Hurdle Modeling in R Using Bayesian Inference

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Motivation

- **Need:** Effective modeling methods for zero-inflated and/or over-dispersed count data.
- **Goal:** Develop a package of user-friendly functions, utilizing MCMC sampling, that will best model problematic count data that cannot be fit to any typical distribution.

Discription

- **hurdle(...):** Used to fit single or double-hurdle regression models to count data via Bayesian inference.
- **hurdle_control(...):** Various parameters for fitting control of hurdle model regression.

Usage

- hurdle(y, x = NULL, hurdle = Inf, dist = c("poisson", "nb", "gpd"), dist.2 = c("none", "gpd", "poisson", "nb"), control = hurdle_control(...), iters = 1000, burn = 500, nthin = 1, plots = T, progress.bar = T)
- hurdle_control(a = 1, b = 1, size = 1, beta.prior.mean = 0, beta.prior.sd = 1000, beta.tune = 1, pars.tune = 0.2, lam.start = 1, mu.start = 1, sigma.start = 1, xi.start = 1)

Arguments

- **hurdle(...)**
  - **y:** numeric response vector.
  - **x:** optional numeric predictor matrix.
  - **hurdle:** numeric threshold (ψ) for 'extreme' observations of two-hurdle models. NULL for one-hurdle models.
  - **dist:** character specification of response distribution.
  - **dist.2:** character specification of response distribution for 'extreme' observations of two-hurdle models.
  - **control:** list of parameters for controlling the fitting process, specified by hurdle_control().
  - **iters:** number of iterations for the Markov chain to run.
  - **burn:** numeric burn-in length.
  - **nthin:** numeric thinning rate.
  - **plots:** logical operator. TRUE to print plots.
  - **progress.bar:** logical operator. TRUE to print progress bar.
- **hurdle_control(...)**
  - **a:** shape parameter for Gamma(a, b) prior distributions.
  - **b:** rate parameter for Gamma(a, b) prior distributions.
  - **size:** size (r) parameter for NB(r, μ) likelihood distributions.
  - **beta.prior.mean:** mean (μ) for Normal(μ, σ^2) prior distributions.
  - **beta.prior.sd:** st. deviation (σ) for Normal(μ, σ^2) prior distributions.
  - **beta.tune:** MCMC tuning for regression coefficient estimation.
  - **pars.tune:** MCMC tuning for parameter estimation.
  - **lam.start, mu.start, sigma.start, xi.start:** initial value(s) for parameter(s) of 'extreme' observations distribution.

Functionality & Applications

Response data:
Surveys → Boat/aerial continuous-time strip transects.

Avian Counts: Sooty Shearwater

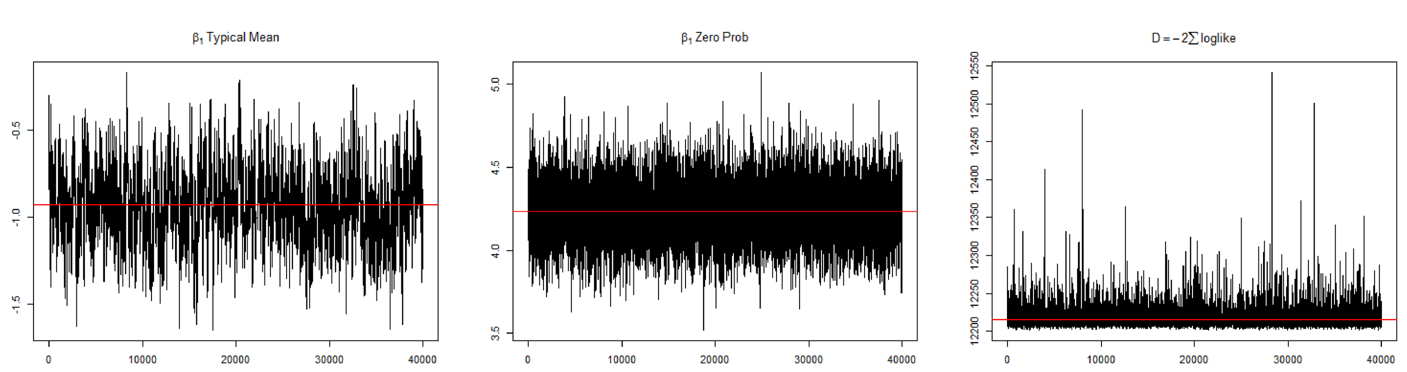
Count	Frequency
0	33503
1 – 10	412
11 – 100	88
101 – 500	16
501 +	3

Environmental covariates:
x_1 = Sea surface temperature.
x_2 = Ocean depth.
x_3 = Chlorophyll-a level.
x_4 = Distance-to-shore.

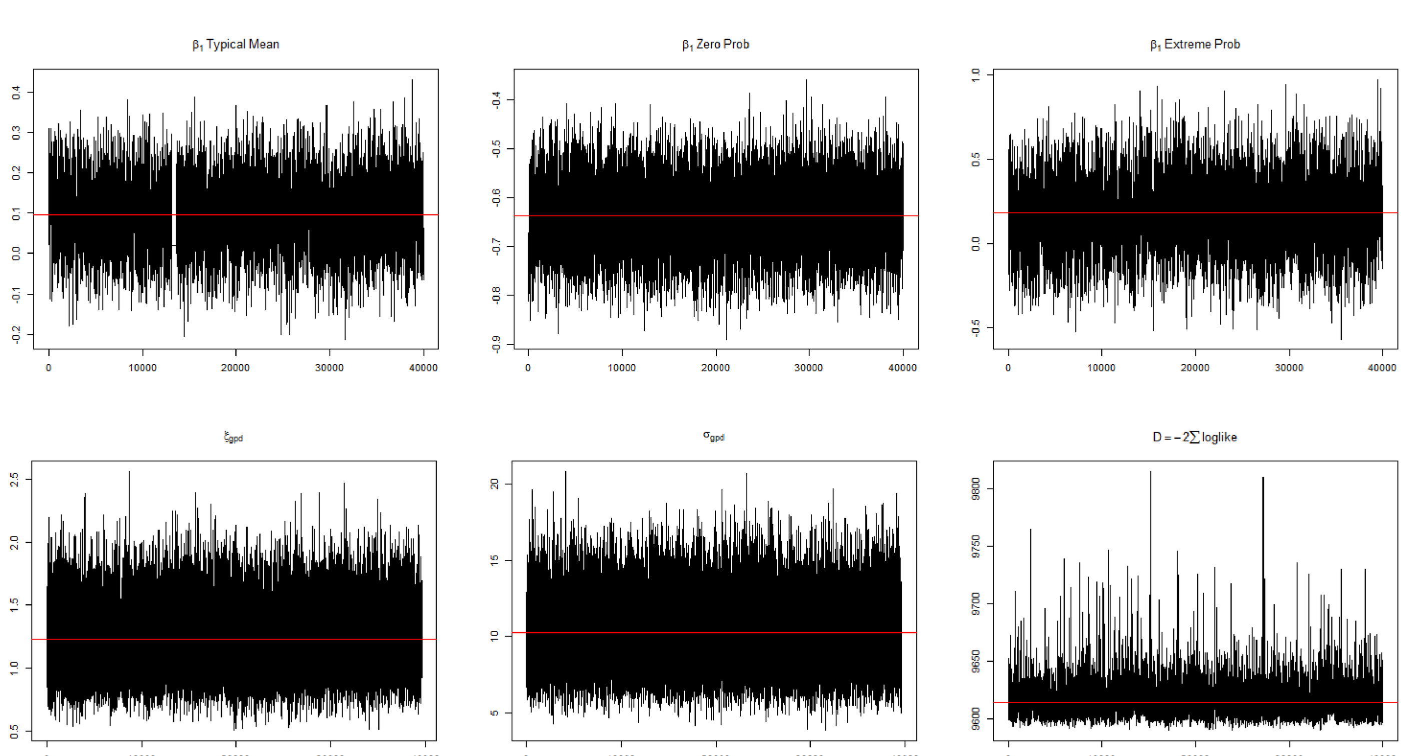
Temporal effects (Fourier basis):
$x_5 = sin(\frac{\pi}{6} \cdot \text{Month})$.
$x_6 = cos(\frac{\pi}{6} \cdot \text{Month})$.

Fit a model to the data in R using hurdlr package functions:

```
f <- hurdle(y, x = x, hurdle = Inf,
            dist = "nb",
            dist.2 = "none",
            control = control,
            iters = 45000, burn = 5000, nthin = 10,
            plots = T, progress.bar = T)
```



```
f <- hurdle(y, x = x, hurdle = hurd,
           dist = "nb",
           dist.2 = "gpd",
           control = control,
           iters = 45000, burn = 5000, nthin = 10,
           plots = T, progress.bar = T)
```



Compare Single vs Double Hurdle model:
→ Improved convergence of model parameters.
→ Decrease in deviance (supported by DIC and pD).
→ Increase in predictive power (based on predictive ordinates PPO and CPO).

Model

- Single-Hurdle modeling is used to fit zero-inflated data.

Likelihood of observing count y_i:

$$f(y_i|\boldsymbol{\theta}) = \begin{cases} p_i, & y_i = 0, \\ [1-p_i] \cdot \text{NB}(\mu_i, r), & 1 \le y_{ij} < \psi, \end{cases}$$

- Double-Hurdle modeling may account for both excessive zero-inflation and extreme over-dispersion.

Likelihood of observing count y_i:

$$f(y_i|\boldsymbol{\theta}) = \begin{cases} p_i, & y_i = 0, \\ [1-p_i] \cdot [1-q_i] \cdot \text{NB}(\mu_i, r), & 1 \le y_{ij} < \psi, \\ [1-p_i] \cdot q_i \cdot \text{GPD}(\psi, \sigma, \xi), & y_{ij} \ge \psi. \end{cases}$$

- Negative binomial (NB) for small, "typical" counts.
  - Left-truncated at 0 and right-truncated at threshold ψ.
    - Single-hurdle models are truncated only at 0.
    - ZIP, ZINB, Poisson-hurdle, NB-hurdle distributions are common.
- Generalized Pareto (GPD) for large, right-tail counts.
  - GPD density is > 0 at threshold ψ or above.

Bayesian Regression

- A series of linear regressions are run to estimate:
  - $\mathbf{p}$ = *P(zero-count)*
    $\text{logit}(\mathbf{p}) = \mathbf{X}\gamma$
  - $\boldsymbol{\mu}$ = *mean of typical-count distribution.*
    $\log(\boldsymbol{\mu}) = \mathbf{X}\beta$
  - $\mathbf{q}$ = *P(large-count | nonzero-count)*
    $\text{logit}(\mathbf{q}) = \mathbf{X}\delta$
- A Bayesian approach to linear regression allows for the user to characterize the uncertainty in the response vector $\mathbf{y}$ through a probability distribution f($\mathbf{y}|\boldsymbol{\theta}$).
- Parameters are updated using a home-grown Markov chain Monte Carlo algorithm utilizing Metropolis sampling.

Current Work & Future Considerations

- Incorporate other distributions; i.e., log-normal models.
- Treat threshold parameter ψ as unknown.
- Create similar functions for applying models to zero-inflated (ZIP, ZINP) count distributions.
- Increase functionality to allow for hierarchical regression of nested data.
- Expand on function output to include clean and variable plots, convergence and coverage diagnostics, predictive power, etc.
- Release hurdlr R package to CRAN for public use.

Acknowledgements

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