HW#1

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Due Date: February 4, 2019

Course Code: ECON 623 Forecasting Financial Markets

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Exercise 1

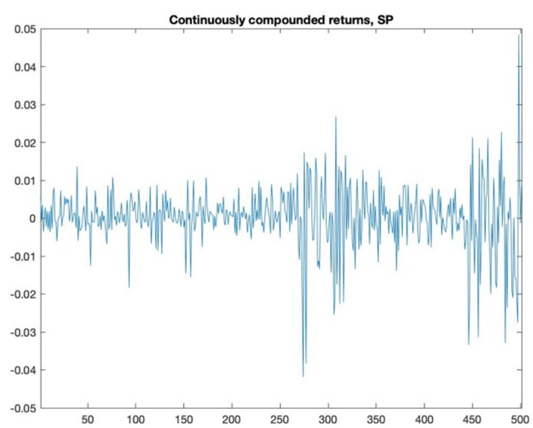
```
%Import a time series of daily prices on two financial assets (S&P500 and APPLE)
SP = csvread('^GSPC.csv', 1, 0);
AAPL = csvread('AAPL.csv', 1, 0);
```

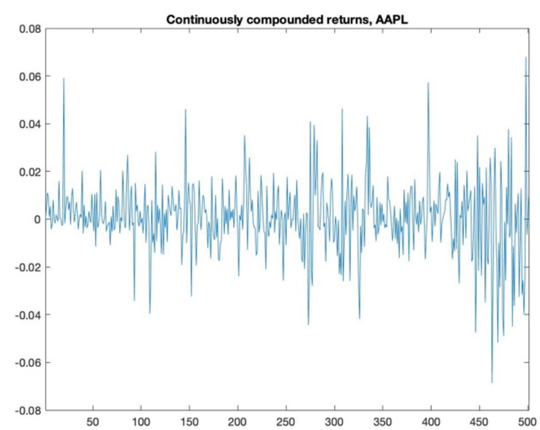
(a)

Code:

```
%Calculate the continuously compounded returns of S&P500
P_SP = SP(:, 6);
ret_SP = log(P_SP(2:end)./P_SP(1:end-1));

%Calculate the continuously compounded returns of APPLE
P_AAPL = AAPL(:, 6);
ret_AAPL = log(P_AAPL(2:end)./P_AAPL(1:end-1))

% Add xlabel and ylabel
figure(1)
plot(ret_SP)
title('Continuously compounded returns, SP')
xlim([1 length(ret_SP)]) % Set the x axis limit

% Add xlabel and ylabel
figure(2)
plot(ret_AAPL)
title('Continuously compounded returns, AAPL')
xlim([1 length(ret_AAPL)]) % Set the x axis limit
```

Output:





(b)

```
function [MeanOut, MedianOut, MaxOut, MinOut, StDOut, SkewOut, KurtOut, JB_t, JB_p]= sum_stats(X)
MeanOut = mean(X);
MedianOut = median(X);
MaxOut = max(X);
MinOut = min(X);
StDOut = std(X);
SkewOut = skewness(X);
KurtOut = kurtosis(X);
[~,JB_p, JB_t,~] = jbtest(X);
end
```

Code:

```
% Calculate mean, median, ... for two financial assets series data
[MeanOut, MedianOut, MaxOut, MinOut, StDOut, SkewOut, KurtOut, JB_t, JB_p] =
sum_stats(P_SP)
sum_SP = [MeanOut, MedianOut, MaxOut, MinOut, StDOut, SkewOut, KurtOut, JB_t,
JB_p]'

[MeanOut, MedianOut, MaxOut, MinOut, StDOut, SkewOut, KurtOut, JB_t, JB_p] =
sum_stats(P_AAPL)
sum_AAPL = [MeanOut, MedianOut, MaxOut, MinOut, StDOut, SkewOut, KurtOut,
JB_t, JB_p]'

% Save data into one table
sum_stats = table(sum_SP, sum_AAPL)
sum_stats.Properties.RowNames = {'Mean' 'Median' 'Max' 'Min'
'StandardDeviation' 'Skewness' 'Kurtosis' 'JB_t' 'JB_p'};
```

Output:

	1 sum_SP	2 sum_AAPL
1 Mean	2.5976e+03	167.5436
2 Median	2.6294e+03	166.5774
3 Max	2.9308e+03	231.2631
4 Min	2.2578e+03	112.4949
5 StandardDeviation	181.9941	27.1985
6 Skewness	-0.0723	0.3952
7 Kurtosis	1.8362	2.6392
8 JB_t	28.7681	15.7938
9 JB_p	1.0000e-03	0.0031

Exercise 2

(a)

Code:

```
% Construct the regression
Intercept = ones(size(ret_AAPL(1:end-1), 1), 1);
X = [Intercept ret_AAPL(1:end-1)];
Y = ret_SP(2:end);

results = ols(Y,X);

% Report the estimated parameters, their standard errors, their t-statistics, and
the R2 and
% R2 adj from this regression
Beta = results.beta
StandardError = results.bstd
Tstatistics = results.tstat
Rsquared = results.rsqr
Rbarsquared = results.rbar
```

Output:

```
Beta =

   1.0e-03 *

    0.1977
    0.2235


StandardError =

    0.0004
    0.0244


Tstatistics =

    0.5388
    0.0092


Rsquared =

   1.6855e-07


Rbarsquared =

   -0.0020
```

(b)

Code:

```
%Calculate Tstatistics
tstat_b = (results.beta(2) - 1)/results.bstd(2)
%Calculate Critical Value for 5% significance level
Critical_values = [norminv(0.025) norminv(0.975)]
%Test if absolute value of Tstatistics is less than the Critical Value
abs(tstat_b) < norminv(0.975)
```

Output:

```
tstat_b =

  -40.9902


Critical_values =

   -1.9600    1.9600


ans =

  logical

   0
```

Interpretation:

Since logical = 0 means False, which also means that the absolute value of T statistics is large than the critical value. Therefore, we can reject the null hypothesis at 5% significance level. β_1 is significant.

(c)

Code:

```
% Construct the regression
Indep = [ret_AAPL(3:end-1) ret_AAPL(2:end-2) ret_AAPL(1:end-3)];
Intercept = ones(size(Indep, 1), 1);
X = [Intercept Indep];
Y = ret_SP(4:end);

results_c = ols(Y,X);

% Report the estimated parameters, their standard errors, their t-statistics, and
the R2 and
% R2 adj from this regression
Beta_c = results_c.beta
StandardError_c = results_c.bstd
Tstatistics_c = results_c.tstat
Rsquared_c = results_c.rsqr
Rbarsquared_c = results_c.rbar
```

Output:

```
Beta_c =

    0.0002
    0.0021
   -0.0326
    0.0108


StandardError_c =

    0.0004
    0.0245
    0.0245
    0.0245


Tstatistics_c =

    0.5583
    0.0851
   -1.3320
    0.4412


Rsquared_c =

    0.0039


Rbarsquared_c =

   -0.0022
```

(d)

Code:

```
% Calculate needed value for chi^2 test
Betas_c = results_c.beta(2:end);
XXinv = (X'*X)\eye(4);
CovarianceMatrix = XXinv*var(results_c.resid);
CovarianceMatrixBetas = CovarianceMatrix(2:end, 2:end); % Drop the constant
R = [1 0 0
     0 1 0
     0 0 1];
%Calculate test statistics
 TestValue_d = (R*Betas_c - [0; 0; 0])'*((R'*CovarianceMatrixBetas*R)\(R*Betas_c -
[0; 0; 0]))
%Calculate Critical Value for 5% significance level
CriticalValue_d = chi2inv(1-0.05,3)
%Test if test statistics is less than the Critical Value
TestValue_d < chi2inv(1-0.05,3)
```

Output:

```
TestValue_d =

    1.9277


CriticalValue_d =

    7.8147


ans =

  logical

   1
```

Interpretation:

Since logical = 1 means True, which also means that the test statistics is less than the critical value. Therefore, we cannot reject the null hypothesis at 5% significance level.

(e)

Code:

```
Betas_d = results_c.beta(2:end);
XXinv = (X'*X)\eye(4);
CovarianceMatrix = XXinv*var(results_c.resid);
CovarianceMatrixBetas = CovarianceMatrix(2:end, 2:end); % Drop the constant
R = [1 -1 0
     1 0 -1];
%Calculate test statistics
TestValue_d = (R*Betas_d - [0; 0])'*((R*CovarianceMatrixBetas*R')\(R*Betas_d
- [0; 0]))
%Calculate Critical Value for 5% significance level
CriticalValue_d = chi2inv(1-0.05,2)
%Test if test statistics is less than the Critical Value
TestValue_d < chi2inv(1-0.05,2)
```

Output:

```
TestValue_e =

    1.6460


CriticalValue_e =

    5.9915


ans =

  logical

   1
```

Interpretation:

Since logical = 1 means True, which also means that the test statistics is less than the critical value. Therefore, we cannot reject the null hypothesis at 5% significance level.

Exercise 3

(a)

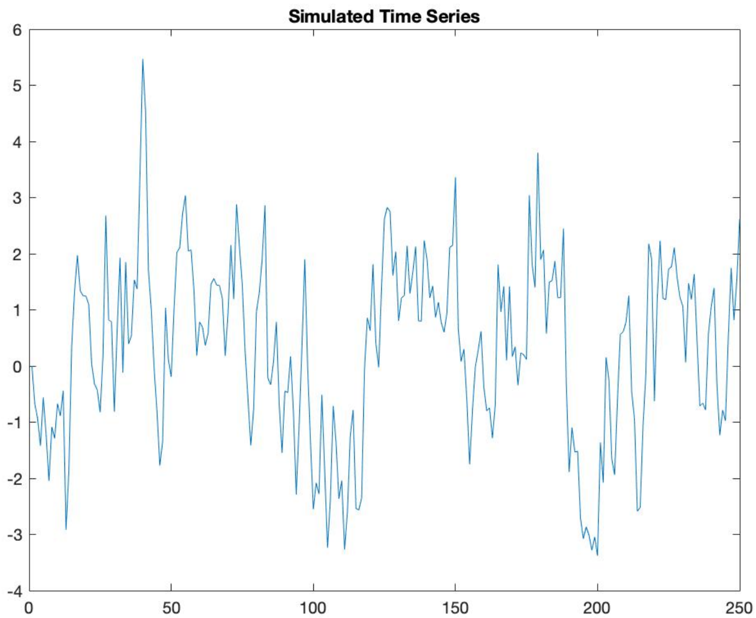
```
function ar1 = AR1(theta, T)

    phi0 = theta(1);
    phi1 = theta(2);
    sig2eps = theta(3);

    ar1 = zeros(T, 1);
    ar1(1) = phi0/(1-phi1);

    eps = sqrt(sig2eps)*randn(T, 1);

    for i = 2:T
        ar1(i) = phi0 + phi1*ar1(i-1) + eps(i);
    end
end
```

(b)

Code:

```
%Assign value to theta and T, and plot the result
theta = [0 0.8 1];
T = 250;
Y = AR1(theta, T);
plot(Y)
title('Simulated Time Series')
```

Output:

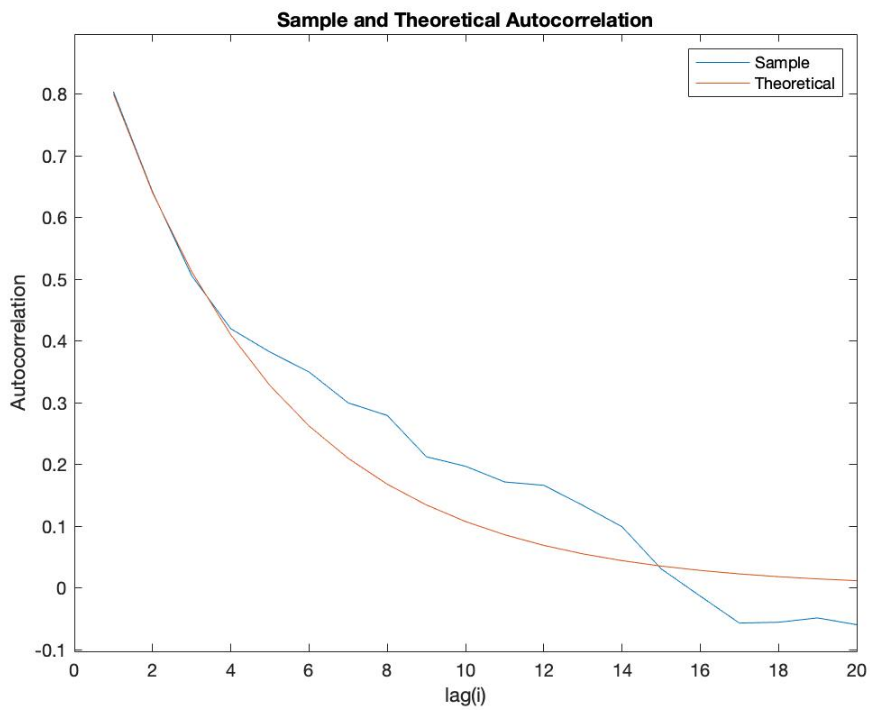


(c)

Code:

```
%Calculate sample autocorrelation
autocorrY = autocorr(Y, 20);
autocorrY_1 = autocorrY(2:21,:);

%Calculate theoretical autocorrelation
for i = 1:20
    phi1 = 0.8;
    autocorrY_2(i) = phi1^(i);
end

%Plotting
plot(autocorrY_1)
hold on
plot(autocorrY_2)
hold off
title('Sample and Theoretical Autocorrelation')
xlabel('lag(i)')
ylabel('Autocorrelation')
```

Output:



Exercise 4

(a)

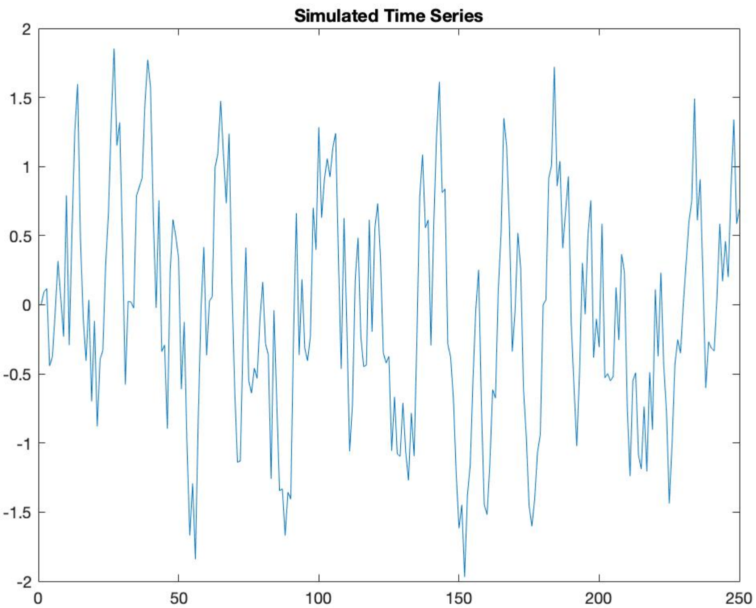
```
function ar1s = AR1simU(theta, T)

    phi0 = theta(1);
    phi1 = theta(2);
    L = theta(3);
    U = theta(4);

    ar1s = zeros(T, 1);
    ar1s(1) = 0;

    epss = L+(U-L).*rand(T, 1);

    for i = 2:T
        ar1s(i) = phi0 + phi1*ar1s(i-1) + epss(i);
    end
end
```

(b)

Code:

```
%Assign value to theta and T, and plot the result
theta = [0 0.8 -1 1];
T = 250;
X = AR1simU(theta, T);
plot(X)
title('Simulated Time Series')
```

Output:



(c)

$$
\begin{aligned}
E[X_t] &= E[\phi_0 + \phi_1 X_{t-1} + \varepsilon_t] \\
&= \phi_0 + \phi_1 E[X_{t-1}] + E[\varepsilon_t] \\
&= \phi_0 + \phi_1 E[X_t] + \frac{L+U}{2}
\end{aligned}
$$

$\Rightarrow E[X_t] = \frac{\phi_0 + \frac{L+U}{2}}{1-\phi_1} = \frac{2\phi_0 + L + U}{2(1-\phi_1)}$

(d)

(i) Code:

```
% (i)Assign value to theta and T, and compute the sample mean
theta = [0 0.8 -1 1];
T = 100000;
X1 = AR1simU(theta, T);
mean(X1)
```

Output:

```
mean(X1)

ans =

  -9.9308e-04
```

Analysis:

$\widehat{E[X_t]} = -0.0039$

$$E[X_t] = \frac{2\phi_0 + L + U}{2(1-\phi_1)} = \frac{2*0 + (-1) + 1}{2*(1-0.8)} = 0$$

=>Verified ✅

(ii) Code:

```
% (ii) Assign value to theta and T, and compute the sample mean
theta = [0 0.8 -2 1];
T = 100000;
X2 = AR1simU(theta, T);
mean(X2)
```

Output:

```
mean(X2)

ans =

  -2.4988
```

Analysis:

$\widehat{E[X_t]} = -2.4973$

$$E[X_t] = \frac{2\phi_0 + L + U}{2(1-\phi_1)} = \frac{2*0+(-2)+1}{2*(1-0.8)} = -2.5$$

=>Verified ✅

(iii) Code:

```
% (iii) Assign value to theta and T, and compute the sample mean
theta = [1 0.8 -5 2];
T = 100000;
X3 = AR1simU(theta, T);
mean(X3)
```

Output:

```
mean(X3)

ans =

  -2.4917
```

Analysis:

$\widehat{E[X_t]} = -2.4879$

$$E[X_t] = \frac{2\phi_0 + L + U}{2(1-\phi_1)} = \frac{2*1+(-5)+2}{2*(1-0.8)} = -2.5$$

=>Verified ✅