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Mixed Coalitional Stabilities With Full Participation of Sanctioning Opponents Within the Graph Model for Conflict Resolution

Shinan Zhao, Haiyan Xu, Keith W. Hipel, *Fellow, IEEE*, and Liping Fang, *Fellow, IEEE*

***Abstract*—Mixed coalition analysis approaches are developed within the framework of the graph model for conflict resolution (GMCR) for analyzing the heterogeneous multicoalitional opponents having mixed coalitional sanctions, in which some sanctioning coalitions may only invoke coalitional improvements (CIs) while others may go to any reachable states to jointly sanction a focal coalition's CIs. To accomplish this, the concept of a full coalition set is defined in which each participating decision maker (DM) is represented once either as an individual player or part of a coalition. All possible scenarios in which a full coalition set could be formed is called the universal coalition set. When calculating the stability of a specific state for a particular coalition in a given conflict, the remaining DMs form the universal coalition set whose moves have the potential to block any CIs by the focal coalition within four specified solution concepts having full participation of sanctioning opponents. To handle heterogeneous opponents with mixed CIs (MCIs), the mixed coalition analysis approach is proposed, which provides a more general coalition analysis framework than existing coalition analysis approaches. Moreover, the interrelationships among coalitional stabilities and mixed coalitional stabilities with full participation are investigated followed by their corresponding matrix representations which can significantly improve their computational efficiency and make the computer implementation possible. Finally, a case study is investigated to demonstrate how to employ the proposed mixed coalition analysis approaches to address a real-world environmental conflict.**

***Index Terms*—Coalition analysis, graph model for conflict resolution (GMCR), heterogeneous opponents, mixed coalition stability.**

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I. Introduction

COALITIONS are very common in conflicts because a decision maker (DM) may gain more benefits by forming a coalition with others. Moreover, the equilibria of a conflict can be greatly affected by coalitions. The well-designed graph model for conflict resolution (GMCR) approach provides a powerful and flexible set of tools to address challenging conflicts involving coalitions in a way that is distinct from classical game theory [1], [2]. Most importantly, GMCR requires only relative preference information for each DM instead of cardinal preferences. In this article, the authors mainly focus on coalition analysis approaches within the GMCR paradigm.

Research related to coalition analyses includes coalition formation, preference of a coalition, definitions of coalition stabilities, and so on. Within the framework of conflict analysis [3], [4], coalition methods were built under the assumption the coalition will last for the entirety of the dispute understudy. In particular, based on the similarity of individual DMs' preferences, Kuhn *et al.* [5] proposed a state-based metric to combine two or more DMs' preferences into one single coalition preference. Subsequently, Meister *et al.* [6], [7] and Hipel and Meister [8] constructed an option-based metric to determine which coalitions are more likely to form among DMs and to calculate the overall preferences of a coalition by using the preference tree. Within the GMCR paradigm, Kilgour *et al.* [9], [10] developed a new approach for coalition analysis based on the impact of a coalition on the equilibria of a given conflict rather than the similarity of preferences, in which the coalitional improvements (CIs) and coalitional Nash (CNash) stability were defined. Inohara and Hipel [11], [12] then extended CNash to coalitional general metarationality (CGMR), coalitional symmetric metarationality (CSMR), and coalitional sequential stability (CSEQ) within the framework of GMCR.

Afterward, the aforementioned four classical coalitional stabilities were extended to handle attitudes [13], [14], unknown preference [15], fuzzy preference [16], and Pareto coalition improvements (PCIs) [17]. To analyze a conflict with both attitudes and coalitions, Walker *et al.* [13], [14] defined coalition relational stabilities, including coalition relational Nash (CRNash), coalition relational GMR (CRGMR), coalition relational SEQ (CRSEQ), and coalition relational SMR (CRSMR). These studies on coalition analysis, however, are based on the assumption of transitive graphs, in which a DM may move twice consecutively. To enhance the computer implementation of coalition analysis, explicit algebraic or matrix expressions of coalitional stabilities were developed by Xu *et al.* [15], in which they took into account both transitive and intransitive graphs. Later, Xu *et al.* [18] defined *CSEQ* for the situations in which the opponents invoked sanctions independently ($CSEQ_2$) and as members of a grand coalition of all sanctioning DMs ($CSEQ_1$). Bashar *et al.* [16] proposed coalition fuzzy stability concepts for the purpose of coalition analysis under fuzzy preference. To allow for a counterresponse by a focal coalition after the credible countermoves by sanctioning opponents, Rêgo and Vieira [19] proposed coalitional symmetric SEQ stability (CSSEQ). Zhu *et al.* [17] extended coalition improvements to PCIs and defined Pareto coalitional stabilities, including Pareto Nash (PNash), Pareto GMR (PGMR), Pareto SMR (PSMR), and Pareto SEQ (PSEQ). However, Zhu *et al.* [17] defined the Pareto reachable list of the sanctioning opponents as the union of all possible coalitional moves (CMs) among the opponents, which does not consider the sequentially coalitional movements by multiple coalitions.

The aforementioned coalitional stabilities can be classified into two branches in terms of the sanctioning opponents. The first one takes into account all possible CMs by the sanctioning opponents, which includes CGMR [11], CSEQ [11], coalition fuzzy GMR (CFGMR) [16], coalition fuzzy SMR (CFSMR) [16], coalition fuzzy SEQ (CFSEQ) [16], CSSEQ [19], PGMR [17], PSMR [17], and PSEQ [17]. Specifically, a possible coalition by the opponents is a subset of all opponents and involves some of the sanctioning opponents. The second branch was proposed by Xu *et al.* [18], [20] from the perspective of full participation, in which the sanctioning opponents are regarded as a full coalition set, consisting of either one grand coalition in $CSEQ_1$ or individual DMs in $CSEQ_2$. In many actual conflicts, however, there exist many possible full coalition sets containing all of the sanctioning opponents but a focal coalition does not know which scenario will occur because it does not control the actions of the opponents. Hence, when the sanctioning coalitions are unknown, it is more general and reasonable for a focal coalition to consider every possible full coalition set in which each opponent is represented once.

When sanctioning, if a sanctioning coalition only invokes CIs, then it is a credible coalition; if it uses any CMs regardless of preference, then it is an noncredible coalition. In existing coalition analysis approaches, all of the sanctioning opponents are assumed to be either credible (CSEQ) or noncredible (CGMR and CSMR), which means that they are homogeneous. In a conflict with multiple coalitions, however, there exists a mixed situation in which some of the sanctioning coalitions are credible while others are noncredible. Alternatively, the sanctioning opponents are heterogeneous when having mixed coalitional sanctions.

This article contains three main contributions.

1) All possible coalition scenarios with full participation of the sanctioning opponents are considered in the newly defined coalitional stabilities for analyzing the situation in which the sanctioning coalitions are unknown for a focal coalition. The coalition analysis approach developed by Xu *et al.* [18], [20] can be regarded as being a special case of the new coalition stabilities proposed in this article.
2) Mixed coalitional stabilities are developed for analyzing heterogeneous multicoalitional opponents, in which the sanctioning moves by the opponents could be mixed, either CIs or CMs, which provides a more general coalition analysis framework than existing coalition analysis approaches.
3) The matrix representations of the proposed coalitional stabilities are provided to facilitate their computational implementation.

The reminder of this article is organized as follows. The existing coalition analysis approaches based on the framework of GMCR are presented in Section II. In Section III, the coalitional stabilities with full participation of sanctioning opponents are developed within the GMCR paradigm to analyze unknown sanctioning coalitions, in which each possible full coalition set containing all of the sanctioning opponents is taken into account. Subsequently, the mixed coalitional stabilities are constructed in Section IV for handling heterogeneous sanctioning opponents having mixed CIs (MCIs). To address the high computational complexity of coalitional stabilities with full participation and mixed coalitional stabilities, their matrix representations are established in Section V. In Section VI, the Changzhou Foreign Language School (CFLS) environmental contamination conflict is studied to demonstrate how to model and analyze an actual dispute by using the new coalition analysis methodology developed in this article.

II. Coalition Analysis Approaches Based on the Graph Model

A. Graph Model for Conflict Resolution

The GMCR can be modeled as $G = \langle N, S, \{A_i, \succsim_i : i \in N\}\rangle$, in which four elements are included [20]–[22].

1) $N = \{1, 2, \ldots, i, \ldots, n\}$ is a finite nonempty set of two or more DMs involved in this conflict where n is the total number of DMs.
2) $S = \{s_1, s_2, \ldots, s_l, \ldots, s_m\}$ is a finite nonempty set of feasible states, in which s_l is the lth state and m is the total number of states.
3) A_i is a finite nonempty set of DM i's oriented arcs, which contains all of the moves in one step controlled by DM i.
4) $\succsim_i$ is DM i's preference relations, where $s_1 \succ_i s_2$ means that DM i prefers state s_1 to state s_2, $s_1 \sim_i s_2$ indicates that DM i equally prefers these two states, and $s_1 \succsim_i s_2$ means that $s_1 \succ_i s_2$ or $s_1 \sim_i s_2$.

Definition 1: Let $s, q \in S$. The set of states more preferred to state s by DM $i \in N$ can be defined as

$$\Phi_i^+(s) = \{q \in S : q \succ_i s\}. \tag{1}$$

Definition 2: Let A_i be the arc set of unilateral moves (UMs) among states for DM i. The set of reachable states by UMs and unilateral improvements (UIs) from state s by DM i can be, respectively, expressed by

$$R_i(s) = \{q \in S : (s, q) \in A_i\}, \text{ and} \tag{2}$$
$$R_i^+(s) = \{q \in S : (s, q) \in A_i \text{ and } q \succ_i s\}. \tag{3}$$

Note that $R_i^+(s) = R_i(s) \cap \Phi_i^+(s)$.

Let a nonempty coalition $H \subseteq N$. The reachable list or the set of UMs by some or all DMs in H from state $s \in S$ is expressed by $R_H(s) \subseteq S$, in which any DM could move more than one step, but not twice consecutively [18], [21], [23], [24]. For state $s_1 \in R_H(s)$, let $\Omega_H(s, s_1)$ be the set of all last DMs in legal sequences by H from s to s_1. The reachable list of H can be formally defined as follows.

Definition 3 (Reachable List of H): Let $s \in S$, and a nonempty coalition $H \subseteq N$. The reachable list of H at state s, $R_H(s)$, can be inductively obtained by the following.

1) $\Omega_H(s, s_1) = \emptyset$ for all $s_1 \in S$.
2) If DM $j \in H$ and $s_1 \in R_j(s)$, then $s_1 \in R_H(s)$ and $\Omega_H(s, s_1) = \Omega_H(s, s_1) \cup \{j\}$.
3) If $s_1 \in R_H(s)$, DM $j \in H$, and $s_2 \in R_j(s_1)$, then $s_2 \in R_H(s)$ and $\Omega_H(s, s_2) = \Omega_H(s, s_2) \cup \{j\}$, provided $\Omega_H(s, s_1) \neq \{j\}$.

Subsequently, a legal sequence of UIs for H can be defined [21], [23], [24]. Let $R_H^+(s) \subseteq S$ be the set of states that can be reached by any legal sequence of UIs, by some or all DMs in H at state s. For $s_1 \in R_H^+(s)$, let $\Omega_H^+(s, s_1)$ be the set of all last DMs in H by UIs from s to s_1.

Definition 4 (UIs by H): Let state $s \in S$, and a nonempty coalition $H \subseteq N$. The set of UIs for H, $R_H^+(s)$, at state s can be inductively determined by the following.

1) $\Omega_H^+(s, s_1) = \emptyset$ for all $s_1 \in S$.
2) If DM $j \in H$ and $s_1 \in R_j^+(s)$, then $s_1 \in R_H^+(s)$ and $\Omega_H^+(s, s_1) = \Omega_H^+(s, s_1) \cup \{j\}$.
3) If $s_1 \in R_H^+(s)$, DM $j \in H$, and $s_2 \in R_j^+(s_1)$, then $s_2 \in R_H^+(s)$ and $\Omega_H^+(s, s_2) = \Omega_H^+(s, s_2) \cup \{j\}$, provided $\Omega_H^+(s, s_1) \neq \{j\}$.

In Definition 4, each UM by a focal DM in H is a UI for this DM, which is different from that in Definition 3.

Within the framework of GMCR, a UI for an individual DM is a move from a state to a more preferred state, which is beneficial for the DM itself. The concept of CIs is presented in Definition 5 in which a terminal state reached by H from a starting state is more preferred by each DM in H [10]–[12], [20]. In other words, a CI for coalition H requires that the terminal state reached by H from a starting state is favorable to the starting state for all of the players in the alliance.

Let a coalition $H = \{1, \ldots, i, \ldots, h\}$, in which "$i$" is the ith DM and "h" is the total number of DMs in H. Then, the set of states more preferred to state $s \in S$ by each DM in H can be written as

$$\Phi_H^+(s) = \bigcap_{i \in H} \Phi_i^+(s). \tag{4}$$

Definition 5 (CIs by H): Let $s \in S$ and a nonempty coalition $H \subseteq N$. The set of CIs for H from state s can be defined as

$$CR_H^+(s) = R_H(s) \cap \Phi_H^+(s). \tag{5}$$

Note that $CR_H^+(s)$ in Definition 5 differs from $R_H^+(s)$ in Definition 4. The former means that a terminal state by H is more preferred to the initial state by every DM in H while the latter indicates that each one-step UM by a focal DM in H must be a UI for the DM.

B. Existing Coalition Analysis Approaches

The comparisons between existing coalition approaches [10]–[12], [17], [18], [20] and this article are presented in Table I. As shown in the right-hand column in Table I, the coalition research contained in [11], [12], and [17] takes into account all nonempty possible sanctioning coalitions by the opponents, in which a possible sanctioning coalition includes only some of the opponents. Different from the above studies, Xu *et al.* [18], [20] considered the sanctioning opponents as being either a grand coalition or individual DMs from the perspective of full participation. When it is unknown, however, which sanctioning coalitions will definitely participate, many possible full coalition sets actually exist, in which each set contains all of the opponents. Hence, it is more reasonable and necessary for a conservative focal coalition to consider each possible coalition scenario with all opponents represented when the coalitions among the sanctioning opponents are unknown. In this article, the coalitional stabilities with full participation of sanctioning opponents are constructed, including coalitional *GMR* (C_F*GMR*), coalitional *SMR* (C_F*SMR*), coalitional *SEQ* (C_F*SEQ*), and coalitional *SSEQ* (C_F*SSEQ*) with full participation, which is more general than the other approaches.

Furthermore, all of the sanctioning coalitions, including a particular coalition with only one opponent are assumed to only invoke CIs in *CSEQ*, *PSEQ*, *CSEQ*$_1$, and *CSEQ*$_2$ whereas all of them can utilize any CMs regardless of preference in *CGMR*, *PGMR*, *PSMR*, and *CSMR*. In brief, the sanctioning opponents in existing coalition approaches [10]–[12], [17], [18], [20] are thought of as being homogeneous. In actual disputes, however, a mixed or heterogeneous situation may exist with respect to the sanctioning moves by the opponents, in which some sanctioning coalitions may only invoke CIs while others may utilize any CMs to jointly block a focal coalition's CIs. In this case, the sanctioning opponents can be regarded as being heterogeneous. To handle heterogeneous sanctioning opponents with MCIs, the logical and matrix representations of the mixed coalitional stabilities, consisting of mixed coalitional two-step stability (MCTS) and mixed coalitional SMR (MCSMR), are developed in this article.

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TABLE I
COMPARISONS BETWEEN EXISTING COALITION APPROACHES AND THIS ARTICLE

References	Coalitional Stabilities	Transitive Graphs?	Matrix Algorithms?	Sanctioning Opponents
Kilgour et al. [10]	$CNash$	Yes	No	No
Inohara and Hipel [11], [12]	$CNash$, $CGMR$, $CSMR$, $CSEQ$	Yes	No	Homogeneous (All moves)
Xu et al. [15], [18], [20]	$CNash$, $CGMR$, $CSEQ_1$, $CSEQ_2$, $CSMR$	No	Yes	Homogeneous (A grand coalition or individual DMs)
Zhu et al. [17]	$PNash$, $PGMR$, $PSMR$, $PSEQ$	No	No	Homogeneous (All moves by each coalition)
This research	C_FGMR, C_FSMR, C_FSEQ, C_FSSEQ	No	Yes	Homogeneous (All coalitions with full participation)
	$MCTS$, $MCSMR$	No	Yes	Heterogeneous (All coalitions with full participation)

III. Coalition Analysis Based on Full Participation of Sanctioning Opponents

A. Full Coalitional Moves

Let "N" denote the set of DMs in a conflict, a nonempty subset $H \subseteq N$, and "$h = |H|$" be the cardinality of "H." A possible coalition with full participation of DMs in H is called a full coalition set.

Definition 6 (Full Coalition Set): A full coalition set containing all of the DMs in H in which each DM is only represented once can be defined as

$$F_H = \{H_1, H_2, \ldots, H_i, \ldots, H_f\} \tag{6}$$

where every H_i is an nonempty coalition, "f" is the total number of coalitions in H ($f \leq h$), and $H_i \cap H_j = \emptyset$ for any $i \neq j$, $i, j = 1, 2, \ldots, f$.

According to Definition 6, a full coalition set F_H includes all DMs in H, but every DM can only be a member of one coalition. Alternatively, all elements in F_H are exclusive. Note that the full coalition set F_H could consist of multiple nonempty coalitions.

For any $H_i \in F_H$, let $|H_i|$ be the cardinality of $H_i \in F_H$, then $|H_i| \geq 1$. Since F_H includes all DMs in H, one can obtain $\sum_{i=1}^{f} |H_i| = h$. If $|H_i| = 1$, then H_i consists of only one DM. In particular, if $|H_i| = 1$ for any $H_i \in F_H$, then $f = h$, which means that each $H_i \in F_H$ reduces to an individual DM. If $f = 1$, then there is only a grand coalition in F_H, which contains all of the DMs in H.

When the coalitions among DMs are unknown, there exist many possible full coalition sets because a DM could form a coalition with any DMs and a coalition may include two or more DMs.

Definition 7 (Universal Coalition Set): Let $\mathbb{F}_H$ be the universal coalition set including all possible full coalition sets which could be formed by H. Then, $\mathbb{F}_H$ can be defined as

$$\mathbb{F}_H = \{F_H^1, F_H^2, \ldots, F_H^j, \ldots, F_H^k\} \tag{7}$$

where F_H^j is the jth full coalition set and "k" is the total number of all possible full coalition sets.

For instance, if there are three DMs in H denoted by $H = \{1, 2, 3\}$, then $F_H = \{1, (2, 3)\}$ is a full coalition set containing all of the DMs in H, in which DMs 2 and 3 in round brackets form a coalition and DM 1 is an individual player. Mathematically, there are total five possible full coalition sets which could be formed by DMs 1–3

$$F_H^1 = \{1, 2, 3\}, F_H^2 = \{(1, 2), 3\}, F_H^3 = \{1, (2, 3)\}$$
$$F_H^4 = \{(1, 3), 2\}, \quad \text{and} \quad F_H^5 = \{(1, 2, 3)\}.$$

The universal coalition set consisting of all possible full coalition sets for H is

$$\mathbb{F}_H = \{F_H^1, F_H^2, F_H^3, F_H^4, F_H^5\}. \tag{8}$$

Note that in (8) each full coalition set $F_H^j \in \mathbb{F}_H$ ($j = 1, 2, 3, 4, 5$) includes all of the DMs in H, and is therefore called full participation.

For a given full coalition set $F_H \in \mathbb{F}_H$, if $s \in S$ can be reached by some coalitions in F_H in which each coalition can move to any reachable states regardless of preference but cannot move twice consecutively, then s is called a full CM (FCM) by F_H. The set of FCMs by F_H from state s is denoted by $CR_{F_H}(s)$. Let the set of all last coalitions in F_H by FCMs from s to s_1 be $\Omega_{F_H}(s, s_1)$.

Definition 8 (The Set of FCMs by F_H): For $F_H \in \mathbb{F}_H$, the set of FCMs by F_H from state $s \in S$, $CR_{F_H}(s)$, can be determined inductively by the following.

1) $\Omega_{F_H}(s, s_1) = \emptyset$ for all $s_1 \in S$.
2) For each $H_i \in F_H$, if $s_1 \in R_{H_i}(s)$, then $s_1 \in CR_{F_H}(s)$ and $\Omega_{F_H}(s, s_1) = \Omega_{F_H}(s, s_1) \cup H_i$.
3) If $s_1 \in CR_{F_H}(s)$, $H_i \in F_H$, and $s_2 \in R_{H_i}(s_1)$, then $s_2 \in CR_{F_H}(s)$ and $\Omega_{F_H}(s, s_2) = \Omega_{F_H}(s, s_2) \cup H_i$, provided $\Omega_{F_H}(s, s_1) \neq H_i$.

Using 2), every FCM by each coalition $H_i \in F_H$ from s is included in $CR_{F_H}(s)$. Then, all other FCMs reachable from every state $s_1 \in CR_{F_H}(s)$ by some coalitions in F_H can be inductively obtained and added to $CR_{F_H}(s)$ by repeating 3) until no further states can be identified or there is no change in $\Omega_{F_H}(s, s_2)$ for any $s_2 \in CR_{F_H}(s)$. There is an important constraint in Definition 8 whereby each coalition $H_i \in F_H$ cannot move twice consecutively, which is expressed

TABLE II
THREE CLASSICAL REACHABLE LISTS

Types	Definitions	DMs
$R_H(s_1) = \{s_2, s_3, s_4, s_5, s_6\}$	Definition 3	Individual non-credible DMs $\{1, 2, 3, 4\}$
$R_H^+(s_1) = \{s_2, s_3, s_4\}$	Definition 4	Individual credible DMs $\{1, 2, 3, 4\}$
$CR_H^+(s_1) = \{s_2\}$	Definition 5	A grand coalition $\{(1, 2, 3, 4)\}$

by $\Omega_{F_H}(s, s_1) \neq H_i$. Alternatively, it is not necessary to identify the states reached by a coalition $H_i \in F_H$ again in step 3) since all reachable states by each coalition $H_i \in F_H$ from s have been added to $CR_{F_H}(s)$ by using 2).

Definition 9 (Class Reachable Moves for $\mathbb{F}_H$): The set of class reachable moves for $\mathbb{F}_H$ from state s is defined as

$$CR_{\mathbb{F}_H}(s) = \bigcup_{j=1}^{k} CR_{F_H^j}(s). \tag{9}$$

For a given full coalition set $F_H \in \mathbb{F}_H$, if $s \in S$ can be reached by some coalitions in F_H in which each coalition moves only to CIs but cannot move twice consecutively, then s is called a full CI (FCI) by F_H. Let the set of all last coalitions in F_H by FCIs from s to s_1 be $\Omega^+_{F_H}(s, s_1)$.

Definition 10 (Set of FCIs by F_H): For $F_H \in \mathbb{F}_H$, the set of FCIs by F_H from state $s \in S$, $CR^+_{F_H}(s)$, can be determined inductively by

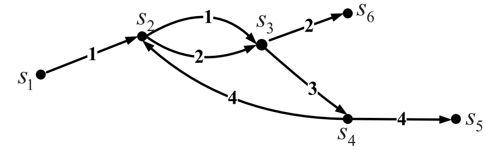
1) $\Omega^+_{F_H}(s, s_1) = \emptyset$ for all $s_1 \in S$.
2) For each $H_i \in F_H$, if $s_1 \in CR^+_{H_i}(s)$, then $s_1 \in CR^+_{F_H}(s)$ and $\Omega^+_{F_H}(s, s_1) = \Omega^+_{F_H}(s, s_1) \cup H_i$.
3) If $s_1 \in CR^+_{F_H}(s)$, $H_i \in F_H$, and $s_2 \in CR^+_{H_i}(s_1)$, then $s_2 \in CR^+_{F_H}(s)$ and $\Omega^+_{F_H}(s, s_2) = \Omega^+_{F_H}(s, s_2) \cup H_i$, provided $\Omega^+_{F_H}(s, s_1) \neq H_i$.

In particular, if $|H_i| = 1$ for every $H_i \in F_H$ ($f = h$), then Definition 10 is the same as Definition 4, which means that $CR^+_{F_H}(s) = R^+_H(s)$; if there is only one grand coalition in F_H ($f = 1$), then Definition 10 is equal to Definition 5, which indicates that $CR^+_{F_H}(s) = CR^+_H(s)$. Therefore, Definition 10 provides a more general framework than Definitions 4 and 5. In other words, both $R^+_H(s)$ and $CR^+_H(s)$ are special cases of $CR^+_{F_H}(s)$.

Definition 11 (Class Reachable Improvements for $\mathbb{F}_H$): The set of class reachable improvements for $\mathbb{F}_H$ from state s is defined as

$$CR^+_{\mathbb{F}_H}(s) = \bigcup_{j=1}^{k} CR^+_{F_H^j}(s). \tag{10}$$

Example 1: Let $H = \{1, 2, 3, 4\}$ and the set of states be $S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$. As presented in Fig. 1, each DM's preference is listed in the bottom part of the directed graph and a label on an arc in the graph refers to the DM who controls this move. If s_1 is selected as the status quo state, using the information provided in Fig. 1, three classical reachable lists, $R_H(s_1)$, $R^+_H(s_1)$, and $CR^+_H(s_1)$, can be constructed as shown in Table II. Note that in the third column all DMs in a pair of parentheses form a coalition.



$$s_2 \succ_1 s_1 \succ_1 s_4 \succ_1 s_3 \succ_1 s_6 \succ_1 s_5$$
$$s_3 \succ_2 s_2 \succ_2 s_6 \succ_2 s_5 \succ_2 s_1 \succ_2 s_4$$
$$s_5 \succ_3 s_4 \succ_3 s_3 \succ_3 s_2 \succ_3 s_1 \succ_3 s_6$$
$$s_4 \succ_4 s_5 \succ_4 s_2 \succ_4 s_3 \succ_4 s_1 \succ_4 s_6$$

Fig. 1. Graph model with four DMs and six states.

As shown in Table II, if there is no coalition in H, then the reachable list of H from s_1 is $R_H(s_1) = \{s_2, s_3, s_4, s_5, s_6\}$, and the set of UIs of H from s_1 is $R^+_H(s_1) = \{s_2, s_3, s_4\}$; if there is one grand coalition, $\{(1, 2, 3, 4)\}$, then the set of CIs of H from s_1 is $CR^+_H(s_1) = \{s_2\}$. However, there exist some other scenarios, such as DM 1 and DM 2 form a coalition and the remaining DMs are individual, denoted by $\{(1, 2), 3, 4\}$.

Using Definitions 6 and 7, one can obtain the universal coalition set of all possible full coalition sets with full participation of all DMs in H

$$\mathbb{F}_H = \left\{F_H^1, F_H^2, \ldots, F_H^j, \ldots, F_H^{15}\right\} \tag{11}$$

in which

$$\begin{aligned}
&F_H^1 = \{1, 2, 3, 4\},\ F_H^2 = \{(1, 2), 3, 4\},\ F_H^3 = \{1, (2, 3), 4\}\\
&F_H^4 = \{1, 2, (3, 4)\},\ F_H^5 = \{2, (1, 3), 4\},\ F_H^6 = \{1, 3, (2, 4)\}\\
&F_H^7 = \{(1, 4), 2, 3\},\ F_H^8 = \{(1, 2), (3, 4)\},\ F_H^9 = \{(1, 3), (2, 4)\}\\
&F_H^{10} = \{(1, 4), (2, 3)\},\ F_H^{11} = \{(1, 2, 3), 4\},\ F_H^{12} = \{(1, 2, 4), 3\}\\
&F_H^{13} = \{(1, 3, 4), 2\}, F_H^{14} = \{1, (2, 3, 4)\}, \text{and } F_H^{15} = \{(1, 2, 3, 4)\}.
\end{aligned}$$

Take $F_H^5 = \{2, (1, 3), 4\}$ for example. In this case, DMs 1 and 3 form a coalition while DMs 2 and 4 are independent. The set of FCIs for F_H^5, $CR^+_{F_H^5}(s_1)$, can be calculated as follows.

1) s_2 is a CI for both DMs 1 and DM 3 from s_1, so $s_2 \in CR^+_{F_H^5}(s_1)$.
2) s_3 can be reached by DM 2 from s_2, which is a UI for DM 2, so $s_3 \in CR^+_{F_H^5}(s_1)$.
3) s_4 is a CI for both DMs 1 and 3 from s_3, so $s_4 \in CR^+_{F_H^5}(s_1)$.

Hence, $CR^+_{F_H^5}(s_1) = \{s_2, s_3, s_4\}$.

Similarly, one can easily obtain $CR^+_{F_H^1}(s_1) = \{s_2, s_3, s_4\}$ and $CR^+_{F_H^{15}}(s_1) = \{s_2\}$.

Note that $CR^+_{F_H^1}(s_1) = R^+_H(s_1)$ since all DMs in F_H^1 are independent and $CR^+_{F_H^{15}}(s_1) = CR^+_H(s_1)$ because all DMs in F_H^{15} form a grand coalition.

B. Coalitional Stabilities With Full Participation of Opponents

For a focal coalition $H_i \subseteq N$, the set of its opponents can be denoted by $N \backslash H_i$, which consists of all of the DMs in N except H_i. Let $s, s_1 \in S$, $F_{N \backslash H_i}$ be a possible full coalition set in which each opponent in $N \backslash H_i$ is represented once, and $\mathbb{F}_{N \backslash H_i}$ denote

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the universal coalition set including all possible full coalition sets in $N\backslash H_i$. From the perspective of H_i, there exist many full coalition sets among the sanctioning opponents but H_i does not know which scenario will occur. Let $F_{N\backslash H_i} \in \mathbb{F}_{N\backslash H_i}$, the set of FCIs from state s_1 by $F_{N\backslash H_i}$ be $CR^{+}_{F_{N\backslash H_i}}(s_1)$, and the set of states less or equally preferred to state s by H_i be $\Phi^{\precsim}_{H_i}(s) = \{q \in S \mid \exists\, j \in H_i, s \succsim_j q\}$. Since no sanctioning opponents are considered in *CNash*, it is not necessary to define *CNash* in this section.

Definition 12: State $s \in S$ is coalitional general metarationally stable with full participation of sanctioning opponents ($C_F GMR$) for H_i, denoted by $s \in S^{C_F GMR}_{H_i}$, iff for every $s_1 \in CR^{+}_{H_i}(s)$, there exist $F_{N\backslash H_i} \in \mathbb{F}_{N\backslash H_i}$ and $s_2 \in CR_{F_{N\backslash H_i}}(s_1)$ such that $s_2 \in \Phi^{\precsim}_{H_i}(s)$.

In Definition 12, from the viewpoint of H_i, if there exists at least one full coalition set including all of its opponents who can jointly sanction any of its CIs by any FCMs, then H_i will stay at s. In other words, state s is $C_F GMR$ stable for H_i.

After the adverse countermoves by a particular $F_{N\backslash H_i}$ in Definition 12, H_i could try to escape the sanction by further counterresponses. Hence, the CSMR stability with full participation of sanctioning opponents is presented below.

Definition 13: State $s \in S$ is coalitional symmetric metarationally stable with full participation of sanctioning opponents ($C_F SMR$) for H_i, denoted by $s \in S^{C_F SMR}_{H_i}$, iff for every $s_1 \in CR^{+}_{H_i}(s)$, there exist $F_{N\backslash H_i} \in \mathbb{F}_{N\backslash H_i}$ and $s_2 \in CR_{F_{N\backslash H_i}}(s_1)$ such that $s_2 \in \Phi^{\precsim}_{H_i}(s)$ and $s_3 \in \Phi^{\precsim}_{H_i}(s)$ for all $s_3 \in R_{H_i}(s_2)$.

In Definitions 12 and 13, all of the sanctioning coalitions in each $F_{N\backslash H_i} \in \mathbb{F}_{N\backslash H_i}$ are assumed to be noncredible, in which they may move to any reachable states to block each CI by a focal coalition. In some cases, the sanctioning moves by the opponents could be credible as defined in Definitions 14 and 15.

Definition 14: State $s \in S$ is coalitional sequentially stable with full participation of sanctioning opponents ($C_F SEQ$) for H_i, denoted by $s \in S^{C_F SEQ}_{H_i}$, iff for every $s_1 \in CR^{+}_{H_i}(s)$, there exist $F_{N\backslash H_i} \in \mathbb{F}_{N\backslash H_i}$ and $s_2 \in CR^{+}_{F_{N\backslash H_i}}(s_1)$ such that $s_2 \in \Phi^{\precsim}_{H_i}(s)$.

Definition 15: State $s \in S$ is coalitional symmetric sequentially stable with full participation of sanctioning opponents ($C_F SSEQ$) for H_i, denoted by $s \in S^{C_F SSEQ}_{H_i}$, iff for every $s_1 \in CR^{+}_{H_i}(s)$, there exist $F_{N\backslash H_i} \in \mathbb{F}_{N\backslash H_i}$ and $s_2 \in CR^{+}_{F_{N\backslash H_i}}(s_1)$ such that $s_2 \in \Phi^{\precsim}_{H_i}(s)$ and $s_3 \in \Phi^{\precsim}_{H_i}(s)$ for all $s_3 \in R_{H_i}(s_2)$.

Let $CS \in \{C_F GMR, C_F SMR, C_F SEQ, C_F SSEQ\}$, the universally coalitional stabilities (UCSs) within a specified solution concept with full participation of opponents can be defined.

Definition 16: State $s \in S$ is universally *CS* stable, denoted by $s \in S^{UCS}$, iff s is *CS* stable for every nonempty set $H \subseteq N$.

Compared to Definitions 12 and 13, all of the sanctioning coalitions in Definitions 14 and 15 are regarded as being credible, in which each sanctioning coalition only invokes CIs for sanctioning purposes. Furthermore, if there exists only one full coalition set with all of the opponents being a grand coalition, then $C_F SEQ$ is the same as $CSEQ_1$ [18], [20]. On the other hand, if there exists only one full coalition set with all of individual opponents, then $C_F SEQ$ is equivalent to $CSEQ_2$ [18], [20]. Therefore, the coalitional stabilities with full participation of sanctioning opponents developed in this article are more general than those proposed by Xu *et al.* [18], [20].

IV. Mixed Coalition Analysis With Full Participation of Heterogeneous Opponents

A. Mixed Coalitional Improvements

As mentioned before, in the coalitional stability definitions of CGMR, CSMR, PGMR, PSMR, $C_F GMR$, and $C_F SMR$, each sanctioning coalition can go to any reachable states by CMs, which could be more, equally, or less preferred to the initial state by the coalition. In this case, the sanctioning coalition is called noncredible since some states by CMs may be equally or less preferred to the initial state by the sanctioning coalition. In *CSEQ*, $CSEQ_1$, $CSEQ_2$, *PSEQ*, $C_F SEQ$, and $C_F SSEQ$, however, each sanctioning coalition can only invoke CIs (CIs) to reach states which are more preferred by every member in this coalition. Hence, the sanctioning coalition is regarded as being credible. However, there exists a mixed situation in which some sanctioning coalitions may use any CMs while others may only utilize CIs to jointly block a focal coalition's CIs. In this article, these mixed sanctioning moves are defined as MCIs.

Let $F_{N\backslash H_i} \in \mathbb{F}_{N\backslash H_i}$, $C \subseteq F_{N\backslash H_i}$ be the set of credible sanctioning coalitions in $F_{N\backslash H_i}$ who move only to CIs, and $C' = F_{N\backslash H_i} \setminus C$ be the set of noncredible sanctioning coalitions in $F_{N\backslash H_i}$ who can move to any CMs even if these moves are equally or less preferred. The set of MCIs by $F_{N\backslash H_i}$ with the set of credible sanctioning coalitions being C from state s can be denoted by $CR^{\oplus}_{F_{N\backslash H_i}(C)}(s) \subseteq S$, in which a sanctioning coalition can either move to a CM or CI from state s. If $s_1 \in CR^{\oplus}_{F_{N\backslash H_i}(C)}(s)$, then $\Omega^{\oplus}_{F_{N\backslash H_i}}(s, s_1)$ denotes the set of all last coalitions in $F_{N\backslash H_i}$ by MCIs from s to s_1.

Definition 17 (MCIs): The set of MCIs by a full coalition set $F_{N\backslash H_i}$ from state s, $CR^{\oplus}_{F_{N\backslash H_i}(C)}(s)$, can be defined inductively by the following.

1) Assuming $\Omega^{\oplus}_{F_{N\backslash H_i}}(s, s_1) = \emptyset$ for all $s_1 \in S$.
2) If $H_j \in C$ and $s_1 \in CR^{+}_{H_j}(s)$, then $s_1 \in CR^{\oplus}_{F_{N\backslash H_i}(C)}(s)$ and $\Omega^{\oplus}_{F_{N\backslash H_i}}(s, s_1) = \Omega^{\oplus}_{F_{N\backslash H_i}}(s, s_1) \cup H_j$; otherwise, if $H_j \in C'$ and $s_1 \in R_{H_j}(s)$, then $s_1 \in CR^{\oplus}_{F_{N\backslash H_i}(C)}(s)$ and $\Omega^{\oplus}_{F_{N\backslash H_i}}(s, s_1) = \Omega^{\oplus}_{F_{N\backslash H_i}}(s, s_1) \cup H_j$.
3) If $s_1 \in CR^{\oplus}_{F_{N\backslash H_i}(C)}(s)$, $H_j \in C$, and $s_2 \in CR^{+}_{H_j}(s_1)$, then, provided $\Omega^{\oplus}_{F_{N\backslash H_i}}(s, s_1) \neq H_j$, $s_2 \in CR^{\oplus}_{F_{N\backslash H_i}(C)}(s)$ and $\Omega^{\oplus}_{F_{N\backslash H_i}}(s, s_2) = \Omega^{\oplus}_{F_{N\backslash H_i}}(s, s_2) \cup H_j$; otherwise, if $s_1 \in CR^{\oplus}_{F_{N\backslash H_i}(C)}(s)$, $H_j \in C'$, and $s_2 \in R_{H_j}(s_1)$, then $s_2 \in CR^{\oplus}_{F_{N\backslash H_i}(C)}(s)$ and $\Omega^{\oplus}_{F_{N\backslash H_i}}(s, s_2) = \Omega^{\oplus}_{F_{N\backslash H_i}}(s, s_1) \cup H_j$, provided $\Omega^{\oplus}_{F_{N\backslash H_i}}(s, s_1) \neq H_j$.

According to Definitions 8, 10, and 17, one can obtain that $CR^{+}_{F_{N\backslash H_i}}(s) \subseteq CR^{\oplus}_{F_{N\backslash H_i}(C)}(s) \subseteq CR_{F_{N\backslash H_i}}(s)$. In particular, if all of the sanctioning coalitions in $F_{N\backslash H_i}$ are noncredible ($C = \emptyset$), then $CR^{\oplus}_{F_{N\backslash H_i}(C)}(s) = CR_{F_{N\backslash H_i}}(s)$; if all of the sanctioning coalitions in $F_{N\backslash H_i}$ are credible ($C = F_{N\backslash H_i}$), then $CR^{\oplus}_{F_{N\backslash H_i}(C)}(s) = CR^{+}_{F_{N\backslash H_i}}(s)$.

Definition 18 (Class MCIs): Let $\mathbb{F}_{N\setminus H_i}$ be the universal coalition set including all possible full coalition sets in $N \setminus H_i$. The set of class MCIs for $\mathbb{F}_{N\setminus H_i}$ from state s is defined as

$$CR^{\oplus}_{\mathbb{F}_{N\setminus H_i}(\mathbb{C})}(s) = \bigcup_{j=1}^{k} CR^{\oplus}_{F^j_{N\setminus H_i}(C_j)}(s) \tag{12}$$

in which C_j is the set of credible coalitions in $F^j_{N\setminus H_i}$, and $\mathbb{C} = \bigcup_{j=1}^{k} C_j$ is the set of all credible sanctioning coalitions in $\mathbb{F}_{N\setminus H_i}$.

To illustrate how to calculate the set of MCIs for $F_{N\setminus H_i}$ with the set of credible coalitions be C, denoted by $CR^{\oplus}_{F_{N\setminus H_i}(C)}(s)$, Example 1 is utilized here. In Example 1, $F^5_H = \{2, (1, 3), 4\}$ and the set of FCIs by F^5_H from state s_1 is $CR^+_{F^5_H}(s_1) = \{s_2, s_3, s_4\}$. If only DM 4 is noncredible, then $C_5 = \{2, (1, 3)\}$ and $C_5^{'} = \{4\}$, which means DM 4 can move to any UMs instead of only UIs. Then, the set of MCIs for F^5_H is $CR^{\oplus}_{F^5_H(C_5)}(s_1) = \{s_2, s_3, s_4, s_5\}$. Different from $CR^+_{F^5_H}(s_1)$, s_5 can be reached by F^5_H because DM 4 is noncredible who can move to s_5 from s_4.

B. Mixed Coalitional Stabilities

Let $H_i \in N$ be a focal coalition, $N \setminus H_i$ denotes the set of its opponents, and $\mathbb{F}_{N\setminus H_i}$ be the universal coalition set of $N \setminus H_i$. For a particular full coalition set $F_{N\setminus H_i} \in \mathbb{F}_{N\setminus H_i}$, let C be the set of credible sanctioning coalitions in $F_{N\setminus H_i}$. To analyze the complex behavior of the sanctioning opponents with MCIs, mixed coalitional stabilities are designed below based on Definitions 12 and 15.

Definition 19: Let C be the set of credible sanctioning coalitions in $F_{N\setminus H_i}$. State $s \in S$ is MCTS stable for H_i, denoted by $s \in S^{MCTS}_{H_i}$, iff for every $s_1 \in CR^+_{H_i}(s)$, there exist $F_{N\setminus H_i} \in \mathbb{F}_{N\setminus H_i}$ and $s_2 \in CR^{\oplus}_{F_{N\setminus H_i}(C)}(s_1)$ such that $s_2 \in \Phi^{\preceq}_{H_i}(s)$.

In Definition 19, H_i considers the mixed sanctioning countermoves by all possible full coalition sets of its opponents after it goes to a CI, in which the credible sanctioning coalitions will only invoke CIs to block any CI by H_i while noncredible sanctioning coalitions will move to any reachable states, even if some of them may be equally or less preferred to the initial state by the sanctioning coalitions, to sanction any CIs by H_i. If there exists at least one full coalition set of the sanctioning opponents who can block any of H_i's CIs from s, then s is *MCTS* stable for H_i.

Definition 20: Let C be the set of credible sanctioning coalitions in $F_{N\setminus H_i}$. State $s \in S$ is mixed CSMR stable (*MCSMR*) for H_i, denoted by $s \in S^{MCSMR}_{H_i}$, iff for every $s_1 \in CR^+_{H_i}(s)$, there exist $F_{N\setminus H_i} \in \mathbb{F}_{N\setminus H_i}$ and $s_2 \in CR^{\oplus}_{F_{N\setminus H_i}(C)}(s_1)$ such that $s_2 \in \Phi^{\preceq}_{H_i}(s)$ and $s_3 \in \Phi^{\preceq}_{H_i}(s)$ for all $s_3 \in R_{H_i}(s_2)$.

In Definition 20, H_i takes into account not only the mixed sanctioning countermoves by its opponents but also its further counterresponse to escape the sanction in comparison with *MCTS* in Definition 19.

In particular, if $CR^{\oplus}_{F_{N\setminus H_i}(C)}(s) = CR_{F_{N\setminus H_i}}(s)$ for each $F_{N\setminus H_i} \in \mathbb{F}_{N\setminus H_i}$, then Definitions 19 and 20 are equal to Definitions 12 and 13, respectively; if $CR^{\oplus}_{F_{N\setminus H_i}(C)}(s) = CR^+_{F_{N\setminus H_i}}(s)$ for each $F_{N\setminus H_i} \in \mathbb{F}_{N\setminus H_i}$, then Definitions 19 and 20 are equal to Definitions 14 and 15, respectively. Hence, the mixed coalitional stabilities provides a more general framework for coalition analyses in comparison with existing coalition analysis approaches.

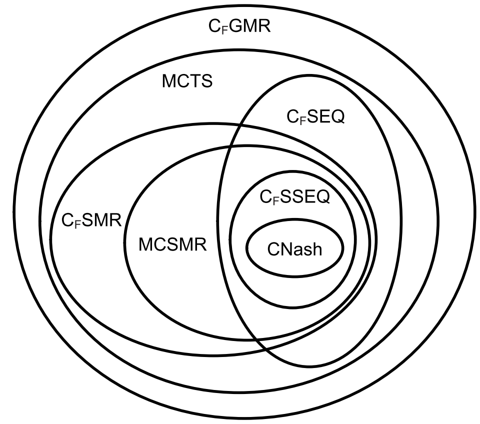

Fig. 2. Inter-relationships among the newly defined coalitional stabilities.

Similar to Definition 16, the universally mixed coalitional stabilities (UMCSs) can be formally defined.

Definition 21: Let $MCS \in \{MCTS, MCSMR\}$. State $s \in S$ is UMCS stable, denoted by $s \in S^{UMCS}$, iff s is MCS stable for every nonempty set $H \subseteq N$.

By using Definitions 12–15, 19, and 20, one can easily determine the interrelationships among *CNash*, $C_F GMR$, $C_F SMR$, $C_F SEQ$, $C_F SSEQ$, *MCTS*, and *MCSMR* as shown in Fig. 2.

As illustrated in Fig. 2, the following relationships can be obtained:

$$\begin{aligned} &S^{CNash}_{H_i} \subseteq S^{C_F SSEQ}_{H_i} \subseteq S^{C_F SEQ}_{H_i} \subseteq S^{MCTS}_{H_i} \subseteq S^{C_F GMR}_{H_i} \\ &S^{CNash}_{H_i} \subseteq S^{C_F SSEQ}_{H_i} \subseteq S^{MCSMR}_{H_i} \subseteq S^{C_F SMR}_{H_i} \\ &\qquad \subseteq S^{MCTS}_{H_i} \subseteq S^{C_F GMR}_{H_i}. \end{aligned}$$

The proofs of these relationships are similar to those of the relationships among Nash, GMR, and SMR as given in [20] and [21].

Proof: One can prove $S^{CNash}_{H_i} \subseteq S^{C_F SSEQ}_{H_i} \subseteq S^{C_F SEQ}_{H_i}$ first. If $s \in S^{CNash}_{H_i}$, then $CR^+_{H_i}(s) = \emptyset$. By using Definition 15, one can further obtain that $s \in S^{C_F SSEQ}_{H_i}$. Hence, $S^{CNash}_{H_i} \subseteq S^{C_F SSEQ}_{H_i}$. For any $s \in S^{C_F SSEQ}_{H_i}$, if $CR^+_{H_i}(s) = \emptyset$, then one can find $s \in S^{C_F SEQ}_{H_i}$ according to Definition 14. Otherwise, for any $s_1 \in CR^+_{H_i}(s)$, there exist at least one $F_{N\setminus H_i} \in \mathbb{F}_{N\setminus H_i}$ and $s_2 \in CR^+_{F_{N\setminus H_i}}(s_1)$ such that $s_2 \in \Phi^{\preceq}_{H_i}(s)$ and $s_3 \in \Phi^{\preceq}_{H_i}(s)$ for all $s_3 \in R_{H_i}(s_2)$ as given in Definition 15. That indicates that there exist $F_{N\setminus H_i} \in \mathbb{F}_{N\setminus H_i}$ and $s_2 \in CR^+_{F_{N\setminus H_i}}(s_1)$ such that $s_2 \in \Phi^{\preceq}_{H_i}(s)$, namely, $S^{C_F SSEQ}_{H_i} \subseteq S^{C_F SEQ}_{H_i}$. Therefore, $S^{CNash}_{H_i} \subseteq S^{C_F SSEQ}_{H_i} \subseteq S^{C_F SEQ}_{H_i}$ holds.

Other relationships among the coalitional stabilities defined in this section can be similarly proven. ■

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V. Matrix Representation of Coalitional Stabilities and Mixed Coalitional Stabilities

A. Basic Matrices in GMCR

Considering the high computational complexity of logical representations [20], [23], [25], the matrix representation is constructed in this section to make the computer implementation of coalitional stabilities and mixed coalitional stabilities possible. Three matrix operations are first defined which will be used later.

Definition 22 (Hadamard Product): Let M and K be $m \times m$ matrices. The Hadamard product of the two matrices can be denoted by $W = M \circ K$ with entry (s, q)

$$W(s, q) = M(s, q) \cdot K(s, q) \tag{13}$$

in which $W(s, q)$, $M(s, q)$, and $K(s, q)$ refer to the value of entry (s, q) for matrices W, M, and K, respectively.

Definition 23 (Disjunction Operator): Let M and K be $m \times m$ matrices. The disjunction operator of the two matrices can be denoted by $B = M \vee K$ with entry (s, q)

$$B(s, q) = \begin{cases} 1 & \text{if } M(s, q) + K(s, q) \neq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

Definition 24 (Sign Function): For an $m \times m$ matrix M, the sign function of M is defined as follows with entry (s, q):

$$\text{sign}[M(s, q)] = \begin{cases} 1 & M(s, q) > 0 \\ 0 & M(s, q) = 0 \\ -1 & M(s, q) < 0. \end{cases}$$

B. Matrix Representation of Coalitional Moves

Let $s, q \in S$, $H_i \subseteq N$, $H = N \setminus H_i$, F_H be a full coalition set of H, and $\mathbb{F}_H$ be the universal coalition set of H.

Definition 25: The FCM reachability matrix for F_H is an $m \times m$ matrix, denoted by CM_{F_H}, with entry (s, q)

$$CM_{F_H}(s, q) = \begin{cases} 1 & \text{if } q \in CR_{F_H}(s) \\ 0 & \text{otherwise.} \end{cases}$$

Definition 26: The FCI reachability matrix for F_H is an $m \times m$ matrix, denoted by $CM^+_{F_H}$, with entry (s, q)

$$CM^+_{F_H}(s, q) = \begin{cases} 1 & \text{if } q \in CR^+_{F_H}(s) \\ 0 & \text{otherwise.} \end{cases}$$

If $CR_{F_H}(s)$ and $CR^+_{F_H}(s)$ are written as an 0-1 row vector, then one can find that

$$CR_{F_H}(s) = e_s^{\mathrm{T}} \cdot CM_{F_H} \text{ and } CR^+_{F_H}(s) = e_s^{\mathrm{T}} \cdot CM^+_{F_H}$$

where e_s^{T} denotes the transpose of e_s, the sth standard basis vector of m-dimensional Euclidean space.

Definition 27 (UM Matrix): Let $H_l \subseteq H$ and $s, q \in S$. The UM reachability matrix of H_l is an $m \times m$ 0-1 matrix, denoted by M_{H_l}, with entry (s, q)

$$M_{H_l}(s, q) = \begin{cases} 1 & \text{if } q \in R_{H_l}(s) \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

Definition 28 (UI Matrix): Let $H_l \subseteq H$ and $s, q \in S$. The UI reachability matrix of H_l is an $m \times m$ 0-1 matrix, denoted by $M^+_{H_l}$, with entry (s, q)

$$M^+_{H_l}(s, q) = \begin{cases} 1 & \text{if } q \in R^+_{H_l}(s) \\ 0 & \text{otherwise.} \end{cases} \tag{16}$$

Definition 29 (CI Matrix): Let $H_l \in F_H$ and $s, q \in S$. The CI reachability matrix of H_l is an $m \times m$ 0-1 matrix, denoted by $CM^+_{H_l}$, with entry (s, q)

$$CM^+_{H_l}(s, q) = \begin{cases} 1 & \text{if } q \in CR^+_{H_l}(s) \\ 0 & \text{otherwise.} \end{cases} \tag{17}$$

According to Definitions 8 and 10, each coalition $H_l \in F_H$ cannot move twice consecutively, which is called the rule of legal FCMs or FCIs.

Definition 30: For $H_l \in F_H$, and $t = 1, 2, 3, \ldots$, define the $m \times m$ matrices $CM^{(t)}_{H_l}$ and $CM^{(t,+)}_{H_l}$ with (s, q) entries as

$$CM^{(t)}_{H_l}(s, q) = \begin{cases} 1 & \text{if } q \in S \text{ is reachable from } s \in S \text{ in} \\ & \text{exactly } t \text{ legal FCMs by coalitions} \\ & \text{in } F_H \text{ with last coalition } H_l \\ 0 & \text{otherwise} \end{cases}$$

$$CM^{(t,+)}_{H_l}(s, q) = \begin{cases} 1 & \text{if } q \in S \text{ is reachable from } s \in S \text{ in} \\ & \text{exactly } t \text{ legal FCIs by coalitions} \\ & \text{in } F_H \text{ with last coalition } H_l \\ 0 & \text{otherwise.} \end{cases}$$

Based on Definition 30, the following lemma can be obtained.

Lemma 1: For $H_l \in F_H$, the two matrices $CM^{(t)}_{H_l}$ and $CM^{(t,+)}_{H_l}$ satisfy

$$CM^{(1)}_{H_l}(s, q) = M_{H_l}(s, q), \text{ and for } t = 2, 3, \ldots$$

$$CM^{(t)}_{H_l}(s, q) = \text{sign}\left[\left(\bigvee_{H_k \in F_H \setminus H_l} CM^{(t-1)}_{H_k}\right) \cdot M_{H_l}\right] \tag{18}$$

and

$$CM^{(1,+)}_{H_l}(s, q) = CM^+_{H_l}(s, q), \text{ and for } t = 2, 3, \ldots$$

$$CM^{(t,+)}_{H_l}(s, q) = \text{sign}\left[\left(\bigvee_{H_k \in F_H \setminus H_l} CM^{(t-1,+)}_{H_k}\right) \cdot CM^+_{H_l}\right]. \tag{19}$$

Because proofs for (18) and (19) are very similar, only (19) is verified here.

Proof: For $t = 2$, the definition of matrix multiplication shows that $G(s, q)$, the (s, q) entry of the matrix $G = (\bigvee_{H_k \in \{F_H \setminus H_l\}} CM^+_{H_k}) \cdot CM^+_{H_l}$, is nonzero iff state q is reachable from state s in exactly two CIs, with the last coalition H_l. The condition $H_k \in \{F_H \setminus H_l\}$ implies that coalition H_l cannot make two CIs consecutively. Hence, $G(s, q) \neq 0$ iff state q is reachable from state s in exactly two legal CIs. Then

$$\begin{aligned} &\text{sign}\left[\left(\bigvee_{H_k \in \{F_H \setminus H_l\}} CM^+_{H_k}\right) \cdot CM^+_{H_l}\right] \\ &= \text{sign}\left[\left(\bigvee_{H_k \in \{F_H \setminus H_l\}} CM^{(1,+)}_{H_k}\right) \cdot CM^+_{H_l}\right] = CM^{(2,+)}_{H_l}. \end{aligned}$$

Now suppose that $t > 2$. Since

$$CM_{H_k}^{(t-1,+)}(s,q) = \begin{cases} 1 & \text{if } q \in S \text{ is reachable from } s \in S \\ & \text{in exactly } t-1 \text{ legal FCIs by} \\ & F_H \text{ with the last coalition } H_l \\ 0 & \text{otherwise.} \end{cases}$$

Using matrix multiplication, matrix $B = \text{sign}[(\bigvee_{H_k \in \{F_H \setminus H_l\}} CM_{H_k}^{(t-1,+)}) \cdot CM_{H_l}^{+}]$ has (s,q) entry

$$B(s,q) = \begin{cases} 1 & \text{if } q \in S \text{ is reachable from } s \in S \text{ in exactly} \\ & t \text{ legal FCIs by } F_H \text{ with last coalition } H_l \\ 0 & \text{otherwise} \end{cases}$$

which confirms (19). ■

For $s \in S$, let $A_{F_H}(s)$ be the FCM arc set of F_H from state s, and $A_{F_H} = \bigcup_{s\in S} A_{F_H}(s)$. Then, the number of elements in A_{F_H} is $l_0 = |A_{F_H}|$. Similarly, let $A_{F_H}^{+}(s)$ be the FCI arc set of F_H from state s, and $A_{F_H}^{+} = \bigcup_{s\in S} A_{F_H}^{+}(s)$. Then, the number of elements in $A_{F_H}^{+}$ is $l_1 = |A_{F_H}^{+}|$.

Lemma 2: For $F_H \in \mathbb{F}_H$, let δ_0 and δ_1 be the number of iteration steps to find $CR_{F_H}(s)$ and $CR_{F_H}^{+}(s)$, respectively. Then $\delta_0 \le l_0$ and $\delta_1 \le l_1$.

According to Lemmas 1 and 2, the following theorem can be obtained.

Theorem 1: Let state $s \in S$, and a full coalition set $F_H \in \mathbb{F}_H$. The FCM reachability matrix CM_{F_H} and FCI reachability matrix $CM_{F_H}^{+}$ of F_H can be, respectively, defined as

$$CM_{F_H} = \bigvee_{t=1}^{l_0} \bigvee_{H_l \in F_H} CM_{H_l}^{(t)}, \text{ and} \tag{20}$$

$$CM_{F_H}^{+} = \bigvee_{t=1}^{l_1} \bigvee_{H_l \in F_H} CM_{H_l}^{(t,+)}. \tag{21}$$

Since proofs for (20) and (21) are similar, only the proof for (21) is given in this section.

Proof: Assume that $D = \bigvee_{t=1}^{l_1} \bigvee_{H_l \in F_H} CM_{H_l}^{(t,+)}$. Based on Definition 26, $CM_{F_H}^{+}(s,q) = 1$ iff $q \in CR_{F_H}^{+}(s)$. Since $\delta_1 \le l_1$, by Definition 30, $q \in CR_{F_H}^{+}(s)$ implies that there exist $1 \le t_0 \le \delta_1$ and $H_0 \in F_H$ such that $CM_{H_0}^{(t_0,+)}(s,q) = 1$. This implies that matrix D has (s,q) entry 1. Therefore, $CM_{F_H}^{+}(s,q) = 1$ iff $D(s,q) = 1$. Since $CM_{F_H}^{+}$ and D are 0-1 matrices, it follows that $CM_{F_H}^{+} = D$. ■

Using Definitions 9 and 11, the universal FCM and FCI reachability matrices for $\mathbb{F}_H$ can be, respectively, denoted by

$$CM_{\mathbb{F}_H} = \bigvee_{F_H \in \mathbb{F}_H} CM_{F_H}, \text{ and}$$

$$CM_{\mathbb{F}_H}^{+} = \bigvee_{F_H \in \mathbb{F}_H} CM_{F_H}^{+}.$$

Let a focal coalition $H_i \subseteq N$, the set of its opponents be $N \setminus H_i$, and $m = |S|$ be the number of feasible states.

Definition 31 (Coalitional Disimprovement Matrix): Let $H_i \subseteq N$ and $s, q \in S$. The coalitional disimprovement matrix of H_i is an $m \times m$ 0-1 matrix, denoted by $P_{H_i}^{-,=}$, with entry (s,q)

$$P_{H_i}^{-,=}(s,q) = \begin{cases} 1 & \text{if } q \in \Phi_{H_i}^{\preceq}(s) \\ 0 & \text{otherwise.} \end{cases} \tag{22}$$

C. Matrix Representation of Coalitional Stabilities

According to Definitions 12–15 in Section III, the matrix representations for *C_FGMR*, *C_FSMR*, *C_FSEQ*, and *C_FSSEQ* can be constructed as follows.

Define the $m \times m$ *C_FGMR* stability matrix as

$$M_{H_i}^{C_F GMR} = CM_{H_i}^{+} \cdot \left[E - \text{sign}\left(CM_{\mathbb{F}_{N\setminus H_i}} \cdot (P_{H_i}^{-,=})^{\mathrm{T}}\right)\right]. \tag{23}$$

Theorem 2: State $s \in S$ is *C_FGMR* stable for H_i, denoted by $s \in S_{H_i}^{C_F GMR}$, iff $M_{H_i}^{C_F GMR}(s,s) = 0$.

Proof: Since $M_{H_i}^{C_F GMR}(s,s)$

$$= \left(e_s^{\mathrm{T}} \cdot CM_{H_i}^{+}\right) \cdot \left[\left(E - \text{sign}\left(CM_{\mathbb{F}_{N\setminus H_i}} \cdot \left(P_{H_i}^{-,=}\right)^{\mathrm{T}}\right)\right) \cdot e_s\right]$$
$$= \sum_{s_1=1}^{m} CM_{H_i}^{+}(s,s_1)\left[1 - \text{sign}\left(e_{s_1}^{\mathrm{T}} \cdot CM_{\mathbb{F}_{N\setminus H_i}} \cdot \left(e_s^{\mathrm{T}} \cdot P_{H_i}^{-,=}\right)^{\mathrm{T}}\right)\right]$$

then $M_{H_i}^{C_F GMR}(s,s) = 0$ holds iff $\forall s_1 \in S$

$$CM_{H_i}^{+}(s,s_1)\left[1 - \text{sign}\left(\left(e_{s_1}^{\mathrm{T}} \cdot CM_{\mathbb{F}_{N\setminus H_i}}\right) \cdot \left(e_s^{\mathrm{T}} \cdot P_{H_i}^{-,=}\right)^{\mathrm{T}}\right)\right] = 0. \tag{24}$$

Note that (24) is equivalent to

$$\left(e_{s_1}^{\mathrm{T}} \cdot CM_{\mathbb{F}_{N\setminus H_i}}\right) \cdot \left(e_s^{\mathrm{T}} \cdot P_{H_i}^{-,=}\right)^{\mathrm{T}} \ne 0 \text{ for any } s_1 \in CR_{H_i}^{+}(s).$$

This implies that for any $s_1 \in CR_{H_i}^{+}(s)$, there exist at least one $F_{N\setminus H_i} \in \mathbb{F}_{N\setminus H_i}$ and $s_2 \in CR_{F_{N\setminus H_i}}(s_1)$ such that $s_2 \in \Phi_{H_i}^{\preceq}(s)$. The proof of Theorem 2 follows using Definition 12. ■

Define the *C_FSMR* stability matrix as

$$M_{H_i}^{C_F SMR} = CM_{H_i}^{+} \cdot [E - \text{sign}(Q)] \tag{25}$$

in which

$$Q = CM_{\mathbb{F}_{N\setminus H_i}} \cdot \left[\left(P_{H_i}^{-,=}\right)^{\mathrm{T}} \circ \left(E - \text{sign}\left(M_{H_i} \cdot \left(E - P_{H_i}^{-,=}\right)^{\mathrm{T}}\right)\right)\right].$$

Theorem 3: State $s \in S$ is *C_FSMR* stable for H_i, denoted by $s \in S_{H_i}^{C_F SMR}$, iff $M_{H_i}^{C_F SMR}(s,s) = 0$.

Proof: Since

$$M_{H_i}^{C_F SMR}(s,s) = \left(e_s^{\mathrm{T}} \cdot CM_{H_i}^{+}\right) \cdot [(E - \text{sign}(Q)) \cdot e_s]$$
$$= \sum_{s_1=1}^{m} CM_{H_i}^{+}(s,s_1)\left[1 - \text{sign}(Q(s_1,s))\right], \text{ with}$$
$$Q(s_1,s) = \sum_{s_2=1}^{m} CM_{\mathbb{F}_{N\setminus H_i}}(s_1,s_2) \cdot W(s_2,s), \text{ and}$$
$$W(s_2,s) = P_{H_i}^{-,=}(s,s_2) \times \left[1 - \text{sign}\left(\sum_{s_3=1}^{m}\left(M_{H_i}(s_2,s_3) \cdot \left(1 - P_{H_i}^{-,=}(s,s_3)\right)\right)\right)\right]$$

then $M_{H_i}^{C_F SMR}(s,s) = 0$ holds iff $Q(s_1,s) \ne 0$, for every $s_1 \in CR_{H_i}^{+}(s)$, which indicates that, for every $s_1 \in CR_{H_i}^{+}(s)$, there exists $s_2 \in CR_{\mathbb{F}_{N\setminus H_i}}(s_1)$ such that

$$P_{H_i}^{-,=}(s,s_2) \ne 0 \tag{26}$$

and

$$\sum_{s_3=1}^{m} M_{H_i}(s_2,s_3) \cdot \left(1 - P_{H_i}^{-,=}(s,s_3)\right) = 0. \tag{27}$$

Equation (26) is equivalent to $s_2 \in \Phi^{\precsim}_{H_i}(s)$ and (27) indicates that

$$P^{-,=}_{H_i}(s, s_3) = 1 \text{ for any } s_3 \in R_{H_i}(s_2). \tag{28}$$

Obviously, (26) and (27) hold iff for every $s_1 \in CR^{+}_{H_i}(s)$ there exist $F_{N\setminus H_i} \in \mathbb{F}_{N\setminus H_i}$ and $s_2 \in CR_{F_{N\setminus H_i}}(s_1)$ such that $s_2 \in \Phi^{\precsim}_{H_i}(s)$ and $s_3 \in \Phi^{\precsim}_{H_i}(s)$ for all $s_3 \in R^{+}_{H_i}(s_2)$. Hence, the proof of Theorem 3 follows using Definition 13. ■

Similar to (23), the C_FSEQ stability matrix $M^{C_FSEQ}_{H_i}$ can be defined as

$$M^{C_FSEQ}_{H_i} = CM^{+}_{H_i} \cdot \left[E - \text{sign}\left(CM^{+}_{\mathbb{F}_{N\setminus H_i}} \cdot \left(P^{-,=}_{H_i}\right)^{\mathrm{T}}\right)\right]. \tag{29}$$

Theorem 4: State $s \in S$ is C_FSEQ stable for H_i, denoted by $s \in S^{C_FSEQ}_{H_i}$, iff $M^{C_FSEQ}_{H_i}(s, s) = 0$.

Corollary 1: If $CM^{+}_{\mathbb{F}_{N\setminus H_i}} = CM^{+}_{N\setminus H_i}$, and $M^{C_FSEQ}_{H_i}(s, s) = 0$, then s is $CSEQ_1$ stable for H_i. If $CM^{+}_{\mathbb{F}_{N\setminus H_i}} = M^{+}_{N\setminus H_i}$, and $M^{C_FSEQ}_{H_i}(s, s) = 0$, then s is $CSEQ_2$ stable for H_i.

Define the C_FSSEQ stability matrix as

$$M^{C_FSSEQ}_{H_i} = CM^{+}_{H_i} \cdot [E - \text{sign}(Q)]$$

in which

$$Q = CM^{+}_{\mathbb{F}_{N\setminus H_i}} \cdot \left[\left(P^{-,=}_{H_i}\right)^{\mathrm{T}} \circ \left(E - \text{sign}\left(M_{H_i} \cdot (E - P^{-,=}_{H_i})^{\mathrm{T}}\right)\right)\right].$$

Theorem 5: State $s \in S$ is C_FSSEQ stable for H_i, denoted by $s \in S^{C_FSSEQ}_{H_i}$, iff $M^{C_FSSEQ}_{H_i}(s, s) = 0$.

The proofs for Theorems 4 and 5 are similar to those for Theorems 2 and 3, respectively.

D. Matrix Representation of Mixed Coalitional Stabilities

Let $F_H \in \mathbb{F}_H$, $C \subseteq F_H$ denote the set of credible sanctioning coalitions in F_H who move only to CIs, and $C' = F_H \setminus C$ be the set of noncredible coalitions in F_H who can move to any CMs even if these moves are equally or less preferred.

Definition 32: The MCI reachability matrix for F_H is an $m \times m$ matrix, denoted by $CM^{\oplus}_{F_H(C)}$, with entry (s, q)

$$CM^{\oplus}_{F_H(C)}(s, q) = \begin{cases} 1 & \text{if } q \in CR^{\oplus}_{F_H(C)}(s) \\ 0 & \text{otherwise.} \end{cases}$$

It is clear that $CR^{\oplus}_{F_H(C)}(s) = \{q \in S : CM^{\oplus}_{F_H(C)}(s, q) = 1\}$. If $CR^{\oplus}_{F_H(C)}(s)$ is written as an 0-1 row vector, then

$$CR^{\oplus}_{F_H(C)}(s) = e^{\mathrm{T}}_s \cdot CM^{\oplus}_{F_H(C)}.$$

Definition 33: For $H_l \in F_H$, the reachability matrix for H_l can be defined as an $m \times m$ matrix

$$M^{\oplus}_{H_l} = \begin{cases} CM^{+}_{H_l} & \text{if } H_l \in C \\ M_{H_l} & \text{if } H_l \in C'. \end{cases} \tag{30}$$

Definition 34: For $H_l \in F_H$, and $t = 1, 2, 3, \ldots$, define the $m \times m$ matrix $CM^{(t,\oplus)}_{H_l}$ with (s, q) entries as follows:

$$CM^{(t,\oplus)}_{H_l}(s, q) = \begin{cases} 1 & \text{if } q \in S \text{ is reachable from } s \in S \\ & \text{in exactly t legal MCIs by some} \\ & \text{coalitions in } F_H \text{ with last} \\ & \text{coalition } H_l \\ 0 & \text{otherwise.} \end{cases}$$

Based on Definition 34, the following lemma can be obtained.

Lemma 3: For $H_l \in F_H$, the matrix $CM^{(t,\oplus)}_{H_l}$ satisfies $CM^{(1,\oplus)}_{H_l}(s, q) = CM^{\oplus}_{H_l}(s, q)$, and for $t = 2, 3, \ldots$

$$CM^{(t,\oplus)}_{H_l}(s, q) = \text{sign}\left[\left(\bigvee_{H_k \in \{F_H \setminus H_l\}} CM^{(t-1,\oplus)}_{H_k}\right) \cdot CM^{\oplus}_{H_l}\right]. \tag{31}$$

For $s \in S$, let $A^{\oplus}_{F_H}(s)$ be the MCI arc set of H from state s, and $A^{\oplus}_{F_H} = \bigcup_{s\in S} A^{\oplus}_{F_H}(s)$. Then, the number of elements of $A^{\oplus}_{F_H}$ can be denoted by $l_2 = |A^{\oplus}_{F_H}|$.

Lemma 4: For $F_H \in \mathbb{F}_H$, let δ_2 be the number of iteration steps to find $CR^{\oplus}_{F_H(C)}(s)$. Then $\delta_2 \leq l_2$.

According to Lemmas 3 and 4, the following theorem can be obtained.

Theorem 6: Let state $s \in S$, a full coalition set $F_H \in \mathbb{F}_H$, and the set of credible coalitions in F_N be C. The MCI reachability matrix, $CM^{\oplus}_{F_H(C)}$, can be expressed by

$$CM^{\oplus}_{F_H(C)} = \bigvee_{t=1}^{l_2} \bigvee_{H_l \in F_H} CM^{(t,\oplus)}_{H_l}. \tag{32}$$

The proof of Theorem 6 is similar to that of Theorem 1.

Let $F^j_H \in \mathbb{F}_H$, C_j be the set of credible coalitions in F^j_H, and $\mathbb{C} = \bigcup_{j=1}^{k} C_j$ be the set of credible coalitions in the universal coalition set $\mathbb{F}_H$. Using Definition 18, the universal MCI reachability matrix $CM^{\oplus}_{\mathbb{F}_H(\mathbb{C})}$ for $\mathbb{F}_H$ is

$$CM^{\oplus}_{\mathbb{F}_H(\mathbb{C})} = \bigvee_{F^j_H \in \mathbb{F}_H} CM^{\oplus}_{F^j_H(C_j)}. \tag{33}$$

Let a focal coalition $H_i \in N$, the MCTS matrix for H_i can be defined as

$$M^{MCTS}_{H_i} = CM^{+}_{H_i} \cdot \left[E - \text{sign}\left(CM^{\oplus}_{\mathbb{F}_{N\setminus H_i}(\mathbb{C})} \cdot (P^{-,=}_{H_i})^{\mathrm{T}}\right)\right]. \tag{34}$$

Theorem 7: State $s \in S$ is MCTS stable for H_i, denoted by $s \in S^{MCTS}_{H_i}$, iff $M^{MCTS}_{H_i}(s, s) = 0$.

Corollary 2: If $CM^{\oplus}_{\mathbb{F}_{N\setminus H_i}(\mathbb{C})} = CM_{\mathbb{F}_{N\setminus H_i}}$, and $M^{MCTS}_{H_i}(s, s) = 0$, then s is C_FGMR stable for H_i. If $CM^{\oplus}_{\mathbb{F}_{N\setminus H_i}(\mathbb{C})}(s) = CM^{+}_{\mathbb{F}_{N\setminus H_i}}(s)$, and $M^{MCTS}_{H_i}(s, s) = 0$, then s is C_FSEQ stable for H_i.

Define the MCSMR stability matrix as

$$M^{MCSMR}_{H_i} = CM^{+}_{H_i} \cdot \left[E - \text{sign}(Q)\right]$$

in which

$$Q = CM^{\oplus}_{\mathbb{F}_{N\setminus H_i}(\mathbb{C})} \cdot \left[\left(P^{-,=}_{H_i}\right)^{\mathrm{T}} \circ \left(E - \text{sign}\left(M_{H_i} \cdot \left(E - P^{-,=}_{H_i}\right)^{\mathrm{T}}\right)\right)\right].$$

Theorem 8: State $s \in S$ is MCSMR stable for H_i, denoted by $s \in S^{MCSMR}_{H_i}$, iff $M^{MCSMR}_{H_i}(s, s) = 0$.

Corollary 3: If $CM^{\oplus}_{\mathbb{F}_{N\setminus H_i}(\mathbb{C})} = CM_{\mathbb{F}_{N\setminus H_i}}$, and $M^{MCSMR}_{H_i}(s, s) = 0$, then s is C_FSMR stable for H_i. If $CM^{\oplus}_{\mathbb{F}_{N\setminus H_i}(\mathbb{C})}(s) = CM^{+}_{\mathbb{F}_{N\setminus H_i}}(s)$, and $M^{MCSMR}_{H_i}(s, s) = 0$, then s is C_FSSEQ stable for H_i.

The proofs for Theorems 7 and 8 are similar to Theorems 2 and 3, respectively.

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

TABLE III
OPTIONS AND FEASIBLE STATES FOR THE CFLS ENVIRONMENTAL CONTAMINATION CONFLICT

DMs	Options	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}	s_{16}	s_{17}	s_{18}	s_{19}	s_{20}	s_{21}	s_{22}	s_{23}	s_{24}
EPA	1. Monitor	N	N	N	N	N	N	N	N	N	N	N	N	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
HC	2. Supervise	N	N	N	N	N	N	Y	Y	Y	Y	Y	Y	N	N	N	N	N	N	Y	Y	Y	Y	Y	Y
TW	3. Obey	N	N	N	Y	Y	Y	N	N	N	Y	Y	Y	N	N	N	Y	Y	Y	N	N	N	Y	Y	Y
MoE	4. Relocate	N	Y	Y	N	Y	Y	N	Y	Y	N	Y	Y	N	Y	Y	N	Y	Y	N	Y	Y	N	Y	Y
CFLS	5. Agree	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y

VI. CASE STUDIES

A. *Environmental Dispute of Changzhou Foreign Languages School (CFLS)*

According to the China Central Television (CCTV) report in April 2016, nearly 500 students of the CFLS, located in Changzhou, China, were diagnosed with serious health problems, such as dermatitis, eczema, bronchitis, leukemia, and lymphoma after this school relocated to its new campus in September 2015 [26], [27]. Actually, this campus is next to the original site of three toxic chemical factories where the level of pollutants detected in the groundwater and soil were excessively above safety standards. Not surprisingly, this brownfield event made headline news across China, and tens of thousands of people, including the parents of those students, were outraged enough to strongly demand that local authorities be held accountable, prompting the Ministry of Environmental Protection of China and the Jiangsu provincial government to launch a joint investigation.

In August 2016, a report issued by the investigation team explained why the students contracted serious diseases. First, the Environmental Protection Agency (EPA) in the city of Changzhou, China, failed to monitor the quality of the environment around the CFLS and the Ministry of Education (MoE) in Changzhou still relocated CFLS to its new campus where the air and underlying aquifers were seriously polluted. Additionally, the Heimudan Company (HC), which was responsible for the contaminated land remediation project, did not supervise the construction operations by Tianma Wangxiang Construction Company, Ltd. (TW). Furthermore, soil and underground water around the CFLS were contaminated due to the open-air remediation by TW during the land restoration process, thereby directly contributing to serious health problems. However, it is still a difficult but significant problem to systematically explain how the serious environmental conflict happened and the complicated interactions among EPA, HC, TW, MoE, and CFLS during the process of land restoration.

In this dispute, there are five key DMs each of whom has one option as presented below.

1) EPA in Changzhou, which is responsible for providing the public with information about air quality and potential health impacts. In this conflict, EPA has one option: whether or not to monitor the quality of the environment around CFLS during the land remediation process.
2) HC, which is in charge of the contaminated land remediation project. HC has one option: whether or not to supervise the land remediation process.
3) TW, which is responsible for the construction operations for the land remediation project. TW has one option: whether or not to obey land remediation standards to prevent human exposure to contaminants during the land remediation.
4) MoE in Changzhou. MoE has one option: whether or not to relocate the students and teachers of CFLS to the new campus in Xinbei District.
5) CFLS. If MoE issues an order for the school relocation program, then CFLS has to decide whether or not to agree to this plan.

A state is formed when the option selections of each DM are determined. As shown in Table III, there are twenty-four feasible states. Note that "*Y*" means that an option in the same row is selected and "*N*" indicates that the option is not chosen.

In this dispute, each DM has its own preference over states in Table III. For instance, EPA prefers HC to strengthen the supervision and administration of the land remediation project. Meanwhile, EPA hopes that TW can abide by land remediation guidelines and take safeguard measures such as the installation of pollution abatement equipment. Furthermore, EPA does not want to enhance the environmental monitoring around CFLS during the land remediation process due to the limited budget and resources. Both of HC and TW do not want EPA to regularly monitor the environmental quality. Moreover, HC is not willing to supervise the land remediation conducted by TW and TW also prefers HC not to oversee the project. To reduce the remediation costs, TW does not want to adopt pollution abatement equipment. On the other hand, MoE wants to relocate CFLS to a new campus as soon as possible and prefers CFLS to agree because the old campus of CFLS is located in downtown. From the perspective of CFLS, it will agree with MoE if MoE adopts the strategy of relocation. In addition, CFLS hopes that TW can obey the land remediation standards, EPA can enhance environmental monitoring around CFLS, and HC can oversee the project if it decides to move to the new campus. By using the direct ranking method [21], the preference information of each DM can be determined from most to least preferred as given in Table IV.

State transitions can be illustrated by arcs in a directed graph. The integrated graph model of this conflict is presented as shown in Fig. 3. A node in this figure stands for a given feasible state from Table III. An arc with arrow heads indicates a move controlled by the DM written on the arc in one step between two nodes or states. Two arrow heads on an arc means the move is reversible.

TABLE IV
PREFERENCES OF DMs FOR CFLS ENVIRONMENTAL CONTAMINATION CONFLICT

DMs	Preference
EPA	$s_{10} \sim s_{11} \sim s_{12} \succ s_4 \sim s_5 \sim s_6 \succ s_7 \succ s_8 \succ s_9 \succ s_1 \succ s_2 \succ s_3 \succ s_{22} \sim s_{23} \sim s_{24} \succ s_{16} \sim s_{17} \sim s_{18} \succ s_{19} \succ s_{20} \succ s_{21} \succ s_{13} \succ s_{14} \succ s_{15}$
HC	$s_1 \succ s_2 \succ s_3 \succ s_4 \sim s_5 \sim s_6 \succ s_{10} \sim s_{11} \sim s_{12} \succ s_7 \succ s_8 \succ s_9 \succ s_{13} \succ s_{14} \succ s_{15} \succ s_{16} \sim s_{17} \sim s_{18} \succ s_{22} \sim s_{23} \sim s_{24} \succ s_{19} \succ s_{20} \succ s_{21}$
TW	$s_1 \sim s_2 \sim s_3 \succ s_7 \sim s_8 \sim s_9 \succ s_{13} \sim s_{14} \sim s_{15} \succ s_{19} \sim s_{20} \sim s_{21} \succ s_4 \sim s_5 \sim s_6 \succ s_{10} \sim s_{11} \sim s_{12} \succ s_{16} \sim s_{17} \succ s_{18} \succ s_{22} \sim s_{23} \sim s_{24}$
MoE	$s_{24} \succ s_{18} \succ s_{12} \succ s_6 \succ s_{21} \succ s_{15} \succ s_9 \succ s_3 \succ s_{23} \succ s_{17} \succ s_{11} \succ s_5 \succ s_{20} \succ s_{14} \succ s_8 \succ s_2 \succ s_{22} \succ s_{16} \succ s_{10} \succ s_4 \succ s_{19} \succ s_{13} \succ s_7 \succ s_1$
CFLS	$s_{24} \succ s_{18} \succ s_{12} \succ s_6 \succ s_{21} \succ s_{15} \succ s_9 \succ s_3 \succ s_{22} \succ s_{16} \succ s_{10} \succ s_4 \succ s_{19} \succ s_{13} \succ s_7 \succ s_1 \succ s_{23} \succ s_{17} \succ s_{11} \succ s_5 \succ s_{20} \succ s_{14} \succ s_8 \succ s_2$

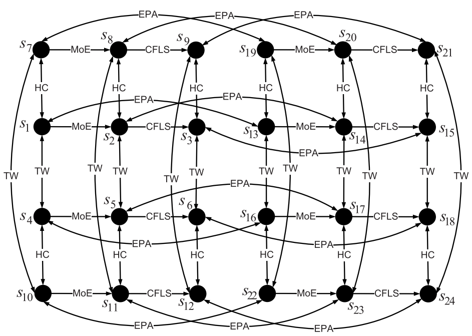

Fig. 3. Integrated graph model for the CFLS environmental contamination conflict.

B. Coalition Analyses With Full Participation

In the CFLS environmental contamination dispute, EPA is an independent player and can be regarded as a special coalition. HC and TW could form a coalition with close relationships and shared interests and MoE and CFLS could belong to an alliance because CFLS is regulated by MoE. Therefore, three coalitions are formed in this conflict: $H_1 = \{EPA\}$, $H_2 = \{HC, TW\}$, and $H_3 = \{MoE, CFLS\}$. For each coalition, the knowledge of exactly which coalitions will form among its opponents for levying sanctions is unknown.

From the viewpoint of EPA (H_1), its opponents are HC, TW, MoE, and CFLS, but the coalitions among them are unknown. Theoretically, there is a total of 15 full coalition sets by HC, TW, MoE, and CFLS. Some of them, however, are infeasible in this dispute, such as $F_{N\backslash H_1} = \{(HC, CFLS), TW, MoE\}$ because it is impossible that HC and CFLS form a coalition in this case. In fact, EPA knows that HC and TW or MoE and CFLS could form a coalition but it is unclear. Hence, there are four feasible full coalition sets that could be formed by its opponents ($N\backslash H_1$)

$$\mathbb{F}_{N\backslash H_1} = \left\{F^1_{N\backslash H_1}, F^2_{N\backslash H_1}, F^3_{N\backslash H_1}, F^4_{N\backslash H_1}\right\}$$

in which

$$\begin{aligned} F^1_{N\backslash H_1} &= \{HC, TW, MoE, CFLS\} \\ F^2_{N\backslash H_1} &= \{(HC, TW), MoE, CFLS\} \\ F^3_{N\backslash H_1} &= \{HC, TW, (MoE, CFLS)\} \\ F^4_{N\backslash H_1} &= \{(HC, TW), (MoE, CFLS)\}. \end{aligned} \tag{35}$$

From the viewpoint of H_2, it has three opponents: 1) EPA; 2) MoE; and 3) CFLS. But H_2 does not know the actual coalitions among them. Hence, there are five feasible full coalition sets in total that could be created by its opponents ($N\backslash H_2$)

$$\mathbb{F}_{N\backslash H_2} = \left\{F^1_{N\backslash H_2}, F^2_{N\backslash H_2}, F^3_{N\backslash H_2}, F^4_{N\backslash H_2}, F^5_{N\backslash H_2}\right\}$$

in which

$$\begin{aligned} F^1_{N\backslash H_2} &= \{EPA, MoE, CFLS\} \\ F^2_{N\backslash H_2} &= \{(EPA, MoE), CFLS\} \\ F^3_{N\backslash H_2} &= \{(EPA, CFLS), MoE\} \\ F^4_{N\backslash H_2} &= \{EPA, (MoE, CFLS)\} \\ F^5_{N\backslash H_2} &= \{(EPA, MoE, CFLS)\}. \end{aligned} \tag{36}$$

Similarly, five full coalition sets could be formed by the opponents of H_3 ($N\backslash H_3$)

$$\mathbb{F}_{N\backslash H_3} = \left\{F^1_{N\backslash H_3}, F^2_{N\backslash H_3}, F^3_{N\backslash H_3}, F^4_{N\backslash H_3}, F^5_{N\backslash H_3}\right\}$$

in which

$$\begin{aligned} &F^1_{N\backslash H_3} = \{EPA, HC, TW\},\ F^2_{N\backslash H_3} = \{(EPA, HC), TW\} \\ &F^3_{N\backslash H_3} = \{(EPA, TW), HC\},\ F^4_{N\backslash H_3} = \{EPA, (HC, TW)\} \\ &F^5_{N\backslash H_3} = \{(EPA, HC, TW)\}. \end{aligned} \tag{37}$$

Using Definitions 14 and 15, the coalitional stabilities with full participation for the CFLS environmental contamination conflict can be determined as shown in Table V. One can find that state s_3 is an equilibrium under *CNash*, $C_F SEQ$, and $C_F SSEQ$ in the CFLS environmental contamination conflict, in which EPA does not monitor the quality of the environment around CFLS, HC neglects to supervise the contaminated land remediation process, TW does not obey the land remediation standards, MoE issues an order to relocate the CFLS to the new campus and CFLS agrees to that. As a result, many students of CFLS were diagnosed with serious health problems because the air surrounding the school was heavily polluted by TW's illegal construction operations.

C. Mixed Coalition Analyses With Full Participation

In this conflict, EPA (H_1) could move to any reachable states regardless of preference in sanctioning. For example, when a focal coalition H_2 moves to a CI, H_2 will consider H_1 and H_3 as the sanctioning coalitions, in which H_1 could move to any reachable states with H_3 instead of only CIs to jointly block any of H_2's CIs. Hence, as sanctioning coalitions, H_1 can be regarded as being noncredible while both H_2 and H_3 are credible.

According to Definitions 19 and 20, mixed coalition analyses for the CFLS environmental contamination conflict can be executed as shown in Table VI.

In Table VI, one can find that state s_3 is an equilibrium under both CNash, MCTS, and MCSMR stabilities. Moreover, states s_6, s_9, and s_{12} are equilibria under both MCTS and MCSMR stabilities. State s_6 means that CFLS accepts the relocation

TABLE V
COALITION ANALYSES WITH FULL PARTICIPATION IN THE CFLS ENVIRONMENTAL CONTAMINATION CONFLICT

State	$CNash$				C_FSEQ				C_FSSEQ			
	H_1	H_2	H_3	Eq	H_1	H_2	H_3	Eq	H_1	H_2	H_3	Eq
s_1	√	√			√	√			√	√		
s_2	√	√			√	√			√	√		
s_3	√	√	√	*	√	√	√	*	√	√	√	*
s_4	√				√				√			
s_5	√				√				√			
s_6	√		√		√		√		√		√	
s_7	√				√				√			
s_8	√				√				√			
s_9	√		√		√		√		√		√	
s_{10}	√				√				√			
s_{11}	√				√				√			
s_{12}	√		√		√		√		√		√	
s_{13}		√				√				√		
s_{14}		√				√				√		
s_{15}		√	√			√	√			√	√	
s_{16}												
s_{17}												
s_{18}			√				√				√	
s_{19}												
s_{20}												
s_{21}			√				√				√	
s_{22}												
s_{23}												
s_{24}			√				√				√	

TABLE VI
MIXED COALITION ANALYSES IN THE CFLS ENVIRONMENTAL CONTAMINATION CONFLICT

State	CNash				$MCTS$				$MCSMR$			
	H_1	H_2	H_3	Eq	H_1	H_2	H_3	Eq	H_1	H_2	H_3	Eq
s_1	√	√			√	√			√	√		
s_2	√	√			√	√			√	√		
s_3	√	√	√	*	√	√	√	*	√	√	√	*
s_4	√				√	√			√	√		
s_5	√				√	√			√	√		
s_6	√		√		√	√	√	*	√	√	√	*
s_7	√				√	√			√	√		
s_8	√				√	√			√	√		
s_9	√		√		√	√	√	*	√	√	√	*
s_{10}	√				√	√			√	√		
s_{11}	√				√	√			√	√		
s_{12}	√		√		√	√	√	*	√	√	√	*
s_{13}		√				√				√		
s_{14}		√				√				√		
s_{15}		√	√			√	√			√	√	
s_{16}												
s_{17}												
s_{18}			√				√				√	
s_{19}												
s_{20}												
s_{21}			√				√				√	
s_{22}												
s_{23}												
s_{24}			√				√				√	

plan proposed by MoE, and meanwhile, TW is willing to adopt pollution abatement equipment during the land remediation process even though EPA fails to monitor the environmental quality and HC does not supervise the project. State s_{12} indicates that CFLS and MoE agree to the relocation plan, and meanwhile, HC supervises the land remediation project and TW adopts pollution abatement equipment, but EPA fails to monitor the environmental quality. In state s_9, both MoE and CFLS decide to relocate CFLS to its new campus, but EPA does not execute environmental monitoring and TW does not obey the soil remediation standards to cleanse up the excavated contaminants although HC supervises the remediation of contaminated land. This may lead to air pollution and pose a great threat to human health. Note that state s_9 can be guided to an attractive win/win state, s_{24}, if EPA takes stringent measures to constantly monitor the environment during the process of the land remediation which can force TW to adopt safeguard measures for avoiding the potential spread of the contaminations. Alternatively, the globally known tragedy in which many students in the CFLS were diagnosed with having serious health problems due to the open-air land remediation could have been avoided if the EPA in Changzhou had assumed its responsibility for the environmental monitoring surrounding the CFLS.

VII. CONCLUSION

The mixed coalitional stabilities with heterogeneous sanctioning opponents proposed in this article provide a more general coalition analysis framework than existing coalition analysis approaches. Specifically, $CSEQ_1$ and $CSEQ_2$ [15], [18], [20] are special cases of C_FSEQ since all possible full coalition sets containing all of the sanctioning opponents are taken into account in C_FSEQ. Moreover, in existing coalitional stabilities, the sanctioning opponents can utilize either CIs or any CM regardless of preference when sanctioning. However, they do not consider the mixed sanctioning moves, including both CIs and CMs. To handle heterogeneous sanctioning opponents with MCIs, the mixed coalitional stabilities, consisting of *MCTS* and *MCSMR*, are developed in this article, in which C_FSEQ and C_FSSEQ are subsets of *MCTS* and *MCSMR*, respectively.

In the future, mixed coalitional analysis approach can be expanded in various ways, such as considering the strength of preference [28], unknown preference [29], grey preference [30], inverse engineering preferences [31], hierarchical conflicts [32], or $Maximin_h$ stability [33]. In addition, one may wish to utilize the new coalition analysis approach developed in this article to account misperceptions [34], [35] or interactive unawareness [36] of DMs.

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