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用 python 中 scipy.integrate 中的 dplquad 函数可以数值求解该积分，给出的结果为

$$0.12584400 \pm 1.5 \times 10^{-8}$$

```
f0 <- function(x, y) {
    exp(-45 * (x + 0.4)^2 - 60 * (y - 0.5)^2) + 0.5 * exp(-90 * (x - 0.5)^2 -
        45 * (y + 0.1)^4)
}

MeanMC <- function(f = f0, N = 1e+05, seed = 1) {
    set.seed(seed)
    x <- runif(N, -1, 1)
    y <- runif(N, -1, 1)
    4 * mean(f(x, y))
}

g0 <- function(x, y, p) {
    p * dnorm(x, -0.4, sqrt(1/90)) * dnorm(y, 0.5, sqrt(1/120)) + (1 - p) *
        dnorm(x, 0.5, sqrt(1/180)) * dnorm(y, -0.1, sqrt(1/20))
}

ImportanceMC <- function(f = f0, g = g0, N = 1e+05, seed = 1) {
    set.seed(seed)
    p <- 0.5358984
    K <- rbinom(N, size = 1, prob = p)
    x <- ifelse(K, rnorm(N, -0.4, sqrt(1/90)), rnorm(N, 0.5, sqrt(1/120)))
```

```
    y <- ifelse(K, rnorm(N, 0.5, sqrt(1/180)), rnorm(N, -0.1, sqrt(1/20)))
    mean(f(x, y)/g(x, y, p))
}

B <- 100
start <- proc.time()
MeanMC.record <- vapply(1:B, function(k) MeanMC(seed = k), FUN.VALUE = 1)
MeanMC.time <- proc.time()
MeanMC.time - start
```

```
##    user  system elapsed
##    4.37    0.12    4.50
```

```
mean(MeanMC.record)
```

```
## [1] 0.1259145
```

```
sd(MeanMC.record)
```

```
## [1] 0.001302427
```

```
ImportanceMC.record <- vapply(1:B, function(k) ImportanceMC(seed = k), FUN.VALUE = 1)
ImportanceMC.time <- proc.time()
ImportanceMC.time - MeanMC.time
```

```
##    user  system elapsed
##   19.75    0.92   20.89
```

```
mean(ImportanceMC.record)
```

```
## [1] 0.1258534
```

```
sd(ImportanceMC.record)
```

```
## [1] 8.771702e-05
```

可以看到，两者结果都是正确的（与真值之差小于标准差），但是采用重要采样后，在总采样数相同的情况下（1×10^5），虽然花费的总时间更多，但是与真值的偏差更小，而且估计得标准差也更小，即重要采样法得精度更高。