

Assignment 6 (Cbse 12 ex 13.2 12)

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Abstract—This document contains the solution to Cbse 12 ex 13.2 12

PROBLEM

Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

- the youngest is a girl
- at least one is a girl

SOLUTION

Let the random variables X_i map to the set $\{0, 1\}$ as described in Table where Event A denotes first child or older child is girl and Event B denotes second child or youngest is girl and also Event C where at least one girl is born

variable	event
$X_1 = 1$	A
$X_2 = 1$	B
$X_3 = 1$	C

TABLE I
RANDOM VARIABLES

as given girl and boy are likely to born so the below probability can be written

$$\Pr(X_1 = 1) = \frac{1}{2} \quad (1)$$

$$\Pr(X_2 = 1) = \frac{1}{2} \quad (2)$$

$$\Pr(X_1 = 1, X_2 = 1) = \frac{1}{4} \quad (3)$$

as both events are independent because one doesn't depend on other so

- here the youngest is a girl and let us denote the event as C where conditional probability that both are girls given that youngest is a girl

$$\Pr(C) = \Pr((X_1 = 1, X_2 = 1) | X_2 = 1) \quad (4)$$

$$= \frac{\Pr(X_1 = 1, X_2 = 1)}{\Pr(X_2 = 1)} \quad (5)$$

$$= \Pr(X_1 = 1) \quad (6)$$

$$= \frac{1}{2} \quad (7)$$

\therefore the conditional probability that both are girls given that the youngest is a girl is 0.5

- here given that at least one girl is born find conditional probability that both are girls lets us denote that event by D as is given by

$$\Pr(D) = \Pr((X_1 = 1, X_2 = 1) | X_3 = 1) \quad (8)$$

$$= \frac{\Pr(X_1 = 1, X_2 = 1, X_3 = 1)}{\Pr(X_3 = 1)} \quad (9)$$

$$= \frac{\Pr(X_1 = 1, X_2 = 1)}{\Pr(X_3 = 1)} \quad (10)$$

Eq(10) numerator is same as Eq(9) numerator because the event where both are girls is same as both are girls and atleast one girl

$$\Pr(X_3 = 1) = 1 - \Pr(X_1 = 0, X_2 = 0) \quad (11)$$

$$= 1 - \frac{1}{4} \quad (12)$$

$$= \frac{3}{4} \quad (13)$$

on substituting values in Eq(10) we will get

$$\Pr(D) = \frac{\frac{1}{4}}{\frac{3}{4}} \quad (14)$$

$$= \frac{1}{3} \quad (15)$$

\therefore the conditional probability that both are girls given that the there is atleast one girl is born is $\frac{1}{3}$