Assignment 5 (Cbse 12 ex 13.2 12)

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***Abstract*—This document contains the solution to Cbse 12 ex 13.2 12**

PROBLEM

Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

i) the youngest is a girl
ii) at least one is a girl

SOLUTION

Let the random variables X_i map to the set $\{0, 1\}$ as described in Table where Event A denotes first child or older child is girl and Event B denotes second child or youngest is girl and also Event C where at least one girl is born

variable	event
X_1 = 1	A
X_2 = 1	B
X_3 = 1	C

TABLE I
RANDOM VARIABLES

as given girl and boy are likely to born so the below probability can be written

$$\Pr(X_1 = 1) = \frac{1}{2} \tag{1}$$

$$\Pr(X_2 = 1) = \frac{1}{2} \tag{2}$$

$$\Pr(X_1 = 1, X_2 = 1) = \frac{1}{4} \tag{3}$$

as both events are independent because one doesn't depend on other so

(i) here the youngest is a girl and let us denote the event as C where conditional probability that both are girls given that youngest is a girl

$$\Pr(C) = \Pr((X_1 = 1, X_2 = 1)|X_2 = 1) \tag{4}$$

$$= \frac{\Pr(X_1 = 1, X_2 = 1)}{\Pr(X_2 = 1)} \tag{5}$$

$$= \Pr(X_1 = 1) \tag{6}$$

$$= \frac{1}{2} \tag{7}$$

$\therefore$ the conditional probability that both are girls given that the youngest is a girl is 0.5

(ii) here given that at least one girl is born find conditional probability that both are girls lets us denote that event by D as is given by

$$\Pr(D) = \Pr((X_1 = 1, X_2 = 1)|X_3 = 1) \tag{8}$$

$$= \frac{\Pr(X_1 = 1, X_2 = 1, X_3 = 1)}{\Pr(X_3 = 1)} \tag{9}$$

$$= \frac{\Pr(X_1 = 1, X_2 = 1)}{\Pr(X_3 = 1)} \tag{10}$$

Eq(10) numerator is same as Eq(9) numerator because the event where both are gorls is same as both are girls and alleast one girl

$$\Pr(X_3 = 1) = 1 - \Pr(X_1 = 0, X_2 = 0) \tag{11}$$

$$= 1 - \frac{1}{4} \tag{12}$$

$$= \frac{3}{4} \tag{13}$$

on substituting values in Eq(10) we will get

$$\Pr(D) = \frac{\frac{1}{4}}{\frac{3}{4}} \tag{14}$$

$$= \frac{1}{3} \tag{15}$$

$\therefore$ the conditional probability that both are girls given that the there is atleast one girl is born is $\frac{1}{3}$