

2. Taylor

Taylor series

$\{a_n\}$: a. seq.

$\sum_{n=1}^{\infty} a_n$: a series.

$\sum_{n=0}^{\infty} C_n x^n$: a power series $= f(x)$, $C_n = \frac{f^{(n)}(0)}{n!}$
 $\Rightarrow C_0 + C_1 x + C_2 x^2 + \dots$

$$\text{ex)} \frac{1}{1-x} = 1 + x + x^2 + \dots + x^n + \dots$$

$$, |x| < 1$$

$$\sum_{n=0}^{\infty} C_n (x-a)^n = f(x), \quad C_n = \frac{f^{(n)}(a)}{n!}$$

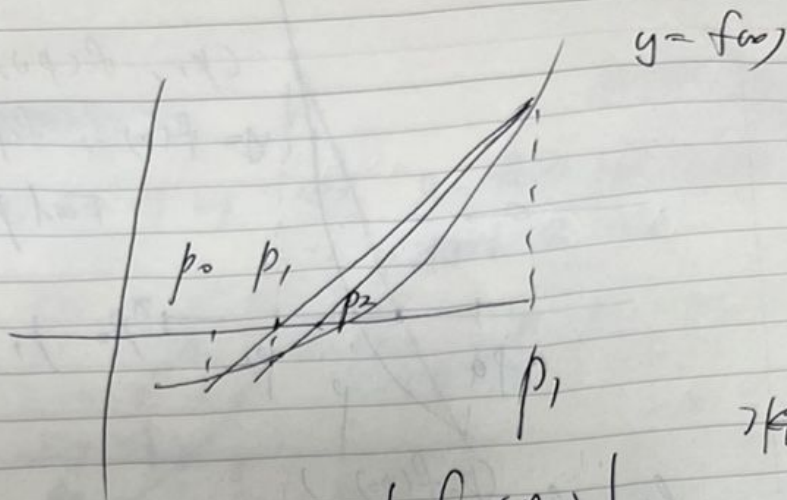
$$= \sum_{k=0}^n \frac{f^{(k)}(a)}{k!} (x-a)^k + \sum_{k=n+1}^{\infty} \frac{f^{(k)}(a)}{k!} (x-a)^k$$

$$= \underbrace{T_n(x)}_{\text{Taylor polynomial}} + \underbrace{R_n(x)}_{\text{Remainder}} \quad R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} (x-a)^{n+1}$$

Taylor series vs Taylor integral

esp, $n=0$

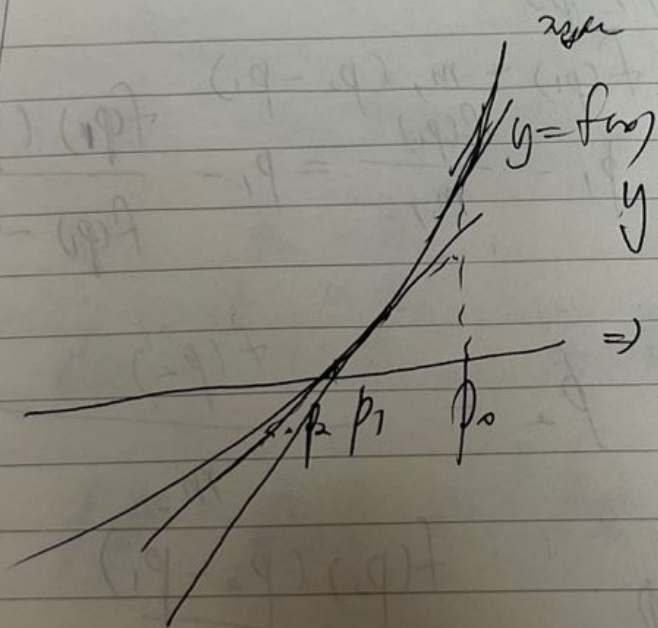
$$\begin{aligned} f(x) &= T_0(x) + R_0(x) \\ &= f(a) + f'(\xi)(x-a) \\ \Leftrightarrow \frac{f(x) - f(a)}{x-a} &= f'(\xi) \end{aligned}$$



× $f'(x)$

$$|f(p_0)| < |f(p_1)|$$

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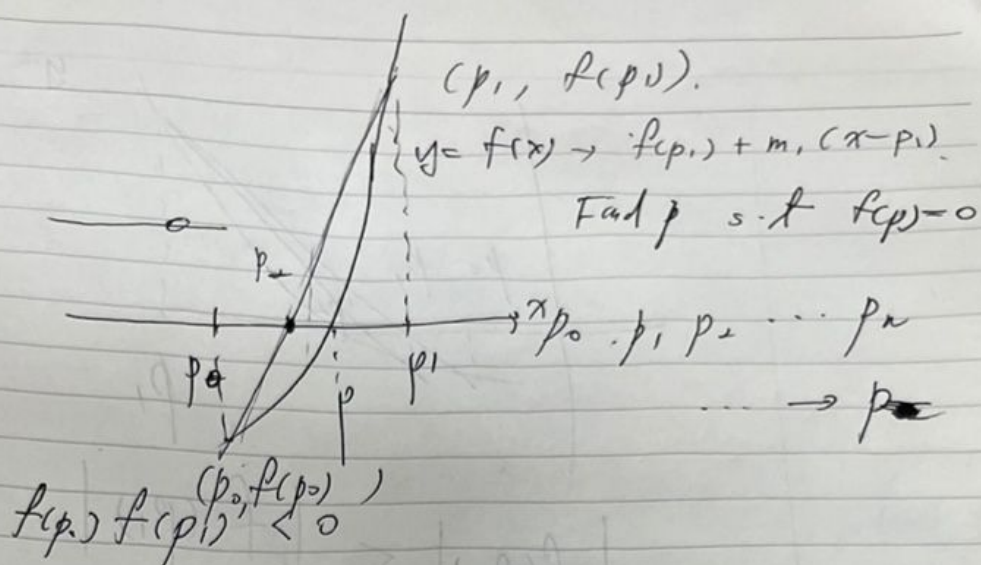
$$y = f(p_0) + f'(p_0)(x - p_0)$$

$$\Rightarrow 0 = f(p_0) + f'(p_0)(p_1 - p_0)$$

$$\Rightarrow p_1 = p_0 - \frac{f(p_0)}{f'(p_0)}$$

$$p_{n+1} = p_n - \frac{f(p_n)}{f'(p_n)}$$

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$$m_1 = \frac{f(p_1) - f(p_0)}{p_1 - p_0}$$

$$\Rightarrow 0 = f(p_1) + m_1(p_2 - p_1)$$

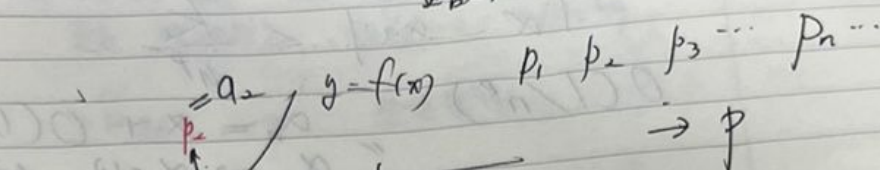
$$\Rightarrow p_2 = p_1 - \frac{f(p_1)}{m_1} = p_1 - \frac{f(p_1)(p_1 - p_0)}{f(p_1) - f(p_0)}$$

$$p_3 = p_2 - \frac{f(p_2)}{m_2}$$

$$= p_2 - \frac{f(p_2)(p_2 - p_1)}{f(p_2) - f(p_1)}$$

2.4.4 二分法 3.1

$$f(x) = 0 \text{ 的根} \Leftrightarrow y = f(x) \text{ 与 } y = 0 \text{ 的交点}$$



$$|p - p_n| < \text{TOL}$$

or $f(a_0) f(b_0) < 0$

$$\text{sgn}(f(a_0)) \text{sgn}(f(b_0)) < 0$$

$$\text{if } f(a_0) f(p_1) < 0, y = f(x)$$

$$a_1 = a_0$$

$$b_1 = p_1$$

$$b_1 = b_0$$

$$\text{else } a_1 = p_1$$

$$|p - p_n| < \frac{|b_0 - a_0|}{2^n} < \text{TOL}$$