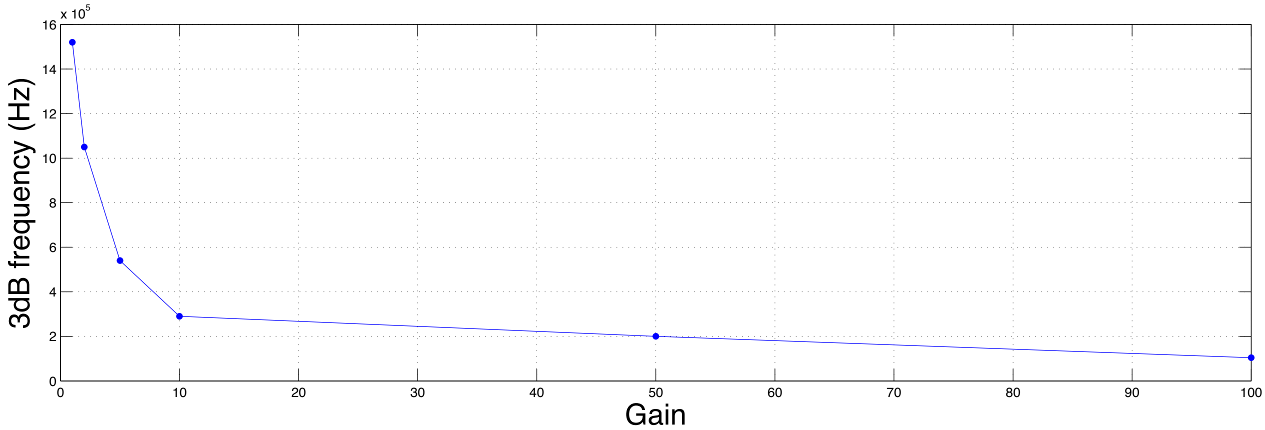

Figure 23: 3 dB frequency as a function of the gain of the preamp

Like it can be seen in figure 23 the 3 dB frequency of the preamp decreases exponentially for growing gain settings. It looks like it would tend to zero for high gains.

If the gain is chosen too high the output signal gets clipped off because the oscilloscope cannot deal with too high voltages. Clipping is shown in figure 24.

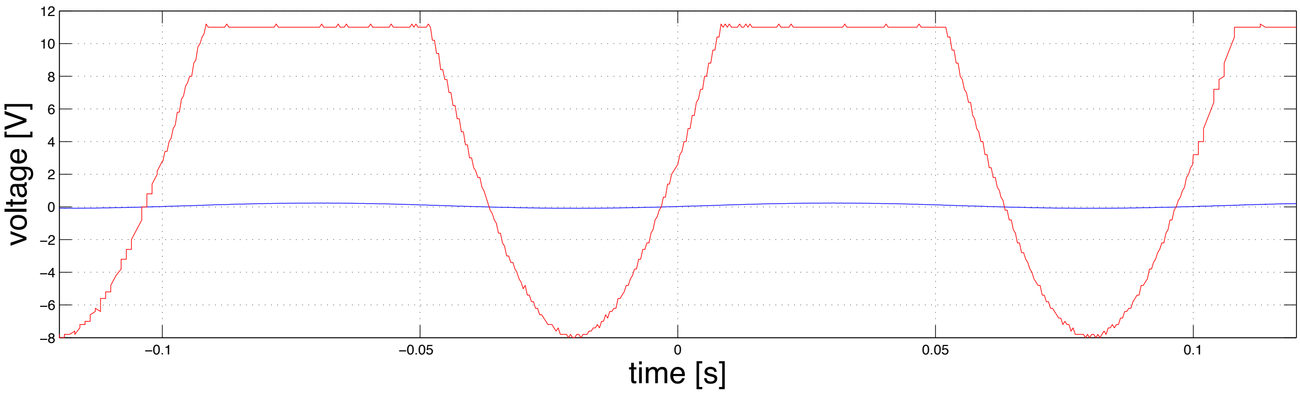

Figure 24: Clipping of the output signal at too high gain

4.2 Filters

In this section the different filters will be examined. First the frequency scale of the band pass had to be calibrated on 300 Hz. That was done by varying the resonance frequency of the band pass until the phase shift of a 300 Hz input signal disappeared. The reason why the band pass instead of the low pass or high pass was used for that can be understood by having at the behavior of its phase shift (figure 7). A phase shift of precise zero can easily be reached from both sides while for the high pass (low pass) it only occurs at a frequency of zero (at a theoretic frequency of infinity).

Next the gain/loss and phase shift of the 300 Hz signal passing the band pass was measured for different quality factors 2, 10, 50 and the Chebyshev, Butterworth and Bessel configurations which equal quality factors of 1, 0.707 and 0.577.

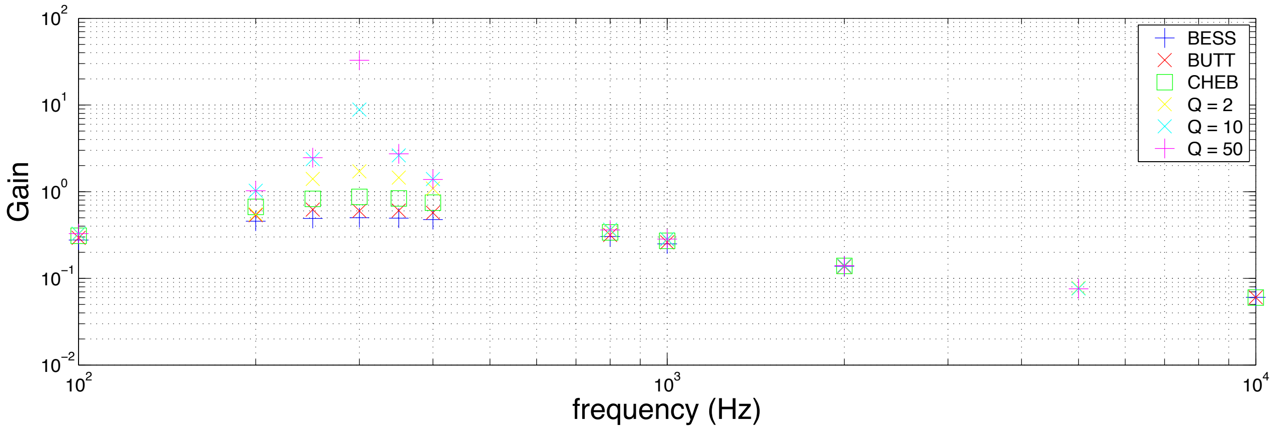

Figure 25: Bode plot of the band pass gain for different q factors

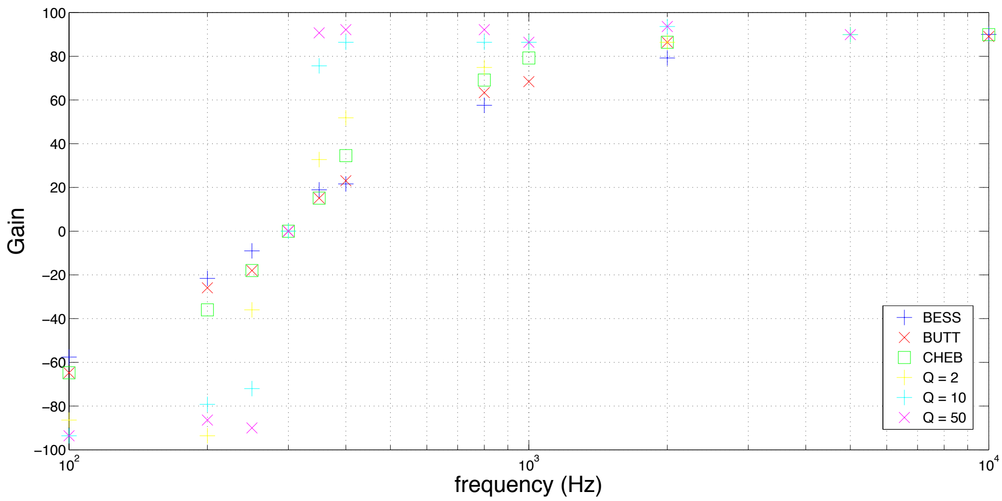

Figure 26: Bode plot of the band pass phase shift for different q factors

Figure 25 shows that the gain peak at the resonance frequency (300 Hz) grows with increasing frequency. For high frequencies the gain converges to zero for all quality factors. It also gets sharper, its full-width-half-maximum decreases. That is what we expected regarding figure 19 and the formula of the quality factor, as it depends reciprocal on the frequency bandwidth. The phase shift also shows the expected behavior. Converging towards 90 ° for high frequencies and towards -90° for very low frequencies the phase shift approaches a step function for high quality factors, having the step at the resonance frequency. For all quality factors the phase shift is zero at the resonance frequency.

With that knowledge we would expect the phase shift of a low pass (figure 3) for higher quality factors to approach a step function which converges to zero for low frequencies an to -90 ° for high frequencies. The gain would decrease more sharply with growing quality factors. The same considerations apply to the corresponding bode plots of the high pass.

After that we want to compare the outputs of different low pass filter types for an input square wave (f = 200 Hz).

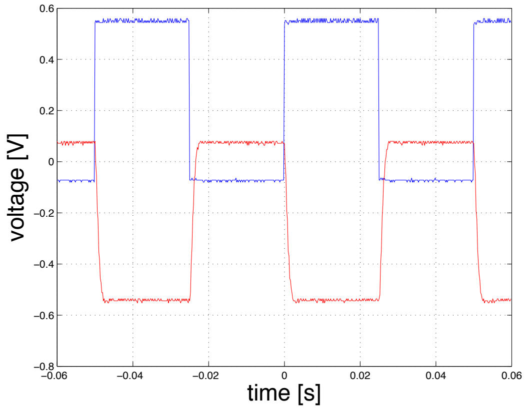

Figure 27: Low pass square wave output, BESSEL

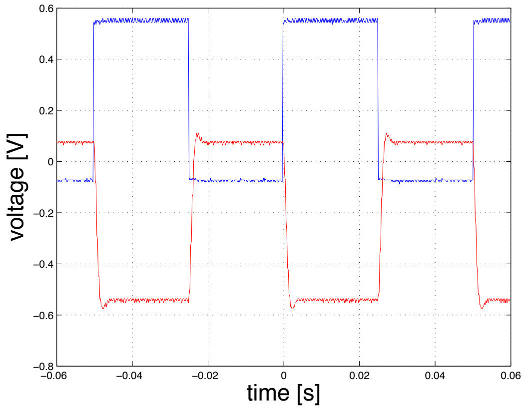

Figure 28: Low pass square wave output, BUTTERWORTH

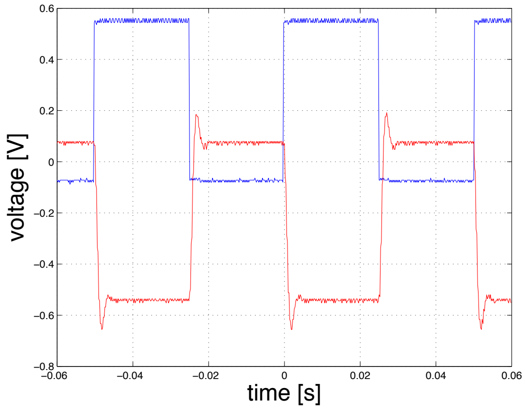

Figure 29: Low pass square wave output, CHEBYSHEV

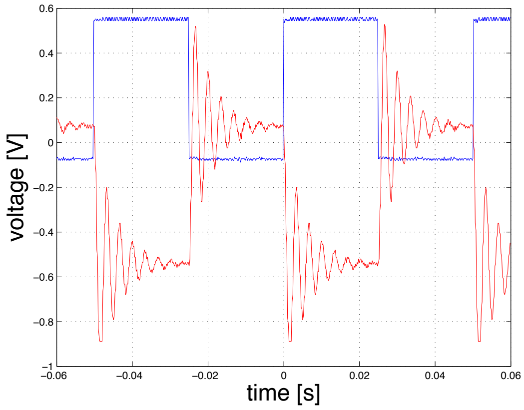

Figure 30: Low pass square wave output, Q = 5

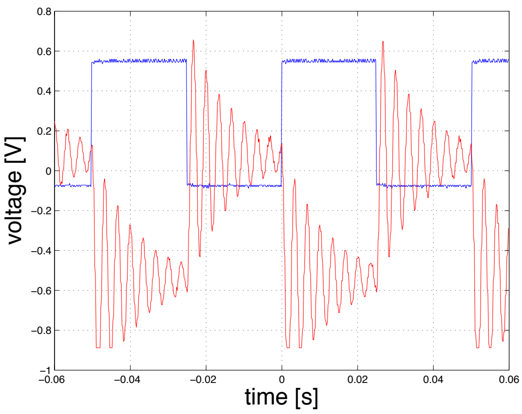

Figure 31: Low pass square wave output, Q = 10

Figure 27 – 31 show that with a growing quality factor the waviness of the step responses increases. Whereas a low pass filter with the Bessel configuration (figure 27) shows a very good step response (which we already discussed in die basics), the

Chebyshev filter (figure 29) already begins to overshoot slightly. The high quality factors 5 (figure 30) and 10 (figure 31) lead to a strongly oscillating step response.

As a last step the gain behavior of the three low pass types Bessel, Butterworth and Chebyshev will be compared.

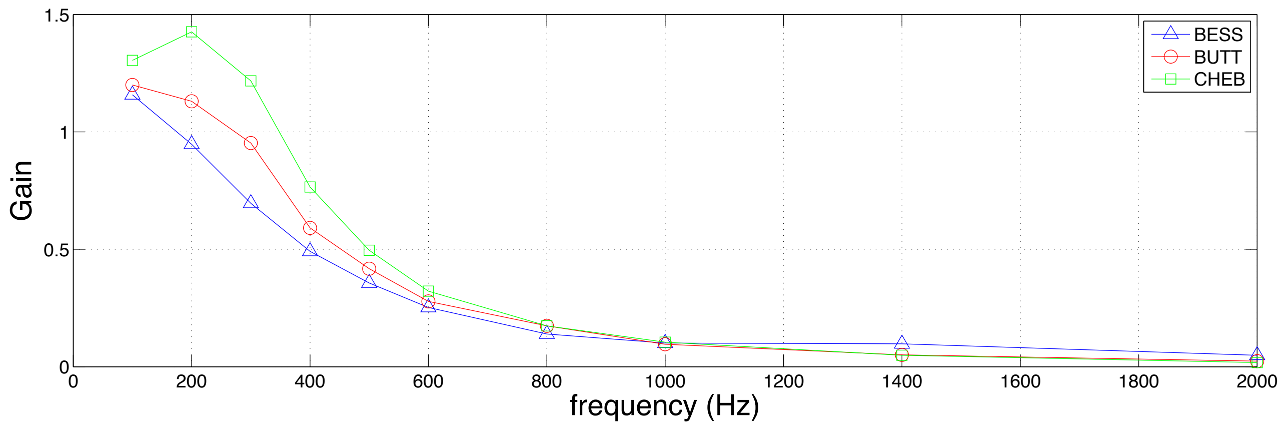

Figure 32: Gain over frequency for Bessel, Butterworth and Chebyshev configuration

As one can see in figure 32, the Bessel low pass has the lowest gain at the pass band. The gain of the Bessel filter decreases quite linearly until it approaches zero, whereas the gain of the Chebychev filter shows a slight overshooting before falling down and converging to zero. The gain of the Butterworth filter stays between the two others. It decreases sharper than it does for the Bessel filter, but not exactly as steep as the gain of the Chebyshev filter. All in all the three types show the same behavior as they were theoretically expected to do (figures 8, 11 and 14).

4.3 Phase Shifter

The next device we look at is the phase shifter. It is simply used to vary the shift the phase of an incoming signal. Later we need it to adjust the phase of the reference signal to the one of the input signal of the lock-in detector. The following figure shows the dependency of the phase shift on the frequency for a fixed phase shift setting on the phase shifter.

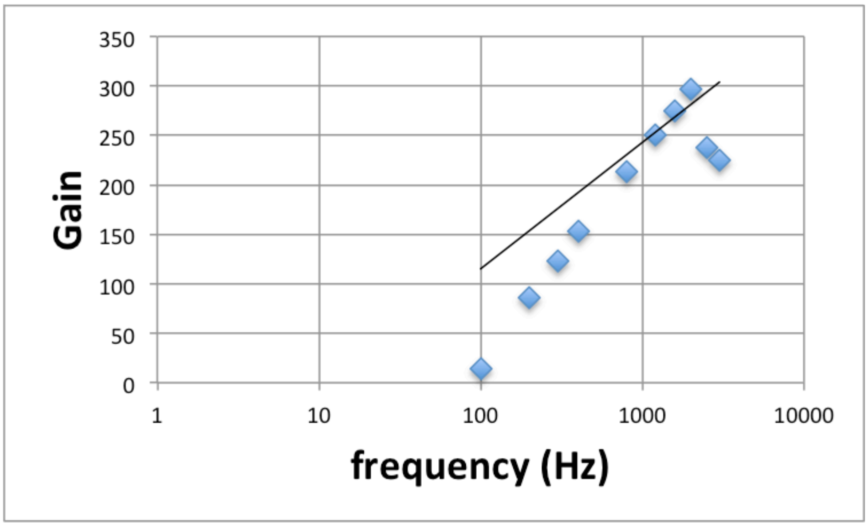

Figure 33: Phase shift as a function of the frequency at a fixed phase shift setting

Figure 33 shows that once the phase shift is set on the phase shifter, the total phase shift is not constant at all. It depends linearly on the frequency of the incoming signal. Two two last values at 2.5 kHz and 3.0 kHz do not seem to fit very well in the linear regression. It might be possible that we have made mistakes noting them, but this is only an assumption. Perhaps the phase shifter just acts in a strange way at high frequencies.

4.4 Lock-In Detector

As a last point before the real lock-in detection we will have a look at the lock-in detector itself, only connected to the phase shifter. The following four figures feature the plots of the output signals (red) of the lock-in detector for four different phase angles. The input signal was a normal sinusoidal signal (blue).

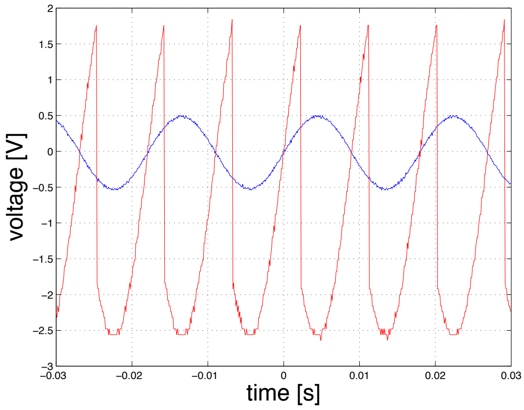

Figure 34: Lock-in detector output, Phase: 30 °

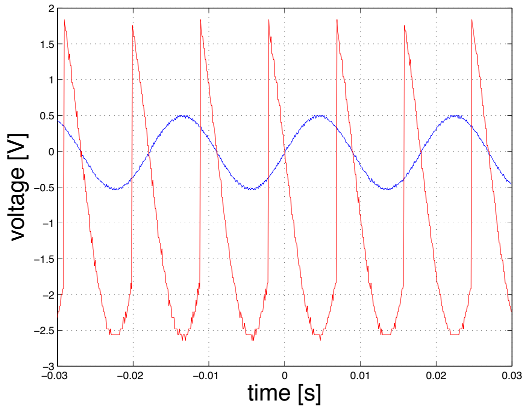

Figure 35: Lock-in detector output, Phase: 120 °

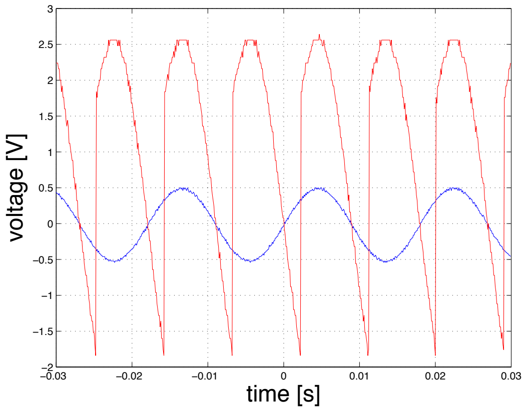

Figure 36: Lock-in detector output, Phase: 210 °

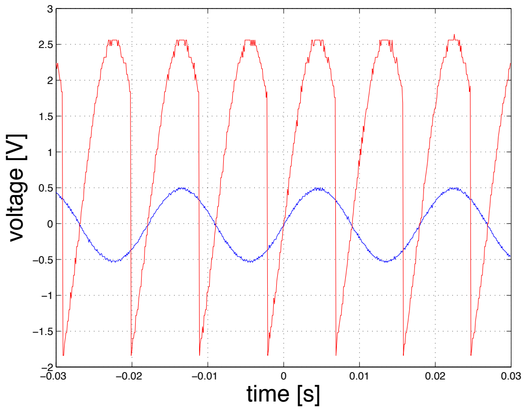

Figure 37: Lock-in detector output, Phase: 300 °

It can be noticed that the output signal is close to a rectified sinusoidal signal whose sides are slightly inclined. Changing the phase by 90 ° the signal gets inclined equally to the other side, another phase shift by 90 ° mirrors the signal horizontally. Another phase shift of 90 ° again flips the signal to the other side.

Matlab is used to calculate the time average of the output signal for each phase shift.

Phase Shift [°]	**Average Output Signal [V]**
30	- 1.11
120	- 1.12
210	1.11
300	1.09

The time average output values confirm our assumptions. The average values of the output signals with a phase angle of 30 ° and 120 ° are exactly the negative values of the output signals with a phase angle of 210 ° and 300 °.

4.5 Lock-In Detector With A Designed Test Signal

This part of the experiment deals with the actual lock-in detection. We used the noise generator to taint the input signal, a 60 Hz sinusoidal signal, with noise. Firstly, we set the signal-to-noise ratio of that designed test signal to 1. We still could recognize the test signal very clearly, although its shape was blurred by noise. After we changed the S/N ratio to 1/10, the test signal disappeared in vast noise.

The designed test signal with S/N = 1/10 was now amplified by the preamp and then sent through the filter module. High pass, low pass and band bass were each tested with different quality factors. The results are featured below.

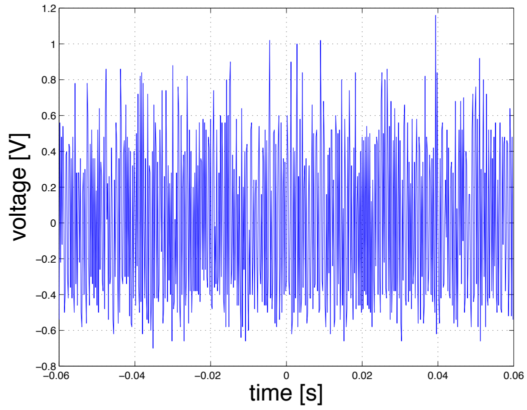

Figure 38: High pass, Bessel

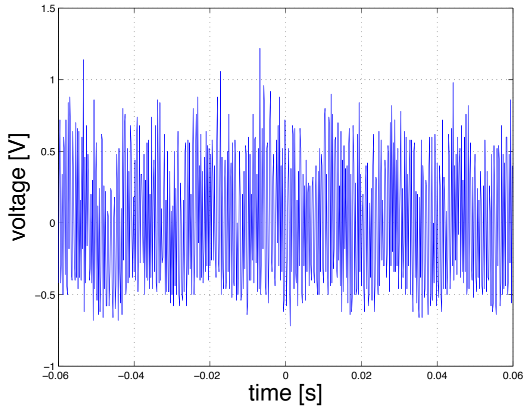

Figure 39: High pass, Chebyshev

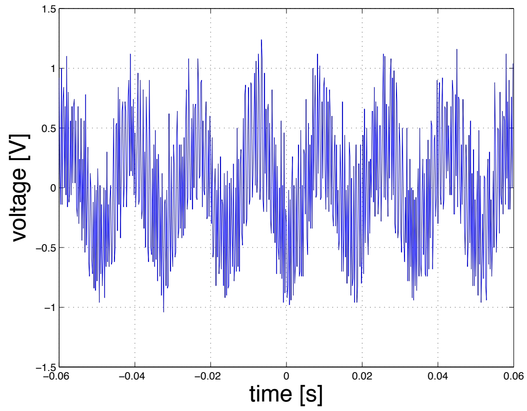

Figure 40: High pass, Q = 5

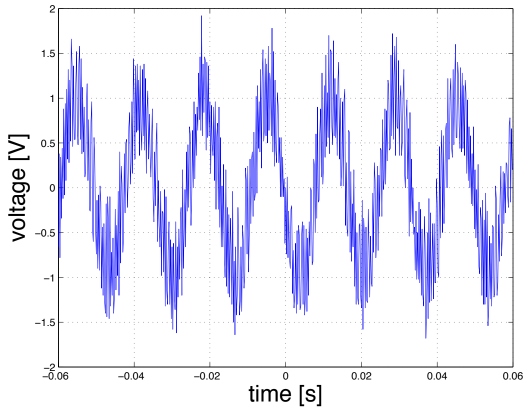

Figure 41: High pass, Q = 50

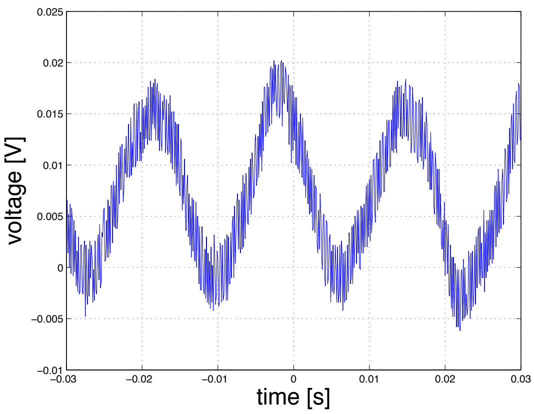

Figure 42: Low pass, Bessel

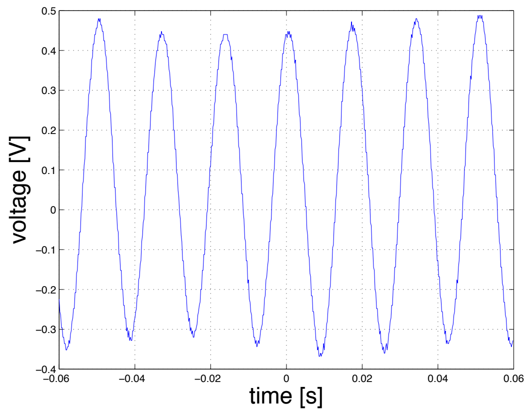

Figure 43: Low pass, Q = 5

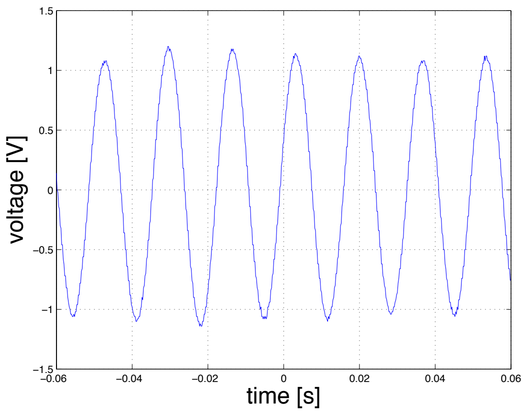

Figure 44: Low pass, Q = 50

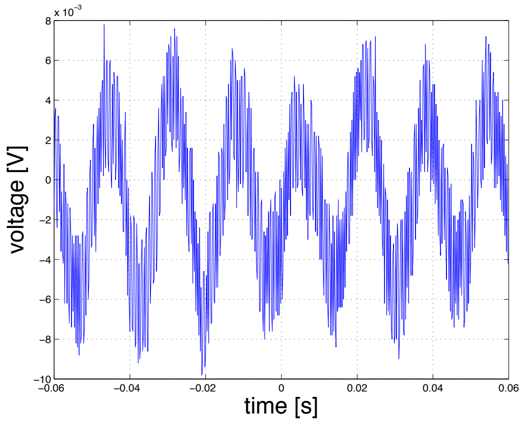

Figure 45: Band pass, Bessel

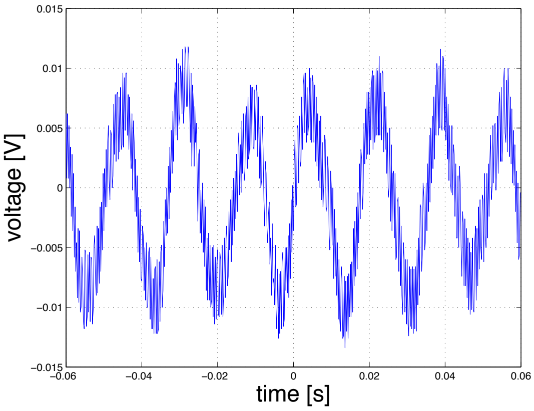

Figure 46: Band pass, Chebyshev

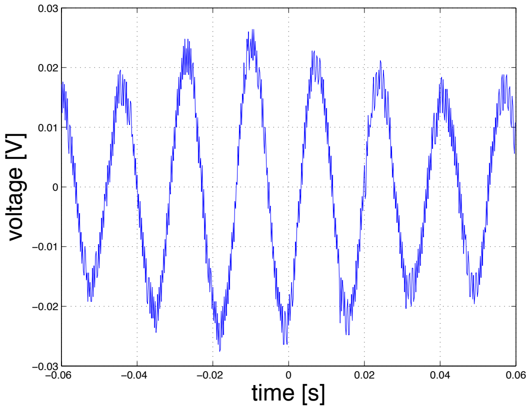

Figure 47: Band pass, Q = 5

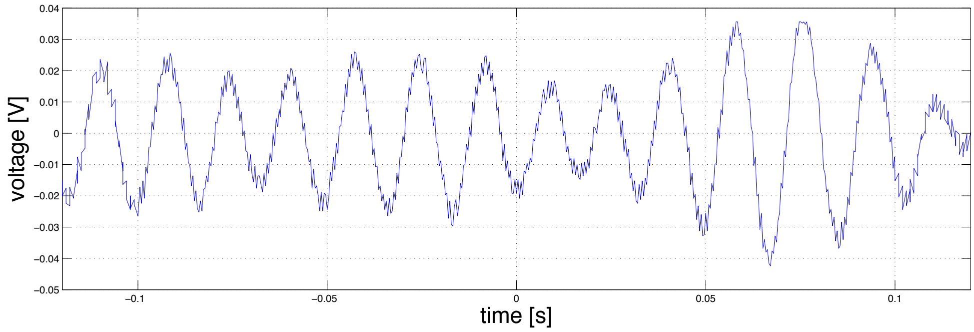

Figure 48: Band pass, Q = 50

We start with having a look at the high pass (figures 38 – 41). Obviously the high pass is a bad choice if we want to detect the designed signal. Using Bessel or Chebyshev configuration, the signal cannot be recognized for the noise is still too overwhelming. One must increase the quality factor to 5 or even to 50 to detect the signal. A high pass with a quality factor of 50 gives an output signal which is only as good as it is using a low pass with the Bessel configuration (figure 42). The low pass is probably the best filter to choose. For a quality factor of 5 one cannot notice any amount of noise anymore (figure 43). The result is already that good that there can be seen no visible improvement anymore increasing the quality factor to 50 (figure 44). We just get the pure test signal out of the filter. The band pass is not as bad as the high pass, but it cannot keep up with the low pass. Using the Bessel (figure 45) or Chebyshev setting (figure 46), the signal can already be detected, but is still noisy. At a quality factor of 5 the result is quite good (figure 47), increasing the quality factor to 50, the signal is only slightly modulated by noise of low frequency (figure 48).

In the following section we use amplitude detection to free the signal from noise. The switch of the detector module is set from lock-in to amplitude, so that we use the detector as a kind of a rectifier.

The first part is to examine the impacts of changing the various parameters. These are the quality factor, the time constant of the low pass, the roll-off and the gain. The table below shows which settings let to which signal-to-noise ratio.

Q	**τ [s]**	**roll-off [dB/oct]**	**gain**	**S/N**
50	0.03	6	5	1.2994
50	0.03	12	5	1.2897
2	0.03	6	5	0.8974
BESS	0.03	6	5	0.9737
BUTT	0.03	6	5	0.9649
50	1	6	5	0.2222
50	10	6	5	0.2453
50	0.03	6	2	0.2195
50	0.03	6	50	1.0179

As there is a large number of parameters that can be varied, it is difficult to do careful analysis out of these few combinations. The measurements let us assume that increasing the quality factor decreases the signal-to-noise ratio. Doubling the roll-off decreases the S/N ratio, but too slightly to definitely say it. Increasing the time constant decreases the S/N ratio, whereas increasing the gain also increases the S/N ratio.

In the second part we examine the change of the DC output when turning off the coded signal, which has a S/N ratio of 1/30. That is done by dividing the total signal by 100 so that the test signal disappears and only noise remains. That happens when the signal is halfway through the sweep, so that we can see the difference. The output signal is red.

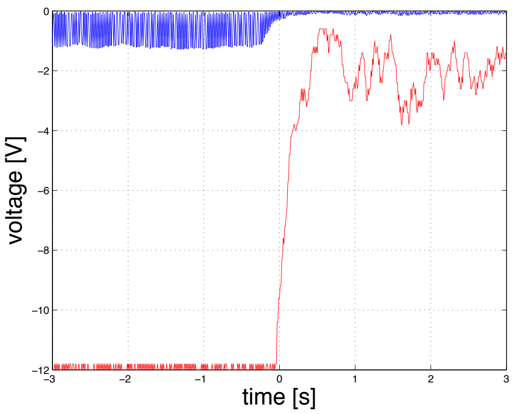

Figure 49: "Half way sweep"; Q = 20, τ = 0.03 s, Gain = 50

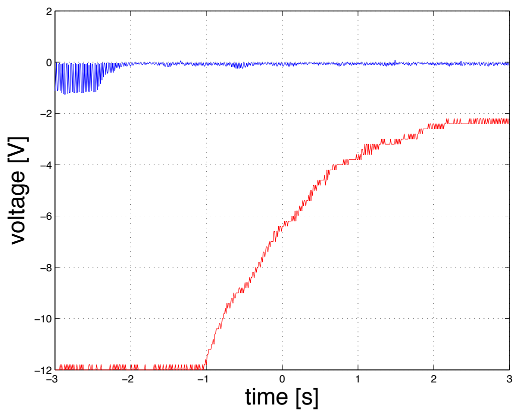

Figure 50: "Half way sweep"; Q = 20, τ = 1 s, Gain = 50

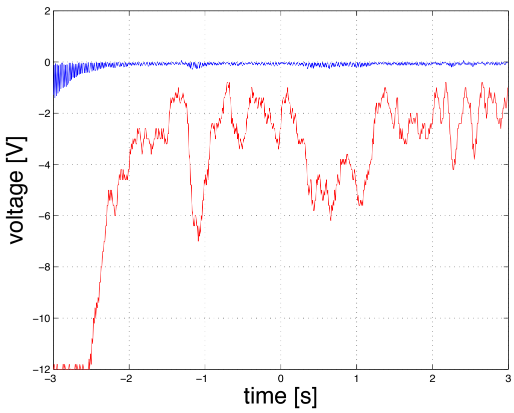

Figure 51: "Half way sweep"; Q = 50, τ = 0.03 s, Gain = 50

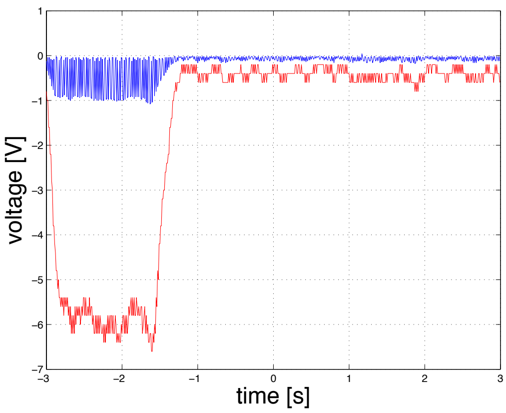

Figure 52: "Half way sweep"; Q = 20, τ = 0.03 s, Gain = 10

Figure 49 shows the result of the basic settings. One can clearly recognize the step function as the signal is turned off but then it is strongly covered with noise. Each of the other figures shows the effect of changing one parameter.

Increasing the time constant slows down the "jump" of the step function and turns it into a kind of logarithmic growth (figure 50). Increasing the quality factor makes the response signal even noisier (figure 51). If the gain is decreased the form of a step function is conserved and it gets flattened (figure 52). The noisy part is attenuated.

For the final part of the experiment the switch of the detector is flipped back to the lock-in setting. The other settings are kept, the signal-to-noise ratio stays 1/30.

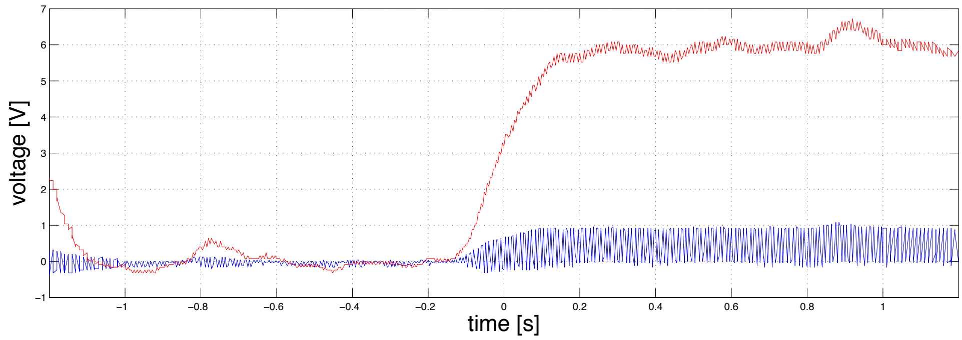

Figure 53: "Half way sweep" - Lock-In Detection; Q = 20, τ = 0.03, Gain = 10, roll-off = 6 dB/oct

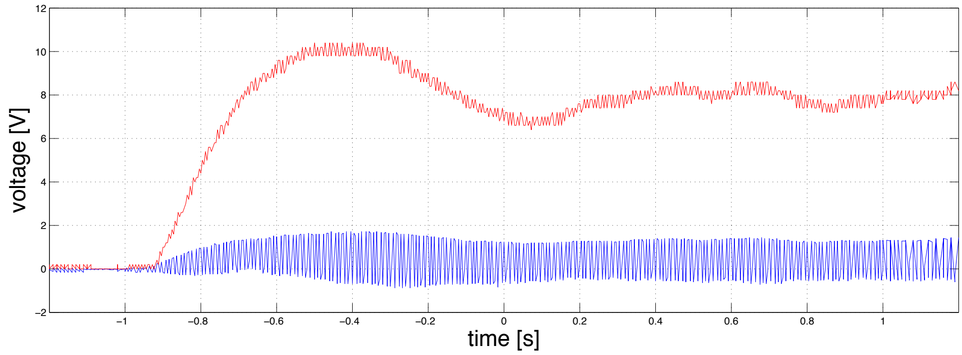

Figure 54: "Half way sweep" - Lock-In Detection; Q = 50, τ = 0.03, Gain = 10, roll-off = 6 dB/oct

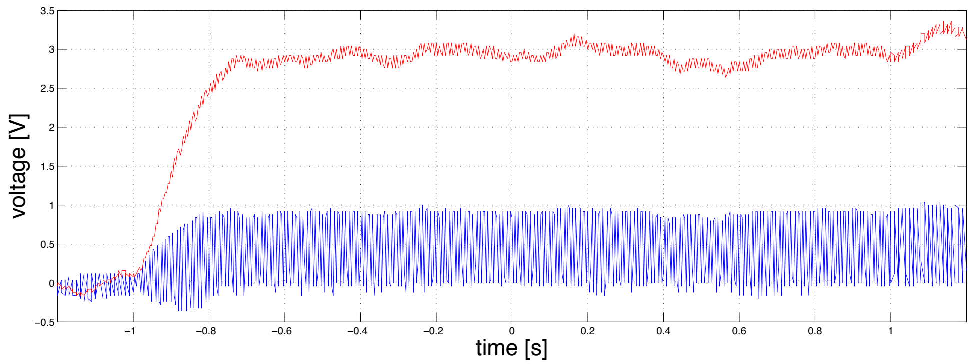

Figure 55: "Half way sweep" - Lock-In Detection; Q = 20, τ = 0.03, Gain = 50, roll-off = 6 dB/oct

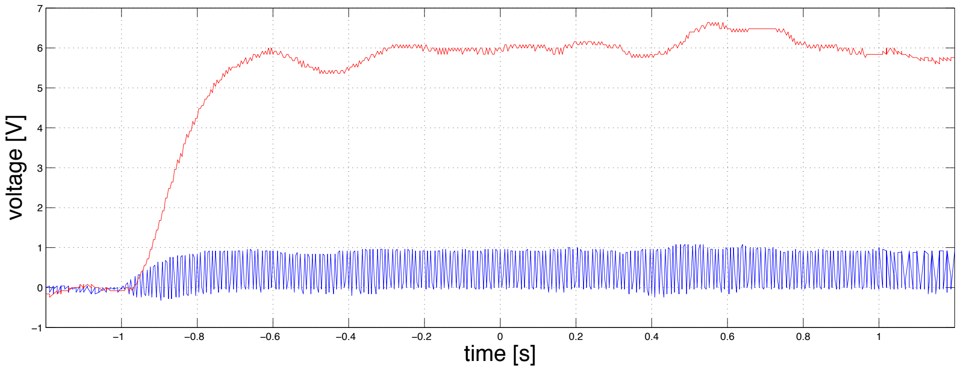

Figure 56: "Half way sweep" - Lock-In Detection; Q = 20, τ = 0.03, Gain = 10, roll-off = 12 dB/oct

Comparing figure 53 and 54 we see that increasing the quality factor of the filter increases the waviness of the step response as well as the total amplitude of the output signal. Figure 55 shows that increasing the gain attenuates the output signal strongly. At the same time the swinging of the signal is slighter. Changing the roll-off from 6 to 12 dB/oct does not have such a strong impact, although it can be said that the signal is not that noisy anymore.
The smallest signal-to-noise ratio that could be detected was 1/300.

The very last thing to do is to examine the influence of the time constant τ on the output signal.

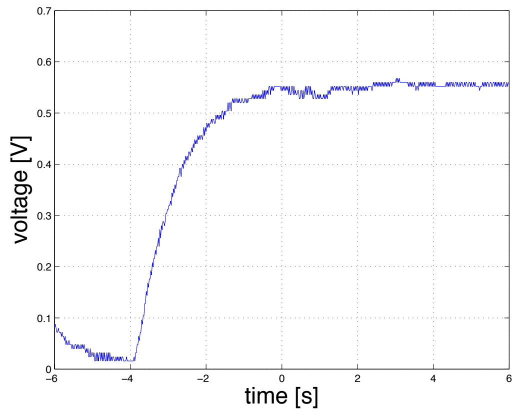

Figure 57: Transient response; τ = 1, roll-off = 6 dB/oct

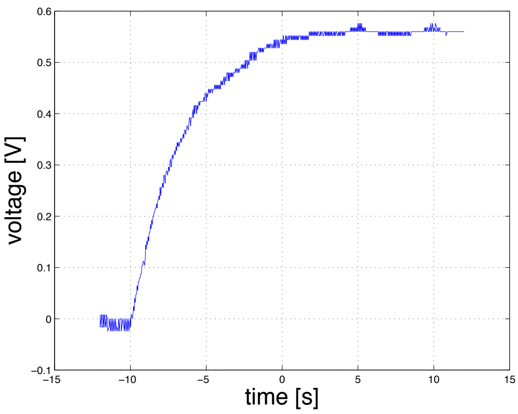

Figure 58: Transient response; τ = 3, roll-off = 6 dB/oct

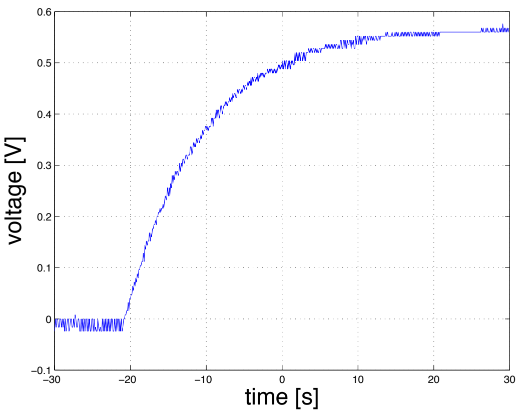

Figure 59: Transient response; τ = 10, roll-off = 6 dB/oct

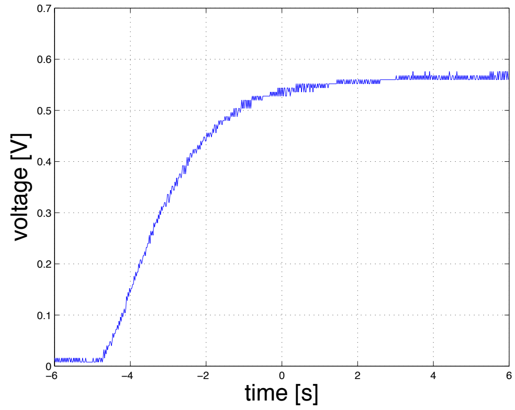

Figure 60: Transient response; τ = 1, roll-off = 12 dB/oct

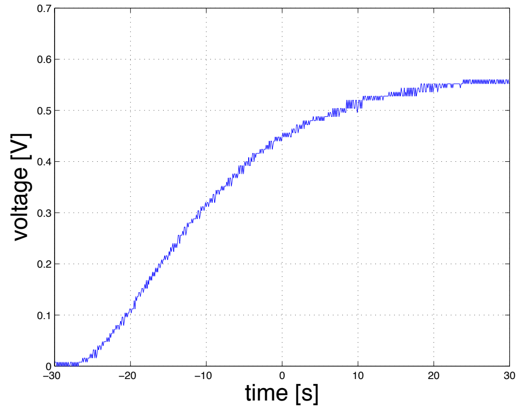

Figure 61: Transient response; τ = 3, roll-off = 12 dB/oct

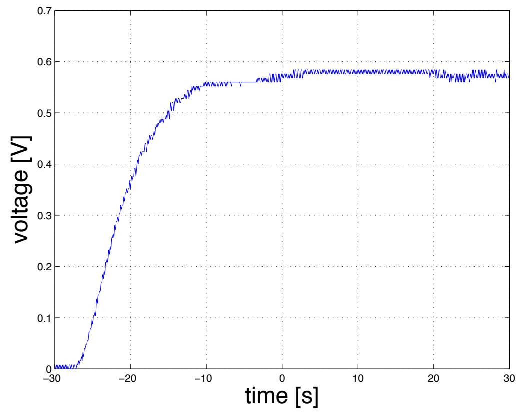

Figure 62: Transient response; τ = 10, roll-off = 12 dB/oct

The time constant is defined to be the time to increase/decrease by the factor 1/e. Thus the response times gets higher for growing time constants. If we have a look at plots with a roll-off of 6 dB/oct (figures 57 – 59) and at those with a roll-off of 12 dB/oct, we hardly notice any other difference than the slightly steeper side inclination of the responses at 6 dB/oct.

List of Figures

The figures 21) to 62) have been plotted by ourselves based on our own measurements.

References

- Teachspin Inc. (2009): “SPLIA1-A Signal Processor / Lock-In Amplifier”
- http://www.et-inf.fho-emden.de/~elmalab/indelek/download/Ind_5.pdf (last accessed on 1/13/2014)
- http://www.itwissen.info/definition/lexikon/Tschebyscheff-Filter-Tschebyscheff-filter.html (last accessed on 1/13/2014)
- http://www.itwissen.info/definition/lexikon/Butterworth-Filter-Butterworth-filter.html (last accessed on 1/13/2014)
- http://www.itwissen.info/definition/lexikon/Bessel-Filter-Bessel-filter.html (last accessed on 1/13/2014)
- http://groups.uni-paderborn.de/physik/studieninfos/praktika/versuche_anleitungen/pm08.pdf (last accessed on 1/13/2014)
- http://waste.org/~msg/ebay/dae/electronics/5207_amp/blk-diag.jpg (last accessed on 1/13/2014)
- http://www.ep1.rub.de/lehre/abschlussarbeiten/thesis/BscJonasHerick.pdf (last accessed on 1/13/2014)