because the gain constant of the integrator was −1000, and the gain constant of the differentiator was −0.001, which results in (−1000) × (−0.001) = 1.

## 5–7 PHASE SHIFT CIRCUITS

A common operation required in electronic circuit design is the process of producing a required phase shift at a given frequency. All reactive networks are capable of producing phase shifts, but the majority of such circuits have amplitude response functions that vary with frequency, which limits their usefulness to carefully controlled situations.

In this section two circuits will be considered that permit the phase shift to be adjusted over a wide range, but in which the amplitude response remains constant. These circuits are examples of a special class of circuits called ***all-pass networks.*** The circuits to be considered will be denoted as the ***all-pass phase lag*** circuit and the ***all-pass phase lead*** circuit. The names suggest the forms of the phase shift functions that can be realized, as will be seen shortly. The two circuits will be considered separately.

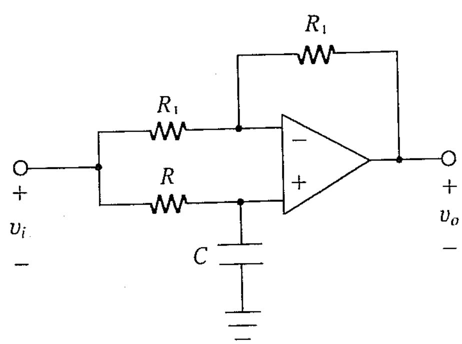
### All-Pass Phase Lag Circuit

The form of the all-pass phase lag circuit is shown in Figure 5–23. The circuit can be analyzed most readily by noting that since the input signal $v_i$ is connected to both signal paths, the circuit is equivalent to one having two identical inputs. The circuit converted to this form, along with the frequency domain representation, is shown in Figure 5–24(a). In this form, the circuit has the same structure as the closed-loop differential amplifier. However, the lower path is a frequency-dependent process as a result of the capacitance. Superposition will be used for the analysis. Let

$$\overline{V}_o = \overline{V}'_o + \overline{V}''_o \tag{5–97}$$

where $\overline{V}'_o$ is produced by the lower equivalent source, and $\overline{V}''_o$ is produced by the upper equivalent source.

**FIGURE 5–23**
All-pass phase lag circuit.



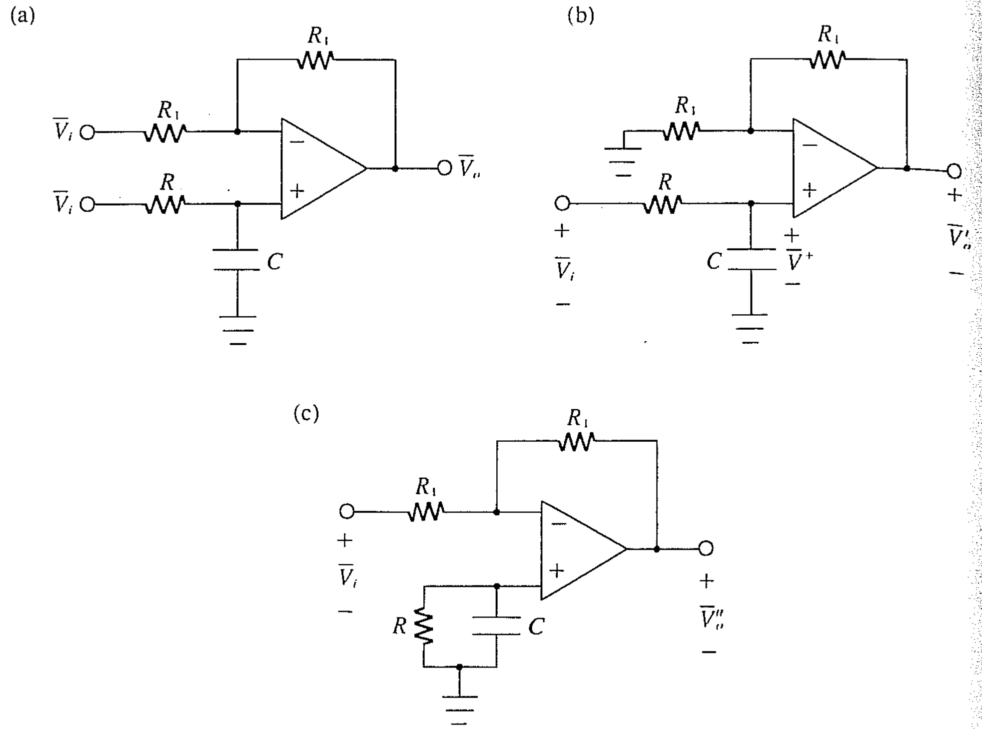



**FIGURE 5–24**
Analysis of all-pass phase lag circuit using superposition.

Consider first the effect due to the lower source, and thus the upper source is deenergized; that is, it is replaced by a short circuit as shown in Figure 5–24(b). The voltage $\overline{V}^+$ at the noninverting input is then determined by the voltage divider rule as

$$\overline{V}^+ = \frac{1/j\omega C}{R + 1/j\omega C} \times \overline{V}_i = \frac{\overline{V}_i}{1 + j\omega RC} \tag{5–98}$$

From the noninverting terminal on to the output, the circuit is just a noninverting amplifier, so the output component $\overline{V}_o'$ is

$$\overline{V}_o' = \left(1 + \frac{R_1}{R_1}\right) \times \overline{V}^+ = 2\overline{V}^+ = \frac{2\overline{V}_i}{1 + j\omega RC} \tag{5–99}$$

The effect due to the upper source is determined by first deenergizing the lower source as shown in Figure 5–24(c). This circuit is simply an inverting amplifier, so the output component $\overline{V}_o''$ is

$$\overline{V}_o'' = \frac{-R_1}{R_1}\overline{V}_i = -\overline{V}_i \tag{5–100}$$

Substituting (5–99) and (5–100) in (5–97), we find that the net output is

$$\overline{V}_o = \frac{2\overline{V}_i}{1 + j\omega RC} - \overline{V}_i = \frac{1 - j\omega RC}{1 + j\omega RC}\overline{V}_i \tag{5–101}$$

The transfer function $H(j\omega)$ is

$$H(j\omega) = \frac{\overline{V}_o}{\overline{V}_i} = \frac{1 - j\omega RC}{1 + j\omega RC} \tag{5–102}$$

The amplitude response $M(\omega)$ corresponding to (5–102) is

$$M(\omega) = \frac{\sqrt{1 + (\omega RC)^2}}{\sqrt{1 + (\omega RC)^2}} = 1 \tag{5–103}$$

The phase response $\theta(\omega)$ is

$$\theta(\omega) = -\tan^{-1}\omega RC - \tan^{-1}\omega RC = -2\tan^{-1}\omega RC \tag{5–104}$$

The forms of the amplitude and phase functions are shown in Figure 5–25.

The amplitude response is observed to have a constant value of unity at all frequencies. The description as an *all-pass* circuit is thus quite appropriate. Actually, the finite open-loop bandwidth of the op-amp causes the amplitude response eventually to drop in accordance with the analysis of Chapter 4, so an op-amp having sufficient bandwidth must be selected in a given application.

From (5–102) and Figure 5–25(b), the phase shift of the circuit is lagging and varies from 0° to −180° over an infinite frequency range. Note that the phase shift is a function of the product $\omega RC$. For a given frequency, the $RC$ product can be determined to provide a given phase shift. Conversely, for a given $RC$ product, a frequency can be determined at which the phase shift will be a specified value. By

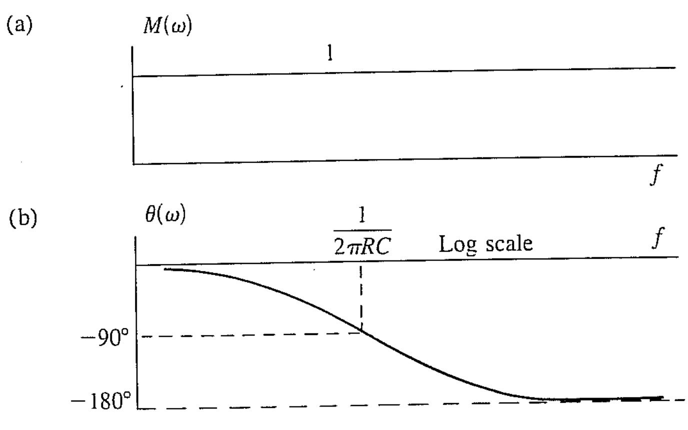



**FIGURE 5–25**
Amplitude and phase of all-pass phase lag circuit.

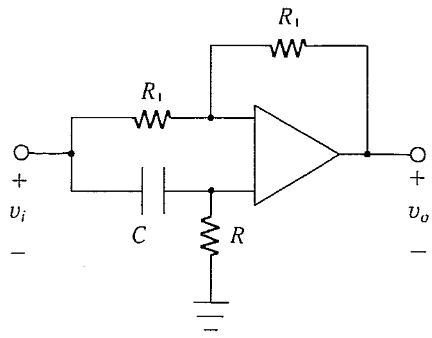



**FIGURE 5–26**
All-pass phase lead circuit.

varying one of the component values, one can adjust the phase shift at a given frequency.

## All-Pass Phase Lead Circuit

The form of the all-pass phase lead circuit is shown in Figure 5–26. An analysis of this circuit follows the same procedure as for the phase lag circuit, and the details will be left as an exercise (Problem 5–28). However, the results will be summarized here.

The transfer function for the phase lead circuit is

$$H(j\omega) = \frac{-1 + j\omega RC}{1 + j\omega RC} \tag{5–105}$$

The amplitude response is the same as for the phase lag circuit:

$$M(\omega) = 1 \tag{5–106}$$

The phase response for this circuit is

$$\theta(\omega) = 180° - 2\tan^{-1}\omega RC \tag{5–107}$$

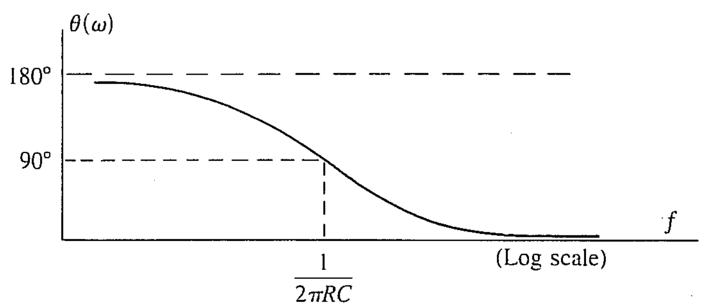



**FIGURE 5–27**
Phase of all-pass lead circuit. (Amplitude is same as in phase lag circuit.)

The form of the phase shift is shown in Figure 5–27. From (5–107) and this figure, we observe that the phase shift is leading and varies from 180° to 0° over an infinite frequency range. Observe also that the general shape and slope of the phase shift for the phase lead circuit are the same as for the phase lag circuit. However, the beginning and end points are quite different.

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**EXAMPLE 5–11** For the circuit of Figure 5–28, determine (a) amplitude response $M(\omega)$, (b) phase response $\theta(\omega)$, and (c) phase shift at 1 kHz.

*Solution*

By comparing the specific circuit of Figure 5–28 to the general form of Figure 5–23, it is deduced that the circuit is an all-pass phase lag circuit with $R_1 = 24\ \text{k}\Omega$, $R = 12\ \text{k}\Omega$, and $C = 0.02\ \mu\text{F}$.

(a) The ideal amplitude response is

$$M(\omega) = 1 \tag{5–108}$$

Finite bandwidth will, of course, eventually cause a roll-off effect.

(b) The phase response is

$$\theta(\omega) = -2\tan^{-1}\omega RC = -2\tan^{-1}2\pi f \times 12 \times 10^3 \times 0.02 \times 10^{-6} = -2\tan^{-1}0.00151f \tag{5–109}$$

(c) The phase shift at the specific frequency of 1 kHz is

$$\theta = -2\tan^{-1}0.00151 \times 10^3 = -2\tan^{-1}1.51 = -2 \times 56.45^\circ = -112.9^\circ \tag{5–110}$$

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**EXAMPLE 5–12** Design an all-pass phase lag circuit to produce a phase shift of $-135^\circ$ at a frequency of 1 kHz.

*Solution*

Refer to the basic form of the phase lag circuit in Figure 5–23 and the equation for the corresponding phase lag as given by (5–104). The choice of $R_1$ is arbitrary, so the common value $R_1 = 10\ \text{k}\Omega$ will be selected. We next equate the desired phase

**FIGURE 5–28**
Circuit of Example 5–11.

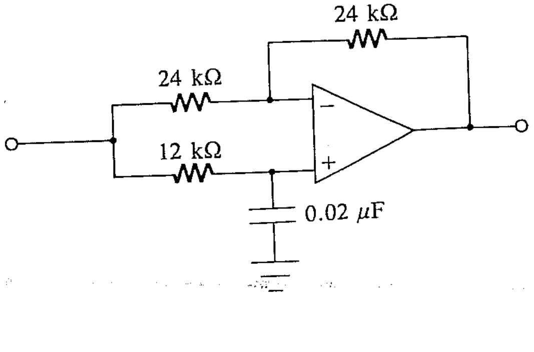
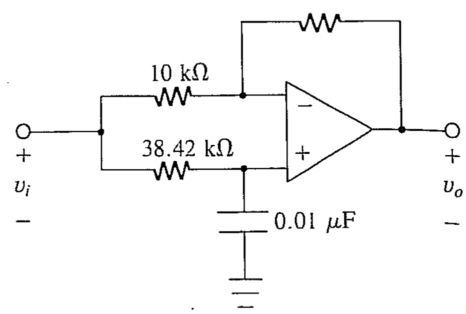


**FIGURE 5–29**
Circuit of Example 5–12.




shift equal to the function of (5–104) with $\omega = 2\pi \times 10^3$ rad/s. The phase shift is considered as $-135°$ in the substitution since it is a phase lag. The details follow:

$$\begin{aligned} -2 \tan^{-1} 2\pi \times 10^3 RC &= -135° \\ \tan^{-1} 2\pi \times 10^3 RC &= 67.5° \\ 2\pi \times 10^3 RC &= \tan 67.5° = 2.414 \end{aligned} \tag{5–111}$$

The required value of the $RC$ product is thus

$$RC = 384.2 \times 10^{-6} \tag{5–112}$$

Theoretically, there is an infinite number of values of $R$ and $C$ that will satisfy the required product. Since it is usually easier to adjust $R$ than $C$ in the given frequency range, the best approach is to select a standard value of $C$ and determine the value of $R$ required. As a reasonable choice, $C = 0.01$ μF will be selected. The corresponding value of $R$ is determined from (5–112) to be $R = 38.420$ kΩ. An adjustable resistance could be used to establish the required value. Actually, since the capacitance value will have some error, the best procedure is to "tweak" the circuit to the correct phase shift by a measurement process with an adjustable resistance. In this manner, the resistance can correct for an error in the capacitance value. The circuit design is shown in Figure 5–29.

## 5–8 SINGLE POWER SUPPLY OPERATION

Operational amplifiers are directly coupled devices (that is, they have no series capacitors) and are capable of amplifying signal frequency components all the way down to dc. In order that the output voltage be zero when no input voltage is applied, most op-amps utilize two power supplies as discussed in Chapter 2. (A few single power supply, special-purpose op-amps having zero output voltage with no signal applied have appeared on the market, but the vast majority conform to the standard pattern noted earlier.)

Recall from Chapter 2 that the standard bias power supply connections involved a positive voltage connected to one op-amp terminal and a negative