Assignment-6 (Cbse 12 ex 13.2 12)

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Abstract

This document contains the solution to Cbse class 12 ex 13.212

Problem

Question

Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

1. the youngest is a girl,
2. at least one is a girl

Solution

Basic Description

Let $X \in \{0, 1\}$ represent the gender where 1 represents a girl. Let $Y_1, Y_2 \in \{0, 1\}$ represent the child in the family, where Y_1 denotes the older child.

1. here we need to find the conditional probability that each born child is girl given that youngest is girl let us denote the event by C
   Since Y_1, Y_2 are independent,

$$\Pr(C) = \Pr((Y_1 = 1, Y_2 = 1) | Y_2 = 1) \tag{1}$$

$$= \frac{\Pr(Y_1 = 1, Y_2 = 1)}{\Pr(Y_2 = 1)} \tag{2}$$

Since Y_1, Y_2 are independent we can further simplify the numerator in Eq(2) as

$$\Pr(Y_1 = 1, Y_2 = 1) = \Pr(Y_1 = 1) \times \Pr(Y_2 = 1) \tag{3}$$

on substituting this Eq(3) in Eq(2) we will get

$$\Pr(C) = \Pr(Y_1 = 1) \tag{4}$$

$$= \frac{1}{2} \tag{5}$$

2. here we need to find the conditional probability that each born child is girl given that at least one girl is born let us denote that event by D and can be wriiten as
$\Pr(D) = \Pr(Y_1 = 1, Y_2 = 1 | 1 - \{Y_2 = 0, Y_1 = 0\})$

$$= \frac{\Pr(\{Y_1 = 1\}\{Y_2 = 1\}\ [1 - \{Y_2 = 0\}\{Y_1 = 0\}])}{1 - \Pr(Y_2 = 0, Y_1 = 0)} \quad (6)$$

$$\Pr(\{Y_1 = 1\}\{Y_2 = 1\}\ [1 - \{Y_2 = 0\}\{Y_1 = 0\}]) = \Pr(\{Y_1 = 1\}\{Y_2 = 1\}) - \Pr(\{Y_1 = 1\}\{Y_2 = 1\}\{Y_1 = 0\}\{Y_2 = 0\}) \quad (7)$$

In Eq(7) negative calculation part value is 0 as a child can't be both boy and girl so

$$\Pr\left(\{Y_1=1\}\{Y_2=1\}\left[1-\{Y_2=0\}\{Y_1=0\}\right]\right)= \Pr\left(\{Y_1=1\}\{Y_2=1\}\right) \quad (8)$$

probability that both are girls and atleast one girl is same as probability that both are girls so numerator of Eq(6) can be written like numerator in Eq(9) based on Eq(8)

$$= \frac{\Pr\left(\{Y_1=1\}\{Y_2=1\}\right)}{1-\Pr\left(Y_2=0,Y_1=0\right)} \quad (9)$$

$$= \frac{\frac{1}{4}}{1-\frac{1}{4}} \quad (10)$$

$$= \frac{1}{3} \quad (11)$$