

데이터 사이언스 과제7

< Linear Regression >

> #1. Packages

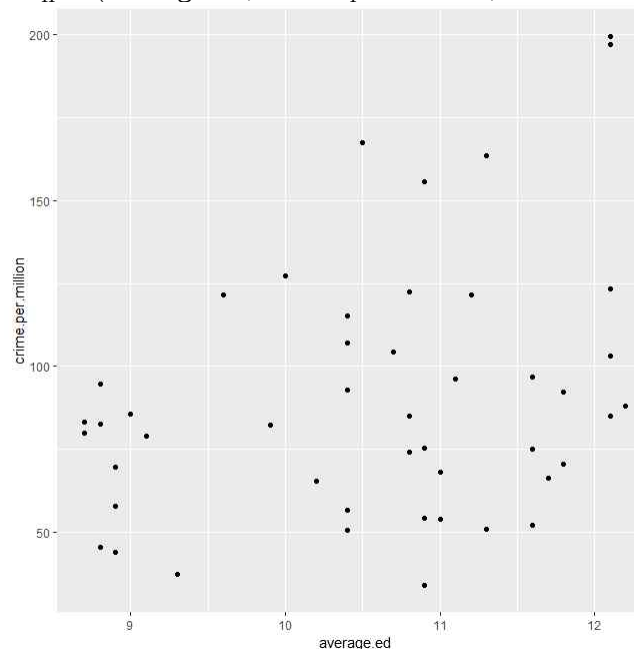
```
> library(MASS)
> library(plyr)
> library(ggplot2)
> library(knitr)
> library(GGally)
>
>
```

> #2. Linear regression

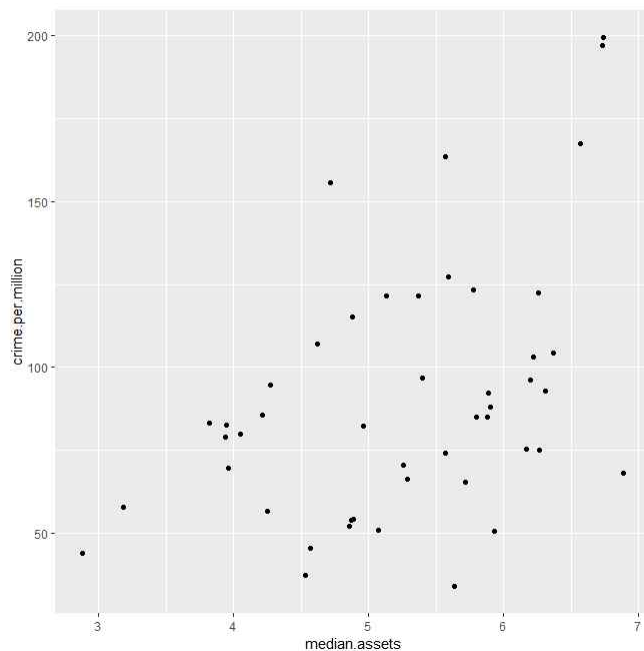
```
> # Import data set
> crime <-
read.table("http://www.andrew.cmu.edu/user/achoulde/94842/data/crime_si
mple.txt", sep = "\t",
+ header = TRUE)
> # Assign more meaningful variable names
> colnames(crime) <- c("crime.per.million", "young.males", "is.south",
"average.ed",
+ "exp.per.cap.1960", "exp.per.cap.1959", "labour.part",
+ "male.per.fem", "population", "nonwhite",
+ "unemp.youth", "unemp.adult", "median.assets", "num.low.salary")
> # Convert is.south to a factor
> # Divide average.ed by 10 so that the variable is actually average
education
> # Convert median assets to 1000's of dollars instead of 10's
> crime <- transform(crime, is.south = as.factor(is.south),
+ average.ed = average.ed / 10,
+ median.assets = median.assets / 100)
> # print summary of the data
> summary(crime)
crime.per.million young.males is.south average.ed
exp.per.cap.1960
Min. : 34.20 Min. :119.0 0:31 Min. : 8.70 Min. : 45.0
1st Qu.: 65.85 1st Qu.:130.0 1:16 1st Qu.: 9.75 1st Qu.: 62.5
Median : 83.10 Median :136.0 Median :10.80 Median : 78.0
Mean : 90.51 Mean :138.6 Mean :10.56 Mean :
85.0
3rd Qu.:105.75 3rd Qu.:146.0 3rd Qu.:11.45 3rd Qu.:104.5
Max. :199.30 Max. :177.0 Max. :12.20 Max. :166.0
exp.per.cap.1959 labour.part male.per.fem population
Min. : 41.00 Min. :480.0 Min. : 934.0 Min. : 3.00
1st Qu.: 58.50 1st Qu.:530.5 1st Qu.: 964.5 1st Qu.: 10.00
Median : 73.00 Median :560.0 Median : 977.0 Median : 25.00
Mean : 80.23 Mean :561.2 Mean : 983.0 Mean : 36.62
3rd Qu.: 97.00 3rd Qu.:593.0 3rd Qu.: 992.0 3rd Qu.: 41.50
```

```
Max. :157.00 Max. :641.0 Max. :1071.0 Max. :168.00
nonwhite unemp.youth unemp.adult median.assets
Min. : 2.0 Min. : 70.00 Min. :20.00 Min. :2.880
1st Qu.: 24.0 1st Qu.: 80.50 1st Qu.:27.50 1st Qu.:4.595
Median : 76.0 Median : 92.00 Median :34.00 Median :5.370
Mean :101.1 Mean : 95.47 Mean :33.98 Mean :5.254
3rd Qu.:132.5 3rd Qu.:104.00 3rd Qu.:38.50 3rd Qu.:5.915
Max. :423.0 Max. :142.00 Max. :58.00 Max. :6.890
num.low.salary
Min. :126.0
1st Qu.:165.5
Median :176.0
Mean :194.0
3rd Qu.:227.5
Max. :276.0
```

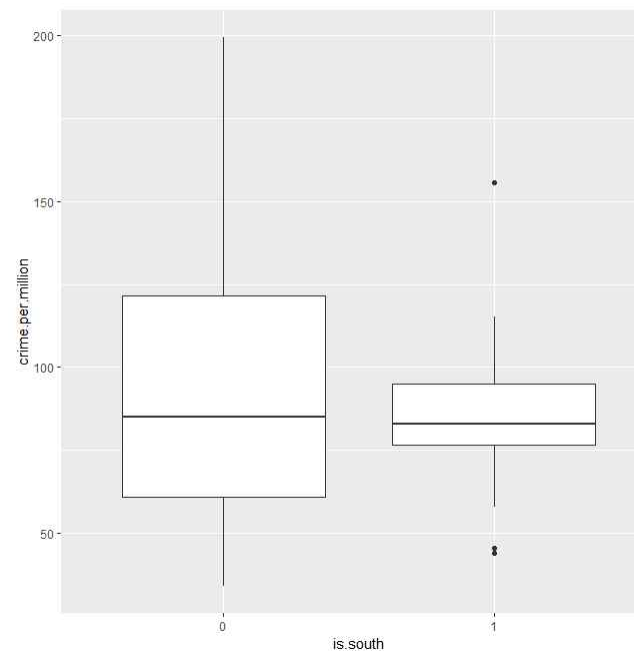
```
>
> # Scatter plot of outcome (crime.per.million) against average.ed
> qplot(average.ed, crime.per.million, data = crime)
```



```
> # correlation between education and crime
> with(crime, cor(average.ed, crime.per.million))
[1] 0.3228349
> # Scatter plot of outcome (crime.per.million) against median.assets
> qplot(median.assets, crime.per.million, data = crime)
```



```
> # correlation between education and crime
> with(crime, cor(median.assets, crime.per.million))
[1] 0.4413199
>
> # Boxplots showing crime rate broken down by southern vs
non-southern state
> qplot(is.south, crime.per.million, geom = "boxplot", data = crime)
```



```
> crime.lm <- lm(crime.per.million ~ ., data = crime)
> # Summary of the linear regression model
> crime.lm
```

Call:

```
lm(formula = crime.per.million ~ ., data = crime)
```

Coefficients:

(Intercept)	young.males	is.south1	average.ed
-6.918e+02	1.040e+00	-8.308e+00	1.802e+01
exp.per.cap.1960	exp.per.cap.1959	labour.part	male.per.fem
1.608e+00	-6.673e-01	-4.103e-02	1.648e-01
population	nonwhite	unemp.youth	unemp.adult
-4.128e-02	7.175e-03	-6.017e-01	1.792e+00
median.assets	num.low.salary		
1.374e+01	7.929e-01		

```
> summary(crime.lm)
```

Call:

```
lm(formula = crime.per.million ~ ., data = crime)
```

Residuals:

Min	1Q	Median	3Q	Max
-34.884	-11.923	-1.135	13.495	50.560

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-6.918e+02	1.559e+02	-4.438	9.56e-05 ***

```

young.males      1.040e+00  4.227e-01  2.460  0.01931 *
is.south1       -8.308e+00  1.491e+01 -0.557  0.58117
average.ed       1.802e+01  6.497e+00  2.773  0.00906 **
exp.per.cap.1960 1.608e+00  1.059e+00  1.519  0.13836
exp.per.cap.1959 -6.673e-01  1.149e+00 -0.581  0.56529
labour.part      -4.103e-02  1.535e-01 -0.267  0.79087
male.per.fem     1.648e-01  2.099e-01  0.785  0.43806
population       -4.128e-02  1.295e-01 -0.319  0.75196
nonwhite         7.175e-03  6.387e-02  0.112  0.91124
unemp.youth      -6.017e-01  4.372e-01 -1.376  0.17798
unemp.adult      1.792e+00  8.561e-01  2.093  0.04407 *
median.assets    1.374e+01  1.058e+01  1.298  0.20332
num.low.salary   7.929e-01  2.351e-01  3.373  0.00191 **

```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 21.94 on 33 degrees of freedom
Multiple R-squared: 0.7692, Adjusted R-squared: 0.6783
F-statistic: 8.462 on 13 and 33 DF, p-value: 3.686e-07

```

>
> options(scipen=4) # Set scipen = 0 to get back to default
> summary(crime.lm)

```

Call:
lm(formula = crime.per.million ~ ., data = crime)

Residuals:

Min	1Q	Median	3Q	Max
-34.884	-11.923	-1.135	13.495	50.560

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-691.837588	155.887918	-4.438	0.0000956 ***
young.males	1.039810	0.422708	2.460	0.01931 *
is.south1	-8.308313	14.911588	-0.557	0.58117
average.ed	18.016011	6.496504	2.773	0.00906 **
exp.per.cap.1960	1.607818	1.058667	1.519	0.13836
exp.per.cap.1959	-0.667258	1.148773	-0.581	0.56529
labour.part	-0.041031	0.153477	-0.267	0.79087
male.per.fem	0.164795	0.209932	0.785	0.43806
population	-0.041277	0.129516	-0.319	0.75196
nonwhite	0.007175	0.063867	0.112	0.91124
unemp.youth	-0.601675	0.437154	-1.376	0.17798
unemp.adult	1.792263	0.856111	2.093	0.04407 *
median.assets	13.735847	10.583028	1.298	0.20332
num.low.salary	0.792933	0.235085	3.373	0.00191 **

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 21.94 on 33 degrees of freedom
Multiple R-squared: 0.7692, Adjusted R-squared: 0.6783
F-statistic: 8.462 on 13 and 33 DF, p-value: 0.0000003686

```

>
> # List all attributes of the linear model
> attributes(crime.lm)
$`names`
 [1] "coefficients" "residuals"      "effects"      "rank"
 [5] "fitted.values" "assign"         "qr"           "df.residual"
 [9] "contrasts"    "xlevels"        "call"         "terms"
[13] "model"

$class
[1] "lm"

> crime.lm$coef
      (Intercept)  young.males      is.south1  average.ed
      -691.837587905      1.039809653      -8.308312889      18.016010601
exp.per.cap.1960 exp.per.cap.1959      labour.part  male.per.fem
      1.607818377      -0.667258285      -0.041031047      0.164794968
      population      nonwhite      unemp.youth      unemp.adult
      -0.041276887      0.007174688      -0.601675298      1.792262901
      median.assets  num.low.salary
      13.735847285      0.792932786

>
> # Pull coefficients element from summary(lm) object
> round(summary(crime.lm)$coef, 3)
      Estimate Std. Error t value Pr(>|t|)
(Intercept)    -691.838    155.888  -4.438    0.000
young.males       1.040      0.423   2.460    0.019
is.south1        -8.308     14.912  -0.557    0.581
average.ed       18.016      6.497   2.773    0.009
exp.per.cap.1960  1.608      1.059   1.519    0.138
exp.per.cap.1959 -0.667      1.149  -0.581    0.565
labour.part      -0.041      0.153  -0.267    0.791
male.per.fem      0.165      0.210   0.785    0.438
population       -0.041      0.130  -0.319    0.752
nonwhite          0.007      0.064   0.112    0.911
unemp.youth      -0.602      0.437  -1.376    0.178
unemp.adult       1.792      0.856   2.093    0.044
median.assets     13.736     10.583   1.298    0.203
num.low.salary     0.793      0.235   3.373    0.002

>
> # Pull the coefficients table from summary(lm)
> crime.lm.coef <- round(summary(crime.lm)$coef, 3)
> # See what this gives
> class(crime.lm.coef)
[1] "matrix"
> attributes(crime.lm.coef)
$`dim`
[1] 14  4

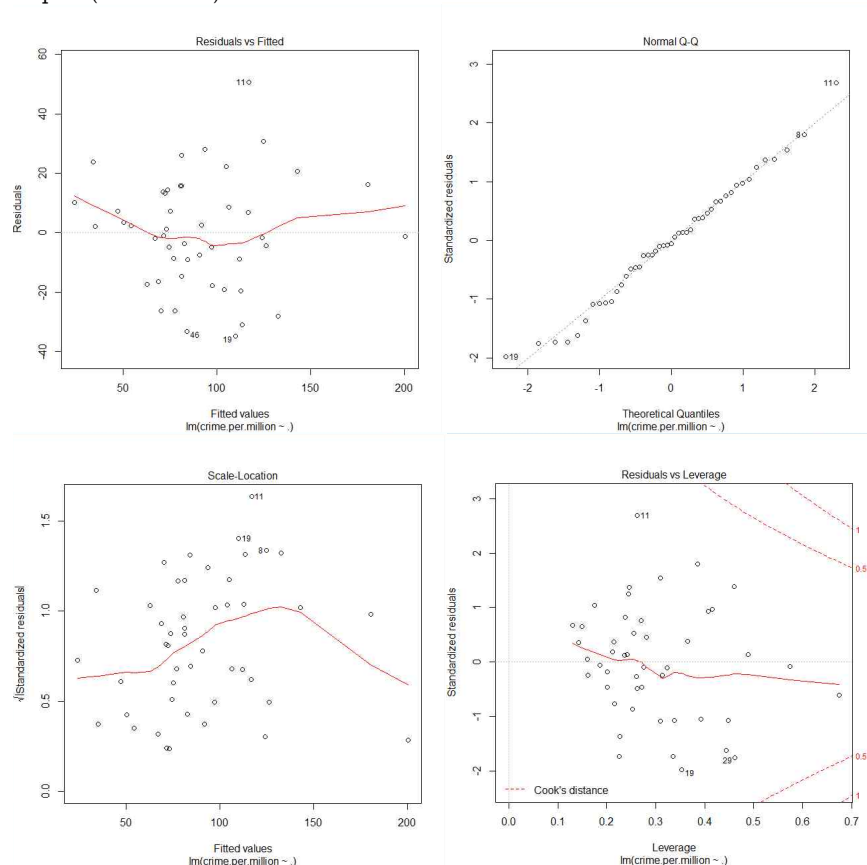
$dimnames
$dimnames[[1]]
 [1] "(Intercept)" "young.males"  "is.south1"

```

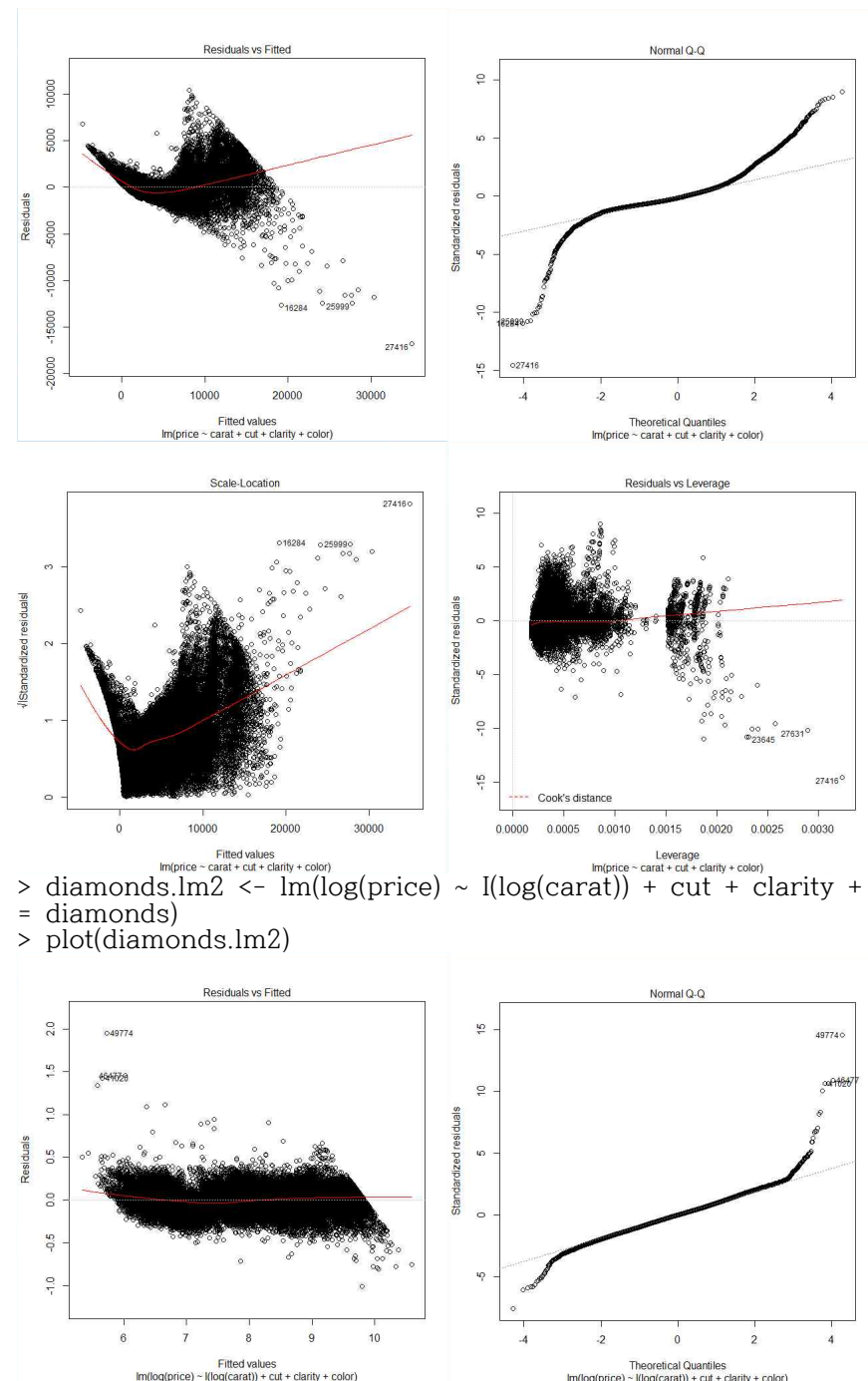
```
[4] "average.ed"      "exp.per.cap.1960" "exp.per.cap.1959"
[7] "labour.part"     "male.per.fem"    "population"
[10] "nonwhite"        "unemp.youth"     "unemp.adult"
[13] "median.assets"   "num.low.salary"
```

```
$dimnames[[2]]
[1] "Estimate" "Std. Error" "t value" "Pr(>|t|)"
```

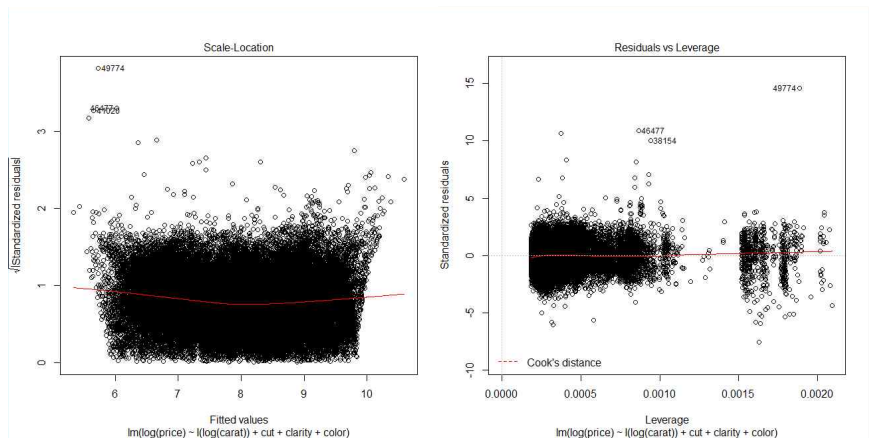
```
> crime.lm.coef["average.ed", "Pr(>|t|)"]
[1] 0.009
>
> plot(crime.lm)
```



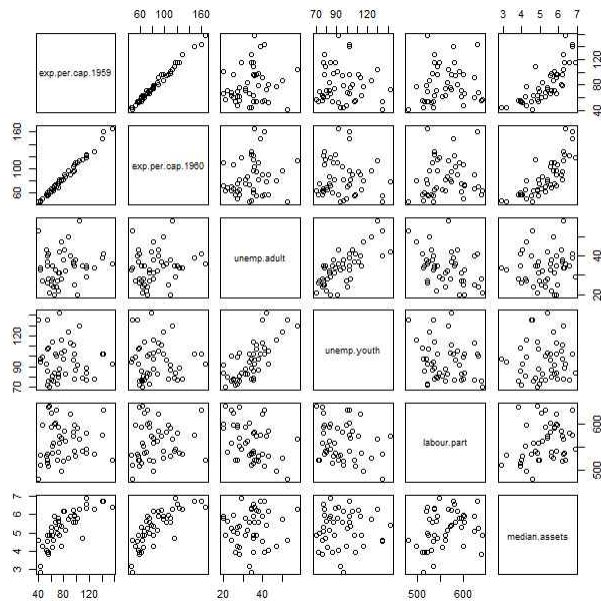
```
> diamonds.lm <- lm(price ~ carat + cut + clarity + color, data =
diamonds)
> plot(diamonds.lm)
```



```
> diamonds.lm2 <- lm(log(price) ~ l(log(carat)) + cut + clarity + color, data
= diamonds)
> plot(diamonds.lm2)
```



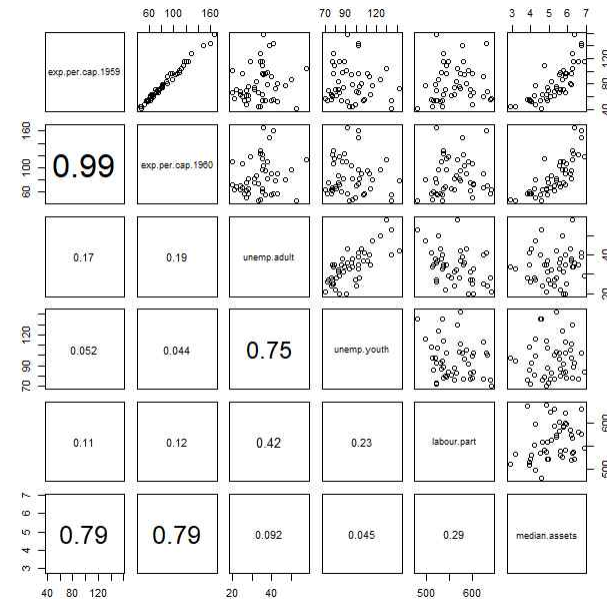
```
> economic.var.names <- c("exp.per.cap.1959", "exp.per.cap.1960",
+ "unemp.adult",
+ "unemp.youth", "labour.part", "median.assets")
> pairs(crime[,economic.var.names])
```



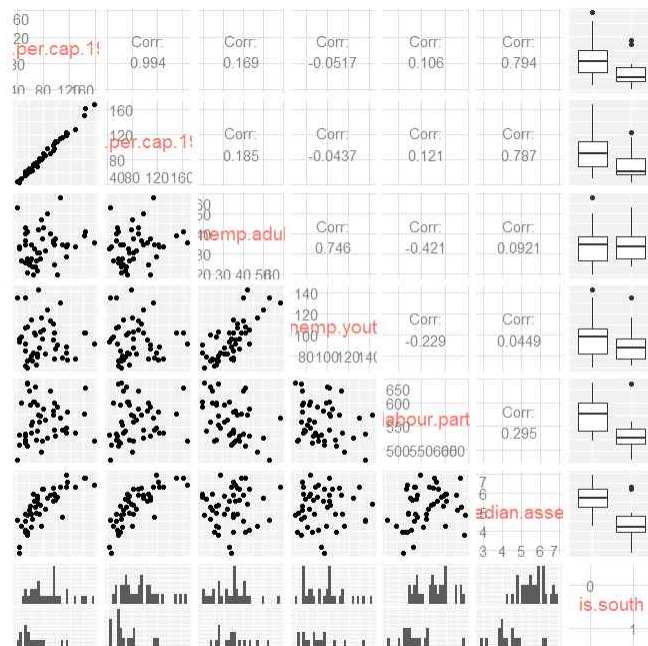
```
> round(cor(crime[,economic.var.names]), 3)
               exp.per.cap.1959 exp.per.cap.1960 unemp.adult
unemp.youth
exp.per.cap.1959      1.000      0.994      0.169      -0.052
exp.per.cap.1960      0.994      1.000      0.185      -0.044
unemp.adult           0.169      0.185      1.000      0.746
unemp.youth          -0.052     -0.044      0.746      1.000
```

```
labour.part      0.106      0.121      -0.421      -0.229
median.assets    0.794      0.787      0.092      0.045
```

```
> # Function taken from ?pairs Example section.
> panel.cor <- function(x, y, digits = 2, prefix = "", cex.cor, ...)
+ {
+   usr <- par("usr"); on.exit(par(usr))
+   par(usr = c(0, 1, 0, 1))
+   r <- abs(cor(x, y))
+   txt <- format(c(r, 0.123456789), digits = digits)[1]
+   txt <- paste0(prefix, txt)
+   if(missing(cex.cor)) cex.cor <- 0.8/strwidth(txt)
+   text(0.5, 0.5, txt, cex = pmax(1, cex.cor * r))
+ }
> # Use panel.cor to display correlations in lower panel.
> pairs(crime[,economic.var.names], lower.panel = panel.cor)
```



```
> # ggpairs from GGally library
> # Unlike pairs(), ggpairs() works with non-numeric
> # predictors in addition to numeric ones.
> # Consider ggpairs() for your final project
> ggpairs(crime[,c(economic.var.names, "is.south")], axisLabels = "internal")
```



```
> crime.lm.2 <- update(crime.lm, . ~ . - exp.per.cap.1959 - unemp.youth)
> summary(crime.lm.2)
```

Call:

```
lm(formula = crime.per.million ~ young.males + is.south + average.ed +
    exp.per.cap.1960 + labour.part + male.per.fem + population +
    nonwhite + unemp.adult + median.assets + num.low.salary,
    data = crime)
```

Residuals:

```
      Min       1Q   Median       3Q      Max
-35.82  -11.57   -1.51   10.63   55.02
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-633.438828	145.470340	-4.354	0.000111 ***
young.males	1.126883	0.418791	2.691	0.010853 *
is.south1	-0.556600	13.883248	-0.040	0.968248
average.ed	15.328028	6.202516	2.471	0.018476 *
exp.per.cap.1960	1.138299	0.226977	5.015	0.0000153 ***
labour.part	0.068716	0.133540	0.515	0.610087
male.per.fem	0.003021	0.173041	0.017	0.986172
population	-0.064477	0.128278	-0.503	0.618367
nonwhite	-0.013794	0.061901	-0.223	0.824960
unemp.adult	0.931498	0.541803	1.719	0.094402 .
median.assets	15.158975	10.524458	1.440	0.158653
num.low.salary	0.825936	0.234189	3.527	0.001197 **

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 21.98 on 35 degrees of freedom
Multiple R-squared: 0.7543, Adjusted R-squared: 0.6771
F-statistic: 9.769 on 11 and 35 DF, p-value: 0.00000009378

```
> crime.lm.summary.2 <- summary(crime.lm.2)
>
> kable(crime.lm.summary.2$coef, digits = c(3, 3, 3, 4), format =
'markdown')
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-633.439	145.470	-4.354	0.0001
young.males	1.127	0.419	2.691	0.0109
is.south1	-0.557	13.883	-0.040	0.9682
average.ed	15.328	6.203	2.471	0.0185
exp.per.cap.1960	1.138	0.227	5.015	0.0000
labour.part	0.069	0.134	0.515	0.6101
male.per.fem	0.003	0.173	0.017	0.9862
population	-0.064	0.128	-0.503	0.6184
nonwhite	-0.014	0.062	-0.223	0.8250
unemp.adult	0.931	0.542	1.719	0.0944
median.assets	15.159	10.524	1.440	0.1587
num.low.salary	0.826	0.234	3.527	0.0012

> #3. Thinking more critically about linear regression

```
> crime.lm <- lm(crime.per.million ~ ., data = crime)
> crime.lm2 <- update(crime.lm, . ~ . - exp.per.cap.1959 - unemp.youth)
>
> kable(summary(crime.lm)$coef, digits = c(3, 3, 3, 4), format =
'markdown')
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-691.838	155.888	-4.438	0.0001
young.males	1.040	0.423	2.460	0.0193
is.south1	-8.308	14.912	-0.557	0.5812
average.ed	18.016	6.497	2.773	0.0091
exp.per.cap.1960	1.608	1.059	1.519	0.1384
exp.per.cap.1959	-0.667	1.149	-0.581	0.5653
labour.part	-0.041	0.153	-0.267	0.7909
male.per.fem	0.165	0.210	0.785	0.4381
population	-0.041	0.130	-0.319	0.7520
nonwhite	0.007	0.064	0.112	0.9112
unemp.youth	-0.602	0.437	-1.376	0.1780
unemp.adult	1.792	0.856	2.093	0.0441
median.assets	13.736	10.583	1.298	0.2033
num.low.salary	0.793	0.235	3.373	0.0019


```
> crime.lm.summary2 <- summary(crime.lm2)
> kable(crime.lm.summary2$coef, digits = c(3, 3, 3, 4), format =
'markdown')
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-633.439	145.470	-4.354	0.0001
young.males	1.127	0.419	2.691	0.0109
is.south1	-0.557	13.883	-0.040	0.9682
average.ed	15.328	6.203	2.471	0.0185
exp.per.cap.1960	1.138	0.227	5.015	0.0000
labour.part	0.069	0.134	0.515	0.6101
male.per.fem	0.003	0.173	0.017	0.9862
population	-0.064	0.128	-0.503	0.6184
nonwhite	-0.014	0.062	-0.223	0.8250
unemp.adult	0.931	0.542	1.719	0.0944
median.assets	15.159	10.524	1.440	0.1587
num.low.salary	0.826	0.234	3.527	0.0012

```
>
> # all 95% confidence intervals
> confint(crime.lm2)
```

	2.5 %	97.5 %
(Intercept)	-928.7593182	-338.1183387
young.males	0.2766914	1.9770739
is.south1	-28.7410920	27.6278928
average.ed	2.7362499	27.9198056
exp.per.cap.1960	0.6775118	1.5990864
labour.part	-0.2023846	0.3398163
male.per.fem	-0.3482706	0.3543119
population	-0.3248958	0.1959409
nonwhite	-0.1394591	0.1118719
unemp.adult	-0.1684209	2.0314168
median.assets	-6.2068096	36.5247604
num.low.salary	0.3505063	1.3013656

```
> # Just for education
> confint(crime.lm2, parm = "average.ed")
```

	2.5 %	97.5 %
average.ed	2.73625	27.91981

```
> # 75% confidence interval
> confint(crime.lm2, parm = "average.ed", level = 0.75)
```

	12.5 %	87.5 %
average.ed	8.072542	22.58351

```
> # How does 2 SE rule compare to confint output?
> # lower endpoint
> coef(crime.lm2)["average.ed"] - 2* summary(crime.lm2)$coef["average.ed",
"Std. Error"]
average.ed
2.922995
> # upper endpoint
> coef(crime.lm2)["average.ed"] + 2* summary(crime.lm2)$coef["average.ed",
"Std. Error"]
average.ed
```

```
27.73306
>
> my.data <- data.frame(y = c(12, 13, 10, 5, 7, 12, 15),
+ x1 = c(6, 6.5, 5, 2.5, 3.5, 6, 7.5),
+ x2 = c(6, 6.5, 5, 2.5, 3.5, 6, 7.5))
> my.data
  y  x1  x2
1 12 6.0 6.0
2 13 6.5 6.5
3 10 5.0 5.0
4  5 2.5 2.5
5  7 3.5 3.5
6 12 6.0 6.0
7 15 7.5 7.5
>
> crime.lm.summary2$coef["exp.per.cap.1960",]
      Estimate      Std. Error      t value      Pr(>|t|)
1.13829907170 0.22697675756 5.01504684417 0.00001532994
>
> crime.lm.summary2$coef["average.ed",]
      Estimate      Std. Error      t value      Pr(>|t|)
15.32802778 6.20251646 2.47125951 0.01847635
>
>
>
> #4. Factors in linear regression
> #추가
> colnames(birthwt) <- c("birthwt.below.2500", "mother.age", "mother.weight",
+ "race", "mother.smokes", "previous.prem.labor", "hypertension",
+ "uterine.irr",
+ "physician.visits", "birthwt.grams")
> birthwt <- transform(birthwt,
+ race = as.factor(mapvalues(race, c(1, 2, 3),
+ c("white", "black", "other"))),
+ mother.smokes = as.factor(mapvalues(mother.smokes,
+ c(0,1), c("no", "yes"))),
+ hypertension = as.factor(mapvalues(hypertension,
+ c(0,1), c("no", "yes"))),
+ uterine.irr = as.factor(mapvalues(uterine.irr,
+ c(0,1), c("no", "yes"))),
+ )
The following `from` values were not present in `x`: 1, 2, 3
The following `from` values were not present in `x`: 0, 1
The following `from` values were not present in `x`: 0, 1
The following `from` values were not present in `x`: 0, 1
>
> # Fit regression model
> birthwt.lm <- lm(birthwt.grams ~ race + mother.age, data = birthwt)
> # Regression model summary
> summary(birthwt.lm)

Call:
lm(formula = birthwt.grams ~ race + mother.age, data = birthwt)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-2131.57	-488.02	-1.16	521.87	1757.07

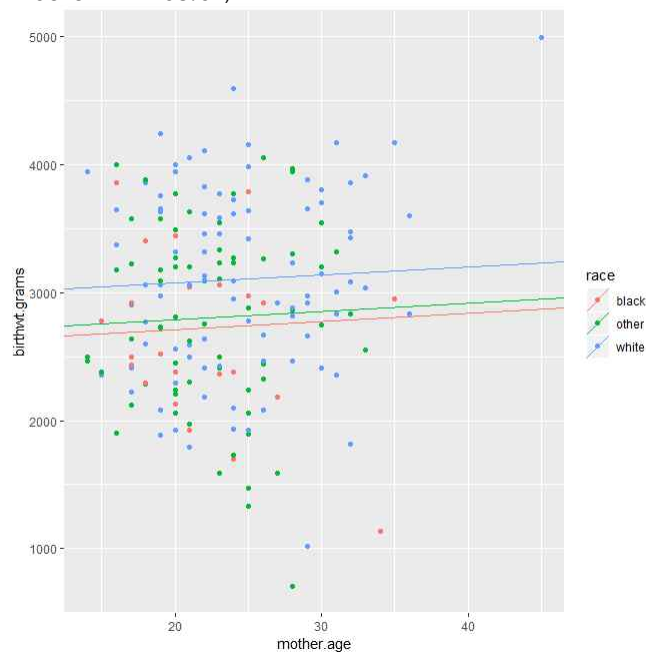
Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	2584.264	258.393	10.001	<2e-16 ***
raceother	80.249	165.582	0.485	0.628
racewhite	365.715	160.636	2.277	0.024 *
mother.age	6.288	10.073	0.624	0.533

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

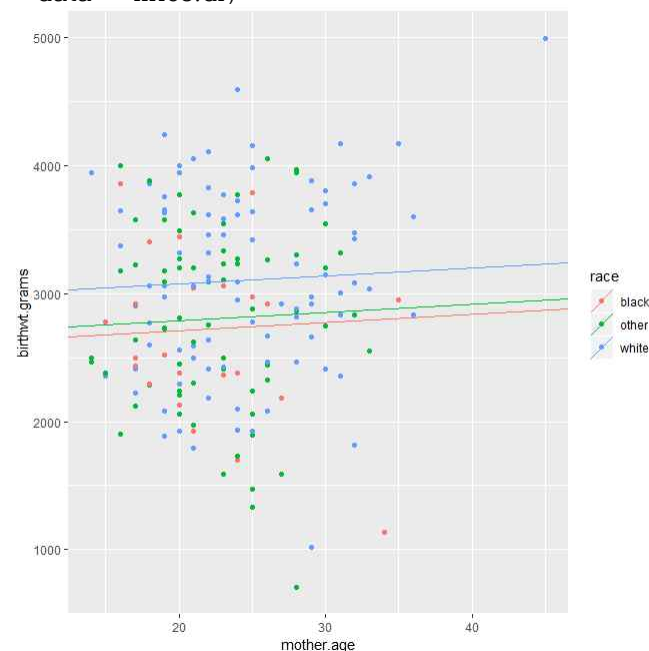
Residual standard error: 715.7 on 185 degrees of freedom
Multiple R-squared: 0.05217, Adjusted R-squared: 0.0368
F-statistic: 3.394 on 3 and 185 DF, p-value: 0.01909

```
>
> # Calculate race-specific intercepts
> intercepts <- c(coef(birthwt.lm)["(Intercept)"],
+   coef(birthwt.lm)["(Intercept)"] + coef(birthwt.lm)["raceother"],
+   coef(birthwt.lm)["(Intercept)"] + coef(birthwt.lm)["racewhite"])
> lines.df <- data.frame(intercepts = intercepts,
+   slopes = rep(coef(birthwt.lm)["mother.age"], 3),
+   race = levels(birthwt$race))
> qplot(x = mother.age, y = birthwt.grams, color = race, data = birthwt) +
+   geom_abline(aes(intercept = intercepts, slope = slopes, color = race),
+   data = lines.df)
```

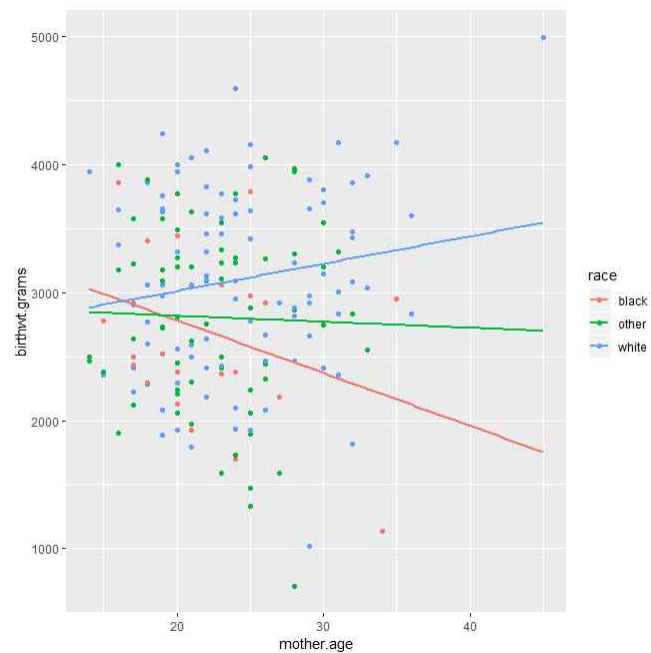


```
> head(model.matrix(birthwt.lm), 20)
      (Intercept) raceother racewhite mother.age
85              1          0          0         19
86              1          1          0         33
87              1          0          1         20
88              1          0          1         21
89              1          0          1         18
91              1          1          0         21
92              1          0          1         22
93              1          1          0         17
94              1          0          1         29
95              1          0          1         26
96              1          1          0         19
97              1          1          0         19
98              1          1          0         22
99              1          1          0         30
100             1          0          1         18
101             1          0          1         18
102             1          0          0         15
103             1          0          1         25
104             1          1          0         20
105             1          0          1         28
```

```
>
> qplot(x = mother.age, y = birthwt.grams, color = race, data = birthwt) +
+   geom_abline(aes(intercept = intercepts, slope = slopes, color = race),
+   data = lines.df)
```



```
> qplot(x = mother.age, y = birthwt.grams, color = race, data = birthwt) +
+   stat_smooth(method = "lm", se = FALSE, fullrange = TRUE)
```

```
> birthwt.lm.interact <- lm(birthwt.grams ~ race * mother.age, data =
birthwt)
> summary(birthwt.lm.interact)
```

Call:

```
lm(formula = birthwt.grams ~ race * mother.age, data = birthwt)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-2182.35	-474.23	13.48	523.86	1496.51

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	3606.33	615.26	5.861	0.000000021 ***
raceother	-696.74	756.65	-0.921	0.3584
racewhite	-1022.79	694.21	-1.473	0.1424
mother.age	-41.17	27.82	-1.480	0.1407
raceother:mother.age	36.51	33.85	1.078	0.2823
racewhite:mother.age	62.54	30.67	2.039	0.0429 *

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 710.7 on 183 degrees of freedom
Multiple R-squared: 0.07541, Adjusted R-squared: 0.05015
F-statistic: 2.985 on 5 and 183 DF, p-value: 0.01291