Assignment I (ICSE Class 10 2018 Problem 3b)

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problem 3b

Prove $\sqrt{\sec^2\theta + \csc^2\theta} = (\tan\theta + \cot\theta)$

Solution

Let's divide the equation into 2 parts those are LHS and RHS and lets simplyfy both LHS and RHS and find their simplyfied value if both are same then statement is true if not then false
What we know before starting proof are

$$\sin^2\theta + \cos^2\theta = 1 \tag{1}$$
$$\sec\theta = \frac{1}{\cos\theta} \tag{2}$$
$$\csc\theta = \frac{1}{\sin\theta} \tag{3}$$
$$\tan\theta = \frac{\sin\theta}{\cos\theta} \tag{4}$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta} \tag{5}$$

we may use these equations in simplyfying RHS ans LHS

Simplyfying LHS

$$LHS \Rightarrow \sqrt{\sec^2\theta + \csc^2\theta} \tag{6}$$
$$\Rightarrow \sqrt{\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}} \tag{7}$$
$$\Rightarrow \sqrt{\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta \times \cos^2\theta}} \tag{8}$$
$$\Rightarrow \sqrt{\frac{1}{\sin^2\theta \times \cos^2\theta}} \tag{9}$$
$$\Rightarrow \sqrt{\sec^2\theta \times \csc^2\theta} \tag{10}$$
$$\Rightarrow \sec\theta \times \csc\theta \tag{11}$$

on simplyfying LHS we got

$$LHS = \sec\theta \times \csc\theta \tag{12}$$

Simplyfying RHS

$$RHS \Rightarrow \tan\theta + \cot\theta \tag{13}$$
$$\Rightarrow \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \tag{14}$$
$$\Rightarrow \frac{\sin^2\theta + \cos^2\theta}{\sin\theta \times \cos\theta} \tag{15}$$
$$\Rightarrow \frac{1}{\sin\theta \times \cos\theta} \tag{16}$$
$$\Rightarrow \sec\theta \times \csc\theta \tag{17}$$

on simplyfying RHS we got

$$RHS = \sec\theta \times \csc\theta \tag{18}$$

conclusion

$$\text{when}(\sec\theta \times \csc\theta > 0) \tag{19}$$

based on eq(12),eq(18) and if eq(19) is true then and we will get down equation

$$LHS = RHS = \sec\theta \times \csc\theta \tag{20}$$

$$\therefore \sqrt{\sec^2\theta + \csc^2\theta} = (\tan\theta + \cot\theta)$$

Hence Proved