

# Team Reference Document

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# 1 数据结构

## 1.1 笛卡尔树

```
1 struct node {
2     array<node *, 2> ch;
3     int val, idx;
4     node(int val, int idx) : val(val), idx(idx) { ch[0] = ch[1] = NULL; }
5 };
6
7 node *build(vector<int> &a) {
8     vector<node *> stk;
9     for (int i = 0; i < a.size(); i++) {
10         node *last = NULL;
11         while (stk.size() && stk.back()->val > a[i]) {
12             last = stk.back();
13             stk.pop_back();
14         }
15         node *u = new node(a[i], i);
16         if (stk.size()) stk.back()->ch[1] = u;
17         if (last) u->ch[0] = last;
18         stk.push_back(u);
19     }
20     return stk[0];
21 }
```

## 1.2 Link-Cut Tree

```
1 struct lct {
2     vector<array<int, 2>> ch;
3     vector<int> fa;
4     vector<int> rev;
5     vector<int> siz;
6     lct(int n)
7         : ch(n + 1, {0, 0}), fa(n + 1, 0), rev(n + 1, false), siz(n + 1, 1) {
8         siz[0] = 0; // initialize nil
9     }
10     void update(int u) { siz[u] = siz[ch[u][0]] + siz[ch[u][1]] + 1; }
11     void reverse(int u) {
12         if (u) rev[u] ^= 1, swap(ch[u][0], ch[u][1]);
13     }
14     void pushdown(int u) {
15         if (rev[u]) reverse(ch[u][0]), reverse(ch[u][1]);
```

```
16         rev[u] = false;
17     }
18     int chid(int u) {
19         if (ch[fa[u]][0] == u) return 0;
20         if (ch[fa[u]][1] == u) return 1;
21         return -1;
22     }
23     bool isroot(int u) { return chid(u) == -1; }
24     void rotate(int u) {
25         int v = fa[u], w = fa[v], k = chid(u), x = ch[u][!k];
26         if (!isroot(v)) ch[w][chid(v)] = u;
27         ch[u][!k] = v, ch[v][k] = x;
28         if (x) fa[x] = v;
29         fa[v] = u, fa[u] = w;
30         update(v), update(u);
31     }
32     void clearup(int u) {
33         if (!isroot(u)) clearup(fa[u]);
34         pushdown(u);
35     }
36     void splay(int u) {
37         clearup(u);
38         while (!isroot(u)) {
39             int f = fa[u];
40             if (!isroot(f)) rotate(chid(u) == chid(f) ? f : u);
41             rotate(u);
42         }
43     }
44     int access(int u) {
45         int v;
46         for (v = 0; u; v = u, u = fa[u]) splay(u), ch[u][1] = v, update(u);
47         return v;
48     }
49     void makeroot(int u) {
50         u = access(u);
51         reverse(u);
52     }
53     void link(int u, int v) {
54         makeroot(u);
55         splay(u);
56         fa[u] = v;
57     }
58     void cut(int u, int v) {
59         makeroot(u);
60         access(v);
```

```

61     splay(v);
62     ch[v][0]=fa[u]=0;
63     update(v);
64 }
65 };

```

### 1.3 李超线段树

```

1 #include <optional>
2
3 #include "bits/stdc++.h"
4 using namespace std;
5 const char el = '\n';
6 typedef long long ll;
7
8 const ll inf = 1e18;
9 struct node {
10     node *ls, *rs;
11     int pl, pm, pr;
12     int xl, xm, xr;
13     ll k, b;
14     int tag;
15     node(int l, int r) {
16         pl = l, pr = r, pm = l + (r - l >> 1);
17         xl = xm = xr = 0;
18         k = 0, b = 0;
19         tag = 0;
20         if (l == r) {
21             ls = rs = NULL;
22             return;
23         }
24         ls = new node(l, pm);
25         rs = new node(pm + 1, r);
26     }
27     void addtag(int t) {
28         tag += t;
29         b -= k * t;
30         xl += t, xm += t, xr += t;
31     }
32     void pushtag() {
33         if (ls) ls->addtag(tag);
34         if (rs) rs->addtag(tag);
35         tag = 0;
36     }

```

```

37     void addfunc(int l, int r, ll _k, ll _b) {
38         if (r < pl || l > pr) return;
39         pushtag();
40         if (l <= pl && r >= pr) {
41             if (k * xm + b < _k * xm + _b) {
42                 swap(k, _k);
43                 swap(b, _b);
44             }
45             if (k * xl + b < _k * xl + _b && ls) ls->addfunc(l, r, _k, _b);
46             if (k * xr + b < _k * xr + _b && rs) rs->addfunc(l, r, _k, _b);
47             return;
48         }
49         if (ls) ls->addfunc(l, r, _k, _b);
50         if (rs) rs->addfunc(l, r, _k, _b);
51     }
52     void addx(int p, int v) {
53         if (pr < p) return;
54         if (pl >= p) {
55             addtag(v);
56             return;
57         }
58         if (pl >= p) xl += v;
59         if (pm >= p) xm += v;
60         if (pr >= p) xr += v;
61         if (ls) ls->addfunc(pl, pr, k, b);
62         if (rs) rs->addfunc(pl, pr, k, b);
63         pushtag();
64         if (ls) ls->addx(p, v);
65         if (rs) rs->addx(p, v);
66         k = 0, b = 0;
67     }
68     optional<pair<int, ll>> query(int p) {
69         if (p < pl || p > pr) return nullopt;
70         if (pl == pr) return pair<int, ll>{xm, k * xm + b};
71         pushtag();
72         auto res = ls->query(p);
73         if (!res) res = rs->query(p);
74         auto [x, y] = *res;
75         return pair<int, ll>{x, max(y, k * x + b)};
76     }
77 };
78 int main() {
79     ios::sync_with_stdio(false);
80     cin.tie(0);
81     cout << setprecision(15);

```

```

82 int tt;
83 cin >> tt;
84 while (tt--) {
85     int n, m;
86     cin >> n >> m;
87     vector<vector<int>> a(m + 1);
88     vector<int> c(m + 1);
89     while (n--) {
90         int l, r;
91         cin >> l >> r;
92         a[l].push_back(r);
93         c[r]++;
94     }
95     vector<ll> p(m + 1);
96     for (int i = 1; i <= m; i++) cin >> p[i];
97     node *rt = new node(0, m + 1);
98     for (int i = 1; i <= m; i++) {
99         auto [x, y] = *rt->query(i);
100         rt->addx(i, -c[i - 1]);
101         sort(a[i].begin(), a[i].end());
102         for (auto j : a[i]) rt->addx(j + 1, 1);
103         rt->addfunc(0, m + 1, p[i], y);
104     }
105     cout << rt->query(m + 1)->second << endl;
106 }
107 return 0;
108 }
109 // https://codeforces.com/contest/1830/problem/F

```

## 1.4 可持久化线段树

```

1 // ver 2.0
2 struct node {
3     int l, r;
4     node *ls, *rs;
5     int tag;
6     node() : l(0), r(0), ls(nullptr), rs(nullptr), tag(0) {}
7     static void* operator new(size_t count) {
8         static node *begin = nullptr, *end = nullptr;
9         if (begin == end) begin = (node*)malloc(count * 1000), end = begin + 1000;
10        return begin++;
11    }
12 };
13 node* build(int l, int r) {

```

```

14     node* u = new node();
15     u->l = l, u->r = r;
16     if (l == r) return u;
17     int m = l + (r - l) / 2;
18     u->ls = build(l, m);
19     u->rs = build(m + 1, r);
20     return u;
21 }
22 node* addtag(node* u, int val) {
23     u = new node(*u);
24     u->tag += val;
25     return u;
26 }
27 node* pushdown(node* u) {
28     u = new node(*u);
29     if (u->tag) {
30         u->ls = addtag(u->ls, u->tag);
31         u->rs = addtag(u->rs, u->tag);
32         u->tag = 0;
33     }
34     return u;
35 }
36 node* add(node* u, int l, int r, int val) {
37     if (u->l > r || u->r < l) return u;
38     if (l <= u->l && u->r <= r) return addtag(u, val);
39     u = pushdown(u);
40     u->ls = add(u->ls, l, r, val);
41     u->rs = add(u->rs, l, r, val);
42     return u;
43 }
44 node* merge(node* u, node* v, int p) {
45     if (u->r <= p) return u;
46     if (v->l > p) return v;
47     u = pushdown(u), v = pushdown(v);
48     node* w = new node(*u);
49     w->ls = merge(u->ls, v->ls, p);
50     w->rs = merge(u->rs, v->rs, p);
51     return w;
52 }
53 int val(node* u, int p) {
54     if (u->r < p || u->l > p) return 0;
55     if (u->l == u->r) return u->tag;
56     return u->tag + val(u->ls, p) + val(u->rs, p);
57 }

```

## 1.5 区间处理

```
1 #include <bits/stdc++.h>
2 using namespace std;
3 typedef long long ll;
4 struct range {
5     int l, r;
6     bool valid() const { return r >= l; }
7     bool cover(range rg) const { return l <= rg.l && r >= rg.r; }
8     bool cross(range rg) const { return l <= rg.r && r >= rg.l; }
9 };
10 bool operator<(range a, range b) {
11     return a.l < b.l || a.l == b.l && a.r > b.r;
12 }
13 struct rngseq {
14     static const int inf = 1e9 + 7;
15     set<range> seq;
16     rngseq() : seq({{-inf, -inf}, {inf, inf}}) {}
17     void add(range rg) {
18         if (!rg.valid()) return;
19         seq.insert(rg);
20         rt->qadd(rg.l, rg.r, 1);
21     }
22     void rmv(range rg) {
23         if (!rg.valid()) return;
24         seq.erase(rg);
25         rt->qadd(rg.l, rg.r, -1);
26     }
27     void insert(range rg) {
28         auto it = seq.upper_bound(rg);
29         auto tmp = *prev(it);
30         if (tmp.cover(rg)) return;
31         if (tmp.cross(rg)) {
32             rg.l = tmp.l;
33             rmv(tmp);
34         }
35         while (rg.cross(*it)) {
36             tmp = *it++;
37             rg.r = max(rg.r, tmp.r);
38             rmv(tmp);
39         }
40         add(rg);
41     }
42     void erase(range rg) {
43         auto it = seq.upper_bound(rg);
```

```
44         auto tmp = *prev(it);
45         if (tmp.cover(rg)) {
46             rmv(tmp);
47             add({tmp.l, rg.l - 1}), add({rg.r + 1, tmp.r});
48             return;
49         }
50         if (tmp.cross(rg)) {
51             rmv(tmp);
52             add({tmp.l, rg.l - 1});
53         }
54         while (rg.cross(*it)) {
55             tmp = *it++;
56             rmv(tmp);
57             add({rg.r + 1, tmp.r});
58         }
59     }
60 };
```

## 1.6 圆方树

```
1 #include <algorithm>
2 #include <cstdio>
3 #include <vector>
4
5 const int MN = 100005;
6
7 int N, M, cnt;
8 std::vector<int> G[MN], T[MN * 2];
9
10 int dfn[MN], low[MN], dfc;
11 int stk[MN], tp;
12
13 void Tarjan(int u) {
14     printf(" Enter : %d\n", u);
15     low[u] = dfn[u] = ++dfc; // low 初始化为当前节点 dfn
16     stk[++tp] = u; // 加入栈中
17     for (int v : G[u]) { // 遍历 u 的相邻节点
18         if (!dfn[v]) { // 如果未访问过
19             Tarjan(v); // 递归
20             low[u] = std::min(low[u], low[v]); // 未访问的和 low 取 min
21             if (low[v] == dfn[u]) { // 标志着找到一个以 u 为根的点双连通分量
22                 ++cnt; // 增加方点个数
23                 printf(" Found a New BCC %d.\n", cnt - N);
24                 // 将点双中除了 u 的点退栈，并在圆方树中连边
```

```

25     for (int x = 0; x != v; --tp) {
26         x = stk[tp];
27         T[cnt].push_back(x);
28         T[x].push_back(cnt);
29         printf("    BCC %d has vertex %d\n", cnt - N, x);
30     }
31     // 注意 u 自身也要连边 (但不退栈)
32     T[cnt].push_back(u);
33     T[u].push_back(cnt);
34     printf("    BCC %d has vertex %d\n", cnt - N, u);
35 }
36 } else
37     low[u] = std::min(low[u], dfn[v]); // 已访问的和 dfn 取 min
38 }
39 printf(" Exit : %d : low = %d\n", u, low[u]);
40 printf(" Stack:\n    ");
41 for (int i = 1; i <= tp; ++i) printf("%d, ", stk[i]);
42 puts("");
43 }
44
45 int main() {
46     scanf("%d%d", &N, &M);
47     cnt = N; // 点双 / 方点标号从 N 开始
48     for (int i = 1; i <= M; ++i) {
49         int u, v;
50         scanf("%d%d", &u, &v);
51         G[u].push_back(v); // 加双向边
52         G[v].push_back(u);
53     }
54     // 处理非连通图
55     for (int u = 1; u <= N; ++u)
56         if (!dfn[u]) Tarjan(u), --tp;
57     // 注意到退出 Tarjan 时栈中还有一个元素即根, 将其退栈
58     return 0;
59 }

```

## 1.7 线段树

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 typedef long long ll;
4
5 const ll inf = 1e18;
6 struct node {

```

```

7     node *ls, *rs;
8     int l, r;
9     ll val, sum;
10    pair<ll, int> mxm, mnm;
11    static node *newnd() {
12        static const int buff = 1000;
13        static node *ptr = new node[buff], *cur = ptr;
14        if (cur == ptr + buff) ptr = new node[buff], cur = ptr;
15        return cur++;
16    }
17    void update() {
18        sum = ls->sum + rs->sum;
19        mxm = max(ls->mxm, rs->mxm);
20        mnm = min(ls->mnm, rs->mnm);
21    }
22    void build(int _l, int _r, vector<ll> &a) {
23        l = _l, r = _r;
24        if (l == r) {
25            val = a[l];
26            mxm = mnm = {val, l};
27            sum = abs(val);
28            return;
29        }
30        int m = (l + r) / 2;
31        ls = new node(), ls->build(l, m, a);
32        rs = new node(), rs->build(m + 1, r, a);
33        update();
34    }
35    void modify(int p, int a) {
36        if (p < l || p > r) return;
37        if (l == r) {
38            val = val + a;
39            mxm = mnm = {val, l};
40            sum = abs(val);
41            return;
42        }
43        ls->modify(p, a), rs->modify(p, a);
44        update();
45    }
46    int ppos(int _l, int _r) {
47        if (_r < l || _l > r) return -1;
48        if (_l <= l && _r >= r) {
49            if (mxm.first < 0) return -1;
50            if (l == r) return l;
51        }

```

```

52     auto res = ls->ppos(_l, _r);
53     if (~res)
54         return res;
55     else
56         return rs->ppos(_l, _r);
57 }
58 pair<ll, int> qmxm(int _l, int _r) {
59     if (_r < 1 || _l > r) return {-inf, -1};
60     if (_l <= 1 && _r >= r) return mxm;
61     return max(ls->qmxm(_l, _r), rs->qmxm(_l, _r));
62 }
63 pair<ll, int> qnmn(int _l, int _r) {
64     if (_r < 1 || _l > r) return {inf, -1};
65     if (_l <= 1 && _r >= r) return mnm;
66     return min(ls->qnmn(_l, _r), rs->qnmn(_l, _r));
67 }
68 ll qsum(int _l, int _r) {
69     if (_r < 1 || _l > r) return 0;
70     if (_l <= 1 && _r >= r) return sum;
71     return ls->qsum(_l, _r) + rs->qsum(_l, _r);
72 }
73 };
74 auto newnd = node::newnd;
75
76 const ll inf = 1e18;
77 int ls(int u) { return u << 1; }
78 int rs(int u) { return u << 1 | 1; }
79 struct segtree {
80     vector<ll> mnm;
81     int l, r;
82     void build(int l_, int r_) {
83         l = l_, r = r_;
84         int k = 1;
85         while (k < r - l + 1) k <<= 1;
86         mnm.resize(k << 1);
87     }
88     void update(int u) { mnm[u] = min(mnm[ls(u)], mnm[rs(u)]); }
89     void init(int mode, int l = -1, int r = -1, int u = 1) {
90         if (!~l) l = this->l, r = this->r;
91         if (l == r) {
92             mnm[u] = (~mode ? mode ? inf : l : -1);
93             return;
94         }
95         int m = l + (r - l) / 2;
96         init(mode, l, m, ls(u));

```

```

97         init(mode, m + 1, r, rs(u));
98         update(u);
99     }
100 void modify(int p, ll a, int l = -1, int r = -1, int u = 1) {
101     if (!~l) l = this->l, r = this->r;
102     if (p < l || p > r) return;
103     if (l == r) {
104         mnm[u] = a;
105         return;
106     }
107     int m = l + (r - l) / 2;
108     modify(p, a, l, m, ls(u));
109     modify(p, a, m + 1, r, rs(u));
110     update(u);
111 }
112 ll qnmn(int l_, int r_, int l = -1, int r = -1, int u = 1) {
113     if (!~l) l = this->l, r = this->r;
114     if (r_ < l || l_ > r) return inf;
115     if (l_ <= l && r_ >= r) return mnm[u];
116     int m = l + (r - l) / 2;
117     return min(qnmn(l_, r_, l, m, ls(u)), qnmn(l_, r_, m + 1, r, rs(u)));
118 }
119 };

```

## 1.8 Treap

```

1 struct node {
2     array<node *, 2> ch;
3     cplx val, sum;
4     int rank;
5     node(cplx val) : val(val) {
6         ch[0] = ch[1] = NULL;
7         sum = val;
8         rank = rand();
9     }
10    void update() {
11        sum = val;
12        if (ch[0]) sum = ch[0]->sum + sum;
13        if (ch[1]) sum = sum + ch[1]->sum;
14    }
15 };
16 bool operator<(cplx a, cplx b) {
17     auto d = det(a, b);
18     if (d) return d < 0;

```



```

19     if (a.x != b.x) return a.x < b.x;
20     if (a.y != b.y) return a.y < b.y;
21     if (a.z != b.z) return a.z < b.z;
22     return false;
23 }
24 void rotate(node *&u, int c) {
25     auto v = u->ch[c];
26     u->ch[c] = v->ch[!c];
27     v->ch[!c] = u;
28     u->update();
29     v->update();
30     u = v;
31 }
32 node *insert(node *rt, cplx val) {
33     if (!rt) return new node(val);
34     auto c = rt->val < val;
35     rt->ch[c] = insert(rt->ch[c], val);
36     if (rt->ch[c]->rank < rt->rank) rotate(rt, c);
37     rt->update();
38     return rt;
39 }
40 void erase(node *&rt, cplx val) {
41     if (val < rt->val) {
42         erase(rt->ch[0], val);
43         rt->update();
44         return;
45     }
46     if (rt->val < val) {
47         erase(rt->ch[1], val);
48         rt->update();
49         return;
50     }
51     auto tmp = rt;
52     if (rt->ch[0]) {
53         if (rt->ch[1]) {
54             auto c = rt->ch[0]->rank > rt->ch[1]->rank;
55             rotate(rt, c);
56             erase(rt->ch[!c], val);
57             rt->update();
58         } else {
59             rt = rt->ch[0];
60             delete tmp;
61         }
62     } else {
63         if (rt->ch[1]) {

```

```

64             rt = rt->ch[1];
65             delete tmp;
66         } else {
67             rt = NULL;
68             delete tmp;
69         }
70     }
71 }
72
73
74 struct node {
75     array<node *, 2> ch;
76     ui p, n, avg;
77     ui sl, sr, tag, sum;
78     ui rank;
79     node(int l, int r) {
80         ch[0] = ch[1] = NULL;
81         sl = p = l, sr = n = r;
82         avg = tag = sum = 0;
83         rank = rand();
84     }
85     void addtag(ui t) {
86         tag += t, avg += t;
87         sum += (sr - sl) * t;
88     }
89     void pushdown() {
90         if (ch[0]) ch[0]->addtag(tag);
91         if (ch[1]) ch[1]->addtag(tag);
92         tag = 0;
93     }
94     void update() {
95         sum = (n - p) * avg;
96         sl = ch[0] ? ch[0]->sl : p;
97         sr = ch[1] ? ch[1]->sr : n;
98         if (ch[0]) sum += ch[0]->sum;
99         if (ch[1]) sum += ch[1]->sum;
100     }
101     void add(ui l, ui r, ui v) {
102         if (l >= sr || r <= sl) return;
103         if (l <= sl && r >= sr) {
104             addtag(v);
105             return;
106         }
107         pushdown();
108         if (l <= p && r >= n) avg += v;

```

```

109     if (ch[0]) ch[0]->add(1, r, v);
110     if (ch[1]) ch[1]->add(1, r, v);
111     update();
112 }
113 ui qsum(ui l, ui r) {
114     if (l >= sr || r <= sl) return 0;
115     if (l <= sl && r >= sr) return sum;
116     pushdown();
117     ui res = 0;
118     if (l <= p && r >= n) res += (n - p) * avg;
119     if (ch[0]) res += ch[0]->qsum(l, r);
120     if (ch[1]) res += ch[1]->qsum(l, r);
121     return res;
122 }
123 };
124 node *merge(node *u, node *v) {
125     if (!u) return v;
126     if (!v) return u;
127     if (u->rank < v->rank) {
128         u->pushdown();
129         u->ch[1] = merge(u->ch[1], v);
130         u->update();
131         return u;
132     } else {
133         v->pushdown();
134         v->ch[0] = merge(u, v->ch[0]);
135         v->update();
136         return v;
137     }
138 }
139 pair<node *, node *> split(node *u, ui p) {
140     if (!u) return {NULL, NULL};
141     u->pushdown();
142     if (u->p <= p) {
143         auto [l, r] = split(u->ch[1], p);
144         u->ch[1] = l;
145         u->update();
146         return {u, r};
147     } else {
148         auto [l, r] = split(u->ch[0], p);
149         u->ch[0] = r;
150         u->update();
151         return {l, u};
152     }
153 }

```

```

154 node *insert(node *u, ui p) {
155     auto [l, r] = split(u, p);
156     auto *v = l;
157     while (v->ch[1]) v = v->ch[1];
158     if (v->p == p) return merge(l, r);
159     auto w = new node(p, v->n);
160     v->n = p;
161     w->avg = v->avg;
162     v->update(), w->update();
163     w = merge(l, w), w = merge(w, r);
164     return w;
165 }

```

## 1.9 Trie

```

1
2 struct node {
3     array<node *, alpha> ch;
4     array<node *, sizf> fa;
5     int dep;
6     ll val;
7     node() {
8         for (auto &v : ch) v = NULL;
9         for (auto &v : fa) v = NULL;
10        dep = 0;
11        val = 0;
12    }
13 };
14 node *rt;
15 struct cmp {
16     bool operator()(node *a, node *b) {
17         if (a == b) return false;
18         if (!a) return true;
19         if (!b) return false;
20         if (a->dep != b->dep) return a->dep > b->dep;
21         for (int i = sizf - 1; i >= 0; i--)
22             if (a->fa[i] != b->fa[i]) a = a->fa[i], b = b->fa[i];
23         node *u = a->fa[0];
24         for (int i = 0; i < alpha; i++) {
25             if (a == u->ch[i]) return false;
26             if (b == u->ch[i]) return true;
27         }
28     }
29     bool operator()(pair<node *, int> a, pair<node *, int> b) {

```

```

30     return cmp()(a.first, b.first);
31 }
32 };
33 node *add(node *u, int n) {
34     if (n / 10) u = add(u, n / 10);
35     n %= 10;
36     if (u->ch[n]) return u->ch[n];
37     u->ch[n] = new node();
38     auto v = u->ch[n];
39     v->fa[0] = u;
40     for (int i = 1; i < sizf; i++) v->fa[i] = v->fa[i - 1]->fa[i - 1];
41     v->dep = u->dep + 1;
42     v->val = (u->val * 10 + n) % mod;
43     return v;
44 }

```

## 1.10 虚树

```

1 inline bool cmp(const int x, const int y) { return id[x] < id[y]; }
2
3 void build() {
4     sort(h + 1, h + k + 1, cmp);
5     sta[top = 1] = 1, g.sz = 0, g.head[1] = -1;
6     // 1 号节点入栈, 清空 1 号节点对应的邻接表, 设置邻接表边数为 1
7     for (int i = 1, l; i <= k; ++i)
8         if (h[i] != 1) {
9             // 如果 1 号节点是关键节点就不要重复添加
10            l = lca(h[i], sta[top]);
11            // 计算当前节点与栈顶节点的 LCA
12            if (l != sta[top]) {
13                // 如果 LCA 和栈顶元素不同, 则说明当前节点不再当前栈所存的链上
14                while (id[l] < id[sta[top - 1]])
15                    // 当次大节点的 Dfs 序大于 LCA 的 Dfs 序
16                    g.push(sta[top - 1], sta[top]), top--;
17                // 把与当前节点所在的链不重合的链连接掉并且弹出
18                if (id[l] > id[sta[top - 1]])
19                    // 如果 LCA 不等于次大节点 (这里的大于其实和不等于是没有区别)
20                    g.head[l] = -1, g.push(l, sta[top]), sta[top] = l;
21                // 说明 LCA 是第一次入栈, 清空其邻接表, 连边后弹出栈顶元素, 并将 LCA
22                // 入栈
23                else
24                    g.push(l, sta[top--]);
25                // 说明 LCA 就是次大节点, 直接弹出栈顶元素
26            }

```

```

27     g.head[h[i]] = -1, sta[++top] = h[i];
28     // 当前节点必然是第一次入栈, 清空邻接表并入栈
29 }
30 for (int i = 1; i < top; ++i)
31     g.push(sta[i], sta[i + 1]); // 剩余的最后一条链连接一下
32 return;
33 }

```

## 2 博弈

### 2.1 国际象棋

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 const char el = '\n';
4 typedef long long ll;
5 typedef unsigned long long ull;
6 array<ull, 64> rook, bishop, knight, queen, arch, chan, maha;
7 void write(ull t, ull p) {
8     for (int i = 0; i < 8; i++, cout << el)
9         for (int j = 0; j < 8; j++) {
10             int x = i << 3 | j;
11             cout << (p >> x & 1 ? "o " : t >> x & 1 ? " " : "x ");
12         }
13     cout << el;
14 }
15 void init() {
16     for (int i = 0; i < 8; i++)
17         for (int j = 0; j < 8; j++) {
18             auto &t = rook[i << 3 | j];
19             t = 0;
20             for (int x = 7; x >= 0; x--)
21                 for (int y = 7; y >= 0; y--) t = t << 1 | (x == i || y == j);
22         }
23     for (int i = 0; i < 8; i++)
24         for (int j = 0; j < 8; j++) {
25             auto &t = bishop[i << 3 | j];
26             t = 0;
27             for (int x = 7; x >= 0; x--)
28                 for (int y = 7; y >= 0; y--)
29                     t = t << 1 | (i + j == x + y || i - j == x - y);
30         }
31     for (int i = 0; i < 8; i++)

```

```

32     for (int j = 0; j < 8; j++) {
33         auto &t = knight[i << 3 | j];
34         t = 0;
35         for (int x = 7; x >= 0; x--)
36             for (int y = 7; y >= 0; y--)
37                 t = t << 1 | ((x != i && y != j && abs(x - i) + abs(y - j) == 3) ||
38                     x == i && y == j);
39     }
40     for (int i = 0; i < 64; i++) queen[i] = rook[i] | bishop[i];
41     for (int i = 0; i < 64; i++) arch[i] = bishop[i] | knight[i];
42     for (int i = 0; i < 64; i++) chan[i] = rook[i] | knight[i];
43     for (int i = 0; i < 64; i++) maha[i] = queen[i] | knight[i];
44 }
45 array<ull, 64> piece(char ch) {
46     switch (ch) {
47         case 'R':
48             return rook;
49         case 'B':
50             return bishop;
51         case 'Q':
52             return queen;
53         case 'A':
54             return arch;
55         case 'C':
56             return chan;
57         case 'M':
58             return maha;
59     }
60 }
61 int cnt = 100;
62 bool win(string s, int p, ull safe, ull used) {
63     if (p == s.size()) return false;
64     bool res = false;
65     const auto &pce = piece(s[p]);
66     for (int i = 0; i < 64 && !res; i++)
67         if ((safe >> i & 1) && !(used & pce[i]))
68             res = res || !win(s, p + 1, safe & ~pce[i], used | (1ull << i));
69     return res;
70 }
71 int main() {
72     ios::sync_with_stdio(false);
73     cin.tie(0);
74     cout << setprecision(15);
75     init();
76     string s;

```

```

77     cin >> s;
78     cout << (win(s, 0, -1ull, 0) ? "Alice" : "Bob") << endl;
79     return 0;
80 }
81 // https://contest.ucup.ac/contest/1399/problem/7635

```

## 2.2 Nimber

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  const char el = '\n';
4  typedef long long ll;
5  typedef unsigned long long ull;
6  array<ull, 64> rook, bishop, knight, queen, arch, chan, maha;
7  void write(ull t, ull p) {
8      for (int i = 0; i < 8; i++, cout << el)
9          for (int j = 0; j < 8; j++) {
10             int x = i << 3 | j;
11             cout << (p >> x & 1 ? "o " : t >> x & 1 ? " " : "x ");
12         }
13     cout << el;
14 }
15 void init() {
16     for (int i = 0; i < 8; i++)
17         for (int j = 0; j < 8; j++) {
18             auto &t = rook[i << 3 | j];
19             t = 0;
20             for (int x = 7; x >= 0; x--)
21                 for (int y = 7; y >= 0; y--) t = t << 1 | (x == i || y == j);
22         }
23     for (int i = 0; i < 8; i++)
24         for (int j = 0; j < 8; j++) {
25             auto &t = bishop[i << 3 | j];
26             t = 0;
27             for (int x = 7; x >= 0; x--)
28                 for (int y = 7; y >= 0; y--)
29                     t = t << 1 | (i + j == x + y || i - j == x - y);
30         }
31     for (int i = 0; i < 8; i++)
32         for (int j = 0; j < 8; j++) {
33             auto &t = knight[i << 3 | j];
34             t = 0;
35             for (int x = 7; x >= 0; x--)
36                 for (int y = 7; y >= 0; y--)

```

```

37         t = t << 1 | ((x != i && y != j && abs(x - i) + abs(y - j) == 3) ||
38             x == i && y == j);
39     }
40     for (int i = 0; i < 64; i++) queen[i] = rook[i] | bishop[i];
41     for (int i = 0; i < 64; i++) arch[i] = bishop[i] | knight[i];
42     for (int i = 0; i < 64; i++) chan[i] = rook[i] | knight[i];
43     for (int i = 0; i < 64; i++) maha[i] = queen[i] | knight[i];
44 }
45 array<ull, 64> piece(char ch) {
46     switch (ch) {
47         case 'R':
48             return rook;
49         case 'B':
50             return bishop;
51         case 'Q':
52             return queen;
53         case 'A':
54             return arch;
55         case 'C':
56             return chan;
57         case 'M':
58             return maha;
59     }
60 }
61 int cnt = 100;
62 bool win(string s, int p, ull safe, ull used) {
63     if (p == s.size()) return false;
64     bool res = false;
65     const auto &pce = piece(s[p]);
66     for (int i = 0; i < 64 && !res; i++)
67         if ((safe >> i & 1) && !(used & pce[i]))
68             res = res || !win(s, p + 1, safe & ~pce[i], used | (1ull << i));
69     return res;
70 }
71 int main() {
72     ios::sync_with_stdio(false);
73     cin.tie(0);
74     cout << setprecision(15);
75     init();
76     string s;
77     cin >> s;
78     cout << (win(s, 0, -1ull, 0) ? "Alice" : "Bob") << el;
79     return 0;
80 }
81 // https://contest.ucup.ac/contest/1399/problem/7635

```

## 2.3 一般博弈

```

1
2 unordered_map<ll, int> result;
3 struct status {
4     array<int, 10> a;
5     status() { fill(a.begin(), a.end(), 0); }
6     status flip() {
7         status b;
8         for (int i = 0; i < 5; i++) b.a[i] = a[i + 5];
9         for (int i = 0; i < 5; i++) b.a[i + 5] = a[i];
10        return b;
11    }
12    ll num() {
13        ll res = 0;
14        for (int i = 0; i < 10; i++) res = (res << 5) + a[i];
15        return res;
16    }
17    vector<status> from() {
18        vector<status> res;
19        for (int i = 0; i < 5; i++)
20            if (a[i])
21                for (int j = 0; j < 5; j++)
22                    if (a[j + 5])
23                        if (i && j - i) {
24                            auto b = *this;
25                            b.a[j + 5]--;
26                            b.a[(j + 5 - i) % 5 + 5]++;
27                            res.push_back(b.flip());
28                        }
29        // cout << "from " << res.size() << el;
30        return res;
31    }
32    vector<status> to() {
33        vector<status> res;
34        for (int i = 0; i < 5; i++)
35            if (a[i])
36                for (int j = 0; j < 5; j++)
37                    if (a[j + 5])
38                        if (i && j) {
39                            auto b = *this;
40                            b.a[i]--;

```

```

41         b.a[(i + j) % 5]++;
42         res.push_back(b.flip());
43     }
44     // cout << "to: " << res.size() << el;
45     return res;
46 }
47 bool win() {
48     if (result.count(num())) return result[num()] == 1;
49     for (auto v : to())
50         if (result.count(v.num()) && result[v.num()] == -1) return true;
51     return false;
52 }
53 bool lose() {
54     if (result.count(num())) return result[num()] == -1;
55     for (auto v : to())
56         if (!result.count(v.num()) || result[v.num()] != 1) return false;
57     return true;
58 }
59 };
60 bool operator<(const status &a, const status &b) { return a.a < b.a; }
61 status read() {
62     status a;
63     for (int i = 0; i < 8; i++) {
64         int t;
65         cin >> t;
66         a.a[t]++;
67     }
68     for (int i = 0; i < 8; i++) {
69         int t;
70         cin >> t;
71         a.a[t + 5]++;
72     }
73     return a;
74 }
75
76 set<status> losestat(int t) {
77     if (!t) {
78         status tmp;
79         tmp.a[5] = 8;
80         return {tmp};
81     }
82     set<status> res;
83     auto tmp = losestat(t - 1);
84     for (auto v : tmp) {
85         for (int i = 0; i < 5; i++) {

```

```

86         auto t = v;
87         t.a[i]++;
88         res.insert(t);
89     }
90 }
91 return res;
92 }
93 set<status> winstat(int t) {
94     if (!t) {
95         status tmp;
96         tmp.a[0] = 8;
97         return {tmp};
98     }
99     set<status> res;
100    auto tmp = winstat(t - 1);
101    for (auto v : tmp) {
102        for (int i = 0; i < 5; i++) {
103            auto t = v;
104            t.a[i + 5]++;
105            res.insert(t);
106        }
107    }
108    return res;
109 }
110 void preprocess() {
111     auto wst = winstat(8), lst = losestat(8);
112     vector<status> win, lose;
113     for (auto v : wst) {
114         win.push_back(v);
115         result[v.num()] = 1;
116     }
117     for (auto v : lst) {
118         lose.push_back(v);
119         result[v.num()] = -1;
120     }
121     while (win.size() || lose.size()) {
122         if (win.size()) {
123             for (auto wn : win)
124                 for (auto ls : wn.from()) {
125                     if (!result.count(ls.num()) && ls.lose()) {
126                         lose.push_back(ls);
127                         result[ls.num()] = -1;
128                     }
129                 }
130             win.clear();

```

```

131     } else {
132         for (auto ls : lose)
133             for (auto wn : ls.from()) {
134                 if (!result.count(wn.num()) && wn.win()) {
135                     win.push_back(wn);
136                     result[wn.num()] = 1;
137                 }
138             }
139         lose.clear();
140     }
141 }
142 }

```

## 3 图论

### 3.1 点分树

```

1 #include "bits/stdc++.h"
2 using namespace std;
3 const char el = '\n';
4 typedef long long ll;
5 const ll inf = 1e18;
6 struct graph {
7     vector<vector<pair<int, ll>>> e;
8     vector<ll> r;
9     graph(int n) : e(n + 1), r(n + 1) {}
10    void adde(int u, int v, ll w) {
11        e[u].push_back({v, w});
12        e[v].push_back({u, w});
13    }
14    vector<int> siz;
15    int calcsiz(int u, int f) {
16        siz[u] = 1;
17        for (auto [v, w] : e[u])
18            if (v != f && ~siz[v]) siz[u] += calcsiz(v, u);
19        return siz[u];
20    }
21    vector<vector<pair<int, ll>>> fa;
22    vector<vector<pair<ll, int>>> ch;
23    void init(int u, int f, int rt, ll d) {
24        fa[u].push_back({rt, d});
25        ch[rt].push_back({r[u] - d, u});
26        for (auto [v, w] : e[u])

```

```

27         if (v != f && ~siz[v]) init(v, u, rt, d + w);
28    }
29    int split(int u) {
30        int n = calcsiz(u, u) / 2;
31        while (true) {
32            bool flg = true;
33            for (auto [v, w] : e[u])
34                if (siz[v] < siz[u] && siz[v] > n) {
35                    u = v;
36                    flg = false;
37                    break;
38                }
39            if (flg) break;
40        }
41        siz[u] = -1;
42        init(u, u, u, 0);
43        sort(ch[u].begin(), ch[u].end());
44        for (auto [v, w] : e[u])
45            if (~siz[v]) v = split(v);
46        return u;
47    }
48    int rt;
49    void ctrdecomp() {
50        int n = e.size();
51        siz.assign(n, 0);
52        fa.assign(n, {});
53        ch.assign(n, {});
54        rt = split(1);
55    }
56    vector<int> solve() {
57        vector<int> dis(e.size(), -1);
58        dis[1] = 0;
59        queue<int> que;
60        que.push(1);
61        while (que.size()) {
62            int u = que.front();
63            que.pop();
64            for (auto [f, d] : fa[u]) {
65                while (ch[f].size()) {
66                    auto [w, v] = ch[f].back();
67                    if (d > w) break;
68                    ch[f].pop_back();
69                    if (~dis[v]) continue;
70                    que.push(v);
71                    dis[v] = dis[u] + 1;

```

```

72     }
73     }
74     }
75     return dis;
76 }
77 };
78 int main() {
79     ios::sync_with_stdio(false);
80     cin.tie(0);
81     cout << setprecision(15);
82     int tt;
83     cin >> tt;
84     while (tt--) {
85         int n;
86         cin >> n;
87         graph g(n);
88         for (int i = 2; i <= n; i++) cin >> g.r[i];
89         for (int i = 1; i < n; i++) {
90             ll u, v, w;
91             cin >> u >> v >> w;
92             g.adde(u, v, w);
93         }
94         g.ctrdecomp();
95         auto res = g.solve();
96         for (int i = 2; i <= n; i++) cout << res[i] << ' ';
97         cout << endl;
98     }
99     return 0;
100 }

```

## 3.2 树链剖分

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 struct graph {
4     vector<vector<int>> e;
5     vector<int> fa, dep, siz, hvs, top, dfn, rnk;
6     vector<bit> seg;
7     graph(int n) : seg(maxd, n) {
8         e.resize(n + 1);
9         fa.resize(n + 1);
10        dep.resize(n + 1);
11        siz.resize(n + 1);
12        hvs.resize(n + 1);

```

```

13        top.resize(n + 1);
14        dfn.resize(n + 1);
15        rnk.resize(n + 1);
16    }
17    void adde(int u, int v) {
18        e[u].push_back(v);
19        e[v].push_back(u);
20    }
21    void subts(int u, int f, int d) {
22        fa[u] = f, dep[u] = d;
23        siz[u] = 1;
24        hvs[u] = -1;
25        for (auto v : e[u])
26            if (v != f) {
27                subts(v, u, d + 1);
28                siz[u] += siz[v];
29                if (!hvs[u] || siz[v] > siz[hvs[u]]) hvs[u] = v;
30            }
31    }
32    int tot;
33    void ordfs(int u, int t) {
34        top[u] = t;
35        tot++, rnk[tot] = u, dfn[u] = tot;
36        if (~hvs[u]) ordfs(hvs[u], t);
37        for (auto v : e[u])
38            if (v != fa[u] && v != hvs[u]) ordfs(v, v);
39    }
40    void hvydecomp(int rt) {
41        subts(rt, -1, 0);
42        tot = 0;
43        ordfs(rt, rt);
44    }
45    void modify(int u, int v, int k, int d) {
46        while (top[u] != top[v]) {
47            if (dep[top[u]] < dep[top[v]]) swap(u, v);
48            seg[d].add(dfn[top[u]], dfn[u], k);
49            u = fa[top[u]];
50        }
51        if (dep[u] < dep[v]) swap(u, v);
52        seg[d].add(dfn[v], dfn[u], k);
53        u = v;
54        while (d--) {
55            seg[d].add(dfn[u], dfn[u], k);
56            u = fa[u];
57            seg[d].add(dfn[u], dfn[u], k);

```



```

58     }
59 }
60 int query(int u) {
61     int ans = 0;
62     for (int i = 0; i < maxd; i++) {
63         ans += seg[i].query(dfn[u]);
64         u = fa[u];
65     }
66     return ans;
67 }
68 };

```

### 3.3 费用流

```

1 #pragma once
2
3 #include <algorithm>
4 #include <optional>
5 #include <queue>
6 #include <vector>
7
8 namespace costflow {
9
10 typedef long long flow_t;
11 typedef long long cost_t;
12
13 const flow_t inf_flow = 1e18;
14 const cost_t inf_cost = 1e18;
15
16 struct CostEdge {
17     int from, to;
18     cost_t cost;
19     flow_t cap, low = 0, flow = 0;
20 };
21
22 int num_node(const std::vector<CostEdge> &edges) {
23     int n = 0;
24     for (const auto &e : edges) n = std::max({n, e.from, e.to});
25     return n;
26 }
27
28 std::pair<flow_t, cost_t> get_flow(const std::vector<CostEdge> &edges, int s) {
29     flow_t flow = 0;
30     cost_t cost = 0;
31     for (const auto &e : edges) {

```

```

31         if (e.from == s) flow += e.flow;
32         cost += e.flow * e.cost;
33     }
34     return {flow, cost};
35 }
36
37 struct CostFlow {
38     struct Edge {
39         int from, to;
40         cost_t cost;
41         flow_t cap;
42     };
43     int n;
44     std::vector<std::vector<int>> eid;
45     std::vector<Edge> edge;
46     void build(const std::vector<CostEdge> &edges) {
47         n = num_node(edges);
48         eid.assign(n + 1, {});
49         edge.clear();
50         int num_edges = 0;
51         for (const auto &e : edges) {
52             eid[e.from].push_back(num_edges++);
53             edge.push_back({e.from, e.to, e.cost, e.cap - e.flow});
54             eid[e.to].push_back(num_edges++);
55             edge.push_back({e.to, e.from, -e.cost, e.flow});
56         }
57     }
58     std::vector<cost_t> dis;
59     std::vector<int> pre;
60     bool spfa(int s, int t) {
61         if (s > n || t > n) return false;
62         dis.assign(n + 1, inf_cost);
63         pre.assign(n + 1, 0);
64         std::vector<bool> inque(n + 1);
65         std::queue<int> que;
66         dis[s] = 0;
67         que.push(s);
68         inque[s] = true;
69         while (que.size()) {
70             int u = que.front();
71             // cerr << 'u' << ' ' << u << endl;
72             que.pop();
73             inque[u] = false;
74             for (auto i : eid[u]) {
75                 const auto &e = edge[i];

```

```

76     if (e.cap && dis[e.to] > dis[u] + e.cost) {
77         dis[e.to] = dis[u] + e.cost;
78         pre[e.to] = i;
79         if (!inque[e.to]) {
80             que.push(e.to);
81             inque[e.to] = true;
82         }
83     }
84 }
85 }
86 return dis[t] < inf_cost;
87 }
88 std::pair<flow_t, cost_t> maxflow(int s, int t) {
89     flow_t flow = 0;
90     cost_t cost = 0;
91     while (spfa(s, t)) {
92         flow_t detf = inf_flow;
93         cost_t detc = 0;
94         for (int u = t, i = pre[u]; u != s; u = edge[i].from, i = pre[u]) {
95             detf = std::min(detf, edge[i].cap);
96             detc += edge[i].cost;
97         }
98         for (int u = t, i = pre[u]; u != s; u = edge[i].from, i = pre[u]) {
99             edge[i].cap -= detf;
100             edge[i ^ 1].cap += detf;
101         }
102         flow += detf;
103         cost += detf * detc;
104     }
105     return {flow, cost};
106 }
107 std::vector<CostEdge> to_edge() {
108     std::vector<CostEdge> edges;
109     for (int i = 0; i < edge.size(); i += 2)
110         edges.push_back({
111             .from = edge[i].from,
112             .to = edge[i].to,
113             .cost = edge[i].cost,
114             .cap = edge[i].cap + edge[i ^ 1].cap,
115             .flow = edge[i ^ 1].cap,
116         });
117     return edges;
118 }
119 };
120

```

```

121 struct Processor {
122     std::vector<bool> neg;
123     std::vector<flow_t> low;
124     std::vector<flow_t> excess;
125     void init(std::vector<CostEdge> &edges) {
126         int n = num_node(edges);
127         neg.clear();
128         neg.reserve(edges.size());
129         low.clear();
130         low.reserve(edges.size());
131         excess.assign(n + 1, 0);
132     }
133     void rmv_low(std::vector<CostEdge> &edges) {
134         for (auto &e : edges) {
135             low.push_back(e.low);
136             if (e.flow >= e.low) {
137                 e.flow -= e.low;
138             } else {
139                 excess[e.from] -= e.low - e.flow;
140                 excess[e.to] += e.low - e.flow;
141                 e.flow = 0;
142             }
143             e.cap -= e.low;
144             e.low = 0;
145         }
146     }
147     void add_low(std::vector<CostEdge> &edges) {
148         reverse(low.begin(), low.end());
149         for (auto &e : edges) {
150             e.low = low.back();
151             e.flow += e.low;
152             e.cap += e.low;
153             low.pop_back();
154         }
155     }
156     void rmv_neg(std::vector<CostEdge> &edges) {
157         for (auto &e : edges) {
158             neg.push_back(e.cost < 0);
159             if (e.cost < 0) {
160                 excess[e.from] -= e.cap - e.flow;
161                 excess[e.to] += e.cap - e.flow;
162                 e.flow = e.cap;
163             }
164             if (e.cost > 0) {
165                 excess[e.from] += e.flow;

```

```

166         excess[e.to] -= e.flow;
167         e.flow = 0;
168     }
169 }
170 }
171 };
172
173 bool excess_flow(std::vector<CostEdge> &edges,
174                 const std::vector<flow_t> &excess) {
175     int n = num_node(edges), m = edges.size();
176     for (int i = 1; i <= n; i++) {
177         if (excess[i] > 0)
178             edges.push_back({.from = n + 1, .to = i, .cost = 0, .cap = excess[i]});
179         if (excess[i] < 0)
180             edges.push_back({.from = i, .to = n + 2, .cost = 0, .cap = -excess[i]});
181     }
182     CostFlow g;
183     g.build(edges);
184     g.maxflow(n + 1, n + 2);
185     edges = g.to_edge();
186     for (int i = m; i < edges.size(); i++)
187         if (edges[i].flow != edges[i].cap) return false;
188     edges.resize(m);
189     return true;
190 }
191
192 std::optional<std::pair<flow_t, cost_t>> feasible_flow(
193     std::vector<CostEdge> &edges, int s = 0, int t = 0) {
194     if (s && t) edges.push_back({.from = t, .to = s, .cost = 0, .cap = inf_flow});
195     Processor p;
196     p.init(edges);
197     p.rmv_low(edges);
198     p.rmv_neg(edges);
199     if (!excess_flow(edges, p.excess)) return std::nullopt;
200     if (s && t) edges.pop_back();
201     p.add_low(edges);
202     return get_flow(edges, s);
203 }
204
205 std::optional<std::pair<flow_t, cost_t>> maximum_flow(
206     std::vector<CostEdge> &edges, int s, int t) {
207     edges.push_back({.from = t, .to = s, .cost = 0, .cap = inf_flow});
208     Processor p;
209     p.init(edges);
210     p.rmv_low(edges);

```

```

211     p.rmv_neg(edges);
212     if (!excess_flow(edges, p.excess)) return std::nullopt;
213     edges.pop_back();
214     CostFlow g;
215     g.build(edges);
216     g.maxflow(s, t);
217     edges = g.to_edge();
218     p.add_low(edges);
219     return get_flow(edges, s);
220 }
221
222 std::optional<std::pair<flow_t, cost_t>> minimum_flow(
223     std::vector<CostEdge> &edges, int s, int t) {
224     edges.push_back({.from = t, .to = s, .cost = 0, .cap = inf_flow});
225     Processor p;
226     p.init(edges);
227     p.rmv_low(edges);
228     p.rmv_neg(edges);
229     if (!excess_flow(edges, p.excess)) return std::nullopt;
230     edges.pop_back();
231     CostFlow g;
232     for (auto &e : edges) e.cost = -e.cost;
233     Processor q;
234     q.rmv_neg(edges);
235     excess_flow(edges, q.excess);
236     g.build(edges);
237     g.maxflow(t, s);
238     edges = g.to_edge();
239     for (auto &e : edges) e.cost = -e.cost;
240     p.add_low(edges);
241     return get_flow(edges, s);
242 }
243
244 } // namespace costflow

```

### 3.4 点分树

```

1 #include "bits/stdc++.h"
2 using namespace std;
3 const char el = '\n';
4 typedef long long ll;
5 const ll inf = 1e18;
6 struct graph {
7     vector<vector<pair<int, ll>>> e;

```

```

8  vector<ll> r;
9  graph(int n) : e(n + 1), r(n + 1) {}
10 void adde(int u, int v, ll w) {
11     e[u].push_back({v, w});
12     e[v].push_back({u, w});
13 }
14 vector<int> siz;
15 int calcsiz(int u, int f) {
16     siz[u] = 1;
17     for (auto [v, w] : e[u])
18         if (v != f && ~siz[v]) siz[u] += calcsiz(v, u);
19     return siz[u];
20 }
21 vector<vector<pair<int, ll>>> fa;
22 vector<vector<pair<ll, int>>> ch;
23 void init(int u, int f, int rt, ll d) {
24     fa[u].push_back({rt, d});
25     ch[rt].push_back({r[u] - d, u});
26     for (auto [v, w] : e[u])
27         if (v != f && ~siz[v]) init(v, u, rt, d + w);
28 }
29 int split(int u) {
30     int n = calcsiz(u, u) / 2;
31     while (true) {
32         bool flg = true;
33         for (auto [v, w] : e[u])
34             if (siz[v] < siz[u] && siz[v] > n) {
35                 u = v;
36                 flg = false;
37                 break;
38             }
39         if (flg) break;
40     }
41     siz[u] = -1;
42     init(u, u, u, 0);
43     sort(ch[u].begin(), ch[u].end());
44     for (auto [v, w] : e[u])
45         if (~siz[v]) v = split(v);
46     return u;
47 }
48 int rt;
49 void ctrdecomp() {
50     int n = e.size();
51     siz.assign(n, 0);
52     fa.assign(n, {});

```

```

53     ch.assign(n, {});
54     rt = split(1);
55 }
56 vector<int> solve() {
57     vector<int> dis(e.size(), -1);
58     dis[1] = 0;
59     queue<int> que;
60     que.push(1);
61     while (que.size()) {
62         int u = que.front();
63         que.pop();
64         for (auto [f, d] : fa[u]) {
65             while (ch[f].size()) {
66                 auto [w, v] = ch[f].back();
67                 if (d > w) break;
68                 ch[f].pop_back();
69                 if (~dis[v]) continue;
70                 que.push(v);
71                 dis[v] = dis[u] + 1;
72             }
73         }
74     }
75     return dis;
76 }
77 };
78 int main() {
79     ios::sync_with_stdio(false);
80     cin.tie(0);
81     cout << setprecision(15);
82     int tt;
83     cin >> tt;
84     while (tt--) {
85         int n;
86         cin >> n;
87         graph g(n);
88         for (int i = 2; i <= n; i++) cin >> g.r[i];
89         for (int i = 1; i < n; i++) {
90             ll u, v, w;
91             cin >> u >> v >> w;
92             g.adde(u, v, w);
93         }
94         g.ctrdecomp();
95         auto res = g.solve();
96         for (int i = 2; i <= n; i++) cout << res[i] << ' ';
97         cout << el;

```

```

98     }
99     return 0;
100 }

```

### 3.5 强连通分量分解

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 struct graph {
4     vector<vector<int>> e, r;
5     graph(int n) {
6         e.resize(n + 1);
7         r.resize(n + 1);
8     }
9     void adde(int u, int v) {
10         e[u].push_back(v);
11         r[v].push_back(u);
12     }
13     vector<bool> vis;
14     vector<int> ord;
15     vector<int> col;
16     void travel(int u, int f) {
17         vis[u] = true;
18         for (auto v : e[u])
19             if (!vis[v]) travel(v, u);
20         ord.push_back(u);
21     }
22     void color(int u, int f, int c) {
23         col[u] = c;
24         for (auto v : r[u])
25             if (!col[v]) color(v, u, c);
26     }
27     vector<int> decomp() {
28         vis.assign(e.size(), false);
29         ord.clear();
30         for (int i = 1; i < e.size(); i++)
31             if (!vis[i]) travel(i, i);
32         reverse(ord.begin(), ord.end());
33         col.assign(e.size(), 0);
34         for (auto v : ord)
35             if (!col[v]) color(v, v, v);
36         return col;
37     }
38 };

```

### 3.6 拓扑排序

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 struct graph {
4     vector<vector<int>> e;
5     graph(int n) : e(n + 1) {}
6     void adde(int u, int v) { e[u].push_back(v); }
7     optional<vector<int>> toposort() {
8         int n = e.size() - 1;
9         vector<int> d(n + 1);
10        queue<int> que;
11        for (int u = 1; u <= n; u++)
12            for (auto v : e[u]) d[v]++;
13        for (int i = 1; i <= n; i++)
14            if (!d[i]) que.push(i);
15        vector<int> res;
16        while (!que.empty()) {
17            auto u = que.front();
18            res.push_back(u);
19            que.pop();
20            for (auto v : e[u]) {
21                if (--d[v]) que.push(v);
22            }
23        }
24        if (res.size() != n) return nullopt;
25        return res;
26    }
27 };

```

### 3.7 二分图匹配

```

1 #pragma once
2
3 #include <vector>
4
5 #include "maxflow.h"
6
7 namespace match {
8
9 std::vector<std::pair<int, int>> max_match(

```

```

10     const std::vector<std::pair<int, int>> &edges) {
11     int n = 1, m = 1;
12     for (auto [l, r] : edges) {
13         n = std::max(n, l);
14         m = std::max(m, r);
15     }
16     std::vector<maxflow::FlowEdge> fedge;
17     int s = n + m + 1, t = n + m + 2;
18     for (int i = 1; i <= n; i++) fedge.push_back({.from = s, .to = i, .cap = 1});
19     for (int i = 1; i <= m; i++)
20         fedge.push_back({.from = i + n, .to = t, .cap = 1});
21     for (auto [l, r] : edges) fedge.push_back({.from = l, .to = r + n, .cap = 1});
22     maxflow::maximum_flow(fedge, s, t);
23     std::vector<std::pair<int, int>> res;
24     for (auto e : fedge)
25         if (e.from != s && e.to != t && e.flow == 1)
26             res.push_back({e.from, e.to - n});
27     return res;
28 }
29
30 void dfs(int u, std::vector<bool> &vl, std::vector<bool> &vr,
31         std::vector<std::vector<int>> &ltr, std::vector<int> &rtl) {
32     if (vl[u]) return;
33     vl[u] = true;
34     for (auto v : ltr[u]) {
35         vr[v] = true;
36         dfs(rtl[v], vl, vr, ltr, rtl);
37     }
38 }
39
40 std::pair<std::vector<int>, std::vector<int>> min_cover(
41     const std::vector<std::pair<int, int>> &edges) {
42     int n = 1, m = 1;
43     for (auto [l, r] : edges) {
44         n = std::max(n, l);
45         m = std::max(m, r);
46     }
47     auto match = max_match(edges);
48     std::vector<std::vector<int>> ltr(n + 1);
49     std::vector<int> rtl(m + 1);
50     std::vector<bool> vis(n + 1);
51     for (auto [l, r] : match) {
52         rtl[r] = l;
53         vis[l] = true;
54     }

```

```

55     for (auto [l, r] : edges) ltr[l].push_back(r);
56     std::vector<bool> vl(n + 1), vr(m + 1);
57     for (int i = 1; i <= n; i++)
58         if (!vis[i]) dfs(i, vl, vr, ltr, rtl);
59     std::pair<std::vector<int>, std::vector<int>> res;
60     for (int i = 1; i <= n; i++)
61         if (!vl[i]) res.first.push_back(i);
62     for (int i = 1; i <= m; i++)
63         if (vr[i]) res.second.push_back(i);
64     return res;
65 }
66
67 } // namespace match

```

### 3.8 最大流

```

1 #pragma once
2
3 #include <algorithm>
4 #include <optional>
5 #include <queue>
6 #include <vector>
7
8 namespace maxflow {
9
10     typedef long long flow_t;
11     const flow_t inf_flow = 1e18;
12     const int inf_dep = 1e9;
13
14     struct FlowEdge {
15         int from, to;
16         flow_t cap, low = 0, flow = 0;
17     };
18
19     int num_node(const std::vector<FlowEdge> &edges) {
20         int n = 0;
21         for (const auto &e : edges) n = std::max({n, e.from, e.to});
22         return n;
23     }
24
25     flow_t get_flow(const std::vector<FlowEdge> &edges, int s) {
26         flow_t flow = 0;
27         for (const auto &e : edges) {
28             if (e.from == s) flow += e.flow;

```

```

29     }
30     return flow;
31 }
32
33 struct MaxFlow {
34     struct Edge {
35         int from, to;
36         flow_t cap;
37     };
38     int n;
39     std::vector<std::vector<int>> eid;
40     std::vector<Edge> edge;
41     void build(const std::vector<FlowEdge> &edges) {
42         n = num_node(edges);
43         eid.assign(n + 1, {});
44         edge.clear();
45         int num_edges = 0;
46         for (const auto &e : edges) {
47             eid[e.from].push_back(num_edges++);
48             edge.push_back({e.from, e.to, e.cap - e.flow});
49             eid[e.to].push_back(num_edges++);
50             edge.push_back({e.to, e.from, e.flow});
51         }
52     }
53
54     std::vector<int> dis;
55     std::vector<int> cur;
56     bool bfs(int s, int t) {
57         if (s > n || t > n) return false;
58         dis.assign(n + 1, inf_dep);
59         cur.assign(n + 1, 0);
60         std::queue<int> que;
61         dis[s] = 0;
62         que.push(s);
63         while (que.size()) {
64             int u = que.front();
65             que.pop();
66             for (auto i : eid[u]) {
67                 const auto &e = edge[i];
68                 if (e.cap && dis[e.to] > dis[u] + 1) {
69                     dis[e.to] = dis[u] + 1;
70                     que.push(e.to);
71                 }
72             }
73         }

```

```

74         return dis[t] < inf_dep;
75     }
76
77     flow_t dfs(int s, int t, flow_t flim) {
78         if (s == t) return flim;
79         flow_t flow = 0;
80         for (int &i = cur[s]; i < eid[s].size() && flow < flim; i++) {
81             auto &e = edge[eid[s][i]];
82             if (dis[e.to] == dis[s] + 1 && e.cap) {
83                 auto detf = dfs(e.to, t, std::min(flim - flow, e.cap));
84                 flow += detf;
85                 e.cap -= detf;
86                 edge[eid[s][i] ^ 1].cap += detf;
87             }
88             if (flow == flim) break;
89         }
90         return flow;
91     }
92     flow_t maxflow(int s, int t) {
93         flow_t flow = 0;
94         while (bfs(s, t)) {
95             flow += dfs(s, t, inf_flow);
96         }
97         return flow;
98     }
99
100     std::vector<FlowEdge> to_edge() {
101         std::vector<FlowEdge> edges;
102         for (int i = 0; i < edge.size(); i += 2)
103             edges.push_back({
104                 .from = edge[i].from,
105                 .to = edge[i].to,
106                 .cap = edge[i].cap + edge[i ^ 1].cap,
107                 .low = 0,
108                 .flow = edge[i ^ 1].cap,
109             });
110         return edges;
111     }
112 };
113
114 struct Processor {
115     std::vector<bool> neg;
116     std::vector<flow_t> low;
117     std::vector<flow_t> excess;
118     void init(std::vector<FlowEdge> &edges) {

```

```

119     int n = num_node(edges);
120     neg.clear();
121     neg.reserve(edges.size());
122     low.clear();
123     low.reserve(edges.size());
124     excess.assign(n + 1, 0);
125 }
126 void rmv_low(std::vector<FlowEdge> &edges) {
127     for (auto &e : edges) {
128         low.push_back(e.low);
129         if (e.flow >= e.low) {
130             e.flow -= e.low;
131         } else {
132             excess[e.from] -= e.low - e.flow;
133             excess[e.to] += e.low - e.flow;
134             e.flow = 0;
135         }
136         e.cap -= e.low;
137         e.low = 0;
138     }
139 }
140 void add_low(std::vector<FlowEdge> &edges) {
141     reverse(low.begin(), low.end());
142     for (auto &e : edges) {
143         e.low = low.back();
144         e.flow += e.low;
145         e.cap += e.low;
146         low.pop_back();
147     }
148 }
149 };
150
151 bool excess_flow(std::vector<FlowEdge> &edges,
152                 const std::vector<flow_t> &excess) {
153     int n = num_node(edges), m = edges.size();
154     for (int i = 1; i <= n; i++) {
155         if (excess[i] > 0)
156             edges.push_back({.from = n + 1, .to = i, .cap = excess[i]});
157         if (excess[i] < 0)
158             edges.push_back({.from = i, .to = n + 2, .cap = -excess[i]});
159     }
160     MaxFlow g;
161     g.build(edges);
162     g.maxflow(n + 1, n + 2);
163     edges = g.to_edge();

```

```

164     for (int i = m; i < edges.size(); i++)
165         if (edges[i].flow != edges[i].cap) return false;
166     edges.resize(m);
167     return true;
168 }
169
170 std::optional<flow_t> feasible_flow(std::vector<FlowEdge> &edges, int s = 0,
171                                   int t = 0) {
172     if (s && t) edges.push_back({.from = t, .to = s, .cap = inf_flow});
173     Processor p;
174     p.init(edges);
175     p.rmv_low(edges);
176     if (!excess_flow(edges, p.excess)) return std::nullopt;
177     if (s && t) edges.pop_back();
178     p.add_low(edges);
179     return get_flow(edges, s);
180 }
181
182 std::optional<flow_t> maximum_flow(std::vector<FlowEdge> &edges, int s, int t) {
183     edges.push_back({.from = t, .to = s, .cap = inf_flow});
184     Processor p;
185     p.init(edges);
186     p.rmv_low(edges);
187     if (!excess_flow(edges, p.excess)) return std::nullopt;
188     edges.pop_back();
189     MaxFlow g;
190     g.build(edges);
191     g.maxflow(s, t);
192     edges = g.to_edge();
193     p.add_low(edges);
194     return get_flow(edges, s);
195 }
196
197 std::optional<flow_t> minimum_flow(std::vector<FlowEdge> &edges, int s, int t) {
198     edges.push_back({.from = t, .to = s, .cap = inf_flow});
199     Processor p;
200     p.init(edges);
201     p.rmv_low(edges);
202     if (!excess_flow(edges, p.excess)) return std::nullopt;
203     edges.pop_back();
204     MaxFlow g;
205     Processor q;
206     excess_flow(edges, q.excess);
207     g.build(edges);
208     g.maxflow(t, s);

```



```

209 edges = g.to_edge();
210 p.add_low(edges);
211 return get_flow(edges, s);
212 }
213
214 } // namespace maxflow

```

### 3.9 换根 DP

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 typedef long long ll;
4 struct graph {
5     vector<vector<int>> e;
6     ll k, c;
7     graph(int n) { e.resize(n + 1); }
8     void adde(int u, int v) {
9         e[u].push_back(v);
10        e[v].push_back(u);
11    }
12    vector<int> dis, dep;
13    ll ans;
14    void setup(int u, int f) {
15        dep[u] = 0;
16        for (auto v : e[u])
17            if (v != f) {
18                dis[v] = dis[u] + 1;
19                setup(v, u);
20                dep[u] = max(dep[u], dep[v] + 1);
21            }
22    }
23    void reroot(int u, int f) {
24        int mxm1 = -1, mxm2 = -1;
25        for (auto v : e[u]) {
26            mxm2 = max(mxm2, dep[v]);
27            if (mxm2 > mxm1) swap(mxm1, mxm2);
28        }
29        dep[u] = mxm1 + 1;
30        ans = max(ans, dep[u] * k - dis[u] * c);
31        for (auto v : e[u])
32            if (v != f) {
33                dep[u] = (dep[v] == mxm1 ? mxm2 : mxm1) + 1;
34                reroot(v, u);
35            }

```

```

36    }
37    ll solve(int s) {
38        dis.resize(e.size());
39        dep.resize(e.size());
40        dep[s] = 0;
41        setup(s, s);
42        ans = 0;
43        reroot(s, s);
44        return ans;
45    }
46 };

```

### 3.10 树哈希

```

1 /**
2  * Author: Boboge adapted from peehs_moorhsum
3  * Date: 23-02-06
4  * Description: return the hash value of every subtree.
5  * Time: O(N).
6  * Status: tested on https://uoj.ac/problem/763
7  */
8 std::mt19937_64 rnd(std::chrono::steady_clock::now().time_since_epoch().count());
9
10 struct treeHash {
11     using ull = unsigned long long;
12     ull bas = rnd();
13
14     ull H(ull x) {
15         return x * x * x * 114514 + 19260817;
16     }
17
18     ull F(ull x) {
19         return H(x & ((1ll << 32) - 1)) + H(x >> 32);
20     }
21
22     std::vector<ull> h;
23
24     treeHash(std::vector<std::vector<int>> &adj, int rt = 0) : h(adj.size()) {
25         auto dfs = [&](auto dfs, int u, int fa) -> void {
26             h[u] = bas;
27             for (int v: adj[u]) {
28                 if (v == fa) continue;
29                 dfs(dfs, v, u);
30                 h[u] += F(h[v]);

```

```

31     }
32     };
33     dfs(dfs, rt, -1);
34 }
35 };

```

## 4 数学

### 4.1 数论分块

```

1 for (ll l = 1, r; l <= n; l = r + 1) {
2     r = n / (n / l);
3     ... // for l<=i<=r, n/i = n/l
4 }
5 /*
6 https://qoj.ac/contest/1103/problem/5503
7 */

```

### 4.2 FWT

```

1
2 const int k = 17, n = 1 << k;
3 const int mod = 1e9 + 7, inv2 = mod / 2 + 1;
4 struct modint {
5     int n;
6     modint(int n = 0) : n(n) {}
7 };
8 modint operator+(modint a, modint b) {
9     return (a.n += b.n) >= mod ? a.n - mod : a.n;
10 }
11 modint operator-(modint a, modint b) {
12     return (a.n -= b.n) < 0 ? a.n + mod : a.n;
13 }
14 modint operator*(modint a, modint b) { return 1ll * a.n * b.n % mod; }
15 vector<modint> fib(n);
16 vector<modint> fwt(vector<modint> a, void (*opr)(modint &, modint &)) {
17     // cout << 1 << endl;
18     int n = a.size();
19     for (int i = 0; 1 << i < n; i++)
20         for (int j = 0; j < n; j++)
21             if (j >> i & 1) opr(a[j - (1 << i)], a[j]);
22     // cout << 2 << endl;

```

```

23     return a;
24 }
25 void fwtand(modint &a, modint &b) { a = a + b; }
26 void revand(modint &a, modint &b) { a = a - b; }
27 void fwtor(modint &a, modint &b) { b = b + a; }
28 void revor(modint &a, modint &b) { b = b - a; }
29 void fwtxor(modint &a, modint &b) {
30     modint x = a + b, y = a - b;
31     a = x, b = y;
32 }
33 void revxor(modint &a, modint &b) {
34     modint x = a + b, y = a - b;
35     a = x * inv2, b = y * inv2;
36 }
37 vector<modint> add(vector<modint> a, vector<modint> b) {
38     for (int i = 0; i < a.size(); i++) a[i] = a[i] + b[i];
39     return a;
40 }
41 vector<modint> mul(vector<modint> a, vector<modint> b) {
42     for (int i = 0; i < a.size(); i++) a[i] = a[i] * b[i];
43     return a;
44 }
45 vector<int> popc(n);
46 vector<modint> filter(vector<modint> a, int k) {
47     for (int i = 0; i < n; i++)
48         if (popc[i] != k) a[i] = 0;
49     return a;
50 }
51 int main() {
52     init();
53     fib[0] = 0, fib[1] = 1;
54     for (int i = 2; i < n; i++) fib[i] = fib[i - 1] + fib[i - 2];
55     popc[0] = 0;
56     for (int i = 1; i < n; i++) popc[i] = popc[i >> 1] + (i & 1);
57     int m;
58     cin >> m;
59     vector<int> s(m);
60     for (auto &v : s) cin >> v;
61     vector<modint> a(n);
62     for (auto v : s) a[v] = a[v] + 1;
63     auto mid = mul(a, fib),
64         rhs = mul(fwt(mul(fwt(a, fwtxor), fwt(a, fwtxor)), revxor), fib);
65     vector b(k + 1, vector<modint>());
66     vector c(k + 1, vector<modint>(n));
67     vector<modint> lhs(n);

```

```

68 for (int i = 0; i <= k; i++) b[i] = fwt(filter(a, i), fwtor);
69 for (int i = 0; i <= k; i++)
70     for (int j = 0; i + j <= k; j++) c[i + j] = add(c[i + j], mul(b[i], b[j]));
71 for (int i = 0; i <= k; i++)
72     lhs = add(lhs, filter(mul(fwt(c[i], revor), fib), i));
73 auto res = fwt(mul(mul(fwt(lhs, fwtand), fwt(mid, fwtand)), fwt(rhs, fwtand)),
74               revand);
75 modint ans;
76 for (int i = 0; i < k; i++) ans = ans + res[1 << i];
77 cout << ans.n << el;
78 return 0;
79 }

```

### 4.3 高斯消元

```

1
2 const ll mod = 1e9 + 7, pri = 5;
3 const array<int, pri> inv = {0, 1, 3, 2, 4};
4 ll qpow(ll a, ll b, ll m = mod) {
5     ll res = 1;
6     while (b) {
7         if (b & 1) res = res * a % m;
8         a = a * a % m, b >>= 1;
9     }
10    return res;
11 }
12 vector<int> read() {
13     string s;
14     cin >> s;
15     vector<int> a(s.size());
16     for (int i = 0; i < s.size(); i++) a[i] = s[i] - 'a';
17     return a;
18 }
19 void flip(vector<vector<int>> &a) {
20     int n = a.size(), m = a[0].size();
21     vector b(m, vector<int>(n));
22     for (int i = 0; i < n; i++)
23         for (int j = 0; j < m; j++) b[j][i] = a[i][j];
24     swap(a, b);
25 }
26 int gaussian(vector<vector<int>> &a) {
27     int n = a.size(), m = a[0].size();
28     int r = 0;
29     for (int j = 0; j < m; j++) {

```

```

30     int p = -1;
31     for (int i = r; i < n; i++)
32         if (a[i][j]) p = i;
33     if (p == -1) continue;
34     swap(a[r], a[p]);
35     int t = inv[a[r][j]];
36     for (int k = 0; k < m; k++) a[r][k] = a[r][k] * t % pri;
37     for (int i = 0; i < n; i++)
38         if (i != r) {
39             int t = a[i][j];
40             for (int k = 0; k < m; k++)
41                 a[i][k] = ((a[i][k] - a[r][k] * t) % pri + pri) % pri;
42         }
43     r++;
44 }
45 a.resize(r);
46 return r;
47 }
48 bool solve(vector<vector<int>> &a, vector<int> &b) {
49     int n = a.size(), m = a[0].size();
50     for (int i = 0; i < n; i++) {
51         int p = 0;
52         while (!a[i][p]) p++;
53         int t = b[p];
54         for (int j = 0; j < m; j++) b[j] = ((b[j] - a[i][j] * t) % pri + pri) % pri;
55     }
56     for (auto v : b)
57         if (v % pri) return false;
58     return true;
59 }

```

### 4.4 Min25 筛

```

1 const int det = 100;
2 const ll mod = 1e9 + 7;
3 template <typename T>
4 struct min25 {
5     ll n, m;
6     vector<ll> p;
7     vector<T> s;
8     void sieve(ll n) {
9         p = {1}, s = {T()};
10        vector<int> minf(n);
11        for (ll i = 2; i < n; i++) {

```

```

12     if (!minf[i]) {
13         minf[i] = p.size();
14         p.push_back(i);
15         s.push_back(s.back() + T::init(i));
16     }
17     for (ll j = 1; j <= minf[i] && i * p[j] < n; j++) minf[i * p[j]] = j;
18 }
19 }
20 vector<ll> lis;
21 vector<int> le, ge;
22 vector<T> g;
23 void init(ll n) {
24     le.resize(m + det), ge.resize(m + det);
25     for (ll i = 1, j; i <= n; i = n / j + 1) {
26         j = n / i;
27         int k = lis.size();
28         lis.push_back(j);
29         (j <= m ? le[j] : ge[n / j]) = k;
30         g.push_back(T::plug(j));
31     }
32 }
33 int id(ll v) { return v <= m ? le[v] : ge[n / v]; }
34 vector<ll> f;
35 void calcp() {
36     for (int k = 1; k < p.size(); k++) {
37         ll pk = p[k], sp = pk * pk;
38         for (int i = 0; i < lis.size() && lis[i] >= sp; i++) {
39             int j = id(lis[i] / pk);
40             g[i] = g[i] - (g[j] - s[k - 1]) * T::item(pk);
41         }
42     }
43     for (auto &v : g) f.push_back(v.val());
44 }
45 min25(ll n) : n(n) {
46     m = sqrt(n);
47     sieve(m + det);
48     init(n);
49     calcp();
50 }
51 ll query(ll n, int k = 1) {
52     if (n < p[k] || n <= 1) return 0;
53     const int i = id(n);
54     ll ans = f[i] - s[k - 1].val();
55     for (int i = k; i < p.size() && 1ll * p[i] * p[i] <= n; i++) {
56         ll mul = p[i];

```

```

57         for (int c = 1; mul * p[i] <= n; c++, mul *= p[i])
58             ans += T::f(p[i], c) * query(n / mul, i + 1) % mod + T::f(p[i], c + 1);
59     }
60     return ans % mod;
61 }
62 };
63 struct ez {
64     ll a, b;
65     ez(ll a = 0, ll b = 0) : a(a % mod), b(b % mod) {}
66     ez operator+(ez r) { return {(a + r.a) % mod, (b + r.b) % mod}; }
67     ez operator-(ez r) { return {(a - r.a) % mod, (b - r.b) % mod}; }
68     ez operator*(ez r) { return {a * r.a % mod, b * r.b % mod}; }
69     ll val() { return (a + b) % mod; }
70     static ll f(ll p, ll c) { return p ^ c; }
71     static ez init(ll p) { return {-1, p}; }
72     static ez plug(ll p) {
73         p %= mod;
74         return {-p + 1, (p + 2) * (p - 1) % mod * (mod + 1 >> 1) % mod};
75     }
76     static ez item(ll p) { return {1, p}; }
77 };

```

## 4.5 Pollard— $\rho$

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  using i64 = long long;
4  using i128 = __int128;
5  i64 fpow(i64 a, i64 t, i64 mod){
6      i64 r = 1;
7      for (; t >= 1, a = (i128)a * a % mod) {
8          if (t & 1) {
9              r = (i128)r * a % mod;
10             }
11         }
12         return r;
13     }
14     i64 gcd(i64 a, i64 b){
15         #define ctz __builtin_ctzll
16         int shift = ctz(a | b);
17         b >>= ctz(b);
18         while (a) {
19             a >>= ctz(a);
20             if (a < b) swap(a, b);

```

```

21         a -= b;
22     }
23     return b << shift;
24 }
25 bool check_prime(i64 n){
26     static const int jp[] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
27     if (n == 1) return false;
28     for (int p : jp) if (n % p == 0) return n == p;
29     i64 r = n - 1, x, y;
30     int e = 0;
31     while (~r & 1) r >>= 1, ++e;
32     for (int p : jp) {
33         x = fpow(p, r, n);
34         for (int t = 0; t < e && x > 1; ++t) {
35             y = (i128)x * x % n;
36             if (y == 1 && x != n - 1) return false;
37             x = y;
38         }
39         if (x != 1) return false;
40     }
41     return true;
42 }
43 i64 find(i64 n){
44     static const int step = 1 << 7;
45     i64 x, y = rand() % n;
46     int c = rand() % n;
47     auto f = [=](i64 x){ return ((i128)x * x + c) % n; };
48     for (int l = 1; ; l <= 1) {
49         x = y;
50         for (int i = 0; i < l; ++i) y = f(y);
51         for (int k = 0; k < l; k += step) {
52             int e = std::min(step, l - k);
53             i64 g = 1, z = y;
54             for (int i = 0; i < e; ++i) g = (i128)g * ((y = f(y)) + n - 1) % n;
55             g = gcd(g, n);
56             if (g == 1) continue;
57             if (g == n) for (g = 1, y = z; g == 1; ) y = f(y), g = gcd(y + n, n);
58             return g;
59         }
60     } //
61 }
62 void rho(i64 n, map<i64,int> &factor){
63     while (~n & 1) n >>= 1, ++factor[2];
64     if (n == 1) return ;
65     if (check_prime(n)) {

```

```

66         ++factor[n];
67         return ;
68     }
69     i64 d;
70     for (d = find(n); d == n; d = find(d));
71     rho(d, factor), rho(n / d, factor);
72 }
73 int T;
74 i64 n;
75 int main(){
76     for (cin >> T; T; --T) {
77         map<i64, int> f;
78         cin >> n;
79         rho(n, f);
80         if (f.size() > 1 || (--f.end())->second > 1) {
81             cout << (--f.end())->first << '\n';
82         } else {
83             cout << "Prime\n";
84         }
85     }
86 }

```

## 4.6 多项式

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 typedef long long ll;
4 const ll mod = 998244353;
5 typedef double ld;
6 typedef complex<ld> cplx;
7 typedef vector<ll> poly;
8
9 ll qpow(ll a, ll b) {
10     ll res = 1;
11     while (b) {
12         if (b & 1) res = res * a % mod;
13         a = a * a % mod;
14         b >>= 1;
15     }
16     return res;
17 }
18 ll inv(ll n) { return qpow(n, mod - 2); }
19
20 const auto pi = acosl(-1);

```

```

21 const int len = 15, mask = (1 << len) - 1;
22
23 struct unitroot {
24     static vector<cplx> w;
25     static vector<cplx> get_root(int n) {
26         n = 1 << 32 - __builtin_clz(n);
27         if (n > w.size()) {
28             w.resize(n);
29             for (int i = 0; i < n; i++)
30                 w[i] = cplx(cos(2 * i * pi / n), sin(2 * i * pi / n));
31         }
32         int m = w.size() / n;
33         vector<cplx> res(n);
34         for (int i = 0, j = 0; i < n; i++, j += m) res[i] = w[j];
35         return res;
36     }
37 };
38 vector<cplx> unitroot::w;
39
40 void fft(vector<cplx> &p, const vector<cplx> &w) {
41     int n = w.size();
42     for (int i = 1, j = 0; i < n - 1; ++i) {
43         int s = n;
44         do {
45             s >>= 1;
46             j ^= s;
47         } while (~j & s);
48         if (i < j) swap(p[i], p[j]);
49     }
50     for (int d = 0; (1 << d) < n; ++d) {
51         int m = 1 << d, m2 = m * 2, rm = n >> (d + 1);
52         for (int i = 0; i < n; i += m2)
53             for (int j = 0; j < m; ++j) {
54                 auto &p1 = p[i + j + m], &p2 = p[i + j];
55                 auto t = w[rm * j] * p1;
56                 p1 = p2 - t;
57                 p2 = p2 + t;
58             }
59     }
60 }
61
62 poly operator+(const poly &a, const poly &b) {
63     poly c(max(a.size(), b.size()));
64     for (int i = 0; i < a.size(); i++) c[i] += a[i];
65     for (int i = 0; i < b.size(); i++) c[i] += b[i];
66     for (auto &v : c) v %= mod;

```

```

66     return c;
67 }
68
69 poly operator-(poly b) {
70     for (auto &v : b) v = v ? mod - v : 0;
71     return b;
72 }
73
74 poly operator-(const poly &a, const poly &b) { return a + -b; }
75
76 poly operator*(const poly &a, const poly &b) {
77     vector<cplx> w = unitroot::get_root(a.size() + b.size() - 1);
78     int n = w.size();
79     vector<cplx> A(n), B(n), C(n), D(n);
80     for (int i = 0; i < a.size(); ++i) A[i] = cplx(a[i] >> len, a[i] & mask);
81     for (int i = 0; i < b.size(); ++i) B[i] = cplx(b[i] >> len, b[i] & mask);
82     fft(A, w), fft(B, w);
83     for (int i = 0; i < n; ++i) {
84         int j = (n - i) % n;
85         cplx da = (A[i] - conj(A[j])) * cplx(0, -0.5),
86                db = (A[i] + conj(A[j])) * cplx(0.5, 0),
87                dc = (B[i] - conj(B[j])) * cplx(0, -0.5),
88                dd = (B[i] + conj(B[j])) * cplx(0.5, 0);
89         C[j] = da * dd + da * dc * cplx(0, 1);
90         D[j] = db * dd + db * dc * cplx(0, 1);
91     }
92     fft(C, w), fft(D, w);
93     poly res(a.size() + b.size() - 1);
94     for (int i = 0; i < res.size(); ++i) {
95         ll da = (ll)(C[i].imag() / n + 0.5) % mod,
96            db = (ll)(C[i].real() / n + 0.5) % mod,
97            dc = (ll)(D[i].imag() / n + 0.5) % mod,
98            dd = (ll)(D[i].real() / n + 0.5) % mod;
99         res[i] = ((dd << (len * 2)) + ((db + dc) << len) + da) % mod;
100     }
101     return res;
102 }
103
104 poly inv(poly a) {
105     int n = a.size();
106     if (a.size() == 1) return {inv(a[0])};
107     poly b = inv({a.begin(), a.end() - n / 2});
108     auto c = a * b;
109     c.resize(n);
110     a = b * (poly{2} - c);
111     a.resize(n);
112     return a;
113 }
114
115 poly operator/(poly a, poly b) {

```

```

111     int n = a.size() + b.size() - 1;
112     b.resize(n);
113     a = a * inv(b);
114     a.resize(n);
115     return a;
116 }

```

## 4.7 GCD

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 unsigned long long gcd(unsigned long long a, unsigned long long b) {
5     int shift = __builtin_ctzll(a | b);
6     b >>= __builtin_ctzll(b);
7     while (a) {
8         a >>= __builtin_ctzll(a);
9         if (a < b) swap(a, b);
10        a -= b;
11    }
12    return b << shift;
13 }

```

# 5 字符串

## 5.1 哈希

```

1 #include "bits/stdc++.h"
2 using namespace std;
3 const char el = '\n';
4 typedef long long ll;
5 const ll mod1 = 1e9 + 7, mod2 = 1e9 + 9;
6 struct graph {
7     vector<vector<pair<int, int>>> e;
8     graph(int n) : e(n + 1) {}
9     void adde(int u, int v, int w) { e[u].push_back({v, w}); }
10    bool check() {
11        vector<int> deg(e.size());
12        for (auto &ed : e)
13            for (auto &[to, wt] : ed) deg[to]++;
14        for (int i = 1; i < e.size(); i++)
15            if (deg[i] != e[i].size()) return false;

```

```

16        return true;
17    }
18    void euler(int u, vector<int> &res) {
19        while (e[u].size()) {
20            auto [v, w] = e[u].back();
21            e[u].pop_back();
22            euler(v, res);
23            res.push_back(w);
24        }
25    }
26 };
27 struct hsh {
28     static const int k = 3;
29     array<ll, k> a;
30 };
31 const hsh zero{0, 0, 0}, one{1, 1, 1}, mul{31, 57, 71},
32     mod{998244353, 1e9 + 7, 1e9 + 9};
33
34 const int mul1 = 31, mul2 = 157;
35 pair<ll, ll> pmul(int n) {
36     static vector<pair<ll, ll>> res = {{1, 1}};
37     while (res.size() <= n) {
38         auto [p, q] = res.back();
39         res.push_back({p * mul1 % mod1, q * mul2 % mod2});
40     }
41     return res[n];
42 }
43 struct hshstr {
44     vector<pair<ll, ll>> hsh;
45     hshstr(string s = "") {
46         hsh.push_back({0, 0});
47         for (auto c : s) {
48             auto [p, q] = hsh.back();
49             hsh.push_back({(p * mul1 + c) % mod1, (q * mul2 + c) % mod2});
50         }
51     }
52     pair<ll, ll> hash(int l, int r) {
53         auto [p1, q1] = hsh[l];
54         auto [p2, q2] = hsh[r];
55         auto [mp, mq] = pmul(r - l);
56         return {(p1 * mp - p2 + mod1) % mod1, (q1 * mq - q2 + mod2) % mod2};
57     }
58 };
59 optional<pair<vector<int>, vector<int>>> solve(vector<hshstr> &a,
60                                             vector<hshstr> &b, int n,

```

```

61         int m) {
62     map<pair<ll, ll>, int> id1, id2;
63     int k = 0;
64     for (auto &s : a) {
65         pair<ll, ll> l = s.hash(0, m), r = s.hash(m, n);
66         if (!id1.count(l)) id1[l] = ++k;
67         if (!id2.count(r)) id2[r] = ++k;
68     }
69     for (auto &s : b) {
70         pair<ll, ll> l = s.hash(0, n - m), r = s.hash(n - m, n);
71         if (!id2.count(l)) return nullopt;
72         if (!id1.count(r)) return nullopt;
73     }
74     graph g(k);
75     int t = 0;
76     for (auto &s : a) {
77         pair<ll, ll> l = s.hash(0, m), r = s.hash(m, n);
78         g.adde(id1[l], id2[r], ++t);
79     }
80     t = 0;
81     for (auto &s : b) {
82         pair<ll, ll> l = s.hash(0, n - m), r = s.hash(n - m, n);
83         g.adde(id2[l], id1[r], ++t);
84     }
85     vector<int> res;
86     if (!g.check()) return nullopt;
87     g.euler(1, res);
88     if (res.size() < 2 * a.size()) return nullopt;
89     reverse(res.begin(), res.end());
90     vector<int> p, q;
91     for (int i = 0; i < res.size(); i += 2) p.push_back(res[i]);
92     for (int i = 1; i < res.size(); i += 2) q.push_back(res[i]);
93     return pair<vector<int>, vector<int>>{p, q};
94 }
95 int main() {
96     ios::sync_with_stdio(false);
97     cin.tie(0);
98     cout << setprecision(15);
99     int tt;
100    cin >> tt;
101    while (tt--) {
102        int n, m;
103        cin >> n >> m;
104        vector<hshstr> a(n), b(n);
105        for (auto &v : a) {

```

```

106        string s;
107        cin >> s;
108        v = hshstr(s);
109    }
110    for (auto &v : b) {
111        string s;
112        cin >> s;
113        v = hshstr(s);
114    }
115    bool flg = false;
116    for (int i = 0; i < m; i++)
117        if (auto res = solve(a, b, m, i)) {
118            flg = true;
119            for (auto v : res->first) cout << v << ' ';
120            cout << el;
121            for (auto v : res->second) cout << v << ' ';
122            cout << el;
123            break;
124        }
125    if (!flg) cout << -1 << el;
126    }
127    return 0;
128 }

```

## 5.2 KMP

```

1  #include <bits/stdc++.h>
2  using namespace std;
3
4  vector<int> kmp(string s) {
5      vector<int> res = {-1};
6      for (auto c : s) {
7          int cur = res.back();
8          while (~cur && c != s[cur]) cur = res[cur];
9          res.push_back(cur + 1);
10     }
11     return res;
12 }

```

## 5.3 后缀自动机

```

1  #include <bits/stdc++.h>

```



```

2 using namespace std;
3 typedef long long ll;
4
5 const int alpha = 26;
6 struct sam {
7     struct node {
8         array<node *, alpha> next;
9         node *link;
10        int len;
11        int cnt;
12        node() {
13            fill(next.begin(), next.end(), (node *)NULL);
14            link = NULL;
15            len = 0;
16        }
17    };
18    node *nd, *end;
19    node *nnode() { return new (end++) node(); }
20    node *root, *last;
21    sam(int n) {
22        end = nd = (node *)new char[sizeof(node) * (n << 1)];
23        last = root = nnode();
24    }
25    void add(int c) {
26        node *cur = nnode();
27        cur->len = last->len + 1;
28        cur->cnt = 1;
29        node *p = last;
30        last = cur;
31        while (p && !p->next[c]) {
32            p->next[c] = cur;
33            p = p->link;
34        }
35        if (!p) {
36            cur->link = root;
37            return;
38        }
39        node *q = p->next[c];
40        if (q->len == p->len + 1) {
41            cur->link = q;
42            return;
43        }
44        node *clone = nnode();
45        clone->next = q->next;
46        clone->link = q->link;

```

```

47        clone->len = p->len + 1;
48        while (p && p->next[c] == q) {
49            p->next[c] = clone;
50            p = p->link;
51        }
52        cur->link = q->link = clone;
53    }
54    ll solve() {
55        vector<int> deg(end - nd);
56        for (auto u = nd + 1; u < end; u++) deg[u->link - nd]++;
57        queue<node *> que;
58        for (auto u = nd; u < end; u++)
59            if (!deg[u - nd]) que.push(u);
60        ll res = 0;
61        while (que.size()) {
62            auto u = que.front();
63            que.pop();
64            if (auto v = u->link) {
65                v->cnt += u->cnt;
66                if (!--deg[v - nd]) que.push(v);
67            }
68            if (u->cnt > 1) res = max(res, 1ll * u->cnt * u->len);
69        }
70        return res;
71    }
72 };

```

## 5.4 字典树

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 const char el = '\n';
4 typedef long long ll;
5 const int alpha = 26;
6 struct node {
7     array<node *, alpha> ch;
8     int cnt;
9     node() : cnt(0) { ch.fill(NULL); }
10 };
11 void add(node *u, const string &s) {
12     for (auto c : s) {
13         c -= 'a';
14         if (!u->ch[c]) u->ch[c] = new node();
15         u = u->ch[c];

```

```

16     u->cnt++;
17 }
18 }
19 pair<node *, ll> run(node *u, const string &s) {
20     ll res = 0;
21     for (auto c : s) {
22         c -= 'a';
23         if (!u->ch[c]) return {NULL, res};
24         u = u->ch[c];
25         res += u->cnt;
26     }
27     return {u, res};
28 }

```

## 6 计算几何

### 6.1 三维计算几何

```

1 #include "bits/stdc++.h"
2 using namespace std;
3 const char el = '\n';
4 typedef long long ll;
5 typedef long double ld;
6 // typedef __int128 double;
7 struct cplx {
8     ld x, y, z;
9     ld abs() { return sqrt(1.0 * x * x + 1.0 * y * y + 1.0 * z * z); }
10 };
11 cplx operator+(cplx a, cplx b) { return {a.x + b.x, a.y + b.y, a.z + b.z}; }
12 cplx operator-(cplx a, cplx b) { return {a.x - b.x, a.y - b.y, a.z - b.z}; }
13 cplx operator*(cplx a, ld b) { return {a.x * b, a.y * b, a.z * b}; }
14 cplx operator*(ld b, cplx a) { return {a.x * b, a.y * b, a.z * b}; }
15 cplx det(cplx a, cplx b) {
16     return {a.y * b.z - b.y * a.z, a.z * b.x - b.z * a.x, a.x * b.y - b.x * a.y};
17 }
18 ld dot(cplx a, cplx b) { return a.x * b.x + a.y * b.y + a.z * b.z; }
19 const ld inf = 1e30, eps = 1e-8;
20 ld dist(array<cplx, 3> a, cplx b) {
21     cplx h = det(a[0] - a[1], a[0] - a[2]);
22     ld dt = dot(b, h);
23     if (dt < eps) return inf;
24     ld cp = dot(a[0], h);
25     cplx o = cp / dt * b;

```

```

26     ld sa = det(a[0] - o, a[1] - o).abs() + det(a[1] - o, a[2] - o).abs() +
27         det(a[2] - o, a[0] - o).abs(),
28     sb = h.abs();
29     if (sa / sb > 1 + eps) return inf;
30     return o.abs();
31 }
32 int main() {
33     ios::sync_with_stdio(false);
34     cin.tie(0);
35     cout << setprecision(3);
36     int n, m;
37     cin >> n >> m;
38     vector a(n, array<cplx, 3>());
39     for (auto &t : a) {
40         for (auto &[x, y, z] : t) cin >> x >> y >> z;
41         if (dot(det(t[0] - t[1], t[0] - t[2]), t[0]) < 0) swap(t[1], t[2]);
42     }
43     while (m--) {
44         cplx b;
45         cin >> b.x >> b.y >> b.z;
46         int p = -1;
47         ld val = inf / 2;
48         for (int i = 0; i < n; i++) {
49             auto tmp = dist(a[i], b);
50             if (tmp < val) {
51                 p = i, val = tmp;
52             }
53         }
54         cout << p + 1 << el;
55     }
56     return 0;
57 }

```

### 6.2 圆凸包

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 const char el = '\n';
4 typedef long long ll;
5 typedef double ld;
6 const ld pi = acosl(-1);
7 const ld eps = 1e-9;
8 struct func {
9     ll x, y, z;

```

```

10 ld val(ld t) { return x * cos(t) + y * sin(t) + z; }
11 pair<ld, ld> zero() {
12     ld l = sqrt(x * x + y * y);
13     if (l < abs(z) - eps) return {0, 2 * pi};
14     ld a = atan2(y, x) + 2 * pi, b = acos(-z / l);
15     ld p = a - b, q = a + b;
16     while (p > 2 * pi) p -= 2 * pi;
17     while (q > 2 * pi) q -= 2 * pi;
18     if (p > q) swap(p, q);
19     return {p, q};
20 }
21 };
22 bool operator==(func a, func b) {
23     return a.x == b.x && a.y == b.y && a.z == b.z;
24 }
25 func operator+(func a, func b) { return {a.x + b.x, a.y + b.y, a.z + b.z}; }
26 func operator-(func a, func b) { return {a.x - b.x, a.y - b.y, a.z - b.z}; }
27 struct pcef {
28     vector<ld> a;
29     vector<func> f;
30     ld val(ld x) {
31         int p = upper_bound(a.begin(), a.end(), x - eps) - a.begin();
32         p = max(min(p - 1, (int)f.size() - 1), 0);
33         return f[p].val(x);
34     }
35     ld fnc(ld x) {
36         return max(val(x) + val(x + pi), val(x + pi / 2) + val(x + pi * 3 / 2));
37     }
38     ld mxm(ld l, ld r) {
39         ld res = min(fnc(l), fnc(r));
40         while (r - l > eps) {
41             ld m1 = (l + r) / 2, m2 = (m1 + r) / 2;
42             ld v1 = fnc(m1), v2 = fnc(m2);
43             res = min({res, v1, v2});
44             if (v1 < v2)
45                 r = m2;
46             else
47                 l = m1;
48         }
49         return res;
50     }
51 };
52 pcef operator+(pcef a, pcef b) {
53     pcef c = {{0}, {}};
54     int p = 0, q = 0;

```

```

55     ld x1 = 0;
56     while (p < a.f.size() && q < b.f.size()) {
57         ld xr = min(a.a[p + 1], b.a[q + 1]);
58         auto [x1, x2] = (a.f[p] - b.f[q]).zero();
59         vector<ld> ax;
60         if (x1 > x1 && x1 < xr) ax.push_back(x1);
61         if (x2 > x1 && x2 < xr) ax.push_back(x2);
62         ax.push_back(xr);
63         for (auto xx : ax) {
64             ld x = (x1 + xx) / 2;
65             auto f = a.f[p].val(x) > b.f[q].val(x) ? a.f[p] : b.f[q];
66             if (c.f.size() && c.f.back() == f)
67                 c.a.back() = xx;
68             else
69                 c.a.push_back(xx), c.f.push_back(f);
70             x1 = xx;
71         }
72         if (a.a[p + 1] < xr + eps) p++;
73         if (b.a[q + 1] < xr + eps) q++;
74     }
75     return c;
76 };
77 pcef comb(const vector<func> &a, int l, int r) {
78     if (r - l == 1) return pcef({{0, 2 * pi}, {a[l]}});
79     int m = (l + r) / 2;
80     return comb(a, l, m) + comb(a, m, r);
81 }

```

## 6.3 凸包

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 typedef long long ll;
4 typedef long double ld;
5 struct cplx {
6     ll x, y;
7     ll abs2() { return x * x + y * y; }
8     ld abs() { return sqrt(abs2()); }
9 };
10 cplx operator+(const cplx &a, const cplx &b) { return {a.x + b.x, a.y + b.y}; }
11 cplx operator-(const cplx &a, const cplx &b) { return {a.x - b.x, a.y - b.y}; }
12 bool operator<(const cplx &a, const cplx &b) {
13     return a.x != b.x ? a.x < b.x : a.y < b.y;
14 }

```

```

15 bool operator!=(const cplx &a, const cplx &b) { return a < b || b < a; }
16 bool operator==(const cplx &a, const cplx &b) { return !(a != b); }
17 ll dot(const cplx &a, const cplx &b) { return a.x * b.x + a.y * b.y; }
18 ll det(const cplx &a, const cplx &b) { return a.x * b.y - a.y * b.x; }
19 void diagsort(vector<cplx> &a, cplx o) {
20     sort(a.begin(), a.end(), [&](const cplx &p, const cplx &q) {
21         if ((o < p) != (o < q)) return o < p;
22         auto d = det(p - o, q - o);
23         if (d) return d > 0;
24         return (p - o).abs2() > (q - o).abs2();
25     });
26     a.resize(unique(a.begin(), a.end(),
27         [&](const cplx &p, const cplx &q) {
28             return !det(p - o, q - o) && dot(p - o, q - o) > 0;
29         }) -
30         a.begin());
31 }
32 vector<cplx> convex(vector<cplx> a) {
33     sort(a.begin(), a.end());
34     auto o = a[0];
35     a.erase(a.begin());
36     diagsort(a, o);
37     vector<cplx> res = {o};
38     for (auto v : a) {
39         while (res.size() >= 2) {
40             auto p = res[res.size() - 2], q = res[res.size() - 1];
41             if (det(q - p, v - q) > 0) break;
42             res.pop_back();
43         }
44         res.push_back(v);
45     }
46     return res;
47 }
48 double diameter(const vector<cplx> &a) {
49     double res = (a.back() - a[0]).abs();
50     for (int i = 1; i < a.size(); i++) res += (a[i] - a[i - 1]).abs();
51     return res;
52 }
53
54 cplx find(vector<cplx> &conv, cplx a) {
55     int l = 0, r = conv.size() - 1;
56     while (r - l) {
57         int m = (l + r) / 2;
58         if (det(conv[m + 1] - conv[m], a) < 0)
59             r = m;

```

```

60     else
61         l = m + 1;
62     }
63     return conv[l];
64 }
65 ll solve(vector<ll> &a) {
66     vector<ll> s = {0};
67     for (auto v : a) s.push_back(s.back() + v);
68     vector<cplx> conv;
69     for (int i = 0; i < s.size(); i++) {
70         cplx r = {i, s[i]};
71         while (conv.size() >= 2) {
72             cplx p = conv[conv.size() - 2], q = conv.back();
73             if (det(q - p, r - q) > 0) break;
74             conv.pop_back();
75         }
76         conv.push_back(r);
77     }
78     ll res = 0;
79     for (int i = 0; i < a.size(); i++) {
80         auto p = find(conv, {1, a[i]});
81         res = max(res, s[i] - p.y + (p.x - i) * a[i]);
82     }
83     return res;
84 }

```

## 6.4 扫描线

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 const char el = '\n';
4 typedef long long ll;
5 typedef __int128 li;
6 const int inf = 3e6;
7 struct frac {
8     li a, b;
9     frac(li _a = 0, li _b = 1) : a(_a), b(_b) {
10         if (b < 0) a = -a, b = -b;
11     }
12 };
13 // note that without gcd plus operation also enlarges the fractor.
14 frac operator+(frac a, frac b) { return {a.a * b.b + b.a * a.b, a.b * b.b}; }
15 frac operator-(frac a, frac b) { return {a.a * b.b - b.a * a.b, a.b * b.b}; }
16 frac operator*(frac a, frac b) { return {a.a * b.a, a.b * b.b}; }

```

```

17 frac operator/(frac a, frac b) { return {a.a * b.b, a.b * b.a}; }
18 bool operator<(frac a, frac b) { return a.a * b.b < b.a * a.b; }
19 bool operator>(frac a, frac b) { return b < a; }
20 bool operator==(frac a, frac b) { return !(a < b); }
21 bool operator<=(frac a, frac b) { return !(b < a); }
22 bool operator!=(frac a, frac b) { return a < b || b < a; }
23 bool operator==(frac a, frac b) { return !(a != b); }
24 frac abs(frac a) { return {a.a < 0 ? -a.a : a.a, a.b}; }
25 struct cplx {
26     frac x, y;
27 };
28 cplx operator+(const cplx &a, const cplx &b) { return {a.x + b.x, a.y + b.y}; }
29 cplx operator-(const cplx &a, const cplx &b) { return {a.x - b.x, a.y - b.y}; }
30 bool operator<(const cplx &a, const cplx &b) {
31     return a.x == b.x ? a.y < b.y : a.x < b.x;
32 }
33 bool operator!=(const cplx &a, const cplx &b) { return a < b || b < a; }
34 bool operator==(const cplx &a, const cplx &b) { return !(a != b); }
35 struct sgmt {
36     cplx a, b;
37     sgmt(cplx _a, cplx _b) : a(_a), b(_b) {
38         if (b < a) swap(a, b);
39     }
40     frac cut(frac x) const {
41         if (x == a.x) return a.y;
42         if (x == b.x) return b.y;
43         auto t1 = (b.x - x) * a.y, t2 = (x - a.x) * b.y;
44         t1.a += t2.a;
45         auto t3 = b.x - a.x;
46         auto res = t1 / t3;
47         return res;
48     }
49 };
50 bool operator<(const sgmt &a, const sgmt &b) {
51     if (a.a != b.a) return a.a < b.a;
52     return a.b < b.b;
53 }
54 map<sgmt, vector<cplx>> solve(vector<sgmt> line, vector<cplx> vtx) {
55     sort(line.begin(), line.end(), [&](sgmt a, sgmt b) { return a.a < b.a; });
56     auto cmp1 = [&](sgmt a, sgmt b) {
57         if (a.b != b.b) return a.b < b.b;
58         return a.a < b.a;
59     };
60     set<sgmt, decltype(cmp1)> curl(cmp1);
61     frac cx;

```

```

62 auto cmp2 = [&](sgmt a, sgmt b) {
63     frac ay = a.cut(cx), by = b.cut(cx);
64     if (ay == by) return a.cut(cx + 1) < b.cut(cx + 1);
65     return ay < by;
66 };
67 set<sgmt, decltype(cmp2)> scan(cmp2);
68 auto findc = [&](cplx v) {
69     auto [x, y] = v;
70     return *scan.lower_bound({x, y}, {x + 1, y - inf});
71 };
72 auto cmp3 = [&](sgmt a, sgmt b) { return a.a.y < b.a.y; };
73 set<sgmt, decltype(cmp3)> hori(cmp3);
74 auto findh = [&](cplx v) -> optional<sgmt> {
75     auto [x, y] = v;
76     auto it = hori.upper_bound({x, y}, {x, y});
77     if (it == hori.begin()) return nullopt;
78     it = prev(it);
79     if (it->b.y < y) return nullopt;
80     return *it;
81 };
82 reverse(line.begin(), line.end());
83 reverse(vtx.begin(), vtx.end());
84 map<sgmt, vector<cplx>> res;
85 while (vtx.size()) {
86     hori.clear();
87     frac x = vtx.back().x;
88     vector<cplx> curv;
89     while (vtx.size() && vtx.back().x <= x)
90         curv.push_back(vtx.back()), vtx.pop_back();
91     while (curl.size() && curl.begin()->b.x <= x)
92         scan.erase(*curl.begin()), curl.erase(curl.begin());
93     cx = x;
94     while (line.size() && line.back().a.x <= x) {
95         auto l = line.back();
96         line.pop_back();
97         if (l.b.x < x) continue;
98         if (l.a.x == l.b.x)
99             hori.insert(l);
100         else
101             scan.insert(l), curl.insert(l);
102     }
103     for (auto v : curv)
104         if (auto t = findh(v))
105             res[*t].push_back(v);
106     else

```

```

107     res[findc(v)].push_back(v);
108 }
109 return res;
110 }
111 const int k = 20;
112 struct graph {
113     vector<vector<int>> e;
114     graph(int n) : e(n + 1) {}
115     void adde(int u, int v) {
116         e[u].push_back(v);
117         e[v].push_back(u);
118     }
119     vector<array<int, k>> f;
120     vector<int> d;
121     void makef(int u, int w) {
122         f[u][0] = w;
123         for (int i = 1; i < k; i++) f[u][i] = f[f[u][i - 1]][i - 1];
124         d[u] = d[w] + 1;
125         for (auto &v : e[u])
126             if (v == w) {
127                 v = e[u].back(), e[u].pop_back();
128                 break;
129             }
130         for (auto v : e[u]) makef(v, u);
131     }
132     int lca(int u, int v) {
133         if (d[u] < d[v]) swap(u, v);
134         for (int i = 0; i < k; i++)
135             if (d[u] - d[v] >> i & 1) u = f[u][i];
136         if (u == v) return u;
137         for (int i = k - 1; i >= 0; i--)
138             if (f[u][i] != f[v][i]) u = f[u][i], v = f[v][i];
139         return f[u][0];
140     }
141     vector<int> s, c;
142     void addup(int u) {
143         for (auto v : e[u]) s[v] += s[u], addup(v), c[u] += c[v];
144     }
145     vector<int> solve(vector<pair<int, int>> q) {
146         int n = e.size();
147         f.resize(n);
148         d.assign(n, 0);
149         makef(1, 0);
150         s.assign(n, 0);
151         c.assign(n, 0);

```

```

152     for (auto [u, v] : q) {
153         auto w = lca(u, v);
154         s[w]++;
155         c[u]++, c[v]++, c[w]--, c[f[w][0]]--;
156     }
157     addup(1);
158     vector<int> res;
159     for (auto [u, v] : q) {
160         auto w = lca(u, v);
161         res.push_back(s[u] + s[v] - 2 * s[w] + c[w] - 1);
162     }
163     return res;
164 }
165 };
166 li read() {
167     ll n;
168     cin >> n;
169     return n;
170 }
171 int main() {
172     ios::sync_with_stdio(false);
173     cin.tie(0);
174     cout << setprecision(3);
175     int n, m;
176     cin >> n >> m;
177     vector<cplx> a(n);
178     for (auto &[x, y] : a) x.a = read(), y.a = read();
179     a.insert(a.begin(), {0, 0});
180     vector<pair<int, int>> edge(n - 1);
181     for (auto &[u, v] : edge) cin >> u >> v;
182     vector<sgmt> line;
183     for (auto [u, v] : edge) line.push_back(sgmt(a[u], a[v]));
184     vector<pair<cplx, cplx>> qry(m);
185     for (auto &[a, b] : qry)
186         a.x.a = read(), a.x.b = read(), a.y.a = read(), a.y.b = read(),
187         b.x.a = read(), b.x.b = read(), b.y.a = read(), b.y.b = read();
188     map<cplx, int> id;
189     for (int i = 1; i <= n; i++) id[a[i]] = i;
190     vector<cplx> vtx;
191     for (auto [a, b] : qry)
192         for (auto v : {a, b})
193             if (!id.count(v)) vtx.push_back(v);
194     sort(vtx.begin(), vtx.end());
195     vtx.resize(unique(vtx.begin(), vtx.end()) - vtx.begin());
196     for (int i = 0; i < vtx.size(); i++) id[vtx[i]] = n + 1 + i;

```

```

197 auto vol = solve(line, vtx);
198 graph g(id.size());
199 for (const auto &[p, q] : line) {
200     auto arr = vol[{p, q}];
201     arr.insert(arr.begin(), p), arr.push_back(q);
202     for (int i = 1; i < arr.size(); i++) g.adde(id[arr[i] - 1], id[arr[i]]);
203 }
204 vector<pair<int, int>> q;
205 for (auto &[a, b] : qry) q.push_back({id[a], id[b]});
206 vector<int> res = g.solve(q);
207 for (auto v : res) cout << v << ' ';
208 cout << el;
209 return 0;
210 }

```

## 7 杂项

### 7.1 随机数生成

```

1
2 namespace myrand {
3     random_device r;
4     default_random_engine e(r());
5     std::uniform_int_distribution<int> g(0, 1e9);
6 } // namespace myrand

```

### 7.2 STL 容器 +Lambda

```

1
2 auto cmp = [&](int u, int v) {
3     if (g.e[u].size() != g.e[v].size()) return g.e[u].size() < g.e[v].size();
4     return u < v;
5 };
6 set<int, decltype(cmp)> st(cmp);

```

### 7.3 子集枚举

```

1

```

```

2 for (int p = 0; p < n; p++)
3     for (int q = p; q; q = q - 1 & p) {
4         int k = ~p & (n - 1);
5         c[k + q] = max(c[k + q], a[k] + b[q]);
6     }

```

### 7.4 二项式反演

$$f(n) = \sum_{i=0}^n \binom{n}{i} g(i) \iff g(n) = \sum_{i=0}^n (-1)^{n-i} \binom{n}{i} f(i)$$

$$f(n) = \sum_{i=m}^n \binom{n}{i} g(i) \iff g(n) = \sum_{i=m}^n (-1)^{n-i} \binom{n}{i} f(i)$$

$f(n)$ :  $n$  选  $i$  个;  $g(n)$ : 恰好  $n$  个

$$f(n) = \sum_{i=n}^m \binom{i}{n} g(i) \iff g(n) = \sum_{i=n}^m (-1)^{i-n} \binom{i}{n} f(i)$$

$f(n)$ : 钦定  $n$  个;  $g(n)$ : 恰好  $n$  个

### 7.5 莫比乌斯反演

$$f(n) = \sum_{d|n} g(d) \iff g(n) = \sum_{d|n} \mu(d) f(n/d)$$

$$f(n) = \sum_{n|d} g(d) \iff g(n) = \sum_{n|d} \mu(d/n) f(d)$$

### 7.6 容斥原理

$$\left| \bigcup_{i=1}^n S_i \right| = \sum_{m=1}^n (-1)^{m-1} \sum_{a_i < a_{i+1}} \left| \bigcap_{i=1}^m S_{a_i} \right|$$

$$\left| \bigcap_{i=1}^n S_i \right| = |U| - \left| \bigcup_{i=1}^n \overline{S_i} \right|$$

$$f(S) = \sum_{S \subseteq T} g(T) \iff g(S) = \sum_{S \subseteq T} (-1)^{|T|-|S|} f(T)$$

### 7.7 Min-Max 容斥

$$\max_{i \in S} x_i = \sum_{T \subseteq S} (-1)^{|T|-1} \min_{j \in T} x_j$$

$$\min_{i \in S} x_i = \sum_{T \subseteq S} (-1)^{|T|-1} \max_{j \in T} x_j$$