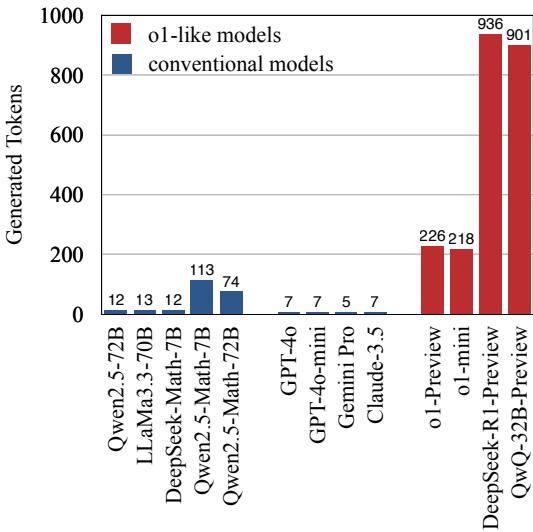


Do NOT Think That Much for 2+3=? On the Overthinking of o1-Like LLMs

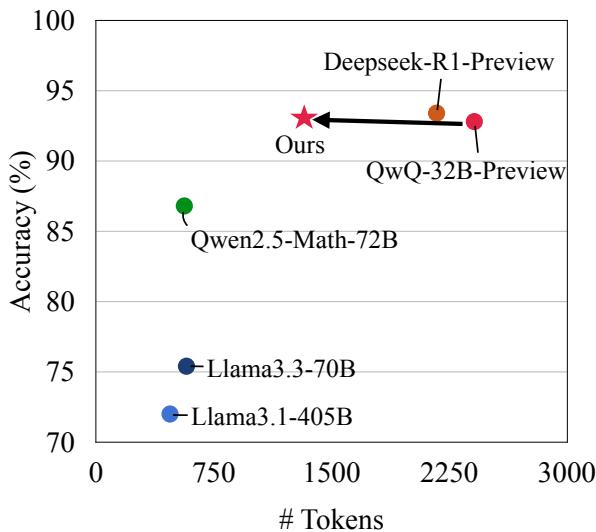
Xingyu Chen^{*1,2}, Jiahao Xu^{*1}, Tian Liang^{*1}, Zhiwei He^{*1,2}, Jianhui Pang¹, Dian Yu¹, Linfeng Song¹, Qiuwei Liu¹, Mengfei Zhou², Zhuosheng Zhang², Rui Wang^{†2}, Zhaopeng Tu^{†1}, Haitao Mi¹, and Dong Yu¹

¹Tencent AI Lab

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(a) Generated tokens on question “what is the answer of 2 plus 3?”



(b) Token-accuracy plot on MATH500

Figure 1: Illustration of **overthinking issue** in Figure (a): o1-like models (right panel) spend much more tokens than conventional LLMs (left and middle panels). Our method reduces the overthinking issue when applied to QwQ-32B-Preview (Figure (b)).

Abstract

The remarkable performance of models like the OpenAI o1 can be attributed to their ability to emulate human-like long-time thinking during inference. These models employ extended chain-of-thought (CoT) processes, exploring multiple strategies to enhance problem-solving capabilities. However, a critical question remains: *How to intelligently and efficiently scale computational resources during testing*. This paper presents the first comprehensive study on the prevalent issue of **overthinking** in these models, where excessive computational resources are allocated for simple problems with minimal benefit. We introduce novel efficiency metrics from both outcome and process perspectives to evaluate the rational use of computational resources by o1-like models. Using a self-training paradigm, we propose strategies to mitigate overthinking, streamlining reasoning processes without compromising accuracy. Experimental results show that our approach successfully reduces computational overhead while preserving model performance across a range of testsets with varying difficulty levels, such as GSM8K, MATH500, GPQA, and AIME.

^{*}Equal Contribution.

[†]Correspondence to: Zhaopeng Tu <zptu@tencent.com> and Rui Wang <wangrui12@sjtu.edu.cn>.

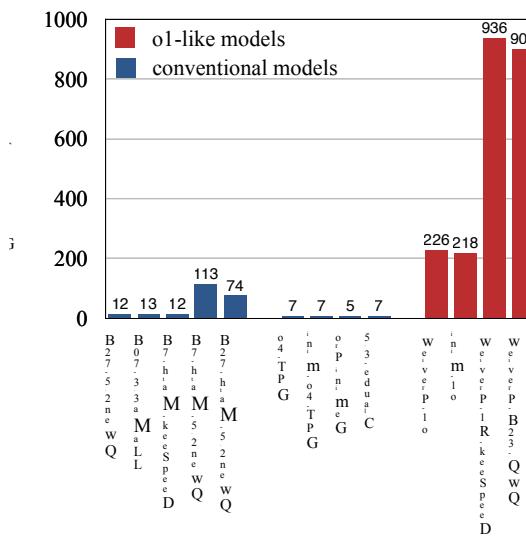
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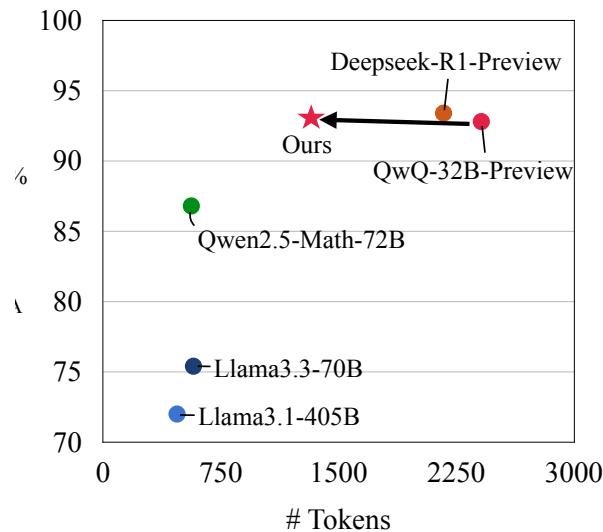
LLM

Xingyu Chen^{*1,2}, Jiahao Xu^{*1}, Tian Liang^{*1}, Zhiwei He^{*1,2}, Jianhui Pang¹, Dian Yu¹, Linfeng Song¹, Qiuwei Liu¹, Mengfei Zhou², Zhuosheng Zhang², Rui Wang⁺², Zhaopeng Tu⁺¹, Haitao Mi¹, and Dong Yu¹

¹Tencent AI Lab
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(a) Generated tokens on question “what is the answer of 2 plus 3?”



(b) Token-accuracy plot on MATH500

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*Equal Contribution.

[†]Correspondence to: Zhaopeng Tu <zptu@tencent.com> and Rui Wang <wangrui12@sjtu.edu.cn>.

1 Introduction

The OpenAI o1 model (OpenAI, 2024) and its replicas (Qwen, 2024; DeepSeek, 2024) exemplify the state-of-the-art in AI reasoning. Their success is largely attributed to mimicking human-like long-time thinking before responding to a question. Specifically, o1-like models cultivate a long chain-of-thoughts (CoT), explore multiple strategies, break down complex steps, and perform double-checking, which ultimately enhance their ability to tackle intricate reasoning tasks. This approach, known as “scaling test-time compute”, involves allocating more computational resources during the model’s inference phase to generally yield more accurate responses.

While effective, a critical yet underexplored question remains: **are we scaling test-time compute efficiently and intelligently?** This study provides an initial exploration of this problem. We first observe that o1-like models exhibit significant **overthinking** issues. Specifically, they tend to expend excessive compute (in terms of tokens or thinking rounds) on questions that are exceptionally simple or for which the answer is already evident. For example, Figure 1(a) compares the token usage of o1-like models with conventional models when answering the question, “what is the answer of 2 plus 3?” On average, o1-like models consumed 1,953% more tokens than conventional models to reach the same answer. Figure 2 illustrates a concrete example where o1-style thinking results in generating 13 solutions for this trivially simple question. Across extensive analyses of mathematical benchmarks, we found these overthinking patterns: (1) contribute minimally to improving accuracy, (2) lack diversity in reasoning strategies, and (3) occur more frequently with simple problems.

The overthinking observed in o1-like models reveals inefficiency in inference and highlights fundamental limitations in their reasoning and decision-making processes. We assert that reasoning involves not only accuracy but also the application of the appropriate level of complexity based on the problem’s requirements. This insight motivates our exploration of studying and mitigating overthinking. To address this, we propose two metrics from both outcome and process perspectives to evaluate o1-like models’ efficiency. These metrics help provide a comprehensive assessment of the **efficiency** of o1-like models, augmenting the commonly-used **effectiveness** metrics.

To mitigate overthinking without introducing external information, we adopt a self-training paradigm. With our proposed efficiency metrics, we streamline the generated responses by removing redundant solutions while maintaining basic reflexivity. Experimental results across testsets of varying difficulty levels (e.g., GSM8K, MATH500, GPQA, and AIME) demonstrate our approach’s effectiveness and robustness in mitigating overthinking issues. For instance, as shown in Figure 1(b), our approach can reduce token output by 48.6% while maintaining accuracy on the widely-used MATH500 testset as applied to QwQ-32B-Preview.

In summary, our contributions are three-fold:

1. We present the first study offering both a definitive explanation and comprehensive analysis of the overthinking issue, showing that o1-like LLMs often expend unnecessary computational resources on redundant solutions that contribute minimally to final outcomes.
2. We introduce metrics considering both outcome and process perspectives to assess the efficiency of o1-like models.
3. We explore several strategies to tackle the overthinking issue, significantly reducing token generation while maintaining model performance across testsets of varying difficulty.

2 Observing Overthinking Issues

In this section, we present a comprehensive analysis of outputs generated by o1-like models. First, we provide a basic illustration of the solution distribution in responses from these models (§ 2.1). We then identify two inefficiencies in long CoT responses: their limited contribution to accuracy (§ 2.2) and diversity (§ 2.3). To evaluate these inefficiencies empirically, we propose two efficiency metrics based on our observations. Finally, we present empirical results in § 2.4 and conclude that *o1-like models often over-think, particularly with easier math problems*.

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§ 2.4

o1-like models often over-think, particularly with easier math problems



Figure 2: An example of overthinking issue for QwQ-32B-Preview model’s output response that consists of 13 solutions. We also list the outputs of other conventional LLMs for reference.

2.1 Solution Distribution of o1-Like Models

Experimental Setup We conduct experiments on three testsets:

- **ASDIV** (Miao et al., 2020): an English math word problem corpus with 2,305 instances, each annotated with its problem type and grade level (1 to 6, indicating difficulty). The test set covers three main problem types (i.e., *basic arithmetic operations*, *aggregative operations*, and *additional domain knowledge required*), typically found in **elementary schools**.
- **GSM8K** (Cobbe et al., 2021): a dataset of high-quality, linguistically diverse **grade school math word problems** created by human problem writers. The test set includes 1,319 problems, with solutions often involving a sequence of elementary calculations using basic arithmetic. A middle school student should be able to solve every problem.
- **MATH500** (Hendrycks et al., 2021): a challenging dataset consisting of problems from **high school math competitions** across seven subjects (e.g., Prealgebra, Algebra, Number Theory) and difficulty levels based on AoPS (ranging from 1 to 5). Problems in these competitions range from level 1, the easiest, often found in AMC 8 exams, to level 5, like those in AIME.

The overall difficulty levels of the test sets are ASDIV < GSM8K < MATH500.

We mainly investigate two widely recognized o1-like models featuring visible long CoT: Qwen-QwQ-32B-Preview and DeepSeek-R1-Preview. QwQ-32B-Preview is an open-source model, while DeepSeek-R1-Preview is accessible only through a web interface. Given DeepSeek-R1-Preview’s daily message limit of 50, we evaluated this model solely on the MATH500 test set.

Solution Distribution In this paper, we define *solution* as part of the full model generation that contains an answer explicitly. For example, in Figure 2, each solution in the QwQ generation contains the answer 5. We use the Llama-3.3-70B model to separate solutions from generated responses. Figure 3 shows the distribution of solution counts in generated responses. Generally, o1-like models produce 2 to 4 solution rounds for most instances, covering 76% to 85% of cases for

Llama3.3-70B (7 tokens)	QwQ-32B-Preview (901 tokens)
The answer to $2 + 3$ is 5.	Solution1 39 tokens 2 plus 3... that's a pretty straightforward arithmetic problem. I think I can handle this. So, let's see, 2 plus 3 means I'm adding two numbers together. I know that when you add two and three, you get five.
Qwen2.5-72B (7 tokens)	Solution2 109 tokens But maybe I should think about it step by step to make sure I understand the process. First, what is addition? Addition is a mathematical operation where you combine two or more numbers to get a total or sum. In this case, I'm dealing with two numbers: 2 and 3. So, starting with 2, if I add 3 to it, I'm essentially moving three steps forward on the number line. Let me visualize that. If I start at 2 and move one step to the right, I reach 3. Another step takes me to 4, and one more step takes me to 5. So, 2 plus 3 equals 5.
Qwen2.5-Math-72B (45 tokens)	Solution3 39 tokens Alternatively, I can think of it in terms of objects. Suppose I have two apples and then I get three more apples. How many apples do I have in total? Well, two apples plus three apples equals five apples.
To find the answer of $2 + 3$, we can follow these steps: 1. Start with the number 2. 2. Add the number 3 to the number 2. When we add 3 to 2, we get: $\boxed{2 + 3 = 5}$ So, the answer is $\boxed{5}$.	Solution4 34 tokens Another way to look at it is using fingers. I can hold up two fingers on one hand and three on the other, then count all the fingers together, which gives me five fingers.
GPT-4o (7 tokens)	Solution5 42 tokens I can also break it down using smaller numbers. For example, 2 plus 2 equals 4, and then adding one more makes it 5. So, 2 plus 2 is 4, plus 1 is 5, which is the same as 2 plus 3.
The answer to $2 + 3$ is 5.	Solution6 26 tokens Moreover, if I consider the number properties, 2 and 3 are both positive integers, and their sum should also be a positive integer, which is 5.
Gemini Pro (5 tokens)	Solution8 29 tokens Furthermore, in algebra, if I have an equation like $x + 3 = 5$, then solving for x would give me $x = 2$, which aligns with 2 plus 3 equals 5.
2 + 3 equals 5.	Solution9 19 tokens Alternatively, using Roman numerals, 2 is II and 3 is III. Adding them together gives V, which is 5.
Claude-3.5 (7 tokens)	In conclusion, the answer to $2 + 3$ is 5.
The answer to $2 + 3$ is 5.	

2: 13

QwQ-32B-Preview
LLM

2.1 o1

가

- ASDIV(Miao et al., 2020): 2,305
(1~6,)
가 (, basic arithmetic operations, aggregative operations, additional domain knowledge required)
- GSM8K(Cobbe et al., 2021):
1,319
가
- MATH500(Hendrycks et al., 2021): 7
AoPS(1~5)
AMC 8
가 1 AIME 5

ASDIV < GSM8K < MATH500

CoT 2B-Preview	DeepSeek-R1-Preview	QwQ-32B-Preview	Qwen- DeepS QwQ-3 DeepSeek-R1-Preview	Llama-3.3
eek-R1-Preview 50	MATH500	가 o1	5 3	2~4
-70B	solution 2 QwQ o1	5 3	5 3	2~4
76%~85%				

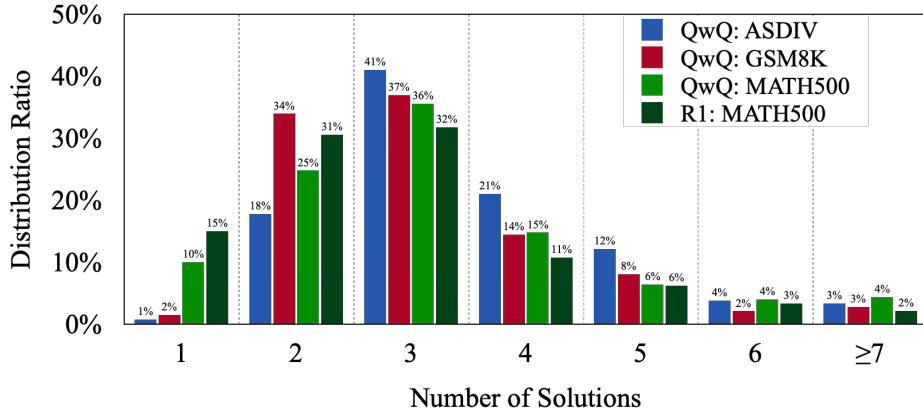


Figure 3: Distribution of solution counts in generated responses for different test sets and models (QwQ-32B-Preview (“QwQ”) and DeepSeek-R1-Preview (“R1”)).

QwQ-32B-Preview across the test sets and 74% for DeepSeek-R1-Preview on the MATH500 test set. Regarding different test sets, QwQ-32B-Preview tends to generate more solutions for easier test sets. For instance, the average number of solutions of QwQ model on the easiest ASDIV test set is 3.6, whereas on the most difficult MATH500 test set, it is 2.8.

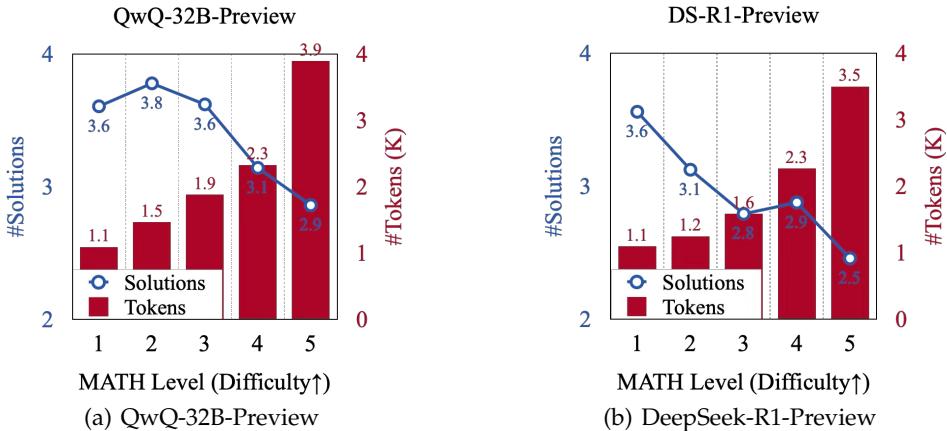
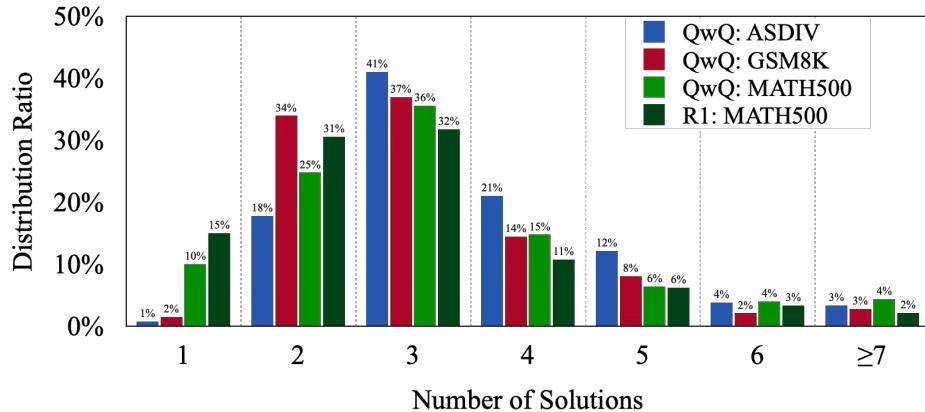


Figure 4: Average rounds of solutions (“#Solutions”) and tokens (“#Tokens”) in responses generated by (a) QwQ-32B-Preview and (b) DeepSeek-R1-Preview across different difficulty levels of the MATH500 test set.

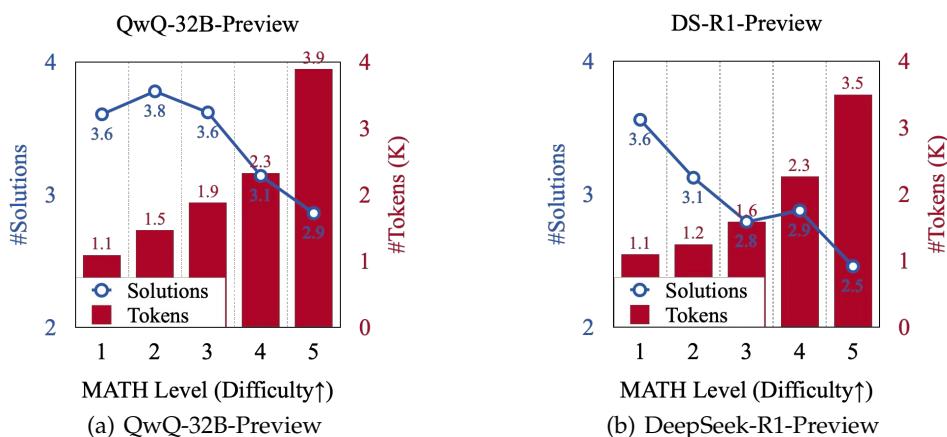
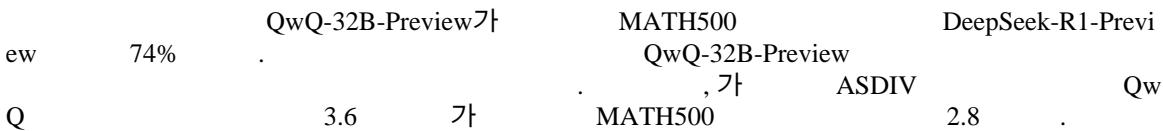
To empirically validate this finding, we conducted an analysis across various difficulty levels in the MATH500 test set, as illustrated in Figure 4. Both QwQ-32B-Preview and DeepSeek-R1-Preview generate more solution rounds for problems at easier levels 1-2 (e.g., averaging 3.75 rounds and 3.35 rounds, respectively) compared to levels 4-5 (e.g., averaging 3.0 rounds and 2.7 rounds, respectively), despite the number of tokens consistently increasing with the difficulty level. These results support our claim that o1-like models tend to generate more solution rounds for easier math problems.

2.2 Efficiency on Accuracy Improvements

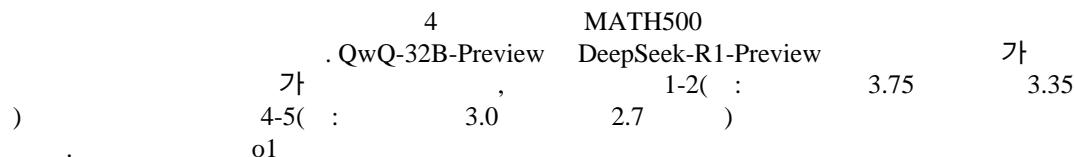
Intuition In the example depicted in Figure 2, we observe that the initial round of solutions already yields the correct answer. Subsequent solutions, which account for the majority of generated tokens, do not enhance accuracy. Based on this observation, we empirically investigate whether



3: (QwQ-32B-Preview("QwQ") DeepSeek-R1-Preview("R1"))



4: (a) QwQ-32B-Preview (b) DeepSeek-R1-Preview ("#Tokens") ("#Solutions") MATH500



2.2

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later solutions contribute to accuracy improvements. Specifically, for all cases where o1-like models produce the correct answer in the response, we calculate the distribution of occurrences for the first correct answer, termed the “first correctness distribution”. If more correct answers appear in earlier solutions, then the subsequent solutions contribute minimally to accuracy improvement, indicating reduced efficiency.

Observation Figure 5 illustrates the first correctness distribution across the test sets and models. In more than 92% of cases, the initial round of solutions produces the correct answer. Notably, the first round generally comprises less than 60% of the total tokens generated, suggesting that the extended CoT might not significantly enhance accuracy. For instance, the average length of the first round of solutions for QwQ-32B-Preview on the ASDIV test set is 287 tokens, constituting only 38.7% of the entire response. These results suggest that **later solutions marginally contribute to improvements in accuracy**.

Outcome Efficiency Metric Based on the above observation, we propose an outcome efficiency metric to empirically evaluate how effectively later solutions contribute to accuracy improvements. The outcome efficiency metric, denoted ξ_O , is defined by the following formula:

$$\xi_O = \frac{1}{N} \sum_{i=1}^N \sigma_i \frac{\hat{T}_i}{T_i} \quad (1)$$

where N is the number of instances in a given test set, T_i is the total number of tokens produced for the i -th instance, and \hat{T}_i denotes the efficient tokens that contribute to reaching the correct answer:

$$\hat{T}_i = \begin{cases} \# \text{tokens to first arrive at correct answer}, & \sigma_i = 1 \\ T_i, & \sigma_i = 0 \end{cases}$$

σ_i denotes whether the evaluated model can produce a correct answer in the response:

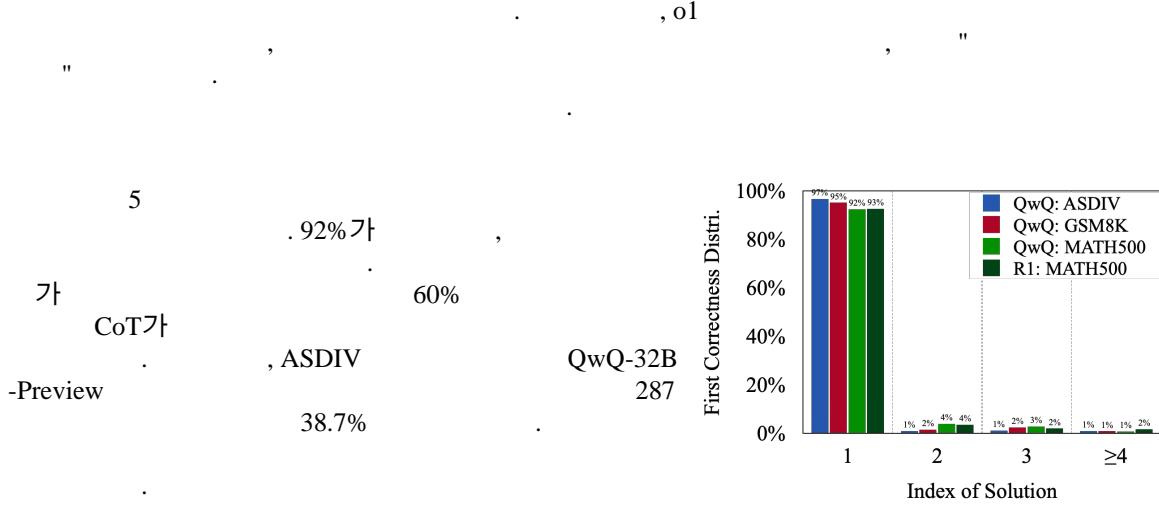
$$\sigma_i = \begin{cases} 1, & \text{if at least one solution in response is correct} \\ 0, & \text{otherwise} \end{cases}$$

Intuitively, if a model correctly answers at an early stage, the tokens generated thereafter do not contribute to improving accuracy and are considered inefficient. Consider Figure 2 as an example: The first solution correctly addresses the problem with $\hat{T} = 39$. Consequently, $\xi_O = \frac{39}{901} = 4.3\%$, which can be considered extremely inefficient.

2.3 Efficiency on Diverse Thinking

Intuition Some researchers might argue that while solving an easy math problem may appear straightforward, approaching it from different perspectives can deepen understanding and build flexibility in mathematical thinking, which is also valuable. Consider the example output of QwQ-32B-Preview in Figure 2: Solution 1 states the basic fact that 2 plus 3 equals 5; Solution 2 breaks the addition into smaller steps; Solution 3 uses a counting objects analogy. These three solutions provide different perspectives. However, Solution 4 repeats Solution 3, and Solution 5 repeats Solution 2 using similar perspectives. In this section, we empirically examine the diversity among solutions within a response.

Observation To empirically evaluate whether later solutions provide new perspectives, we introduce the “distinctness ratio” as the measure for the ratio of distinct solutions for each data index.

5:

↗
ξ_O

$$\xi_O = \frac{1}{N} \sum_{i=1}^N \sigma_i \frac{\hat{T}_i}{T_i} \quad (1)$$

$$N, T_i, i, \hat{T}_i$$

$$\hat{T}_i = \begin{cases} \# \text{tokens to first arrive at correct answer}, & \sigma_i = 1 \\ T_i, & \sigma_i = 0 \end{cases}$$

$\sigma_i \rightarrow$

$$\sigma_i = \begin{cases} 1, & \text{if at least one solution in response is correct} \\ 0, & \text{otherwise} \end{cases}$$

$$\hat{T} = 39 \quad \xi_O = \frac{2}{39} = 4.3\%$$

2.3

2 QwQ-32B-Preview

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2
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5

Consider $R_i = \{s_i^1, \dots, s_i^m, \dots, s_i^{M_i}\}$ as the set of M_i solutions in the i -th instance response. Let $S^m = \{s_1^m, \dots, s_k^m, \dots, s_K^m\}$ be the set of m -th solutions in the responses of all instances in the test subset.¹ The distinctness ratio is defined as:

$$Dis^m = \frac{\sum_{k=1}^K \tau_k^m}{K}$$

where

$$\tau_k^m = \begin{cases} 1, & \text{if } \Phi(s_k^m) \not\subseteq \{\Phi(s_1^1), \dots, \Phi(s_k^{m-1})\} \\ 0, & \text{otherwise} \end{cases}$$

In this context, $\Phi(s_k^m)$ is the solution perspective of s_k^m . We use GPT-4o to cluster the solutions for each instance into groups via a prompt like (Ye et al., 2024).² The clustering results for the QwQ-32B-Preview response in Figure 2 are:

cluster1 [Solution 1, Solution 6, Solution 11] stating or affirming the basic arithmetic fact that 2 plus 3 equals 5.
 cluster2 [Solution 2, Solution 5] breaking down the addition into smaller, simpler steps to reach the result.
 cluster3 [Solution 3, Solution 4] using a practical analogy of counting objects to explain the addition.
 cluster4 [Solution 7] using subtraction as a reverse check to verify the addition result.
 cluster5 [Solution 8] using algebraic manipulation and solving simple equations to confirm the result.
 cluster6 [Solution 9, Solution 10] converting numbers into different systems (binary and Roman numerals) to verify the result.
 cluster7 [Solution 12, Solution 13] considering specific contexts or frameworks like modular arithmetic or programming which could change traditional addition results.

Figure 6 displays the distinctness ratio for each solution index. Intuitively, the ratio for Solution#1 is always 100%, as it has no preceding solutions, thus $\tau \equiv 1$ for all instances. Generally, the ratio decreases with higher indices, indicating that **later solutions often repeat earlier ones**. For example, the distinctness ratios for Solution#4 across test sets are mostly below 30%, lower than Solution#3, which is above 45%. In test sets other than ASDIV, the ratio of Solution#2 significantly decreases, underperforming Solution#3. By reviewing outputs, we find that Solution#2 often double-checks answers from Solution#1 using the same perspective. Subsequently, Solution#3 attempts to solve the problem from a new perspective.

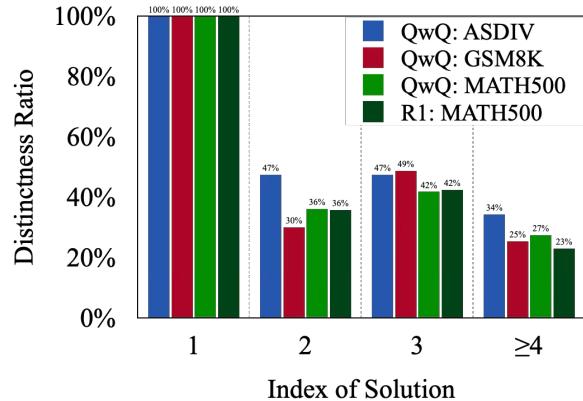


Figure 6: Ratio of whether a solution provides a new perspective for each index.

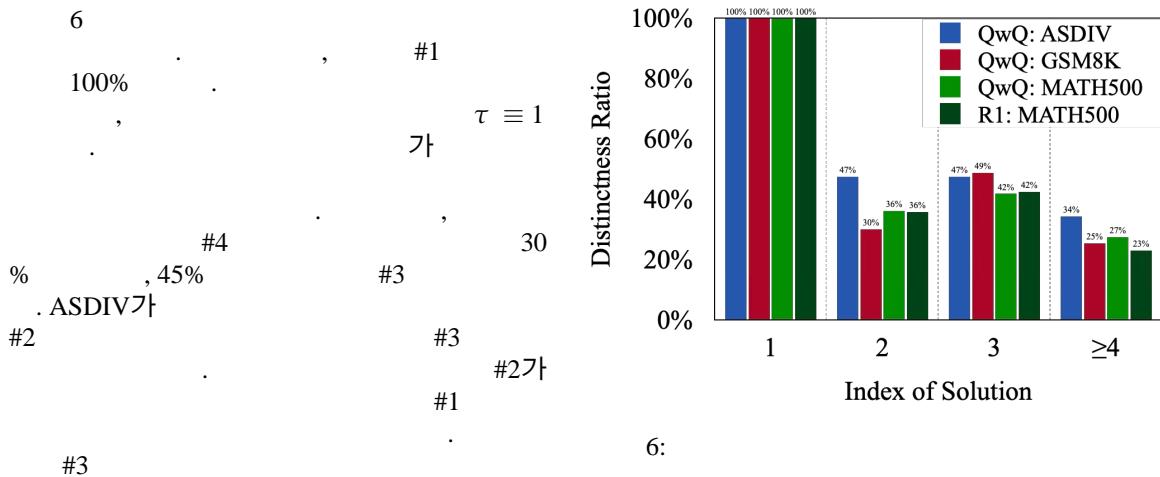
Process Efficiency Metric Based on the above observation, we propose a process efficiency metric to empirically evaluate the contribution of later solutions to solution diversity. The process efficiency metric, denoted ξ_P , is calculated using the formula:

$$\xi_P = \frac{1}{N} \sum_{i=1}^N \frac{D_i}{T_i} \quad (2)$$

¹If a response does not contain the m -th solution (i.e. $M_i < m$), that response is excluded from the set, hence K does not necessarily equal the number of test set instances N .

²Refer to Appendix A.2 for clustering prompt details.

$R_i = \{s_i^1, \dots, s_i^m, \dots, s_i^{M_i}\}$	M_i
$S^m = \{s_1^m, \dots, s_k^m, \dots, s_K^m\}_1$	m
$Dis^m = \frac{\sum_{k=1}^K \tau_k^m}{K}$	
$\tau_k^m = \begin{cases} 1, & \text{if } \Phi(s_k^m) \not\subseteq \{\Phi(s_k^1), \dots, \Phi(s_k^{m-1})\} \\ 0, & \text{otherwise} \end{cases}$	
$\Phi(s_k^m)$	s_k^m
	GPT-4o
	(Ye et al., 2024)
	2
	2 QwQ-32B-Preview



$$\xi_P = \frac{1}{N} \sum_{i=1}^N \frac{D_i}{T_i} \quad (2)$$

¹If a response does not contain the m -th solution (i.e. $M_i < m$), that response is excluded from the set, hence K does not necessarily equal the number of test set instances N .

²Refer to Appendix A.2 for clustering prompt details.

Models	Accuracy	Response		Efficiency	
		#Solution	#Token	Outcome	Process
ASDIV					
Llama-3.3-70B-Instruct	95.6	1.0	167.4	95.6%	100.0%
Qwen2.5-Math-72B-Instruct	96.3	1.0	209.6	96.3%	100.0%
QwQ-32B-Preview	96.2	3.6	714.8	41.8%	66.4%
GSM8K					
Llama-3.3-70B-Instruct	92.6	1.0	235.4	92.6%	100.0%
Qwen2.5-Math-72B-Instruct	95.8	1.0	312.1	95.8%	100.0%
QwQ-32B-Preview	94.3	3.2	745.6	50.4%	67.7%
MATH500					
Llama-3.3-70B-Instruct	75.4	1.0	575.0	75.4%	100.0%
Qwen2.5-Math-72B-Instruct	86.8	1.0	561.5	86.8%	100.0%
QwQ-32B-Preview	92.8	3.3	2409.2	52.2%	72.4%
DeepSeek-R1-Preview	93.4	2.8	2168.6	58.9%	76.0%

Table 1: Model efficiency results of strong LLMs.

where D_i represents the number of efficient tokens that contribute to the solutions' diversity. Here, we intentionally exclude the factor σ_i to concentrate on diversity, **independent of correctness**.

Let T_i^m denote the number of tokens in solution s_i^m . We define:

$$D_i = \sum_{m=1}^M \tau_i^m T_i^m$$

Intuitively, the tokens in a distinct solution are regarded as process efficient tokens. In the example shown in Figure 2, the 13 solutions are categorized into 7 distinct perspectives. Consequently, tokens in Solutions 1, 2, 3, 7, 8, 9, and 12 are efficient, resulting in $\xi_P = \frac{(39+109+39+29+29+19+59)}{901} = 35.8\%$.

2.4 Empirical Efficiency Results

Table 1 presents the results on model efficiency. For comparison, we include two representative conventional large language models (LLMs): Llama-3.3-70B-Instruct and Qwen2.5-Math-72B-Instruct. These conventional LLMs produce only a single solution, meaning that $\frac{D_i}{T_i} = \frac{\hat{T}_i}{T_i} = 1$. Therefore, in these cases, the outcome efficiency metric $\xi_O = \frac{1}{N} \sum_{i=1}^N \sigma_i$ equals accuracy, and the process efficiency metric $\xi_P = 1.0$. In comparison, o1-like models generate significantly longer responses, which are less efficient in improving accuracy and solution diversity. We refer to the inefficient use of generated tokens as the “**overthinking issue**”.

Figure 7 presents the detailed efficiency results across various difficulty levels of the MATH500 test set. DeepSeek-R1-Preview consistently outperforms QwQ-32B-Preview in terms of both outcome and process efficiency on Levels 2-5. Notably, both models perform poorly on the simplest Level 1 problems, achieving less than 50% outcome efficiency, a pattern that corresponds with results observed on the easy ASDIV test set. These findings underscore that the **overthinking issues faced by o1-like models are particularly pronounced with simpler math problems**.

3 Mitigating Overthinking Issues

In this section, we explore several strategies aimed at enhancing the efficiency of o1-like models. We adopt the settings for LLM reasoning tasks and primarily utilize the self-training strategy (Zelikman et al., 2022; Ho et al., 2023), where the model itself generates the training data. Consistent with

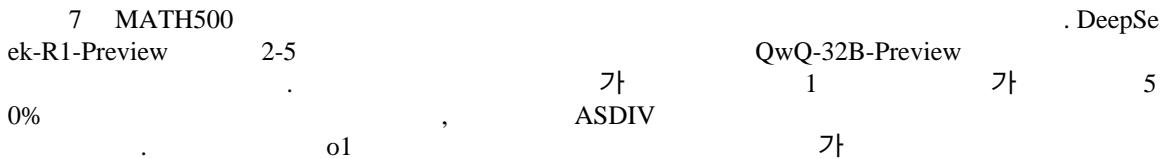
Models	Accuracy	Response		Efficiency	
		#Solution	#Token	Outcome	Process
ASDIV					
Llama-3.3-70B-Instruct	95.6	1.0	167.4	95.6%	100.0%
Qwen2.5-Math-72B-Instruct	96.3	1.0	209.6	96.3%	100.0%
QwQ-32B-Preview	96.2	3.6	714.8	41.8%	66.4%
GSM8K					
Llama-3.3-70B-Instruct	92.6	1.0	235.4	92.6%	100.0%
Qwen2.5-Math-72B-Instruct	95.8	1.0	312.1	95.8%	100.0%
QwQ-32B-Preview	94.3	3.2	745.6	50.4%	67.7%
MATH500					
Llama-3.3-70B-Instruct	75.4	1.0	575.0	75.4%	100.0%
Qwen2.5-Math-72B-Instruct	86.8	1.0	561.5	86.8%	100.0%
QwQ-32B-Preview	92.8	3.3	2409.2	52.2%	72.4%
DeepSeek-R1-Preview	93.4	2.8	2168.6	58.9%	76.0%

1: LLM

$$\xi_P = \frac{7}{901} = 35.8\% \quad 1, 2, 3, 7, 8, 9, 12$$

2.4

$$\text{(LLM)} \quad \text{Llama-3.3-70B-Instruct} \quad \frac{\frac{D_i}{T_i}}{\frac{\hat{T}_i}{T_i}} = 1 \quad \text{Qwen2.5-Math-72B-Instruct} \quad \xi_p = 1.0 \quad \text{LLM} \quad \xi_O = \frac{1}{N} \sum_{i=1}^N \sigma_i$$



3.

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022; Ho et al., 2023) . . Zelikman et al., 2

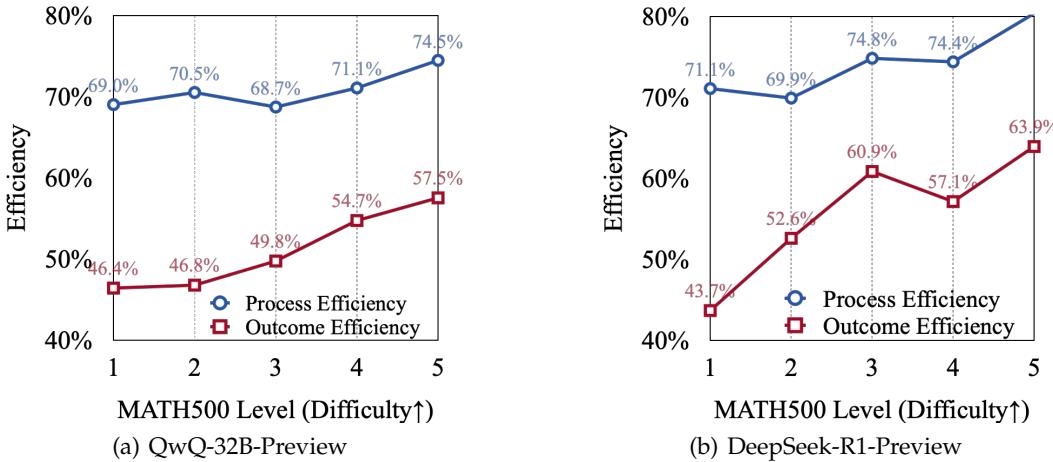


Figure 7: Efficiency results of (a) QwQ-32B-Preview and (b) DeepSeek-R1-Preview across different difficulty levels of the MATH500 testset.

previous studies, we employ the PRM12K dataset (Lightman et al., 2024) as our training dataset to generate self-training data. The QwQ-32B-Preview model serves as our testing platform because it is available for post-training.

3.1 Length Preference Optimization

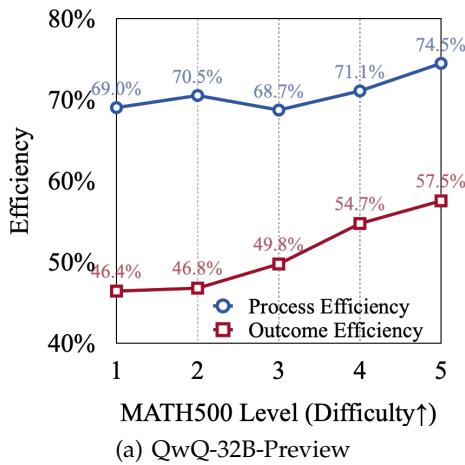
Response	#Solutions	#Tokens	Efficiency	
			Outcome	Process
Greedy	3.1	1434.8	55.6%	72.6%
Shortest	2.5	1051.3	69.8%	80.3%
Longest	4.1	2258.7	46.0%	66.4%

Table 2: Statistics on different types of generated responses based on the training data. “Greedy” denotes responses generated via greedy search; “Shortest” and “Longest” indicate the shortest and longest responses among 10 samples, respectively.

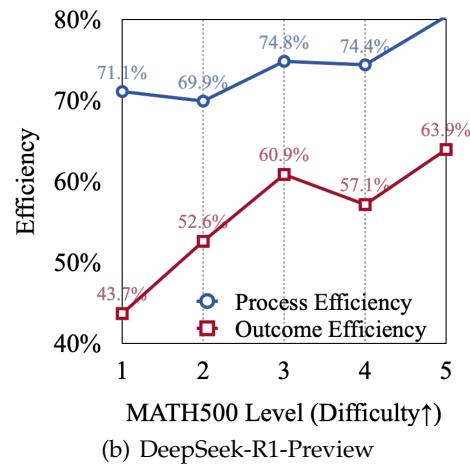
We began by assessing whether the model could produce more efficient responses. We generated 10 samples for each instance in the training dataset with a temperature of 1.0. We discard samples that failed to generate a correct answer. Table 2 presents the statistics of different types of generated responses. Our analysis of these sampled responses reveals that the shortest response performs better in terms of both outcome and process efficiency, using fewer rounds and tokens. These findings support our initiative to enhance model efficiency through self-improvement.

We explore several effective post-training methods for self-improvement:

- **Supervised Fine-Tuning** (SFT; Wei et al. 2022a): This method involves fine-tuning a pre-trained model using positive synthetic data. The model learns to map inputs to preferred outputs by minimizing the cross-entropy loss between predicted and actual outputs. SFT enables the model to mimic the behavior demonstrated in training examples.
- **Direct Preference Optimization** (DPO; Rafailov et al. 2024): This method trains a model directly on human-preferred responses to increase the likelihood of preferred responses over unpreferred ones. DPO implicitly optimizes the objective by aligning reward functions with optimal policies.



(a) QwQ-32B-Preview



(b) DeepSeek-R1-Preview

7: MATH500 eview

(a) QwQ-32B-Preview (b) DeepSeek-R1-Pr

PRM12K (Lightman et al., 2024) . QwQ-32B-Preview

3.1

Response	#Solutions	#Tokens	Efficiency	
			Outcome	Process
Greedy	3.1	1434.8	55.6%	72.6%
Shortest	2.5	1051.3	69.8%	80.3%
Longest	4.1	2258.7	46.0%	66.4%

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(SFT; Wei et al. 2022a);

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(DPO; Rafailev et al. 2024).

. DPO

- **Reasoning Preference Optimization** (RPO; Pang et al. 2024; Liu et al. 2024): This approach modifies the DPO loss by adding a negative log-likelihood (NLL) term on the preferred response. RPO enhances DPO training stability by maintaining desired formatting for generated content and preventing a decrease in the log probability of selected responses.
- **Simple Preference Optimization** (SimPO; Meng et al. 2024): This method addresses the discrepancy between the reward function and the generation metric during inference found in other preference optimization methods. SimPO incorporates techniques like adaptive margin and length regularization into DPO training.

Apart from the SFT method, which uses only the shortest sampled response as training data, the other three preference optimization methods require contrastive instance pairs (positive, negative). It is straightforward to use the response generated by greedy search as the negative example, aligning with the real-time inference scenario. However, in our preliminary experiments, we found it less effective than using the longest sampled response as the negative example. One possible reason is that the longest sampled response provides a clearer contrastive signal.

3.2 Simplifying Responses to Further Enhance Efficiency

Positive Example	#Solutions	#Tokens	Efficiency	
			Outcome	Process
Shortest Response	2.5	1051.3	69.8%	80.3%
First-Correct Solutions (FCS)	1.1	681.0	99.5%	99.1%
FCS + Reflection	1.9	878.7	78.4%	82.4%
Greedily Diverse Solutions (GDS)	1.6	856.8	86.8%	94.2%

Table 3: Statistics on different types of generated responses based on the training data. “Greedy” denotes responses generated via greedy search; “Shortest” and “Longest” indicate the shortest and longest responses among 10 samples, respectively.

Although shorter sampled responses improve the efficiency of o1-like models, they still suffer from overthinking issues. Based on the observations in Section 2, where earlier solutions in the response are more efficient, we further simplify the responses to enhance efficiency. We propose three types of simplification strategies that differ in how they truncate the responses from the beginning:

- **First-Correct Solutions (FCS):** This strategy retains the earliest solutions that first arrive at the correct answer.
- **FCS+Reflection:** Since the majority of responses achieve the correct answer on the first solution (see Figure 5), maintaining only the First-Correct Solutions might cause o1-like models to revert to conventional LLM behavior. To counter this, we extend the approach to include the second solution that reaches the correct answer in positive examples, recalling the model’s long-reflective capability while maintaining efficiency.
- **Greedily Diverse Solutions (GDS):** Figure 6 demonstrates that the distinctiveness of Solution#2 significantly decreases because the second solution often double-checks answers from the first using the same perspective. Consequently, FCS+Reflection may reduce efficiency. To address this issue, we propose a simple heuristic that greedily expands solutions providing new perspectives. Additionally, this strategy includes more solutions when the second solution does not repeat the first, thereby increasing diversity.

For each instance, we select the shortest result of each type from a set of 10 samples. Consequently, the three types of simplified responses may originate from different original responses.

- 가 (NLL) (RPO; Pang et al. 2024; Liu et al. 2024):

가	DPO	. RPO
		DPO
 - (SimPO; Meng et al. 2024):

DPO	. SimPO
-----	---------

3.2

Positive Example	#Solutions	#Tokens	Efficiency	
			Outcome	Process
Shortest Response	2.5	1051.3	69.8%	80.3%
First-Correct Solutions (FCS)	1.1	681.0	99.5%	99.1%
FCS + Reflection	1.9	878.7	78.4%	82.4%
Greedily Diverse Solutions (GDS)	1.6	856.8	86.8%	94.2%

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- (FCS): 가 가
 - FCS+ : o1 LLM (5),
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 - (GDS): 6 #2

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Table 3 presents the statistics for these three types of simplified responses. Notably, all three types of simplified responses enhance efficiency compared to the shortest sampled response. “FCS” is the most efficient, both in terms of outcome and process, using the fewest number of solution rounds and tokens. “FCS+Reflection” incorporates reflection, requiring approximately one additional solution round, which reduces both outcome and process efficiencies. “Greedily Diverse Solutions” serves as a compromise, balancing the number of solutions and tokens, and achieving moderate to high efficiency. Figure 8 shows the distribution of solution counts in the simplified responses.

3.3 Experimental Results

Table 4 displays the results of the proposed efficiency-enhancing methods.

Performance of Length Preference Optimization Methods SFT only slightly reduces the number of solution rounds and tokens compared to the vanilla QwQ-32B-Preview model, underperforming the preference optimization methods. Among these methods, SimPO achieves the best results, reducing the number of generated tokens by 31.1%, 21.8%, and 22.3% on ASDIV, GSM8K, and MATH500, respectively. Consequently, SimPO is used as the default post-training method in the subsequent experiments.

Performance of Response Simplification Methods As anticipated, the First-Correction Solutions strategy achieves the greatest reduction in length. However, this method decreases performance on the difficult MATH500 test set, which may require more rounds of reflection. The “FCS+Reflection” approach alleviates this issue and surpasses the FCS method by 1.4% with an additional round of reflection. The “Greedily Diverse Solutions” strategy balances performance with the number of generated tokens. However, it significantly underperforms compared to “FCS+Reflection”, reinforcing our claim that the difficult MATH500 testset requires the deep inference provided by o1-like models. Hence, we adopt “FCS+Reflection” as the default response simplification method.

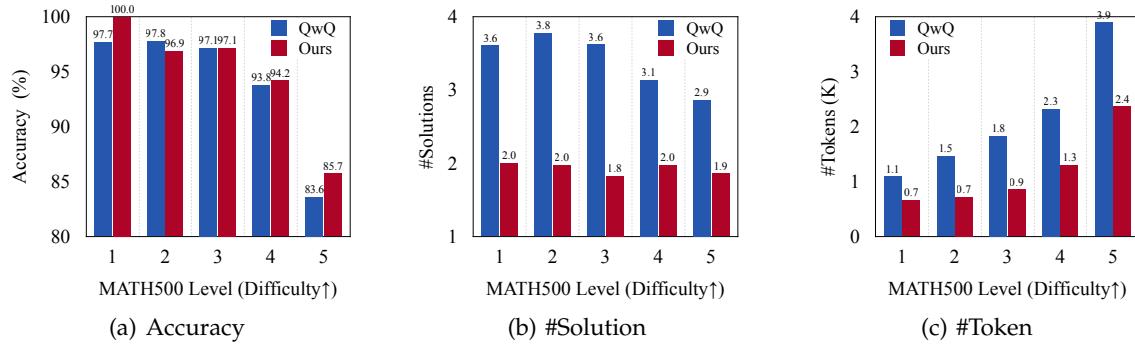
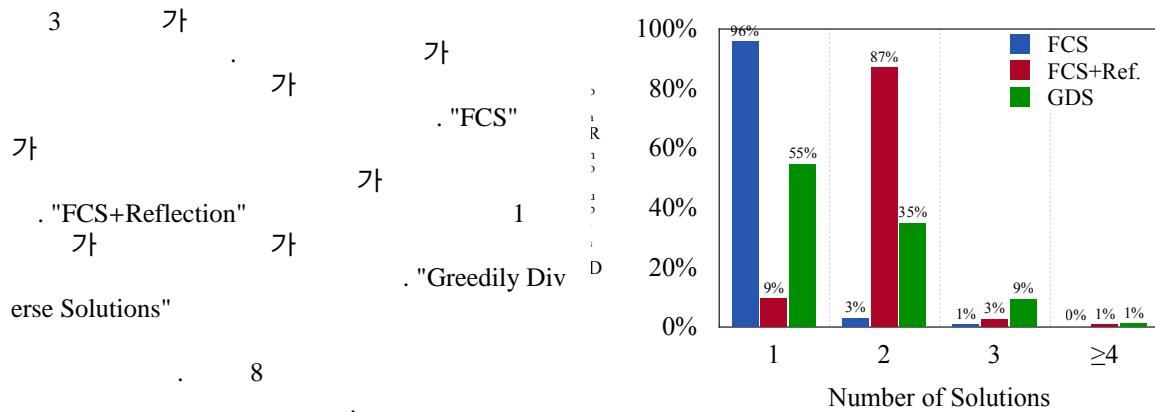


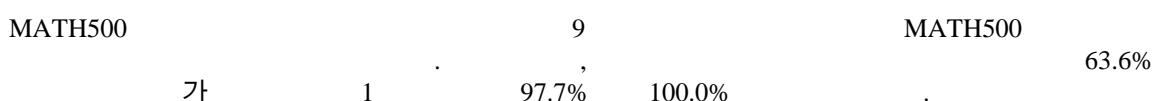
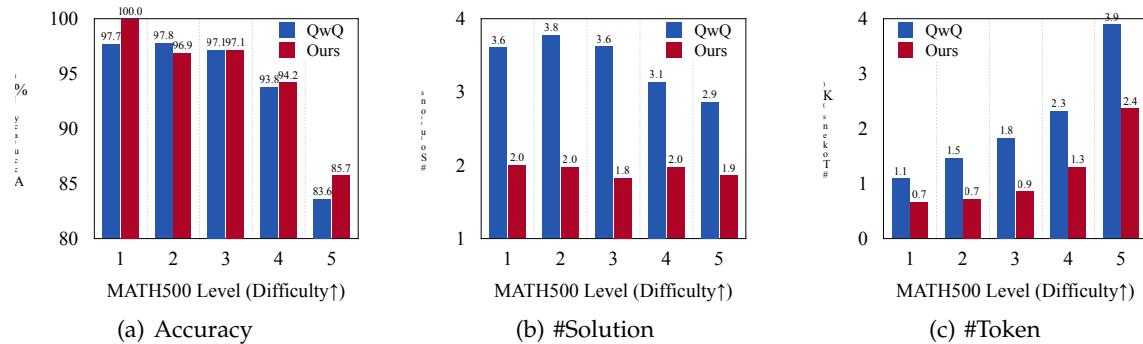
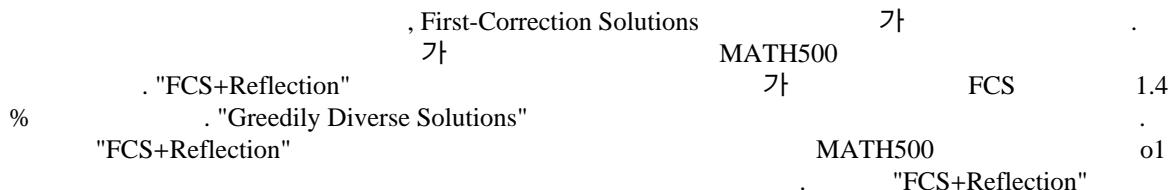
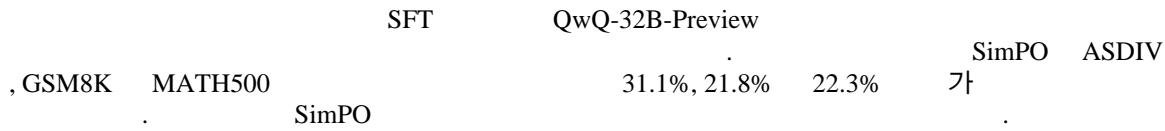
Figure 9: Detailed comparative results on the MATH500 testset across different difficulty levels.

Detailed Results on the MATH500 Testset Figure 9 presents the detailed results on the MATH500 testset across varying difficulty levels. Interestingly, our approach improves the accuracy for the easiest Level 1 from 97.7% to 100.0% while using only 63.6% of the tokens compared to the vanilla



3.3

4



Methods	Accuracy	Response		Efficiency	
		#Solution	#Token	Outcome	Process
ASDIV					
Llama-3.3-70B-Instruct	95.6	1.0	167.4	95.6%	100%
Llama-3.1-405B-Instruct	95.2	1.0	127.0	95.2%	100%
Qwen2.5-Math-72B-Instruct	96.3	1.0	209.6	96.3%	100%
QwQ-32B-Preview	96.2	3.5	741.8	41.9%	66.5%
SFT Shortest Response	96.2	3.4	725.3	44.8%	68.2%
+DPO Shortest Response	96.7	3.0	583.4	49.5%	69.7%
+RPO Shortest Response	96.5	3.0	576.5	49.5%	70.4%
+SimPO Shortest Response	96.6	2.7	511.0	53.5%	71.5%
+SimPO First-Correct Solution	96.7	1.2	271.3	92.8%	96.4%
+SimPO FCS+Reflection	96.6	2.0	381.6	68.5%	78.3%
+SimPO Greedily Diverse Solutions	96.3	1.6	332.5	80.1%	88.1%
GSM8K					
Llama-3.3-70B-Instruct	92.6	1.0	235.4	92.6%	100%
Llama-3.1-405B-Instruct	95.6	1.0	186.7	95.6%	100%
Qwen2.5-Math-72B-Instruct	95.8	1.0	312.1	95.8%	100%
QwQ-32B-Preview	94.3	3.1	772.8	50.7%	67.6%
SFT Shortest Response	94.8	3.0	749.5	53.7%	70.4%
+DPO Shortest Response	94.9	2.6	607.0	58.7%	71.7%
+RPO Shortest Response	94.8	2.6	593.3	59.3%	71.6%
+SimPO Shortest Response	94.2	2.3	554.2	63.5%	74.8%
+SimPO First-Correct Solution	94.8	1.1	326.2	95.7%	98.9%
+SimPO FCS+Reflection	95.1	1.6	416.6	79.4%	85.0%
+SimPO Greedily Diverse Solutions	94.9	1.3	368.4	90.7%	95.0%
MATH500					
Llama-3.3-70B-Instruct	75.4	1.0	575.0	75.4%	100%
Llama-3.1-405B-Instruct	72.0	1.0	470.3	72.0%	100%
Qwen2.5-Math-72B-Instruct	86.8	1.0	561.5	86.8%	100%
QwQ-32B-Preview	92.8	3.2	2407.9	52.3%	71.2%
SFT Shortest Response	92.8	3.0	2359.5	53.4%	73.2%
+DPO Shortest Response	93.4	2.7	1929.5	60.2%	76.4%
+RPO Shortest Response	92.0	2.8	2015.7	57.4%	75.4%
+SimPO Shortest Response	92.4	2.6	1871.8	60.6%	77.0%
+SimPO First-Correct Solution	91.8	1.4	1016.0	87.7%	94.2%
+SimPO FCS+Reflection	93.2	1.9	1330.7	75.8%	83.4%
+SimPO Greedily Diverse Solutions	92.2	1.6	1286.1	79.8%	90.6%

Table 4: Experimental results of the proposed efficiency enhancing methods.

Methods	Accuracy	Response		Efficiency	
		#Solution	#Token	Outcome	Process
ASDIV					
Llama-3.3-70B-Instruct	95.6	1.0	167.4	95.6%	100%
Llama-3.1-405B-Instruct	95.2	1.0	127.0	95.2%	100%
Qwen2.5-Math-72B-Instruct	96.3	1.0	209.6	96.3%	100%
QwQ-32B-Preview	96.2	3.5	741.8	41.9%	66.5%
SFT Shortest Response	96.2	3.4	725.3	44.8%	68.2%
+DPO Shortest Response	96.7	3.0	583.4	49.5%	69.7%
+RPO Shortest Response	96.5	3.0	576.5	49.5%	70.4%
+SimPO Shortest Response	96.6	2.7	511.0	53.5%	71.5%
+SimPO First-Correct Solution	96.7	1.2	271.3	92.8%	96.4%
+SimPO FCS+Reflection	96.6	2.0	381.6	68.5%	78.3%
+SimPO Greedily Diverse Solutions	96.3	1.6	332.5	80.1%	88.1%
GSM8K					
Llama-3.3-70B-Instruct	92.6	1.0	235.4	92.6%	100%
Llama-3.1-405B-Instruct	95.6	1.0	186.7	95.6%	100%
Qwen2.5-Math-72B-Instruct	95.8	1.0	312.1	95.8%	100%
QwQ-32B-Preview	94.3	3.1	772.8	50.7%	67.6%
SFT Shortest Response	94.8	3.0	749.5	53.7%	70.4%
+DPO Shortest Response	94.9	2.6	607.0	58.7%	71.7%
+RPO Shortest Response	94.8	2.6	593.3	59.3%	71.6%
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+SimPO First-Correct Solution	94.8	1.1	326.2	95.7%	98.9%
+SimPO FCS+Reflection	95.1	1.6	416.6	79.4%	85.0%
+SimPO Greedily Diverse Solutions	94.9	1.3	368.4	90.7%	95.0%
MATH500					
Llama-3.3-70B-Instruct	75.4	1.0	575.0	75.4%	100%
Llama-3.1-405B-Instruct	72.0	1.0	470.3	72.0%	100%
Qwen2.5-Math-72B-Instruct	86.8	1.0	561.5	86.8%	100%
QwQ-32B-Preview	92.8	3.2	2407.9	52.3%	71.2%
SFT Shortest Response	92.8	3.0	2359.5	53.4%	73.2%
+DPO Shortest Response	93.4	2.7	1929.5	60.2%	76.4%
+RPO Shortest Response	92.0	2.8	2015.7	57.4%	75.4%
+SimPO Shortest Response	92.4	2.6	1871.8	60.6%	77.0%
+SimPO First-Correct Solution	91.8	1.4	1016.0	87.7%	94.2%
+SimPO FCS+Reflection	93.2	1.9	1330.7	75.8%	83.4%
+SimPO Greedily Diverse Solutions	92.2	1.6	1286.1	79.8%	90.6%

Methods	Accuracy	Response		Efficiency	
		#Solution	#Token	Outcome	Process
GPQA					
Llama-3.3-70B-Instruct	42.4	1.0	831.5	42.4%	100%
Llama-3.1-405B-Instruct	53.5	1.0	604.3	53.5%	100%
Qwen2.5-Math-7B-Instruct	31.8	1.0	762.0	31.8%	100%
Qwen2.5-Math-72B-Instruct	46.5	1.0	682.7	46.5%	100%
QwQ-32B-Preview	58.6	2.5	3098.1	50.9%	79.5%
Ours	59.1	2.2	2085.7	56.2%	83.6%
AIME90					
Llama-3.3-70B-Instruct	26.7	1.0	956.7	26.7%	100%
Llama-3.1-405B-Instruct	22.2	1.0	1099.9	22.2%	100%
Qwen2.5-Math-7B-Instruct	10.0	1.0	1109.8	10.0%	100%
Qwen2.5-Math-72B-Instruct	16.7	1.0	955.4	16.7%	100%
QwQ-32B-Preview	37.8	2.6	5879.8	34.7%	80.4%
Ours	37.8	1.6	5070.0	37.0%	92.2%

Table 5: Results on the challenging GPQA and AIME testsets.

model. These results reaffirm the overthinking issues present in o1-like models, and our approach effectively mitigates them. Encouragingly, our approach also enhances performance on more challenging Levels 4-5 with fewer tokens, demonstrating the effectiveness and versatility of our approach in addressing overthinking issues.

3.4 Results on Challenging Testsets

To address the concerns of some researchers that our approach might weaken the ability of o1-like models to tackle complex problems requiring long-term reasoning, we validate our method using more challenging test sets, specifically GPQA and AIME:

- **GPQA** ([Rein et al., 2023](#)): a **graduate-level** dataset consisting of multiple-choice questions in subdomains of physics, chemistry, and biology. For our experiment, we select the highest quality subset, known as GPQA Diamond (composed of 198 questions).
- **AIME90** ([MAA Committees](#)): a dataset from the American Invitational Mathematics Examination, which tests mathematical problem solving across multiple areas (e.g. algebra, counting, geometry, number theory, and probability). Because AIME 2024 contains only 30 examples, we combined the examples from AIME 2022-2024 to create a larger set, AIME90, with 90 examples.

As listed in Table 5, our approach maintains model performance while using fewer tokens, demonstrating the robustness and generalization capability of our approach.

4 Related Work

4.1 Scaling Test-Time Compute

Scaling test-time compute has been shown to enhance model performance, particularly in reasoning and solving complex tasks. This approach can be broadly categorized into two main lines: 1) scaling search space, and 2) scaling human-like thinking patterns.

Scaling search space One perspective is that current LLMs already possess strong reasoning capabilities ([Li et al., 2024a](#)), such that the correct solution often lies within their probability distribution. However, the auto-regressive decoding process typically fails to explore the full range of possible

Methods	Accuracy	Response		Efficiency	
		#Solution	#Token	Outcome	Process
GPQA					
Llama-3.3-70B-Instruct	42.4	1.0	831.5	42.4%	100%
Llama-3.1-405B-Instruct	53.5	1.0	604.3	53.5%	100%
Qwen2.5-Math-7B-Instruct	31.8	1.0	762.0	31.8%	100%
Qwen2.5-Math-72B-Instruct	46.5	1.0	682.7	46.5%	100%
QwQ-32B-Preview	58.6	2.5	3098.1	50.9%	79.5%
Ours	59.1	2.2	2085.7	56.2%	83.6%
AIME90					
Llama-3.3-70B-Instruct	26.7	1.0	956.7	26.7%	100%
Llama-3.1-405B-Instruct	22.2	1.0	1099.9	22.2%	100%
Qwen2.5-Math-7B-Instruct	10.0	1.0	1109.8	10.0%	100%
Qwen2.5-Math-72B-Instruct	16.7	1.0	955.4	16.7%	100%
QwQ-32B-Preview	37.8	2.6	5879.8	34.7%	80.4%
Ours	37.8	1.6	5070.0	37.0%	92.2%

ble 5: GPQA AIME

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GPQA AIME

- GPQA(Rein et al., 2023): GPQA Diamond(198)
 - AIME90(MAA): (: , , , ,)
American Invitational Mathematics Examination . AIME 2024 3
0 AIME 2022-2024 90 ↗ AIME90

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가 (Li et al., 2024a).
기 LLM

solutions, thus limiting the model’s ability to find the optimal outcome. By scaling the search space, more opportunities are provided for the model to discover and select the correct solution within the probability distribution. A notable example of this approach is the self-consistency method (Wang et al., 2023b), where multiple responses are generated, and the final output is determined through majority voting. When equipped with an external reward function or scorer, this strategy can be adapted to techniques like best-of-n decoding (Lightman et al., 2024), weighted majority voting (Li et al., 2023; Khanov et al., 2024) and minimum bayes risk decoding (Heineman et al., 2024; Wu et al., 2024). Other strategies, such as Tree-of-Thought (Yao et al., 2024), Graph-of-Thought (Besta et al., 2024), and Monte Carlo Tree Search (Luo et al., 2024; Tian et al., 2024; Wan et al., 2024), attempt to scale the search space in different forms and structures.

Scaling human-like thinking patterns LLM reasoning is typically in the form of natural language. Therefore, it is natural for it to reason in human-like thinking patterns. Chain-of-thought (Wei et al., 2022b; Kojima et al., 2022) pioneered this trend by prompting the LLM to think step by step, rather than producing a final answer directly. This approach has been extended by subsequent works such as debating (Liang et al., 2024; Du et al., 2024; Xiong et al., 2023), self-correction (Kumar et al., 2024; Kamoi et al., 2024), self-critique (Ke et al., 2023; Lin et al., 2024; Yu et al., 2024), plan-and-solve (Wang et al., 2023a), etc. Recent works also explore LLM thinking in latent space to further simulate human cognitive processes (Hao et al., 2024; Goyal et al., 2024). Rather than applying these thinking patterns in isolation, o1-like models ensemble and scale these thinking patterns within a single response, forming an extremely long chain-of-thought that might span minutes or even hours during test time. As they reason, these models associate relevant knowledge, test various strategies, and retrace their steps when a particular approach proves unpromising—much like human thinking. Recent progress (OpenAI, 2024) has shown that the more time a model spends on thinking, the more accurate its response tends to be.

4.2 Efficient Thinking

Scaling the search space and scaling human-like thinking involves two distinct aspects of efficiency: efficient search and efficient thinking. However, few works specifically focus on efficient thinking in LLMs. Zhao et al. (2024) encourages the model to terminate reasoning by saying “I don’t know” when the problem is hard to solve. Han et al. (2024) introduces token-budget-aware reasoning, where the model is prompted with a specified token budget to guide its reasoning process. There are also several contributions (Damani et al., 2024; Wang et al., 2024) made to predict the distribution of the computation budget and allocate the computation power based on the prompt’s difficulty. Another line of work emphasizes the early stopping strategy to save computation budget while reasoning (Manvi et al., 2024; Li et al., 2024b). Moreover, multi-agent framework utilize large LLMs for difficult tasks while small LLMs for simple tasks (Kirchner et al., 2024; Damani et al., 2024)

In summary, all the aforementioned works consider conventional models rather than o1-like models with longer chains-of-thought. In contrast, our work first identifies the overthinking problem in o1-like model. Additionally, instead of limiting the reasoning space or leaving the token budget to be specified by the user, we aim to train the model to learn how to think efficiently.

5 Conclusion

This study identifies a key challenge in o1-like models — efficient and intelligent scaling of test-time computational resources. By highlighting the overthinking phenomenon and proposing efficiency metrics, we enhance our understanding of resource utilization in o1-like models. Our self-training based approach effectively mitigates overthinking, reducing unnecessary computation while maintaining performance. This work not only improves model efficiency but also sets the groundwork for future research on optimizing computational resource allocation in AI reasoning tasks. Future directions include exploring adaptive compute strategies that dynamically adjust to problem complexity and refining efficiency metrics for broader model generalization.

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Heineman et al., 2024; Wu et al., 2024) . best-of-n (L
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4), Graph-of-Thought(Besta , 2024), Monte Carlo Tree Search(Luo , 2024; Tian , 2024; Wan , 202
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(Ke , 2023; Lin , 2024; Yu , 2024), (Wang , 2023a)
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Limitation

This study has the following limitations:

- **Model Coverage:** Our analysis of overthinking issues is based solely on two models: QwQ-32B-Preview and DeepSeek-R1-Preview. These are the only two o1-like models that exhibit a visible long chain-of-thought (CoT) process. Additionally, our efficiency-enhancing methods are exclusively validated on QwQ-32B-Preview, the only o1-like model available for further post-training. We plan to validate our findings and approaches on more o1-like models as they become accessible.
- **Diversity Measurement:** Our diversity measurement utilizes the powerful GPT-4o for clustering solutions, which is costly and difficult to replicate. In forthcoming revisions, we aim to employ more open-source LLMs for diversity assessment, such as LLama3.3-70B, to facilitate easier replication of the methodology.
- **Bias in PRM12K Training Data:** Our current validation relies solely on the widely-used PRM12K dataset, which might compromise the robustness of our approach. We intend to explore larger datasets (e.g., with 1 million instances) to address this concern.

This work does not solve the overthinking issue in o1-like models but lays the groundwork for future research.

References

- Maciej Besta, Nils Blach, Ales Kubicek, Robert Gerstenberger, Michal Podstawski, Lukas Gianinazzi, Joanna Gajda, Tomasz Lehmann, Hubert Niewiadomski, Piotr Nyczek, et al. Graph of thoughts: Solving elaborate problems with large language models. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 38, pp. 17682–17690, 2024.
- Karl Cobbe, Vineet Kosaraju, Mohammad Bavarian, Mark Chen, Heewoo Jun, Lukasz Kaiser, Matthias Plappert, Jerry Tworek, Jacob Hilton, Reiichiro Nakano, Christopher Hesse, and John Schulman. Training verifiers to solve math word problems. *arXiv:2110.14168*, 2021.
- Mehul Damani, Idan Shenfeld, Andi Peng, Andreea Bobu, and Jacob Andreas. Learning how hard to think: Input-adaptive allocation of lm computation, 2024. URL <https://arxiv.org/abs/2410.04707>.
- DeepSeek. Deepseek-r1-lite-preview: Unleashing supercharged reasoning power. <https://api-docs.deepseek.com/news/news1120>, 2024. Accessed: 2024-12-29.
- Yilun Du, Shuang Li, Antonio Torralba, Joshua B Tenenbaum, and Igor Mordatch. Improving factuality and reasoning in language models through multiagent debate. In *Forty-first International Conference on Machine Learning*, 2024.
- Sachin Goyal, Ziwei Ji, Ankit Singh Rawat, Aditya Krishna Menon, Sanjiv Kumar, and Vaishnavh Nagarajan. Think before you speak: Training language models with pause tokens. In *The Twelfth International Conference on Learning Representations*, 2024. URL <https://openreview.net/forum?id=ph04CRkPdC>.
- Tingxu Han, Chunrong Fang, Shiyu Zhao, Shiqing Ma, Zhenyu Chen, and Zhenting Wang. Token-budget-aware llm reasoning. *arXiv preprint arXiv:2412.18547*, 2024.
- Shibo Hao, Sainbayar Sukhbaatar, DiJia Su, Xian Li, Zhiting Hu, Jason Weston, and Yuandong Tian. Training large language models to reason in a continuous latent space, 2024. URL <https://arxiv.org/abs/2412.06769>.
- David Heineman, Yao Dou, and Wei Xu. Improving minimum bayes risk decoding with multi-prompt. In *Proceedings of the 2024 Conference on Empirical Methods in Natural Language Processing*, pp. 22525–22545, 2024.

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Maciej Besta, Nils Blach, Ales Kubicek, Robert Gerstenberger, Michal Podstawska, Lukas Gianinazzi, Joan na Gajda, Tomasz Lehmann, Hubert Niewiadomski, Piotr Nyczek, et al. :

. Proceedings of the AAAI Conference on Artificial Intelligence, 38 , 17682~17690 , 2024 .

, , , , , , , . *arXiv:2110.14168*, 2021.

Mehul Damani, Idan Shenfeld, Andi Peng, Andreea Bobu, Jacob Andreas.

:lm , 2024. URL <https://arxiv.org/abs/2410.04707>.

DeepSeek. Deepseek-r1-lite-preview: . https://api-docs.deepseek.com/news/news1120, 2024. : 2024-12-29.

Yilun Du, Shuang Li, Antonio Torralba, Joshua B Tenenbaum, Igor Mordatch.
. Forty-first International Conference on Machine Learning, 2024.

Sachin Goyal, Ziwei Ji, Ankit Singh Rawat, Aditya Krishna Menon, Sanjiv Kumar, Vaishnavh Nagarajan.

International Conference on Learning Representations, 2024. URL <https://openreview.net/forum?id=ab04C8bPdG>

Tingxu Han, Chunrong Fang, Shiyu Zhao, Shiqing Ma, Zhenyu Chen Zhenting Wang.
 IJLMS arXiv preprint arXiv:2412.18547, 2024

Shibo Hao, Sainbayar Sukhbaatar, DiJia Su, Xian Li, Zhiting Hu, Jason Weston, Yuandong Tian., 2024. URL <https://arxiv.org/abs/2412.06769>.

Proceedings of the 2024 Conference on Empirical Methods in Natural Language Processing, pp. 22525–22545, 2024.

Dan Hendrycks, Collin Burns, Saurav Kadavath, Akul Arora, Steven Basart, Eric Tang, Dawn Song, and Jacob Steinhardt. Measuring mathematical problem solving with the MATH dataset. In *NeurIPS*, 2021.

Namgyu Ho, Laura Schmid, and Se-Young Yun. Large language models are reasoning teachers. In *Proceedings of the 61st Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pp. 14852–14882, 2023.

Ryo Kamoi, Yusen Zhang, Nan Zhang, Jiawei Han, and Rui Zhang. When can llms actually correct their own mistakes? a critical survey of self-correction of llms. *Transactions of the Association for Computational Linguistics*, 12:1417–1440, 2024.

Pei Ke, Bosi Wen, Zhuoer Feng, Xiao Liu, Xuanyu Lei, Jiale Cheng, Shengyuan Wang, Aohan Zeng, Yuxiao Dong, Hongning Wang, et al. Critiquellm: Scaling llm-as-critic for effective and explainable evaluation of large language model generation. corr, abs/2311.18702. detection for generative large language models. In *Proceedings of the 2023 Conference on Empirical Methods in Natural Language Processing*, pp. 9004–9017, 2023.

Maxim Khanov, Jirayu Burapachep, and Yixuan Li. Args: Alignment as reward-guided search. In *The Twelfth International Conference on Learning Representations*, 2024.

Jan Hendrik Kirchner, Yining Chen, Harri Edwards, Jan Leike, Nat McAleese, and Yuri Burda. Prover-verifier games improve legibility of llm outputs, 2024. URL <https://arxiv.org/abs/2407.13692>.

Takeshi Kojima, Shixiang Shane Gu, Machel Reid, Yutaka Matsuo, and Yusuke Iwasawa. Large language models are zero-shot reasoners. *Advances in neural information processing systems*, 35: 22199–22213, 2022.

Aviral Kumar, Vincent Zhuang, Rishabh Agarwal, Yi Su, John D Co-Reyes, Avi Singh, Kate Baumli, Shariq Iqbal, Colton Bishop, Rebecca Roelofs, et al. Training language models to self-correct via reinforcement learning. *arXiv preprint arXiv:2409.12917*, 2024.

Chen Li, Weiqi Wang, Jingcheng Hu, Yixuan Wei, Nanning Zheng, Han Hu, Zheng Zhang, and Houwen Peng. Common 7b language models already possess strong math capabilities, 2024a. URL <https://arxiv.org/abs/2403.04706>.

Yifei Li, Zeqi Lin, Shizhuo Zhang, Qiang Fu, Bei Chen, Jian-Guang Lou, and Weizhu Chen. Making language models better reasoners with step-aware verifier. In Anna Rogers, Jordan Boyd-Graber, and Naoaki Okazaki (eds.), *Proceedings of the 61st Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pp. 5315–5333, Toronto, Canada, July 2023. Association for Computational Linguistics. doi: 10.18653/v1/2023.acl-long.291. URL <https://aclanthology.org/2023.acl-long.291>.

Yiwei Li, Peiwen Yuan, Shaoxiong Feng, Boyuan Pan, Xinglin Wang, Bin Sun, Heda Wang, and Kan Li. Escape sky-high cost: Early-stopping self-consistency for multi-step reasoning. In *The Twelfth International Conference on Learning Representations*, 2024b. URL <https://openreview.net/forum?id=ndR8Ytrzh>.

Tian Liang, Zhiwei He, Wenxiang Jiao, Xing Wang, Yan Wang, Rui Wang, Yujiu Yang, Shuming Shi, and Zhaopeng Tu. Encouraging divergent thinking in large language models through multi-agent debate. In Yaser Al-Onaizan, Mohit Bansal, and Yun-Nung Chen (eds.), *Proceedings of the 2024 Conference on Empirical Methods in Natural Language Processing*, pp. 17889–17904, Miami, Florida, USA, November 2024. Association for Computational Linguistics. doi: 10.18653/v1/2024.emnlp-main.992. URL <https://aclanthology.org/2024.emnlp-main.992>.

Hunter Lightman, Vineet Kosaraju, Yuri Burda, Harrison Edwards, Bowen Baker, Teddy Lee, Jan Leike, John Schulman, Ilya Sutskever, and Karl Cobbe. Let’s verify step by step. In *The Twelfth International Conference on Learning Representations*, 2024. URL <https://openreview.net/forum?id=v8L0pN6EOi>.

Dan Hendrycks, Collin Burns, Saurav Kadavath, Akul Arora, Steven Basart, Eric Tang, Dawn Song, Jacob Steinhardt. MATH . NeurIPS, 2021. Namgyu Ho, Laura Schmid, Se-Young Yun.

Proceedings of the 61st Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers), pp. 14852 – 14882, 2023. Ryo Kamoi, Yusen Zhang, Nan Zhang, Jiawei Han, Rui Zhang. llms가 ? llms

Transactions of the Association for Computational Linguistics, 12:1417 – 1440, 2024. Pei Ke, Bosi Wen, Zheuo Feng, Xiao Liu, Xuanyu Lei, Jiale Cheng, Shengyuan Wang, Aohan Zeng, Yuxiao Dong, Hongning Wang, et al. Critiquellm: 가 llm-as-critic corr, abs/2311.18702.

Proceedings of the 2023 Conference on Empirical Methods in Natural Language Processing, pp. 9004 – 9017, 2023. Maxim Khanov, Jirayu Burapachetip, Yixuan Li. Args:

The Twelfth International Conference on Learning Representations, 2024. Jan Hendrik Kirchner, Yining Chen, Harri Edwards, Jan Leike, Nat McAleese, Yuri Burda. llm 가 , 2024. URL <https://arxiv.org/abs/2407.13692>.

Advances in neural information processing systems, 35: 22199 – 22213, 2022. Aviral Kumar, Vincent Zhuang, Rishabh Agarwal, Yi Su, John D Co-Reyes, Avi Singh, Kate Baumli, Shariq Iqbal, Colton Bishop, Rebeca Roelofs . 가

arXiv preprint arXiv:2409.12917, 2024. Chen Li, Weiqi Wang, Jingcheng Hu, Yixuan Wei, Nanning Zheng, Han Hu, Zheng Zhang Houwen Peng. 7b

(2024a). URL <https://arxiv.org/abs/2403.04706>. Yifei Li, Zeqi Lin, Shizhuo Zhang, Qiang Fu, Bei Chen, Jian-Guang Lou Weizhu Chen.

. Anna Rogers, Jordan Boyd-Graber Naoaki Okazaki (eds.), *Proceedings of the 61st Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pp. 5315 – 5333, 2023 7 . doi: 10.18653/v1/2023.acl-long.291. URL <https://aclanthology.org/2023.acl-long.291>. Yiwei Li, Peiwen Yuan, Shaoxiong Feng, Boyan Pan, Xinglin Wang, Bin Sun, Heda Wang Kan Li.

: . The Twelfth *International Conference on Learning Representations*, 2024b. URL <https://openreview.net/forum?id=ndR8Ytrzh>. Tian Liang, Zhiwei He, Wenxiang Jiao, Xing Wang, Yan Wang, Rui Wang, Yujiu Yang, Shuming Shi, Zhaopeng Tu.

. Yaser Al-Onaizan, Mohit Bansal, Yun-Nung Chen(), *Proceedings of the 2024 Conference on Empirical Methods in Natural Language Processing*, pp. 17889 – 17904, 2024 11 . Association for Computational Linguistics. doi: 10.18653/v1/2024.emnlp-main.992. URL <https://aclanthology.org/2024.emnlp-main.992>. Hunter Lightman, Vineet Kosaraju, Yuri Burda, Harrison Edwards, Bowen Baker, Teddy Lee, Jan Leike, John Schulman, Ilya Sutskever, Karl Cobbe. . The Twelfth *International Conference on Learning Representations*, 2024. URL <https://openreview.net/forum?id=v8L0pN6EOi>.

Zicheng Lin, Zhibin Gou, Tian Liang, Ruilin Luo, Haowei Liu, and Yujiu Yang. CriticBench: Benchmarking LLMs for critique-correct reasoning. In Lun-Wei Ku, Andre Martins, and Vivek Srikanth (eds.), *Findings of the Association for Computational Linguistics: ACL 2024*, pp. 1552–1587, Bangkok, Thailand, August 2024. Association for Computational Linguistics. doi: 10.18653/v1/2024.findings-acl.91. URL <https://aclanthology.org/2024.findings-acl.91>.

Zhihan Liu, Miao Lu, Shenao Zhang, Boyi Liu, Hongyi Guo, Yingxiang Yang, Jose Blanchet, and Zhaoran Wang. Provably mitigating overoptimization in rlhf: Your sft loss is implicitly an adversarial regularizer. *arXiv preprint arXiv:2405.16436*, 2024.

Liangchen Luo, Yinxiao Liu, Rosanne Liu, Samrat Phatale, Harsh Lara, Yunxuan Li, Lei Shu, Yun Zhu, Lei Meng, Jiao Sun, et al. Improve mathematical reasoning in language models by automated process supervision. *arXiv preprint arXiv:2406.06592*, 2024.

Rohin Manvi, Anikait Singh, and Stefano Ermon. Adaptive inference-time compute: Llms can predict if they can do better, even mid-generation, 2024. URL <https://arxiv.org/abs/2410.02725>.

MAA Committees. Aime problems and solutions. https://artofproblemsolving.com/wiki/index.php/AIME_Problems_and_Solutions.

Yu Meng, Mengzhou Xia, and Danqi Chen. Simpo: Simple preference optimization with a reference-free reward. In *Advances in Neural Information Processing Systems (NeurIPS)*, 2024.

Shen-Yun Miao, Chao-Chun Liang, and Keh-Yih Su. A diverse corpus for evaluating and developing english math word problem solvers. In *Proceedings of the 58th Annual Meeting of the Association for Computational Linguistics*, 2020.

OpenAI. Learning to reason with llms. <https://openai.com/index/learning-to-reason-with-llms>, 2024.

Richard Yuanzhe Pang, Weizhe Yuan, He He, Kyunghyun Cho, Sainbayar Sukhbaatar, and Jason E Weston. Iterative reasoning preference optimization. In *The Thirty-eighth Annual Conference on Neural Information Processing Systems*, 2024. URL <https://openreview.net/forum?id=4XIKfvNYvx>.

Qwen. Qwq: Reflect deeply on the boundaries of the unknown, November 2024. URL <https://qwenlm.github.io/blog/qwq-32b-preview/>.

Rafael Rafailov, Archit Sharma, Eric Mitchell, Christopher D Manning, Stefano Ermon, and Chelsea Finn. Direct preference optimization: Your language model is secretly a reward model. *Advances in Neural Information Processing Systems*, 36, 2024.

David Rein, Betty Li Hou, Asa Cooper Stickland, Jackson Petty, Richard Yuanzhe Pang, Julien Dirani, Julian Michael, and Samuel R Bowman. Gpqa: A graduate-level google-proof q&a benchmark. *arXiv preprint arXiv:2311.12022*, 2023.

Ye Tian, Baolin Peng, Linfeng Song, Lifeng Jin, Dian Yu, Haitao Mi, and Dong Yu. Toward self-improvement of llms via imagination, searching, and criticizing. *arXiv preprint arXiv:2404.12253*, 2024.

Ziyu Wan, Xidong Feng, Muning Wen, Stephen Marcus McAleer, Ying Wen, Weinan Zhang, and Jun Wang. Alphazero-like tree-search can guide large language model decoding and training. In *Forty-first International Conference on Machine Learning*, 2024.

Lei Wang, Wanyu Xu, Yihuai Lan, Zhiqiang Hu, Yunshi Lan, Roy Ka-Wei Lee, and Ee-Peng Lim. Plan-and-solve prompting: Improving zero-shot chain-of-thought reasoning by large language models. In Anna Rogers, Jordan Boyd-Graber, and Naoaki Okazaki (eds.), *Proceedings of the 61st Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pp. 2609–2634, Toronto, Canada, July 2023a. Association for Computational Linguistics. doi: 10.18653/v1/2023.acl-long.147. URL <https://aclanthology.org/2023.acl-long.147>.

Zicheng Lin, Zhibin Gou, Tian Liang, Ruilin Luo, Haowei Liu, Yujiu Yang. CriticBench: LLM . Lun-Wei Ku, Andre Martins, Vivek Srikumar (eds.), *Findings of the Association for Computational Linguistics: ACL 2024*, pp. 1552 – 1587, 2024. doi: 10.18653/v1/2024.findings-acl.91. URL <https://aclanthology.org/2024.findings-acl.91>. Zhihan Liu, Miao Lu, Shenao Zhang, Boyi Liu, Hongyi Guo, Yingxiang Yang, Jose Blanchet, Zhaoran Wang. rlhf : s ft . arXiv preprint arXiv:2405.16436, 2024. Liangchen Luo, Yinxiao Liu, Rosanne Liu, Samrat Phatale, Harsh Lara, Yunxuan Li, Lei Shu, Yun Zhu, Lei Meng, Jiao Sun . arXiv preprint arXiv:2406.06592, 2024. Rohin Manvi, Anikait Singh, Stefano Ermon. : Llms , 2024. URL <https://arxiv.org/abs/2410.02725>. MAA . Aime . https://artofproblemsolving.com/wiki/index.php/AIME_Problems_and_Solutions. Yu Meng, Mengzhou Xia, Danqi Chen. Simpo: *Advances in Neural Information Processing Systems (NeurIPS)* 2024. Shen-Yun Miao, Chao-Chun Liang, Keh- . Proceedings of the 58th Annual Meeting of the Association for Computational Linguistics, 2020. OpenAI. L LM . https://openai.com/index/learning-to-reason-with-llms, 2024 . Richard Yuanzhe Pang, Weizhe Yuan, He He, , , E . The Thirty-eighth Annual Conference on Neural Information Processing Systems, 2024. URL <https://openreview.net/forum?id=4XIKfvNYvx>. Qwen. Qwq: , 2024 11 . URL <https://qwenlm.github.io/blog/qwq-32b-preview/>. Rafael Rafailov, Archit Sharma, Eric Mitchell, Christopher D Manning, Stefano Ermon, Chelsea Finn. : . Advances in Neural Information Processing Systems , 36, 2024. , , , R . Gpqa: , , , Q&A . arXiv preprint arXiv:2311.12022, 2023. Ye Tian, Baolin Peng, Linfeng Song, Lifeng Jin, Dian Yu, Haitao Mi, and Dong Yu. , , llms . arXiv preprint arXiv:2404.12253 , 2024. Ziyu Wan, Xidong Feng, Muning Wen, Stephen Marcus McAleer, Ying Wen, Weinan Zhang, Jun Wang. Alphazero . Forty-first International Conference on Machine Learning, 2024. , , , , , : . Anna Rogers, Jordan Boyd-Graber, Naoaki Okazaki(), Proceedings of the 61st Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers), pp. 2609 – 2634, , , 2023 7 a. Association for Computational Linguistics. doi: 10.18653/v1/2023.acl-long.147. URL <https://aclanthology.org/2023.acl-long.147>.

Xinglin Wang, Shaoxiong Feng, Yiwei Li, Peiwen Yuan, Yueqi Zhang, Boyuan Pan, Heda Wang, Yao Hu, and Kan Li. Make every penny count: Difficulty-adaptive self-consistency for cost-efficient reasoning, 2024. URL <https://arxiv.org/abs/2408.13457>.

Xuezhi Wang, Jason Wei, Dale Schuurmans, Quoc V Le, Ed H. Chi, Sharan Narang, Aakanksha Chowdhery, and Denny Zhou. Self-consistency improves chain of thought reasoning in language models. In *The Eleventh International Conference on Learning Representations*, 2023b. URL <https://openreview.net/forum?id=1PL1NIMMrw>.

Jason Wei, Maarten Bosma, Vincent Zhao, Kelvin Guu, Adams Wei Yu, Brian Lester, Nan Du, Andrew M Dai, and Quoc V Le. Finetuned language models are zero-shot learners. In *International Conference on Learning Representations*, 2022a.

Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten Bosma, Fei Xia, Ed Chi, Quoc V Le, Denny Zhou, et al. Chain-of-thought prompting elicits reasoning in large language models. *Advances in neural information processing systems*, 35:24824–24837, 2022b.

Ian Wu, Patrick Fernandes, Amanda Bertsch, Seungone Kim, Sina Pakazad, and Graham Neubig. Better instruction-following through minimum bayes risk. *arXiv preprint arXiv:2410.02902*, 2024.

Kai Xiong, Xiao Ding, Yixin Cao, Ting Liu, and Bing Qin. Examining inter-consistency of large language models collaboration: An in-depth analysis via debate. In *Findings of the Association for Computational Linguistics: EMNLP 2023*, pp. 7572–7590, 2023.

Shunyu Yao, Dian Yu, Jeffrey Zhao, Izhak Shafran, Tom Griffiths, Yuan Cao, and Karthik Narasimhan. Tree of thoughts: Deliberate problem solving with large language models. *Advances in Neural Information Processing Systems*, 36, 2024.

Junyi Ye, Jingyi Gu, Xinyun Zhao, Wenpeng Yin, and Guiling Wang. Assessing the creativity of llms in proposing novel solutions to mathematical problems. *arXiv preprint arXiv:2410.18336*, 2024.

Yue Yu, Zhengxing Chen, Aston Zhang, Liang Tan, Chenguang Zhu, Richard Yuanzhe Pang, Yundi Qian, Xuewei Wang, Suchin Gururangan, Chao Zhang, et al. Self-generated critiques boost reward modeling for language models. *arXiv preprint arXiv:2411.16646*, 2024.

Eric Zelikman, Yuhuai Wu, Jesse Mu, and Noah Goodman. Star: Bootstrapping reasoning with reasoning. *Advances in Neural Information Processing Systems*, 35:15476–15488, 2022.

Zirui Zhao, Hanze Dong, Amrita Saha, Caiming Xiong, and Doyen Sahoo. Automatic curriculum expert iteration for reliable llm reasoning, 2024. URL <https://arxiv.org/abs/2410.07627>.

- Xinglin Wang, Shaoxiong Feng, Yiwei Li, Peiwen Yuan, Yueqi Zhang, Boyuan Pan, Heda Wang, Yao Hu, Kan Li. : , 2024.
- URL <https://arxiv.org/abs/2408.13457>. Xuezhi Wang, Jason Wei, Dale Schuurmans, Quoc V Le, Ed H. Chi, Sharan Narang, Aakanksha Chowdhery, Denny Zhou. . *The Eleventh International Conference on Learning Representations* , 2023b.
- URL <https://openreview.net/forum?id=1PL1NIMMrw>. Jason Wei, Maarten Bosma, Vincent Zhao, Kelvin Guu, Adams Wei Yu, Brian Lester, Nan Du, Andrew M Dai, Quoc V Le. . *International Conference on Learning Representations*, 2022a. Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten Bosma, Fei Xia, Ed Chi, Quoc V Le, Denny Zhou, et al. Chain-of-thought prompting elicits reasoning in large language models. *Advances in neural information processing systems*, 35: 24824 – 24837, 2022b. Ian Wu, Patrick Fernandes, Amanda Bertsch, Seungone Kim, Sina Pakazad, and Graham Neubig. - . *arXiv preprint arXiv:2410.02902*, 2024. Kai Xiong, Xiao Ding, Yixin Cao, Ting Liu, and Bing Qin. : . *Findings of the Association for Computational Linguistics: EMNLP 2023*, pp. 7572 – 7590, 2023. Shunyu Yao, Dian Yu, Jeffrey Zhao, Izhak Shafran, Tom Griffiths, Yuan Cao Karthik Narasimhan. : . *Advances in Neural Information Processing Systems*, 36, 2024. Junyi Ye, Jingyi Gu, Xinyun Zhao, Wenpeng Yin Guiling Wang. LLM . *arXiv preprint arXiv:2410.18336*, 2024. Yue Yu, Zhengxing Chen, Aston Zhang, Liang Tan, Chenguang Zhu, Richard Yuanzhe Pang, Yundi Qian, Xuewei Wang, Suchin Gururangan, Chao Zhang . *arXiv preprint arXiv:2411.16646*, 2024. Eric Zelikman, Yuhuai Wu, Jesse Mu, Noah Goodman. Star: . *Advances in Neural Information Processing Systems*, 35:15476 – 15488, 2022. Zirui Zhao, Hanze Dong, Amrita Saha, Caiming Xiong, Doyen Sahoo. llm . *arXiv preprint arXiv:2410.07627*, 2024. URL <https://arxiv.org/abs/2410.07627>.

A Appendix

A.1 Case Overview for Deepseek-R1-Preview

Deepseek-R1-Preview (936 tokens)

Solution1
158 tokens

Alright, so I've got this problem here: "What is the answer of 2 plus 3?" At first glance, it seems pretty straightforward, but I want to make sure I understand it fully. Okay, so addition is one of the basic operations in arithmetic, right? It's combining two quantities to find their total. So, I need to add 2 and 3. Let me think about what 2 and 3 represent. If I have 2 apples and someone gives me 3 more apples, how many apples do I have in total? Let me count: one, two (that's the original 2), and then three, four, five (that's the additional 3). So, altogether, I have five apples. That seems correct.

Solution2
64 tokens

Wait, maybe I should use my fingers to double-check. I'll hold up two fingers on one hand and three on the other. Now, if I count them all together: one, two, three, four, five. Yep, that's five fingers. So, 2 plus 3 equals 5.

Solution3
67 tokens

Is there another way to look at this? Maybe using a number line. If I start at 2 and move 3 units to the right, where do I end up? Starting at 2, one step to 3, another to 4, and another to 5. So again, I'm at 5.

Solution4
154 tokens

Hmm, I'm consistently getting 5 as the answer. I should consider if there's any reason to think otherwise. Could there be a different interpretation of "plus" or the numbers 2 and 3? In standard arithmetic, plus means addition, and 2 and 3 are positive integers. I don't think there's any trick here.

Let me try adding them in a different order, just to see if it changes anything. So, 3 plus 2. Using the same methods: 3 apples plus 2 apples is 5 apples, or 3 fingers plus 2 fingers is 5 fingers, or starting at 3 on the number line and moving 2 units to the right lands me at 5. Still 5.

Solution5
46 tokens

Okay, what if I use objects of different types? Like 2 oranges plus 3 bananas. Do I get 5 fruits? Yes, I do. So, even with different objects, the total is still 5.

.....

Solution11
32 tokens

Just to be thorough, I'll try adding them in my head again: 2 plus 3. 2 and 3 make 5. Yep.

Okay, I'm satisfied with this conclusion. 2 plus 3 equals 5.

Figure 10: Deepseek-R1-Preview response for the query “What is the answer of 2 plus 3?”

A.2 Prompts for Clustering Solutions

Inspired by (Ye et al., 2024), we leverage GPT-4o to cluster the solutions for each instance into groups with the following prompt:

A.1 Deepseek-R1-Preview

Deepseek-R1-Preview (936 tokens)

Solution1
158 tokens

Alright, so I've got this problem here: "What is the answer of 2 plus 3?" At first glance, it seems pretty straightforward, but I want to make sure I understand it fully. Okay, so addition is one of the basic operations in arithmetic, right? It's combining two quantities to find their total. So, I need to add 2 and 3. Let me think about what 2 and 3 represent. If I have 2 apples and someone gives me 3 more apples, how many apples do I have in total? Let me count: one, two (that's the original 2), and then three, four, five (that's the additional 3). So, altogether, I have five apples. That seems correct.

Solution2
64 tokens

Wait, maybe I should use my fingers to double-check. I'll hold up two fingers on one hand and three on the other. Now, if I count them all together: one, two, three, four, five. Yep, that's five fingers. So, 2 plus 3 equals 5.

Solution3
67 tokens

Is there another way to look at this? Maybe using a number line. If I start at 2 and move 3 units to the right, where do I end up? Starting at 2, one step to 3, another to 4, and another to 5. So again, I'm at 5.

Solution4
154 tokens

Hmm, I'm consistently getting 5 as the answer. I should consider if there's any reason to think otherwise. Could there be a different interpretation of "plus" or the numbers 2 and 3? In standard arithmetic, plus means addition, and 2 and 3 are positive integers. I don't think there's any trick here.

Let me try adding them in a different order, just to see if it changes anything. So, 3 plus 2. Using the same methods: 3 apples plus 2 apples is 5 apples, or 3 fingers plus 2 fingers is 5 fingers, or starting at 3 on the number line and moving 2 units to the right lands me at 5. Still 5.

Solution5
46 tokens

Okay, what if I use objects of different types? Like 2 oranges plus 3 bananas. Do I get 5 fruits? Yes, I do. So, even with different objects, the total is still 5.

.....

Solution11
32 tokens

Just to be thorough, I'll try adding them in my head again: 2 plus 3. 2 and 3 make 5. Yep.

Okay, I'm satisfied with this conclusion. 2 plus 3 equals 5.

10: "2

3

? "

Deepseek-R1-Preview

A.2

Inspired by (Ye et al., 2024), we leverage GPT-4o to cluster the solutions for each instance into groups with the following prompt:

Criteria for clustering the mathematical solutions:

1. If the solutions used to arrive at the solutions are fundamentally different from each other, such as algebraic manipulation versus geometric reasoning, they can be considered novel;
2. Even if the results are the same, if the intermediate steps or processes involved in reaching those solutions vary significantly, the solutions can be considered different;
3. If the solutions relies on different assumptions or conditions, they should be considered different from each other;
4. A solution might generalize to a broader class of problems, while another solution might be specific to certain conditions. In such cases, they are considered distinct;
5. If one solution is significantly simpler or more complex than the others, it can be regarded as essentially novel, even if they lead to the same result.

Given the following mathematical problem:

problem

Solutions:

Solution 1: ...

Solution 2: ...

Please output the clusters strictly following the following format, each row containing a cluster, names, and reasons. Do not include any additional text or explanations outside of this format:

cluster1 [solution names] reason for cluster

cluster2 [solution names] reason for cluster

cluster3 [solution names] reason for cluster

...

For example:

cluster1 [Solution 1, Solution 3, Solution 5] similar algebraic approach using the volume formula and canceling terms or directly solving for the height.

cluster2 [Solution 2, Solution 4] verifying the correctness and consistency of the formula and solution and considering unit checks or logical reasoning on how volume relates to base area and height.

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