迹函数求导证明

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给定$f(x) = tr(AX)$,$A \in \mathbb{R}^{p*n}$, $X \in \mathbb{R}^{n*p}$, $AX \in \mathbb{R}^{p*p}$,

且 $\frac{df}{dx} = (\frac{df}{dx_{ij}})$。

证明: $\frac{df}{dX} = A^T$

证:

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{p1} & \cdots & a_{pn} \end{pmatrix}_{p*n}$$

$$X = \begin{pmatrix} x_{11} & \cdots & x_{1p} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{np} \end{pmatrix}_{n*p}$$

$$AX = \begin{pmatrix} \sum_{k=1}^{n} a_{1k}x_{k1} & \sum_{k=1}^{n} a_{1k}x_{k2} & \cdots & \sum_{k=1}^{n} a_{1k}x_{kp} \\ & & \sum_{k=1}^{n} a_{jk}x_{kj} & \\ & & & \sum_{k=1}^{n} a_{pk}x_{pk} \end{pmatrix}_{p*p}$$

$$tr(AX) = \sum_{k=1}^{n} a_{1k}x_{k1} + \sum_{k=1}^{n} a_{2k}x_{k2} + \cdots + \sum_{k=1}^{n} a_{jk}x_{kj} + \cdots + \sum_{k=1}^{n} a_{pk}x_{kp}$$

$$\sum_{k=1}^{n} a_{jk}x_{kj} = a_{j1}x_{1j} + a_{j2}x_{2j} + \cdots + a_{ji}x_{ij} + \cdots + a_{jn}x_{nj}$$

So,

$$\frac{df}{dx_{ij}} = \frac{d(tr(AX))}{dx_{ij}} = \frac{d(\sum_{k=1}^{n} a_{jk}x_{kj})}{d(x_{ij})} = a_{ji}$$

So,

$$\frac{df}{dX} = A^T$$

证明完毕