MSDM5058 Information Science
Computational Project II:
Teamwork in Portfolio Management Using Prediction Rules and Communication in Social Networks

This computational project consists of twelve parts, of which the "extra" ones are optional. You need to complete the project in groups of two to four people. While you will analyze the same data and can share graphs with your groupmates, you need to write an individual report, which will be appraised independently from theirs. Your report must conform to the LNCS format, otherwise 10% of score will be deducted from your report. You need to submit your report onto Canvas at the latest on **2 June, 2020**.

1 Data Preprocessing

Choose a stock available on more than 4000 days from *closing_price.xlsx* or other credible sources. Denote its closing-price time series as $s(t)$, where "today" $t = 0$ is defined so that the length ratio of the past $\{s(t) \mid t \leq 0\}$ to the future $\{s(t) \mid t > 0\}$ is around 3:1.

- Compute its daily return

$$x(t) = \frac{s(t) - s(t-1)}{s(t-1)} \quad . \tag{1}$$

- Digitize $x(t)$ as $d(t)$ with three alphabets, viz. D for "down", U for "up", and H for "hold". You need to decide a sensible value for the positive constant ε.

$$d(t) = \begin{cases} \text{D} & [x(t) < -\varepsilon] \\ \text{U} & [x(t) > +\varepsilon] \\ \text{H} & (\text{otherwise}) \end{cases} \tag{2}$$

- Split all series at $t = 0$ into a learning set ($t \leq 0$) and a testing set ($t > 0$). Work on the learning sets until Section 5, then work on the testing sets from Section 6 onwards.

2 Cumulative Distribution Function

Given the return $x(t)$ on one day, we would like to predict its value $x(t+1)$ one day later. However, a quantitative prediction is too ambitious; it is more realistic to predict $d(t+1)$ qualitatively instead. Let us focus on $d(t+1) = \text{U}$ and $d(t+1) = \text{D}$, which respectively correspond to a strong bullish and a strong bearish market.

- Plot the conditional CDFs $F_U(x) = \mathrm{CDF}[x(t) \mid d(t+1) = \mathrm{U}]$ and $F_D(x) = \mathrm{CDF}[x(t) \mid d(t+1) = \mathrm{D}]$.

3 Probability Density Function

We would then like to extract their corresponding PDFs, viz. $\mathrm{PDF}[x(t) \mid d(t+1) = \mathrm{U}]$ and $\mathrm{PDF}[x(t) \mid d(t+1) = \mathrm{D}]$.

- Fit $F_U(x)$ and $F_D(x)$ with a Fermi-Dirac distribution

$$F(x) = \frac{1}{1+\exp[-b(x-x_0)]}\,. \tag{3}$$

  Then plot the fitted distributions' derivatives $f_U(x)$ and $f_D(x)$ on the same graph. They estimate the desired PDFs.

- On the other hand, consider a Gaussian distribution (let us use this name instead of a normal distribution to parallel a Fermi-Dirac distribution)

$$g(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]. \tag{4}$$

  Compute the means μ and the variances σ^2 of $\{x(t) \mid d(t+1) = \mathrm{U}\}$ and $\{x(t) \mid d(t+1) = \mathrm{D}\}$, then fit the desired PDFs as $g_U(x)$ and $g_D(x)$. Plot them on the same graph.

4 Bayes Detector

We now construct a Bayes detector with the PDFs to predict $d(t+1)$. Formally, we choose between two hypotheses H_U: $d(t+1) = \mathrm{U}$ and H_D: $d(t+1) = \mathrm{D}$ after observing $x(t)$.

- Compute the probabilities $P[d(t+1) = \mathrm{U}]$ and $P[d(t+1) = \mathrm{D}]$. These are the hypotheses' prior probabilities.
- Construct the detector with the Fermi-Dirac PDFs $f_U(x)$ and $f_D(x)$. In general, $x \in (x_1, x_2)$ favors one hypothesis, whereas $x < x_1$ or $x > x_2$ favors the other. Plot $x = x_1$ and $x = x_2$ on the graph of $f_U(x)$ and $f_D(x)$.
- Repeat last step with the Gaussian PDFs $g_U(x)$ and $g_D(x)$.

5 Association Rules

Prediction in U suggests buying stocks, whereas prediction in D suggests selling stocks. In principle, we can simply follow the Bayes detectors and trade accordingly, but we will end up trading too frequently and lose a lot due to transaction costs. Therefore, we impose extra association rules to limit the frequency of trading. For $X = \mathrm{U}$ or $X = \mathrm{D}$, a k-day rule takes the general form of

$$R_X^k : \{d(t-k+1), d(t-k+2), \ldots, d(t)\} \to d(t+1) = X\,. \tag{5}$$

- Mine the two best 1-day rules R_{U}^1 and R_{D}^1, then report their support and confidence. You need to decide how to measure a rule's goodness, e.g. with support, confidence, lift, or other indicators. If you do not use support or confidence, report the rules' measured goodness as well.
- Repeat last step with the two best 5-day rules R_{U}^5 and R_{D}^5.

6 Portfolio Management

Now, invest in the stock to see if the Bayes detectors and association rules really work. Let $M(t)$ be the amount of your money and $N(t)$ the number of shares you own at the end on the tth day. Your portfolio's monetary value is measured with

$$V(t) = M(t) + N(t)\, s(t)\,. \tag{6}$$

Initially, you are given $M(0) = \$100{,}000$ and $N(0) = 0$ shares. Then you start trading according to the following rules at $t = 1$.

- When you observe the antecedent of R_{U}^k on the tth day and your Bayes detector predicts $d(t+1) = \mathrm{U}$, buy stocks at the price $s(t)$ and update

$$\begin{cases} M(t) \leftarrow M(t) - m \\ N(t) \leftarrow N(t) + m/s(t) \end{cases} \quad \text{for} \quad m = \gamma M(t)\,. \tag{7}$$

- When you observe the antecedent of R_{D}^k on the tth day and your Bayes detector predicts $d(t+1) = \mathrm{D}$, sell stocks at the price $s(t)$ and update

$$\begin{cases} M(t) \leftarrow M(t) + ns(t) \\ N(t) \leftarrow N(t) - n \end{cases} \quad \text{for} \quad n = \gamma N(t)\,. \tag{8}$$

The constant parameter $\gamma \in (0, 1)$ quantifies your "greed". The greedier you are, the more you want to earn and thus the more money or stock you trade per transaction. Choose any value γ_0 you like for the time being. Note that this market model has made several unrealistic assumptions for simplicity.

1. The stock's closing price $s(t)$ becomes its price on the entire tth day.
2. The stock's price is measured in dollars regardless of its original market.
3. $M(t)$ and $N(t)$ do not have any smallest unit per transaction, so they can be any real numbers.
4. You trade at most once on one day.

First compare the performance of the two Bayes detectors obtained in Section 4. Consider $k = 1$ and $\gamma = \gamma_0$.

- Trade according to the Fermi-Dirac Bayes detector. Plot the portfolio's monetary value $V_f(t)$.

- Trade again according to the Gaussian Bayes detector. Plot the portfolio's monetary value $V_g(t)$.
- Which Bayes detector performs better? Stick to the better detector in the following sections.
- **Extra.** Repeat the analysis with $k = 5$. Does it change the detectors' performance?

7 Transaction Cost

A stock market usually charges certain execution fee upon transaction. Therefore, we modify the scheme of buying and selling as Eq. (9) and (10) by including some tax ξ.

$$\begin{cases} M(t) \leftarrow M(t) - m \\ N(t) \leftarrow N(t) + (1-\xi)m/s(t) \end{cases} \quad \text{for} \quad m = \gamma M(t) \tag{9}$$

$$\begin{cases} M(t) \leftarrow M(t) + (1-\xi)ns(t) \\ N(t) \leftarrow N(t) - n \end{cases} \quad \text{for} \quad n = \gamma N(t) \tag{10}$$

Intuitively, the tax favors a lower frequency of trading. Consider $\xi = 0.2\%$ and $\gamma = \gamma_0$.

- Trade according to the 1-day rules. Plot the portfolio's monetary value $V^1(t)$. How often do you trade on average?
- Trade again according to the 5-day rules. Plot the portfolio's monetary value $V^5(t)$. How often do you trade on average?
- Which pair of rules perform better? Does this match your expectation? Stick to the better rules in the following sections.
- **Extra.** Repeat the analysis with $\xi = 0.1\%$ and $\xi = 0.5\%$. How does the rules' performance depend on ξ?

8 Risk-Free Interest

In addition to transaction costs, further assume that $M(t)$ grows without risk because it is deposited in a bank or used to invest in a government bond. Anyhow, at the beginning of each day, first update

$$M(t) \leftarrow M(t)\,(1+r) \tag{11}$$

for some daily interest rate r. Consider $\xi = 0.2\%$, $r = 0.001\%$, and $\gamma = \gamma_0$.

- Trade and plot the portfolio's monetary value $V_r(t)$.
- If you do not trade at all, your portfolio's monetary value reduces to

$$M_r(t) = M(0)\,(1+r)^t\,, \tag{12}$$

  which can serve as a portfolio's benchmark value. Plot the ratio $\rho_r(t) = V_r(t)/M_r(t)$.
- **Extra.** Repeat the analysis with $r = 0.005\%$ and $r = 0.01\%$, whereas $r = 0$ in Section 7. How does $\rho_r(t)$ depend on r?

9 Greed

Now let us investigate the effect of γ. Consider $\xi = 0.2\%$ and $r = 0.001\%$.

- Trade and plot the portfolio's monetary value $V_\gamma(t)$ for $\gamma \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$.
- Plot the last monetary value $V_\gamma(\max t)$, the highest monetary value $\max_t V_\gamma(t)$, and the average monetary value $\langle V_\gamma(t)\rangle$ against γ. Hence discuss the effect of γ.
- **Extra.** Repeat the analysis with fifteen more samples of γ.

10 Alternative Portfolio Measure

Instead of its monetary value, we may alternatively measure a portfolio's worth $W(t)$ in terms of number of shares.

$$W(t) = N(t) + M(t) / s(t) \tag{13}$$

Consider $\xi = 0.2\%$ and $r = 0.001\%$.

- Trade with $\gamma \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, then compute each portfolio's monetary value $V_\gamma(t)$ and worth $W_\gamma(t)$.
- Plot $v(t) = \ln[V_\gamma(t)/V(0)]$ and $w(t) = \ln[W_\gamma(t)/W(0)]$. Do they always have the same sign?
- Plot $v_i = \ln[V_\gamma(t_i)/V_\gamma(t_i–1)]$ and $w_i = \ln[W_\gamma(t_i)/W_\gamma(t_i–1)]$ against i, where the ith transaction happens at $t = t_i$. Do they always have the same sign?
- **Extra.** Note that $v(t_j) = \sum_{i=1}^{j} v_i$ and $w(t_j) = \sum_{i=1}^{j} w_i$. Have you gained or lost when the signs of $v(t)$ and $w(t)$ are different?

11 Efficient Frontier

While it seems a bit arbitrary to trade with $m = \gamma M(t)$ and $n = \gamma N(t)$ per transaction, we may justify these amounts with the efficient frontier and hence generalize the scheme of investment.

Denote the expected returns of the portfolio, $M(t)$, and $s(t)$ as $\{u, u_1, u_2\}$ and their risks as $\{\sigma^2, \sigma_1^2, \sigma_2^2\}$. As $M(t)$ is risk-free, $\sigma_1^2 = 0$. The minimum-risk portfolio thus consists of 100% of money and 0% of stock, whereas the maximum-risk portfolio consists of 0% of money and 100% of stock. Consequently, $u_1 \le u \le u_2$: the more money you hold, the less risky your portfolio is, but it gives a lower return.

Suppose we have observed the signal of buying on the tth day. Right before buying, $u = Au_1 + (1–A)u_2$, where $A = M(t)/[M(t) + N(t)s(t)]$ is the proportion of money. Instead of spending $m = \gamma M(t)$, we spend $\tilde{m} = \gamma_U M(t)$ buying $(1–\xi)\tilde{m}/s(t)$ shares so that u becomes $u_U = A_U u_1 + (1–A_U)u_2$ and satisfies

$$\frac{u_U - u}{u_2 - u} = \gamma\,. \tag{14}$$

A greedier investor buys more when he predicts a bullish market, so he pushes his portfolio more towards the maximum-return portfolio along the efficient frontier. The constraint is one of the maps that fulfils this intuition: $\gamma \to 1 \Rightarrow u_U \to u_2$; conversely, $\gamma \to 0 \Rightarrow u_U \to u$.

The case for selling is slightly different. A greedier investor sells more when he predicts a bearish market, so he pushes his portfolio more towards the minimum-return portfolio. Therefore, the constraint is replaced by

$$\frac{u_D - u}{u_1 - u} = \gamma \,, \tag{15}$$

where $u_D = A_D u_1 + (1-A_D)u_2$ is the portfolio's return after selling $\tilde{n} = \gamma_D N(t)$ shares to earn $(1-\xi)\tilde{n}s(t)$.

- Express the proportion of money after buying A_U and after selling A_D in terms of $\{\gamma_U, \gamma_D, M(t), N(t), s(t), \xi\}$.
- Express A, A_U, and A_D in terms of $\{u, u_U, u_D, u_1, u_2\}$, then express A_U/A and $(1-A_D)/(1-A)$ in terms of γ.
- Hence express γ_U and γ_D in terms of $\{\gamma, M(t), N(t), s(t), \xi\}$.

It turns out that $\gamma = \gamma_U = \gamma_D$ for a small tax ξ. This justifies trading with $m = \gamma M(t)$ and $n = \gamma N(t)$. Now consider an absurdly heavy tax $\xi = 20\%$ with $r = 0.001\%$ and $\gamma = \gamma_0$.

- Trade with $m = \gamma M(t)$ and $n = \gamma N(t)$. Plot the portfolio's monetary value $V(t)$.
- Trade again with $\tilde{m} = \gamma_U M(t)$ and $\tilde{n} = \gamma_D N(t)$. Plot the portfolio's monetary value $\tilde{V}(t)$. Which scheme of investment performs better?

12 Adaptive Greed

We have been so far using a constant greed throughout trading. We now investigate an adaptive greed that changes according to the market. Use Section 7's scheme of investment instead of the one derived in last section.

12.1 Posterior Analysis

Consider $\xi = 0.2\%$ and $r = 0.001\%$.

- Trade with $\gamma \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, then compute each portfolio's monetary value $V_\gamma(t)$.
- Plot $\gamma_i^* = \operatorname{argmax}_\gamma V_\gamma(t_i)/V_\gamma(t_i-1)$ against i, where the ith transaction happens at $t = t_i$. Hence discuss how the optimal greed γ_i^* changes.
- **Extra.** Repeat the analysis with fifteen more samples of γ.
- Trade again with γ_i^* in the ith transaction. Plot the portfolio's monetary value $V^*(t)$.

12.2 Extra: Prior Analysis

Practically, γ_i^* is useless because it is obtained a posteriori, while we need to decide γ before trading. Since it is hard to choose its value from a real interval (0, 1), we simplify the problem to choosing between two values $\{\gamma_A, \gamma_C\}$: before each investment, we decide whether it is better to use an aggressive greed γ_A or a conservative greed γ_C.

- You are Alice, Bob, and Charlie's banker. Alice's greed is $\gamma_A = 0.7$, and Charlie's greed is $\gamma_C = 0.3$. Bob wants to strike a balance between them, so he lets you analyze when to invest with $\gamma_B = \gamma_A$ and when to invest with $\gamma_B = \gamma_C$. Discuss how you can decide Bob's greed γ_B.
- Trade for Alice, Bob, and Charlie at $\xi = 0.2\%$ and $r = 0.001\%$. Plot their portfolios' values $V_A(t)$, $V_B(t)$, and $V_C(t)$. Does Bob's portfolio perform better?

Acknowledgments. While your groupmates are not your co-authors, you must acknowledge their contribution at the end of the report for academic etiquette.