

Assignment-9 (Papoulis chap 6 Ex 6.10)

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Abstract

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This document contains the solution to Papoulis chap 6 Ex 6.10

Question: The random variables X and Y are independent and $Z = X + Y$. Find $f_Y(y)$. If ,

$$f_X(x) = ce^{-cx} U(x) \quad (1)$$

$$f_Z(z) = c^2 ze^{-cz} U(z) \quad (2)$$

Solution: Given,

$$Z = X + Y \quad (3)$$

So we know if X and Y are independent variables,

$$f_Z(z) = \int_0^z f_X(x) * f_Y(z - y) dx \quad (4)$$

$$\Rightarrow f_Z(z) = \int_0^z f_X(z - y) * f_Y(y) dy \quad (5)$$

Lets use equation 5 since we have to find y ,

$$c^2 e^{-cz} z U(z) = \int_0^z c e^{-c(z-y)} U(z) * f_Y(y) dy \quad (6)$$

$$\Rightarrow cz = \int_0^z e^{cy} * f_Y(y) dy \quad (7)$$

Now differentiating on both sides w.r.t z we get,

$$c = e^{cz} f_Y(z) \quad (8)$$

$$\Rightarrow f_Y(z) = c e^{-cz} \quad (9)$$

The above derived equation is $f_Y(y)$.