Assignment-8 Papoullis 4 exercise 17 question

Asli

IIT Hyderabad

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Abstract

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This document contains the solution to Papoullis chapter 4 exercise 17 question

Problem

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Show that if $\beta(t) = f(t|x > t)$ is the conditional failure rate of the random variable x and $\beta(t) = kt$, then $f(x)$ is a Rayleigh density

Solution

Given things

given $\beta(t) = f(t|x > t)$ is the conditional failure rate of the random variable x and $\beta(t) = kt$

$$\beta(t) = f(t|x > t) \tag{1}$$
$$= \frac{f(t)}{1 - F(t)} \tag{2}$$
$$= \frac{\frac{d}{dt}F(t)}{1 - F(t)} \tag{3}$$

Solution

Continuation

on substituting $\beta(t)$ in Eq(3) we will get below thing

$$kt = \frac{\frac{d}{dt}F(t)}{1 - F(t)} \tag{4}$$

$$= \frac{d}{dt}\left(-\log[1 - F(t)]\right) \tag{5}$$

Integrating both sides of this equation from 0 to x yields

Solution

Continuation

as $\beta(t) = kt > 0$ so t cannot be negative so we can say like for all $F(x)$ where $x < 0$ or $x = 0$ the value is 0
also for integration we take limits from 0 to t for the same reason

Solution

Continuation

$$\int_0^x (kt)dt = (-\log[1 - F(x)]) - (-\log[1 - F(0)]) \tag{6}$$

since $F(0) = 0$ we can write and simplyfy Eq(6) as

$$\frac{k(x)^2}{2} = -\log[1 - F(x)] \tag{7}$$

Solution

Continuation

on simplyfying Eq(7) we will get

$$1 - F(x) = e^{-\frac{k(x)^2}{2}} \tag{8}$$

on further simplyfying Eq(8) we will get

$$F(x) = 1 - e^{-\frac{k(x)^2}{2}} \tag{9}$$

Solution

on Diffrentiating Eq(9) we will get

$$f(x) = kx\left(e^{-\frac{k(x)^2}{2}}\right) \text{ for } x > 0 \tag{10}$$

which is nothing but in the form of Rayleigh density