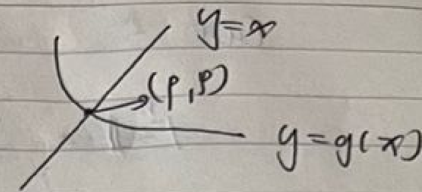
고정점

$g(p) = p$

$y = g(x)$ 와 $y = x$



ex) $f(x) = 0$

$f(p) = 0$

$\Rightarrow g(x) = x - f(x)$

$= x + 3f(x)$

$g(p) = p - f(p) = p - 0$

$= p + 3f(p) - p = p$

$\therefore p$: $g(x)$의 고정점

$g(p) = p - f(p) = p$

$\Rightarrow f(p) = 0$

$\therefore p$: $f(x) = 0$ 의 근(해)

e.g.)

$x^2 - 2 = x$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\therefore x = 2,\ x = -1$

Th 2.3

Def [고정점]

p = $g(x)$의 고정점

if $g(p) = p$.

a) i) $g(a) = a$ $g(b) = b$

$\Rightarrow \exists\, p$ s.t $g(p) = p$

ii) $g(a) \neq a$ $g(b) \neq b$

$(g(a) > a$ and $g(b) <$

$h(x) = g(x) - x \in C[a,b]$

$h(a) = g(a) - a > 0,\ h(b) = g(b) - b < 0$

$\Rightarrow \exists p$ s.t $h(p) = 0$

by 중간값 정리

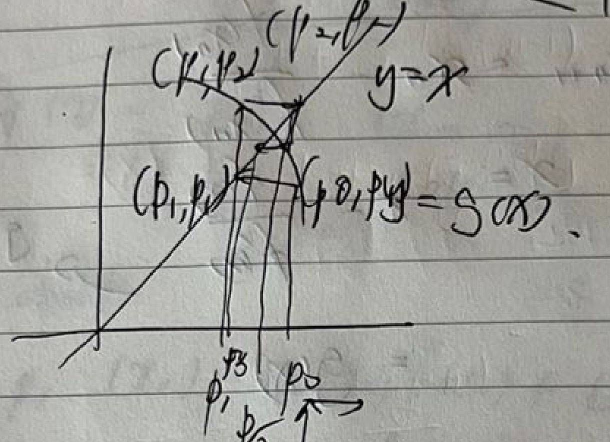
b)

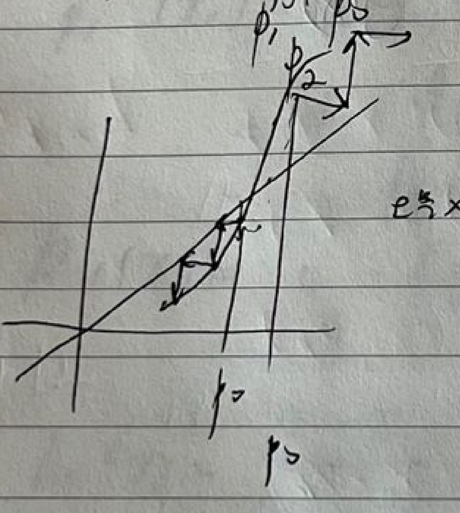
Assume $g(p)=p$, $g(q)=q$ $(p \neq q)$

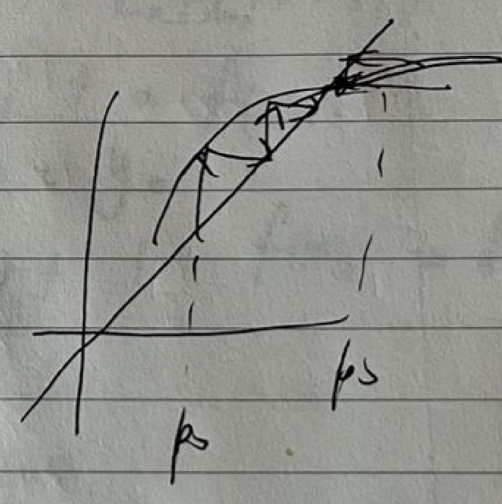
$$\frac{g(p)-g(q)}{p-q} = g'(k) \quad \text{for some } k \in (a,b)$$

by MVT

$$|p-q| = |g(p)-g(q)| = |g'(k)|\,|p-q| < |p-q|$$ 矛盾







$$x^2 + 4x - 10 = 0$$

$$f(x) = 0$$

$$\Rightarrow 0 = -f(x)$$

$$\Rightarrow x = x - f(x) = g(x)^*$$

$$x^3 = 10 - 4x^2 \quad \times \frac{1}{x}$$

$$\Rightarrow x^2 = \frac{10}{x} - 4x$$

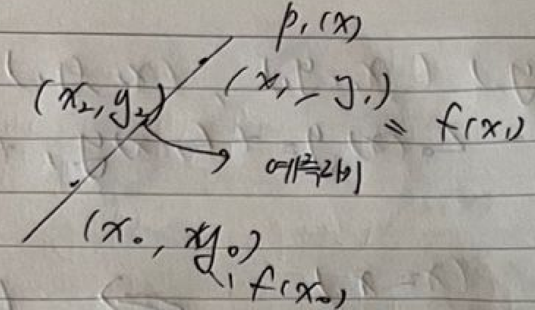
$$\Rightarrow x = \left(\frac{10}{x} - 4x\right)^{1/2}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x = x - \frac{f(x)}{f'(x)} \rightarrow g'(p)?$$

$$= g(x)$$

Lagrange
보간법



$$p_1(x) = L_0(x) f(x_0) + L_1(x) f(x_1)$$

$$L_0(x) = \begin{cases} 1 & (x = x_0) \\ 0 & (else) \end{cases}$$

$$L_1(x) = \begin{cases} 1 & (x = x_1) \\ 0 & (else) \end{cases}$$

check

$$\begin{aligned} p_1(x_0) &= L_0(x_0) f(x_0) + L_0(x_0) f(x_1) \\ &= 1 \cdot f(x_0) + 0 \cdot f(x_1) \\ &= f(x_0) = y_0 \end{aligned}$$

$$p_1(x) = 0 \cdot f(x_0) + 1 \cdot f(x_1) = f(x_1) = y_1$$

$$L_0(x) = \frac{x - x_1}{x_0 - x_1}$$

$$L_1(x) = \frac{x - x_0}{x_1 - x_0}$$

Input (x_0, y_0) (x_1, y_1), (x_2, y_2)

$$p_2(x) = L_0(x)y_0 + L_1(x)y_1 + L_2(x)y_2$$

$$L_0(x) = \begin{cases} 1 & (x = x_0) \\ 0 & (x = x_1, x_2) \end{cases} \Rightarrow L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$$

$$L_1(x) = \begin{cases} 1 & (x = x_1) \\ 0 & (x = x_0, x_2) \end{cases} \qquad L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

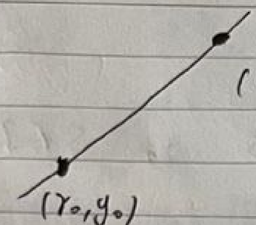
단점: 하나 추가되면 다시 처음부터...

장점: 분석하기 좋음

$$L_{2,0}(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \qquad L_{n,k}(x) = \frac{\prod_{l \neq k}(x - x_l)}{\prod_{l \neq k}(x_k - x_l)}$$

$$= \frac{\prod_{k \neq 0}(x - x_k)}{\prod_{k \neq 0}(x_0 - x_k)}$$

제차분.
(Divided Differences)



$p_0(x)$

$$p_0(x) = y_0$$

$$p_1(x) = p_0(x) + g_1(x)$$

$$p_2(x) = p_1(x) + g_2(x)$$

$$g_1(x) = p_1(x) - p_0(x)$$

$$g_1(x_0) = p_1(x_0) - p_0(x_0)$$

$$= y_0 - y_0 = 0$$

∴ $g_1(x_0) = c_1(x - x_0)$

$$p_1(x) = \left| y_0 + c_1(x - x_0) \right|_{(x_1, y_1)}$$

$y_1 - y_0$

$$\therefore \quad c_1 = \frac{y_1 - y_0}{x_1 - x_0} = f[x_0, x_1]$$

$$\therefore \; p_1(x) = f[x_0] + f[x_0, x]$$

$$g_2(x) = p_2(x) - p_1(x) \Big|_{x = x_0, x_1} = 0$$

$$\Rightarrow \; g_2(x) = c_2(x - x_0)(x - x_1)$$

$$y_2 = p_1(x_2) + c_2(x_2 - x_0)(x_2 - x_1)$$

$$= y_0 + f[x_0, x_1](x_2 - x_0) + c_2(x_2 - x_0)(x_2 - x_1)$$

(x_0, y_0), (x_1, y_1), (x_2, y_2)

$$p_0(x) = y_0$$

$$p_1(x) = p_0(x) + \boxed{g_1(x)}$$ 1차 다항식

$$p_2(x) = p_1(x) + \boxed{g_2(x)}$$ 2차 〃

$\vdots$

n차 〃

$$p_n(x) = p_{n-1}(x) + \boxed{g_n(x)}$$

$$g_2(x) = p_2(x) - p_1(x) \Big|_{[x_0, x_1]} = 0$$

$$\therefore g_2(x) = a_2(x - x_0)(x - x_1)$$

$$a_2(x - x_0)(x - x_1) = p_2(x) - \{f(x) + f(x_0, x_1)(x - x_0)\}$$

$$a_2(x_2 - x_0)(x_2 - x_1) = p_2(x) - \{y + f(x_0, x_1)(x_2 - x_0)\}$$

표 3.7

$$p_2(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)$$

$s(s-1)h^2 = sh \cdot (s-1)h$

$x_0 = x_0$

$x_1 = x + h$

$x_2 = x_0 + 2h$

$x_3 = x_0 + 3h$

$\vdots$

$x_i = x_0 + ih$

$x = x_0 + sh$

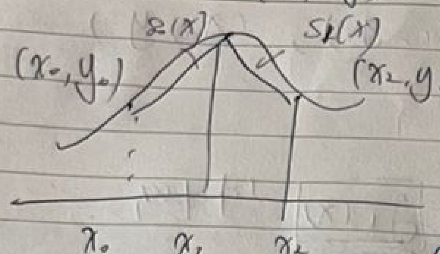
$x - x_i = (x_0 + sh) - (x_0 + ih)$

$= (s - i)h$

$S_0(x) = \overset{=y_0}{a_0} + b_0(x-x_0) + c_0(x-x_0)^2 + d_0(x-x_0)^3$

$S_1(x) = \overset{=y_1}{a_1} + b_1(x-x_1) + c_1(x-x_1)^2 + d_1(x-x_1)^3$

스플라인 보간법



$S(x) = \begin{cases} S_0(x) & (x \in [x_0, x_1]) \\ S_1(x) & x \in [x_1, x_2] \end{cases}$

자연조건

(a) $S_0''(x_0) = 0$, $S_1''(x_2) = 0$

미지수 8개 $S_0(x_0) = y_0$, $S_0(x_1) = y_1$, $S_0'(x_1) = S_1'(x_1)$

$S_1(x_1) = y_1$, $S_1(x_2) = y_2$ (b) $S_0'(x_0) = f'(x_0)$, $S_1'(x_2) = f'(x_2)$

$S_0'(x_1) = y_1$, $S_1(x_2) = y_2$ $S_0''(x_1) = S_1''(x_1)$

+ 예제 3차

$S_0(x) = a_0 + b_0(x-x_0) + c_0(x-x_0)^2 + d_0(x-x_0)^3$

$S_1(x) = a_1 + b_1(x-x_1) + c(x-x_1)^2 + d_1(x-x_1)^3$

$S_0(x_0) = a_0 = y_0$, $S_1(x_1) = a_1 = y_1$

$S_0'(x) = b_0 + 2c_0(x-x_0) + 3d_0(x-x_0)^2$

$S_0''(x) = 2c_0 + 3 \cdot 2 d_0 (x-x_0)$

$S_0''(x_0) = 2c_0 = 0 \quad \therefore c_0 = 0$

$S_1''(x_2) = 2c_1 + 3 \cdot 2 d_1 (x_2 - x_1) = 0$

$d_1 = \frac{-2c_1}{6h}$

$S_0(x_1) = a_0 + b_0(x_1 - x_0) + c_0(x_1 - x_0)^2$
$+ d_0(x_1 - x_0)^3 = y_1$

$\Rightarrow a_0 + b_0 h + c_0 h^2 + d_0 h^3 = y_1$

$S_1(x_2) = a_1 + b_1 h + c_1 h^2 + d_1 h^3$

$= y$