Assignment-11

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Abstract

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This document contains the solution to Chapter 8 Exercise problem 8.19 from Papoulis Book

Question

Question

The readings of a voltmeter introduces an error *nu* with mean 0. We wish to estimate its standard deviation σ. We measure a calibrated source V $=$ 3 V four times and obtain the values $2.90, 3.15, 3.05, 2.96$ Assuming that ν is normal, find the 0.95 confidence interval of σ

Solution

Solution

so here there are 4 observations like $2.90, 3.15, 3.05, 2.96$ where expected values for each are 3.00

Also 0.95 level of confidence for σ is nothing but an interval between $0.025, 0.975$

Formulae

Formulae1

The Confidence interval for the variance is given by:

$$\frac{k}{\chi^2_{0.025}} > \sigma^2 > \frac{k}{\chi^2_{0.975}} \tag{1}$$

Formulae

Formulae2

$\chi^2_{0.025}$ and $\chi^2_{0.975}$ can be calculated respectively from Fig1 and Fig2 for values of v $= 4$ and the critical probability from above

$$\chi^2_{0.025} = 0.484 \tag{2}$$

$$\chi^2_{0.975} = 11.143 \tag{3}$$

Formulae

Formulae3

the value of k is given by n×v
where v is the variance of the observations and n is the no of observations

$$k = 4\left((2.90 - 3.00)^2 + (3.15 - 3.00)^2 + (3.05 - 3.00)^2 + (2.96 - 3.00)^2\right)$$

which on calculating we will get

$$k = 0.0366 \tag{4}$$

Lower-tail critical values of chi-square distribution with ν degrees of freedom

	Probability less than the critical value				
ν	0.10	0.05	0.025	0.01	0.001
1.	.016	.004	.001	.000	.000
2.	.211	.103	.051	.020	.002
3.	.584	.352	.216	.115	.024
4.	1.064	.711	.484	.297	.091
5.	1.610	1.145	.831	.554	.210
6.	2.204	1.635	1.237	.872	.381
7.	2.833	2.167	1.690	1.239	.598
8.	3.490	2.733	2.180	1.646	.857
9.	4.168	3.325	2.700	2.088	1.152
10.	4.865	3.940	3.247	2.558	1.479
11.	5.578	4.575	3.816	3.053	1.834
12.	6.304	5.226	4.404	3.571	2.214
13.	7.042	5.892	5.009	4.107	2.617
14.	7.790	6.571	5.629	4.660	3.041
15.	8.547	7.261	6.262	5.229	3.483
16.	9.312	7.962	6.908	5.812	3.942

Figure: lower tail critical values of χ^2 with v degrees of freedom

Upper-tail critical values of chi-square distribution with ν degrees of freedom

	Probability less than the critical value				
ν	0.90	0.95	0.975	0.99	0.999
1	2.706	3.841	5.024	6.635	10.828
2	4.605	5.991	7.378	9.210	13.816
3	6.251	7.815	9.348	11.345	16.266
4	7.779	9.488	11.143	13.277	18.467
5	9.236	11.070	12.833	15.086	20.515
6	10.645	12.592	14.449	16.812	22.458
7	12.017	14.067	16.013	18.475	24.322
8	13.362	15.507	17.535	20.090	26.125
9	14.684	16.919	19.023	21.666	27.877
10	15.987	18.307	20.483	23.209	29.588
11	17.275	19.675	21.920	24.725	31.264
12	18.549	21.026	23.337	26.217	32.910
13	19.812	22.362	24.736	27.688	34.528
14	21.064	23.685	26.119	29.141	36.123
15	22.307	24.996	27.488	30.578	37.697
16	23.542	26.296	28.845	32.000	39.252

Figure: lower tail critical values of χ^2 with v degrees of freedom

Substituting and solving

Solving

On substituting all values in Eq(1) we will get

$$\frac{0.0366}{0.484} > \sigma^2 > \frac{0.0366}{11.143} \tag{5}$$

on simplyfying Eq(5) we will get

$$0.275 > \sigma > 0.057 \tag{6}$$

or simply

$$0.057 < \sigma < 0.275 \tag{7}$$