实验二十七 交流电桥 实验报告

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1 数据处理

电容、电感盒用的是12号，磁环用的是8号。

1.1 电容桥

$f = 1.000\mathrm{kHz}$，$E = 4.01\mathrm{V}$，$\frac{e_f}{f} = 0.1\%$

1.1.1 纸质电容（$0.2\mu\mathbf{F}$）

$R_1 = R_2 = 800.0\Omega$，$R_0 = 2.8\Omega$，$C_0 = 0.2188\mu\mathrm{F}$，最终$\Delta U = 0.13\mathrm{mV}$。

$$\frac{e_{R_1}}{R_1} = \frac{e_{R_2}}{R_2} = 0.1\%$$

$$\frac{e_{R_0}}{R_0} = \frac{2 \times 0.5\% + 0.8 \times 2\%}{2.8} = 0.9\%$$

$$\frac{e_{C_0}}{C_0} = \frac{0.2 \times 0.5\% + 0.01 \times 0.65\% + 0.008 \times 2\% + 0.0008 \times 5\%}{0.2188} = 0.6\%$$

计算电容C_x：

$$C_x = \frac{R_2}{R_1} C_0 = 0.2188\mu\mathrm{F}$$

$$\sigma_{C_x} = \frac{C_x}{\sqrt{3}} \sqrt{(\frac{e_{R_1}}{R_1})^2 + (\frac{e_{R_2}}{R_2})^2 + (\frac{e_{C_0}}{C_0})^2} = \frac{0.22}{\sqrt{3}} \times 0.6\% = 0.0008\mu\mathrm{F}$$

$$C_x \pm \sigma_{C_x} = (0.2188 \pm 0.0008)\mu\mathrm{F}$$

计算损耗电阻R_C：

$$R_C = \frac{R_1}{R_2} R_0 = 2.8\Omega$$

$$\sigma_{R_C} = \frac{R_C}{\sqrt{3}} \sqrt{(\frac{e_{R_1}}{R_1})^2 + (\frac{e_{R_2}}{R_2})^2 + (\frac{e_{R_0}}{R_0})^2} = \frac{2.8}{\sqrt{3}} \times 0.9\% = 0.02\Omega$$

$$R_C \pm \sigma_{R_C} = (2.80 \pm 0.02)\Omega$$

计算损耗角$\tan\delta$：

$$\tan\delta = 2\pi R_0 C_0 f = 3.85 \times 10^{-3}$$

$$\sigma_{\tan\delta} = \frac{\tan\delta}{\sqrt{3}} \sqrt{(\frac{e_{R_0}}{R_0})^2 + (\frac{e_{C_0}}{C_0})^2 + (\frac{e_f}{f})^2} = \frac{3.9 \times 10^{-3}}{\sqrt{3}} \times 1\% = 0.02 \times 10^{-3}$$

$$\tan\delta \pm \sigma_{\tan\delta} = (3.85 \pm 0.02) \times 10^{-3}$$

1.1.2 电解电容（6.8μF）

$R_1 = 100.0\Omega$，$R_2 = 1000.0\Omega$，$R_0 = 34.3\Omega$，$C_0 = 0.6698\mu$F，最终$\Delta U = 0.00$mV。

$$\frac{e_{R_1}}{R_1} = \frac{e_{R_2}}{R_2} = 0.1\%$$

$$\frac{e_{R_0}}{R_0} = \frac{30 \times 0.1\% + 4 \times 0.5\% + 0.3 \times 2\%}{34.3} = 0.2\%$$

$$\frac{e_{C_0}}{C_0} = \frac{0.6 \times 0.5\% + 0.06 \times 0.65\% + 0.009 \times 2\% + 0.0008 \times 5\%}{0.6698} = 0.5\%$$

计算电容C_x：

$$C_x = \frac{R_2}{R_1} C_0 = 6.698\mu\mathrm{F}$$

$$\sigma_{C_x} = \frac{C_x}{\sqrt{3}} \sqrt{(\frac{e_{R_1}}{R_1})^2 + (\frac{e_{R_2}}{R_2})^2 + (\frac{e_{C_0}}{C_0})^2} = \frac{6.7}{\sqrt{3}} \times 0.5\% = 0.02\mu\mathrm{F}$$

$$C_x \pm \sigma_{C_x} = (6.70 \pm 0.02)\mu\mathrm{F}$$

计算损耗电阻R_C：

$$R_C = \frac{R_1}{R_2} R_0 = 3.43\Omega$$

$$\sigma_{R_C} = \frac{R_C}{\sqrt{3}} \sqrt{(\frac{e_{R_1}}{R_1})^2 + (\frac{e_{R_2}}{R_2})^2 + (\frac{e_{R_0}}{R_0})^2} = \frac{3.4}{\sqrt{3}} \times 0.2\% = 0.005\Omega$$

$$R_C \pm \sigma_{R_C} = (3.430 \pm 0.005)\Omega$$

计算损耗角$\tan\delta$：

$$\tan\delta = 2\pi R_0 C_0 f = 0.1444$$

$$\sigma_{\tan\delta} = \frac{\tan\delta}{\sqrt{3}} \sqrt{(\frac{e_{R_0}}{R_0})^2 + (\frac{e_{C_0}}{C_0})^2 + (\frac{e_f}{f})^2} = \frac{0.14}{\sqrt{3}} \times 0.5\% = 0.0004$$

$$\tan\delta \pm \sigma_{\tan\delta} = 0.1444 \pm 0.0004$$

1.2 麦克斯韦一维恩桥测空心电感（$L = 6$mH，$R_L = 20\Omega$）

$R_1 = R_2 = 200.0\Omega$，$R_0 = 1916.6\Omega$，$C_0 = 0.1500\mu$F，最终$\Delta U = 0.11$mV。

$$\frac{e_{R_1}}{R_1} = \frac{e_{R_2}}{R_2} = \frac{e_{R_0}}{R_0} = 0.1\%$$

$$\frac{e_{C_0}}{C_0} = \frac{0.1 \times 0.5\% + 0.05 \times 0.65\%}{0.15} = 0.6\%$$

计算电感L_x：

$$L_x = C_0 R_1 R_2 = 6.00\mathrm{mH}$$

$$\sigma_{L_x} = \frac{L_x}{\sqrt{3}} \sqrt{(\frac{e_{R_1}}{R_1})^2 + (\frac{e_{R_2}}{R_2})^2 + (\frac{e_{C_0}}{C_0})^2} = \frac{6}{\sqrt{3}} \times 0.6\% = 0.02\mathrm{mH}$$

$$L_x \pm \sigma_{L_x} = (6.00 \pm 0.02)\mathrm{mH}$$

计算损耗电阻R_L：

$$R_L = \frac{R_1 R_2}{R_0} = 20.87\Omega$$

$$\sigma_{R_L} = \frac{R_L}{\sqrt{3}}\sqrt{(\frac{e_{R_1}}{R_1})^2 + (\frac{e_{R_2}}{R_2})^2 + (\frac{e_{R_0}}{R_0})^2} = \frac{21}{\sqrt{3}} \times 0.2\% = 0.02\Omega$$

$$R_L \pm \sigma_{R_L} = (20.87 \pm 0.02)\Omega$$

计算Q值：

$$Q = 2\pi R_0 C_0 f = 1.806$$

$$\sigma_Q = \frac{Q}{\sqrt{3}}\sqrt{(\frac{e_{R_0}}{R_0})^2 + (\frac{e_{C_0}}{C_0})^2 + (\frac{e_f}{f})^2} = \frac{1.8}{\sqrt{3}} \times 0.6\% = 0.006$$

$$Q \pm \sigma_Q = 1.806 \pm 0.006$$

1.3 麦克斯韦桥测空心电感（$L = 6$**mH**，$R_L = 20\Omega$）

$R_1 = 500.0\Omega$，$R_0 = 16.3\Omega$，$R_2 = 500.0\Omega$，$L_0 = 6.0\text{mH}$，$R_{L_0} = 4.48\Omega$，最终$\Delta U = 0.41\text{mV}$。

$$\frac{e_{R_1}}{R_1} = \frac{e_{R_2}}{R_2} = 0.1\%$$

$$\frac{e_{R_0}}{R_0} = \frac{10 \times 0.1\% + 6 \times 0.5\% + 0.3 \times 2\%}{16.3} = 0.3\%$$

$$\frac{e_{L_0}}{L_0} = 2\%$$

$$\frac{e_{R_{L_0}}}{R_{L_0}} = \frac{0.01}{4.48} = 0.2\%$$

计算电感L_x：

$$L_x = \frac{R_1}{R_2} L_0 = 6.00\text{mH}$$

$$\sigma_{L_x} = \frac{L_x}{\sqrt{3}}\sqrt{(\frac{e_{R_1}}{R_1})^2 + (\frac{e_{R_2}}{R_2})^2 + (\frac{e_{L_0}}{L_0})^2} = \frac{6}{\sqrt{3}} \times 2\% = 0.07\text{mH}$$

$$L_x \pm \sigma_{L_x} = (6.00 \pm 0.07)\text{mH}$$

计算损耗电阻R_L：

$$R_L = \frac{R_1}{R_2}(R_0 + R_{L_0}) = 20.78\Omega$$

$$\sigma_{R_L} = \frac{R_L}{\sqrt{3}}\sqrt{(\frac{e_{R_1}}{R_1})^2 + (\frac{e_{R_2}}{R_2})^2 + \frac{e_{R_0}^2 + e_{R_{L_0}}^2}{(R_0 + R_{L_0})^2}} = \frac{21}{\sqrt{3}} \times 0.3\% = 0.04\Omega$$

$$R_L \pm \sigma_{R_L} = (20.78 \pm 0.04)\Omega$$

计算Q值：

$$Q = 2\pi f \frac{L_0}{R_0 + R_{L_0}} = 1.81$$

$$\sigma_Q = \frac{Q}{\sqrt{3}}\sqrt{\frac{e_{R_0}^2 + e_{R_{L_0}}^2}{(R_0 + R_{L_0})^2} + (\frac{e_{L_0}}{L_0})^2 + (\frac{e_f}{f})^2} = \frac{1.8}{\sqrt{3}} \times 2\% = 0.02$$

$$Q \pm \sigma_Q = 1.81 \pm 0.02$$

1.4 比较麦克斯韦－维恩桥和麦克斯韦桥的收敛性（含思考题（2））

麦克斯韦－维恩桥四个臂的阻抗分别为：$\tilde{Z}_1 = R_1$，$\tilde{Z}_2 = \frac{R_0}{1+j\omega C_0 R_0}$，$\tilde{Z}_3 = R_L + j\omega L_x$，$\tilde{Z}_4 = R_2$。

电桥平衡时，有

$$\frac{\tilde{Z}_1}{\tilde{Z}_2} = \frac{\tilde{Z}_3}{\tilde{Z}_4}$$

$$(R_L + j\omega L_x) - (\frac{R_1 R_2}{R_0} + j\omega C_0 R_1 R_2) = 0$$

记复数$\mathbf{A} = R_L + j\omega L_x$，$\mathbf{B} = \frac{R_1 R_2}{R_0} + j\omega C_0 R_1 R_2$，则$\mathbf{A}$是一个常数。调节$R_0$可改变$\mathbf{B}$的实部，调节$C_0$可改变$\mathbf{B}$的虚部。

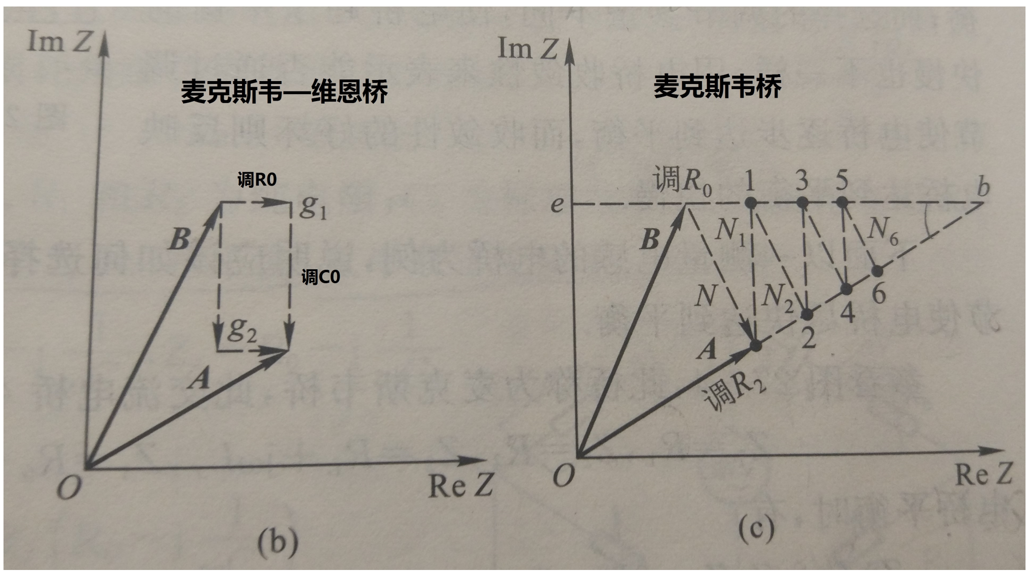

Figure 1: 麦克斯韦－维恩桥和麦克斯韦桥达到平衡的过程图

所以理论上，麦克斯韦－维恩桥由于C_0和R_0是独立的，只需要两步即可调平衡（如左图）；而麦克斯韦桥一般情况下需要很多步逐步趋近平衡点（如右图）。

本次实验中，由于我的待测电感值恰好为6.00mH，与电感箱L_0的值正好相等，也即初始点$\mathbf{A}$与$\mathbf{B}$的虚部相等，所以我只调了一次R_0（$\mathbf{B}$的实部）就直接使电桥趋于平衡。但理论上讲，麦克斯韦桥的收敛性是较差的。

1.5 麦克斯韦－维恩桥测量标准互感器的互感值

顺接时，L顺$= L_1 + L_2 + 2M$；反接时，L反$= L_1 + L_2 - 2M$。所以$M = \frac{1}{4}(L\text{顺} - L\text{反})$。

1.5.1 顺接

$R_1 = R_2 = 640.0\Omega$，$R_0 = 3175.5\Omega$，$C_0 = 0.6155\mu\text{F}$，最终$\Delta U = 0.16\text{mV}$。

$$\frac{e_{R_1}}{R_1} = \frac{e_{R_2}}{R_2} = \frac{e_{R_0}}{R_0} = 0.1\%$$

$$\frac{e_{C_0}}{C_0} = \frac{0.6 \times 0.5\% + 0.01 \times 0.65\% + 0.005 \times 2\% + 0.0005 \times 5\%}{0.62} = 0.5\%$$

计算电感L_x1（顺接）：

$$L_x1 = C_0 R_1 R_2 = 0.252\text{H}$$

$$\sigma_{L_x1} = \frac{L_x1}{\sqrt{3}} \sqrt{(\frac{e_{R_1}}{R_1})^2 + (\frac{e_{R_2}}{R_2})^2 + (\frac{e_{C_0}}{C_0})^2} = \frac{0.62}{\sqrt{3}} \times 0.5\% = 0.002\text{H}$$

$$L_x1 \pm \sigma_{L_x1} = (0.252 \pm 0.002)\text{H}$$

1.5.2 反接

$R_1 = R_2 = 300.0\Omega$，$R_0 = 786.8\Omega$，$C_0 = 0.5895\mu\text{F}$，最终$\Delta U = 0.02\text{mV}$。

$$\frac{e_{R_1}}{R_1} = \frac{e_{R_2}}{R_2} = \frac{e_{R_0}}{R_0} = 0.1\%$$

$$\frac{e_{C_0}}{C_0} = \frac{0.5 \times 0.5\% + 0.08 \times 0.65\% + 0.009 \times 2\% + 0.0005 \times 5\%}{0.59} = 0.5\%$$

计算电感L_x2（反接）：

$$L_x2 = C_0 R_1 R_2 = 0.0531\text{H}$$

$$\sigma_{L_x2} = \frac{L_x2}{\sqrt{3}} \sqrt{(\frac{e_{R_1}}{R_1})^2 + (\frac{e_{R_2}}{R_2})^2 + (\frac{e_{C_0}}{C_0})^2} = \frac{0.053}{\sqrt{3}} \times 0.5\% = 0.0002\text{H}$$

$$L_x2 \pm \sigma_{L_x2} = (0.0531 \pm 0.0002)\text{H}$$

1.5.3 互感值M及其不确定度σ_M

$$M = \frac{1}{4}(L_x1 - L_x2) = 0.0497\text{H}$$

$$\sigma_M = \frac{1}{4}\sqrt{\sigma_{L_x1}^2 + \sigma_{L_x2}^2} = \frac{1}{4} \times 0.002 = 0.0005\text{H}$$

$$M \pm \sigma_M = (0.0497 \pm 0.0005)\text{H}$$

1.6 麦克斯韦一维恩桥测量磁环

电源电压$E = 1.007\text{V}$，$\bar{D} = 8.40\text{cm}$，$S = 1.91\text{cm}^2$，$N = 490$。

$$L = C_0 R_1 R_2$$

$$R_L = \frac{R_1 R_2}{R_0}$$

$$\mu = \frac{\pi \bar{D}}{\mu_0 N^2 S} L = 2.3975\text{mH}^{-1} \cdot L$$

Table 1: 不同频率下测量磁环的电感L与损耗R_L

f/kHz	R_1/Ω	R_2/Ω	$C_0/\mu\text{F}$	R_0/Ω	L/mH	R_L/Ω	μ
0.100	240.0	240.0	0.1255	8849.5	7.23	6.509	17.33
0.400	200.0	200.0	0.0824	3584.5	3.30	11.159	7.90
0.700	200.0	200.0	0.0594	2837.5	2.38	14.097	5.70
1.00	200.0	200.0	0.0479	2460.5	1.92	16.257	4.60
2.00	200.0	200.0	0.0322	1880.5	1.288	21.27	3.09
3.00	200.0	200.0	0.0254	1600.5	1.016	24.99	2.44
4.99	150.0	150.0	0.0352	734.5	0.792	30.63	1.90
7.00	150.0	150.0	0.0294	645.5	0.662	34.86	1.59
9.00	150.0	150.0	0.0262	585.5	0.590	38.43	1.41
10.00	150.0	150.0	0.0248	562.5	0.558	40.00	1.34

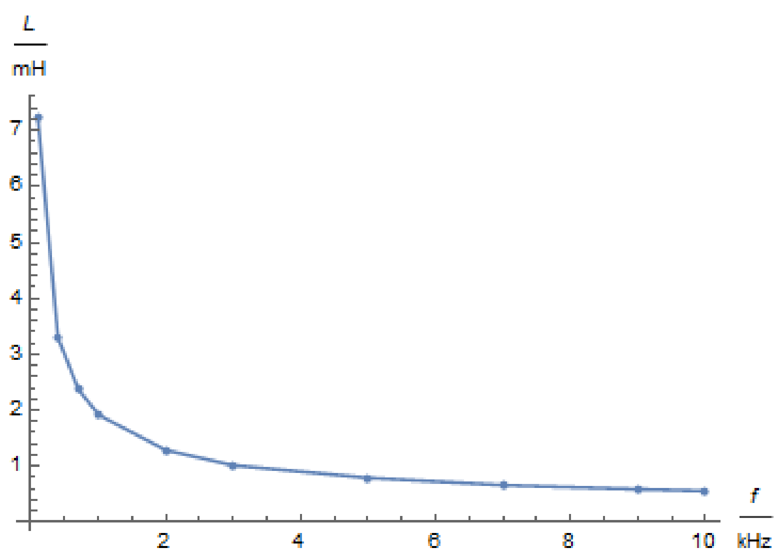

Figure 2: 磁环$L-f$曲线

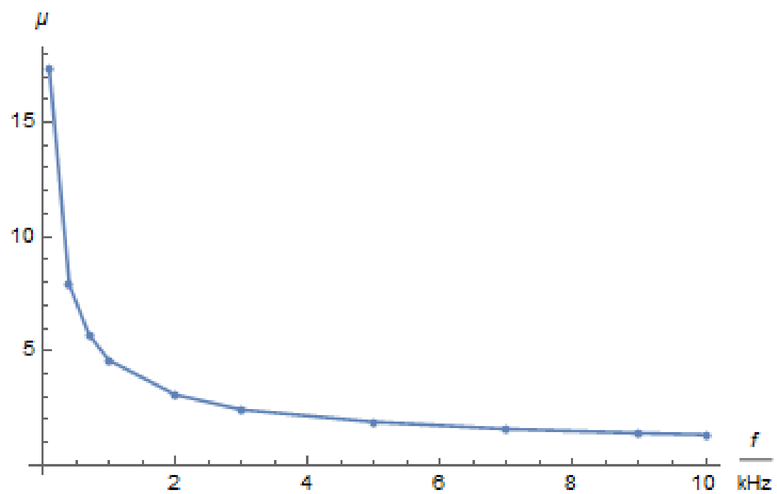

Figure 3: 磁环$\mu-f$曲线

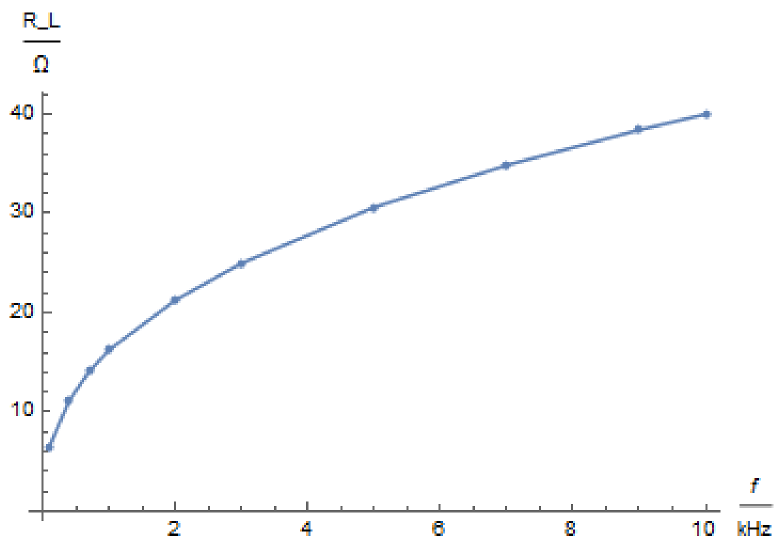

Figure 4: 磁环R_L-f曲线

可见L和μ随频率升高而下降，R_L随频率升高而上升。

当频率升高以后，磁畴极性的变化跟不上电流的变化，这样它的磁性就弱了，磁导率下降，因而电感量下降。

此外，频率升高时，铁芯的损耗（磁滞损耗和涡流损耗）增大，以及线圈的趋肤效应加大，导致电感器的损耗增大。

2　思考题（2）

见1.4“比较麦克斯韦－维恩桥和麦克斯韦桥的收敛性”。