Extraction of Rabi Frequency from Spectra

Fermi's Golden Rule

$$T_{fi}(\omega)=\int \rho(E_f)\,\frac{2\pi}{\hbar}\,\delta(E_f-E_i-\hbar\omega)\,|V_{fi}|^2\,dE_f$$

Natural Linewidth

$$\rho(\omega)=\frac{\hbar\gamma/2\pi}{(\hbar\omega_0-\hbar\omega)^2+(\hbar\gamma/2)}$$

$$\rho(E)=\frac{\hbar\gamma/2\pi}{(E_0-E)^2+(\hbar\gamma/2)^2}$$

$$H=-eE_0 r$$

$$\Omega=\frac{|V_{fi}|}{\hbar}=-eE_0\frac{\langle f|r|i\rangle}{\hbar}$$

Rewrite

$$T_{fi}(E)=\int\frac{\hbar\gamma/2\pi}{(E_0-E)^2+(\hbar\gamma/2)^2}\,\frac{2\pi}{\hbar}\,\delta[E_f-E_i-E]\,\hbar^2\Omega^2\,dE_f$$

$$\boxed{T_{fi}(E)=\frac{\hbar^2\Omega^2\gamma}{(E_f-E_i-E)^2+\left(\frac{\hbar\gamma}{2}\right)^2}}$$

Now, Let's include Broadening

$$\tilde{T}_{fi}(E)=\int F(\tilde{E})\,T_{fi}(\tilde{E})\,d\tilde{E}$$

$$=\int F(\tilde{E})\,d\tilde{E}\,\frac{\hbar\Omega^2\gamma}{(E_f-E_i-E)^2+(\hbar\gamma/2)^2}$$

$$=\int F(\tilde{E})\,d\tilde{E}\;2\hbar\Omega^2\,\frac{(\hbar\gamma/2)}{(E_f-E_i-\tilde{E})^2-(\hbar\gamma/2)^2}$$

$$\pi\delta(x)=_{\varepsilon\to 0}\frac{\varepsilon}{x^2-\varepsilon^2}\qquad \varepsilon=\hbar\gamma/2\quad X=E_f-E_i-\tilde{E}$$

$$\tilde{T}_{fi}=F(E_f-E_i)\,2\hbar\Omega^2\pi$$

Assume Specific Form of $F(E)$

$$F(\tilde{E}) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(E-E_f+E_i)^2}{2\sigma^2}}$$

$$\tilde{T}_{fi}(E) = 2\pi \Omega^2 \hbar \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(E-E_f-E_i)^2}{2\sigma^2}}$$

$$E = \hbar\omega \qquad \sigma = \hbar\sigma_\omega$$

$$\tilde{T}(\omega) = \frac{2\pi\Omega^2\hbar}{\sqrt{2\pi}\,\sigma_\omega\hbar} e^{-(\omega-\omega_0)^2/2\sigma_\omega^2} \qquad \text{where } \omega_f - \omega_i = \omega_0$$

$$\boxed{\tilde{T}(\omega) = \frac{\sqrt{2\pi}\,\Omega^2}{\sigma_\omega} e^{-(\omega-\omega_0)^2/2\sigma_\omega^2}}$$

Trap Loss as function of time

$$N(t,\omega) = N_0 e^{-\tilde{T}(\omega)t}$$

Small time - Taylor Expand

$$N(t,\omega) = N_0 - \tilde{T}(\omega) t N_0$$

$$N(t,\omega) = N_0 - \frac{\sqrt{2\pi}\,\Omega^2 t N_0}{\sigma_0} e^{-(\omega-\omega_0)^2/2\sigma_\omega^2}$$

Experimentally, we use $f = \frac{\omega}{2\pi} \Rightarrow \Omega_f = \frac{\Omega}{2\pi} \Rightarrow \sigma_f = \frac{\sigma_\omega}{2\pi}$

$$N(t,f) = N_0 - \frac{\sqrt{2\pi}\,\Omega_f^2 t N_0}{(2\pi)^2\, \sigma_f/\cancel{2\pi}} e^{-(f-f_0)^2/2\sigma_f^2}$$

$$N(t,f) = N_0 - \frac{N_0 \Omega_f^2 t}{\sqrt{2\pi}\,\sigma_f} e^{-(f-f_0)^2/2\sigma_f^2}$$

Matlab fit function

$$N = N_0 - A\, e^{-(f-B)^2/2C^2}$$

$B = f_0$ (center freq)

$C = \sigma_f$ (width)

$$A = \frac{\Omega_f^2\, t N_0}{\sqrt{2\pi}\,\sigma_f} \Rightarrow \Omega_f = \left[\frac{\sqrt{2\pi}\,\sigma_f A}{N_0 t}\right]^{1/2}$$

$$\Omega = 2\pi\Omega_f = \boxed{(2\pi)^{5/4}\sqrt{\frac{\sigma_f A}{N_0 t}} = \Omega}$$