ECON 310: Dynamic Macroeconomics

Spring 2025

Fulbright University Vietnam

Model: Stochastic Growth Model.

The social planner's problem is

$$\max_{\{c_t\}_{t=1}^{\infty},\{k_{t+1}\}_{t=1}^{\infty}} \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \ ,$$

$$\text{s.t. } y_t = c_t + i_t \ ,$$

$$y_t = A_t k_t^{\alpha} \ ,$$

$$k_{t+1} = (1-\delta)k_t + i_t \ ,$$

$$\log(A_{t+1}) = \mu + \rho \log(A_t) + \epsilon_{t+1} \ ,$$

$$c_t > 0, k_t > 0 \ ,$$

where $|\rho| < 1$, $\epsilon_{t+1} \sim \mathcal{N}(0, \sigma_\epsilon^2)$, and the constraints hold for $t = 1, 2, ...,$. The rationale for the constraints that are similar to those in the deterministic growth model. The productivity term, A_t, is a stationary AR(1) in logs by assumption, but is based on empirical literature. This stochastic process is what the conditional expectations operator, $\mathbb{E}_0(\cdot)$, is applied to.

Recursive Formulation.

The recursive formulation, combining the constraints into one, is

$$V_t(k_t, A_t) = \max_{c_t, k_{t+1}} \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t[V_{t+1}(k_{t+1}, A_{t+1})] \ ,$$

$$\text{s.t. } A_t k_t^{\alpha} = c_t + k_{t+1} - (1-\delta)k_t \ \text{ for } \ t = 1, 2, ... \ ,$$

$$\log(A_{t+1}) = \mu + \rho \log(A_t) + \epsilon_{t+1} \ .$$

$$c_t > 0, k_t > 0 \ .$$

Because of the productivity shocks, the state space has now expanded to include this. Substituting the combined constraint into the utility function gives

$$V_t(k_t, A_t) = \max_{k_{t+1}} \frac{\left(A_t k_t^{\alpha} + (1-\delta)k_t - k_{t+1}\right)^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t[V_{t+1}(k_{t+1}, A_{t+1})] \ ,$$

$$\text{s.t. } \log(A_{t+1}) = \mu + \rho \log(A_t) + \epsilon_{t+1} \ ,$$

$$k_t > 0 \ ,$$

simplifying the problem as before.

Set up directories and paths.

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The directory is C:/Users/xmgb/Dropbox/02_FUV/teaching/spring_2025/dynamic_macro/code/vfi_sgm_matla
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Set the parameters and generate the state space.

Calls: model.m.

We cannot directly compute the steady state capital as before because of the productivity fluctuations, but we can still build the grid for k_t around the deterministic steady state.

Solve the model using Value Function Iteration.

Calls: solve.m, and model.m.

As in the deterministic case, we will solve the model using value function iteration using

$$V(k_t) = \frac{1}{1-\beta} \frac{\left(A_t k_t^\alpha - (1-\delta)k_t\right)^{1-\sigma}}{1-\sigma} \ ,$$

as a guess.

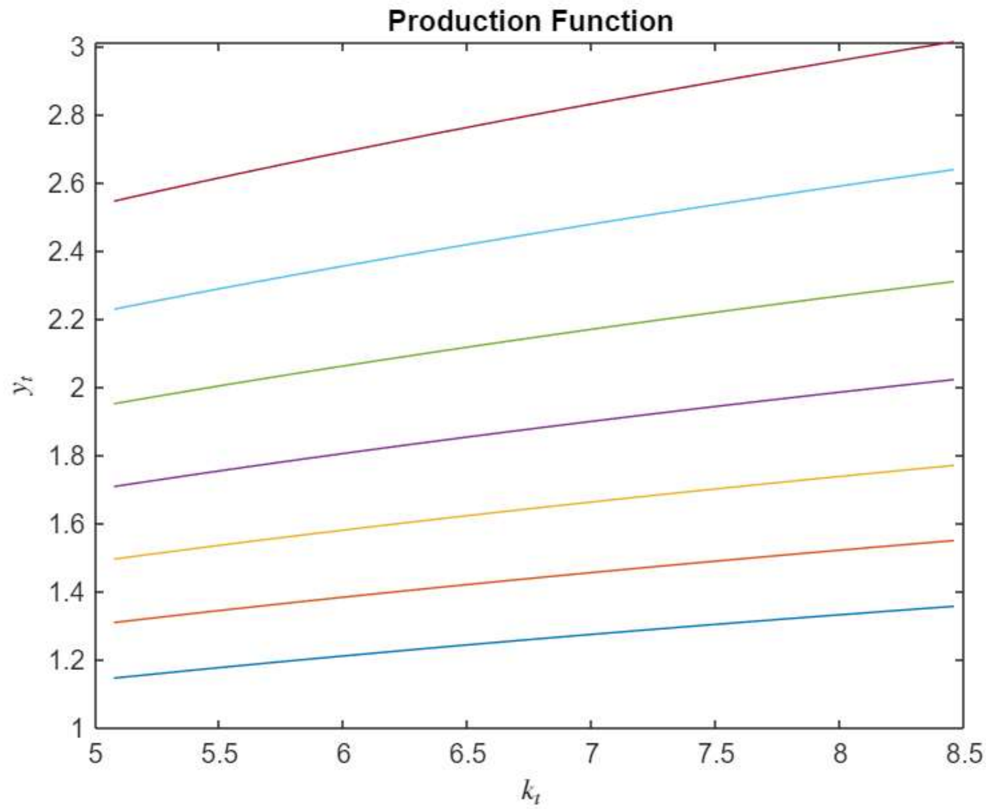
```
------------Beginning Value Function Iteration.------------

Iteration: 25.
Iteration: 50.
Iteration: 75.
Iteration: 100.
Iteration: 125.
Iteration: 150.
Iteration: 175.
Iteration: 200.
Iteration: 225.
Iteration: 250.
Iteration: 275.
Iteration: 300.

Converged in 322 iterations.

Model solved in 1.5625 seconds.
```

Simulate the model and plot the model functions and time series.

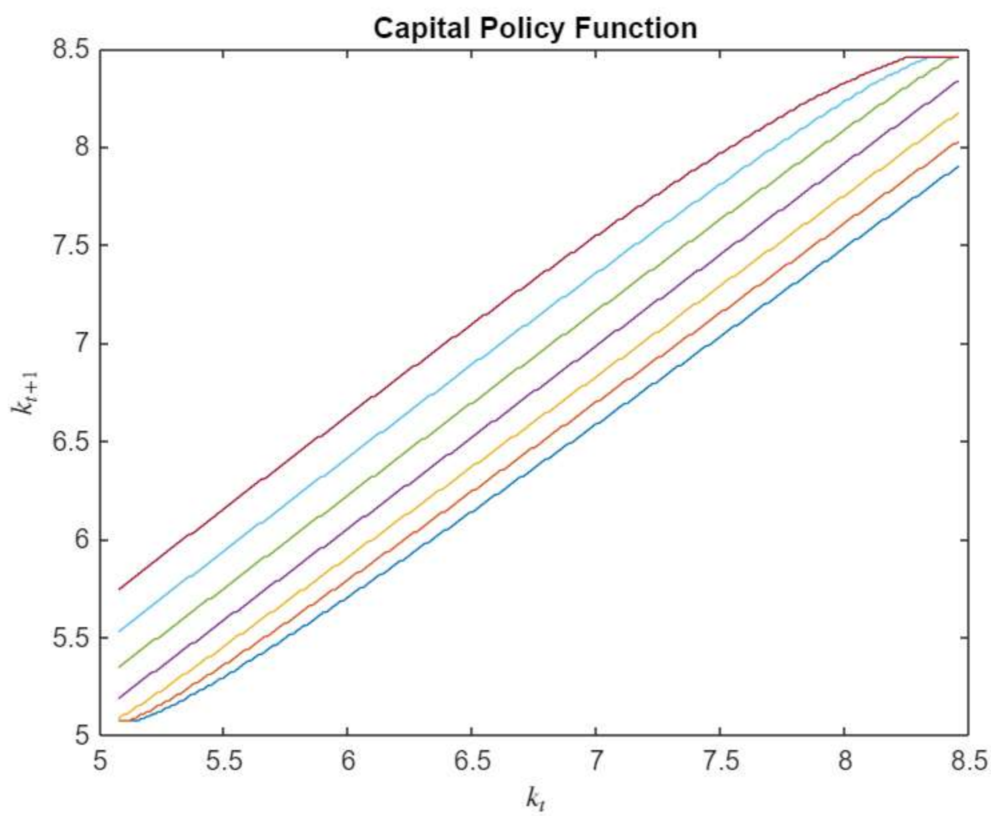
Calls: simulate.m and my_graph.m.

The model functions behave in a similarly to those in the deterministic growth model. The difference now is that because there are different levels of productivity, there will be a separate function for each level. For example, the seven production functions correspond to different levels of A_t. The blue line is production at the lowest level of A_t while the red line is production at the highest level. The general pattern is that the higher A_t is, the more resources there are for consumption, saving, and investment.

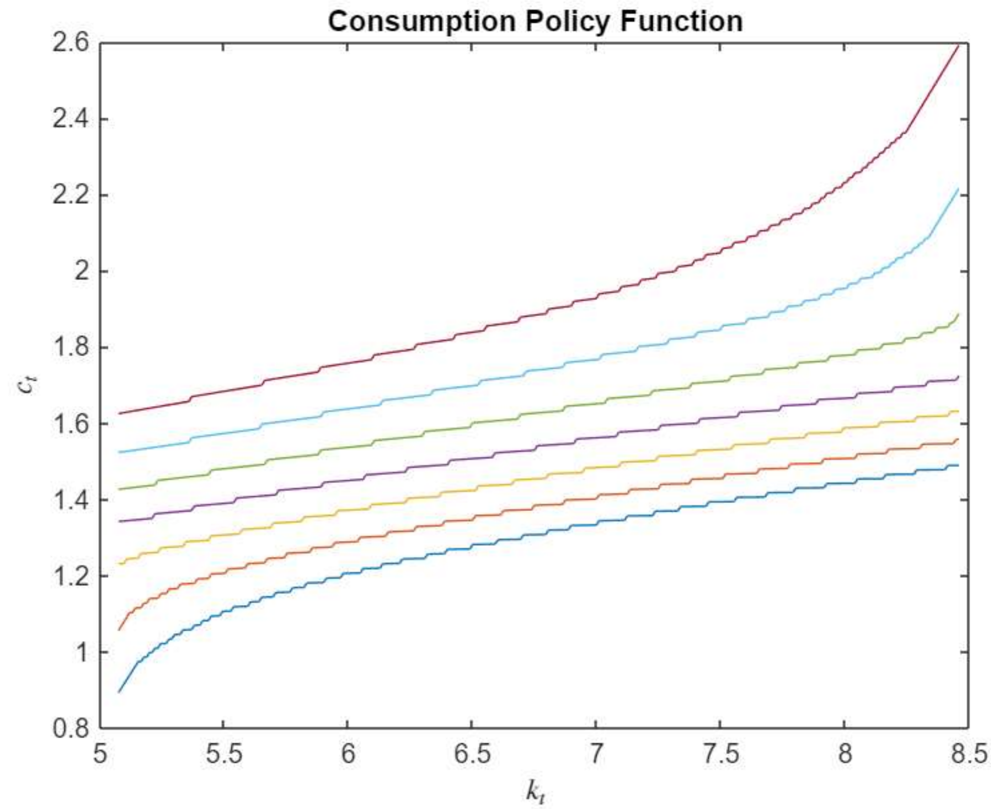
Because of productivity fluctuations, there is not a deterministic steady state level. Instead, the variables co-move with the productivity shocks. When productivity is low, output, consumption, and investment are low. Similarly, these variables are high when there are high productivity shocks. Utility in each period fluctuates with consumption.

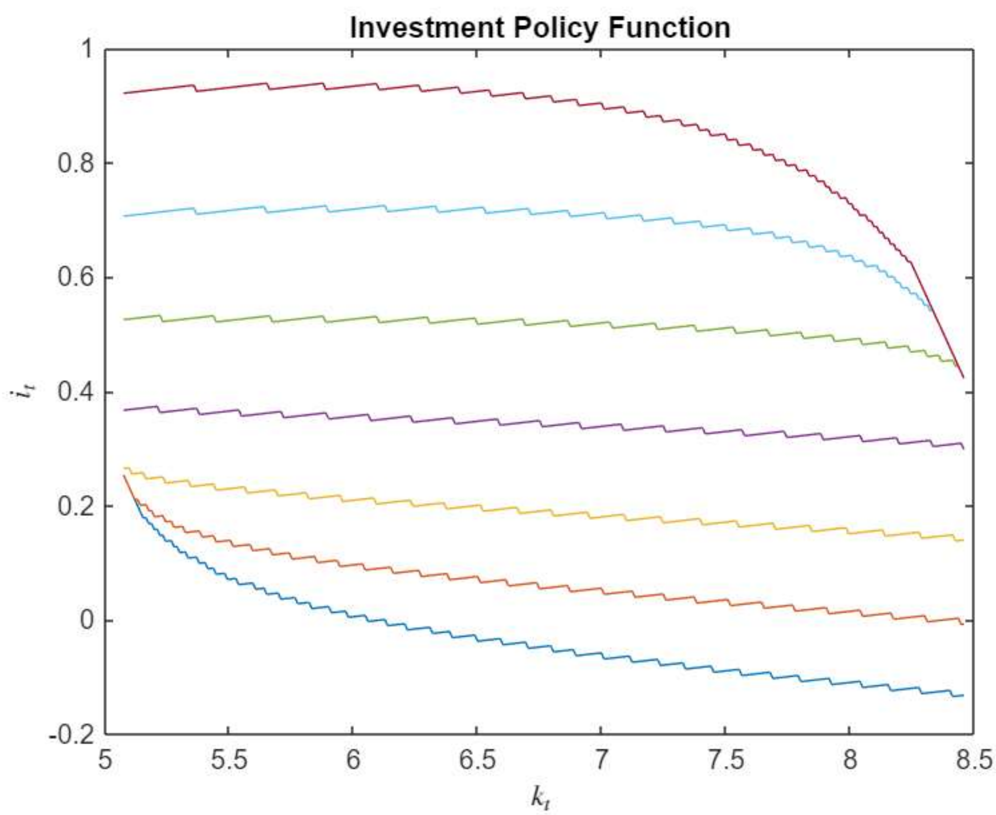


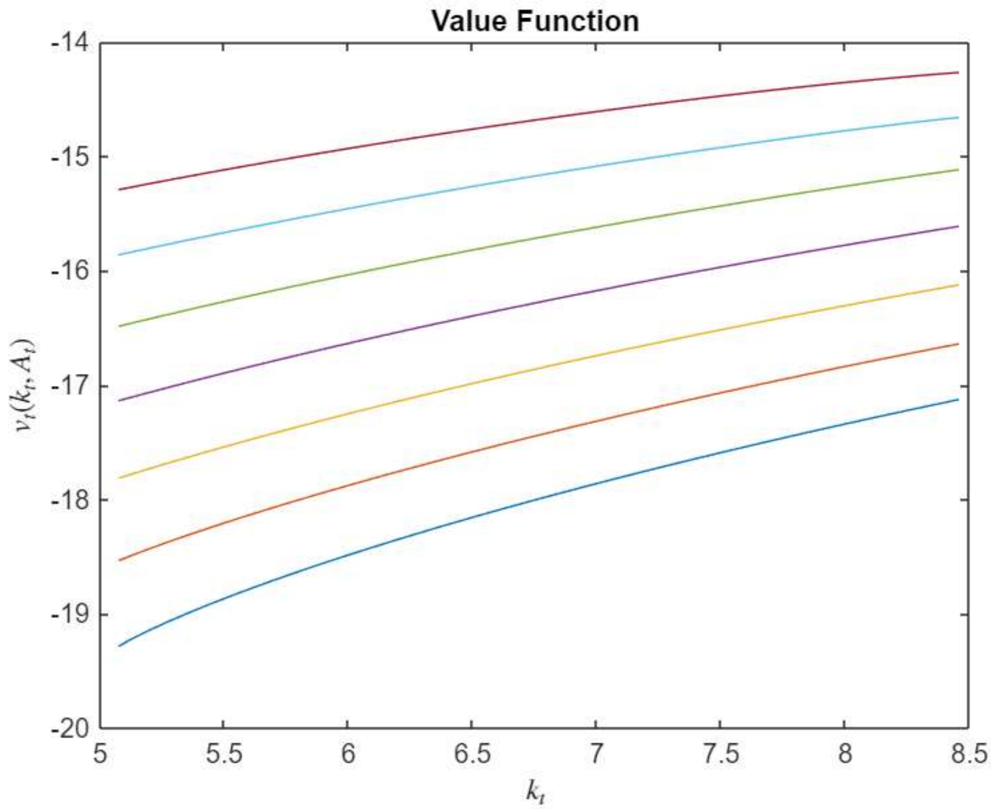
Production Function
y_t
k_t
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
5
5.5
6
6.5
7
7.5
8
8.5

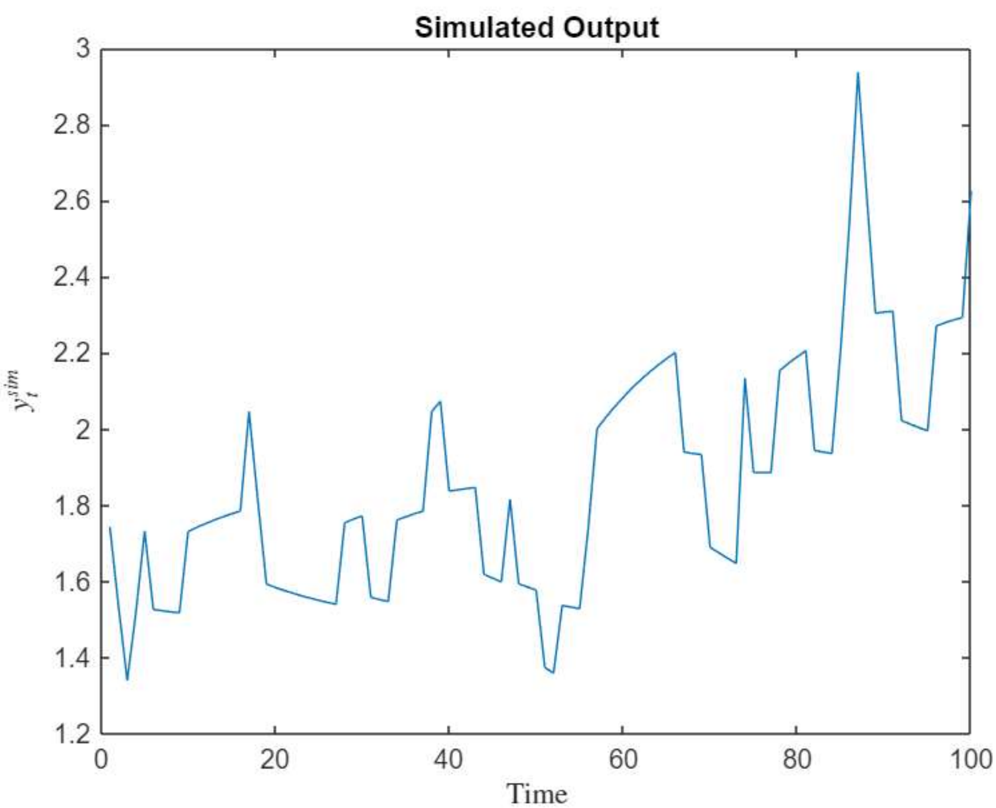


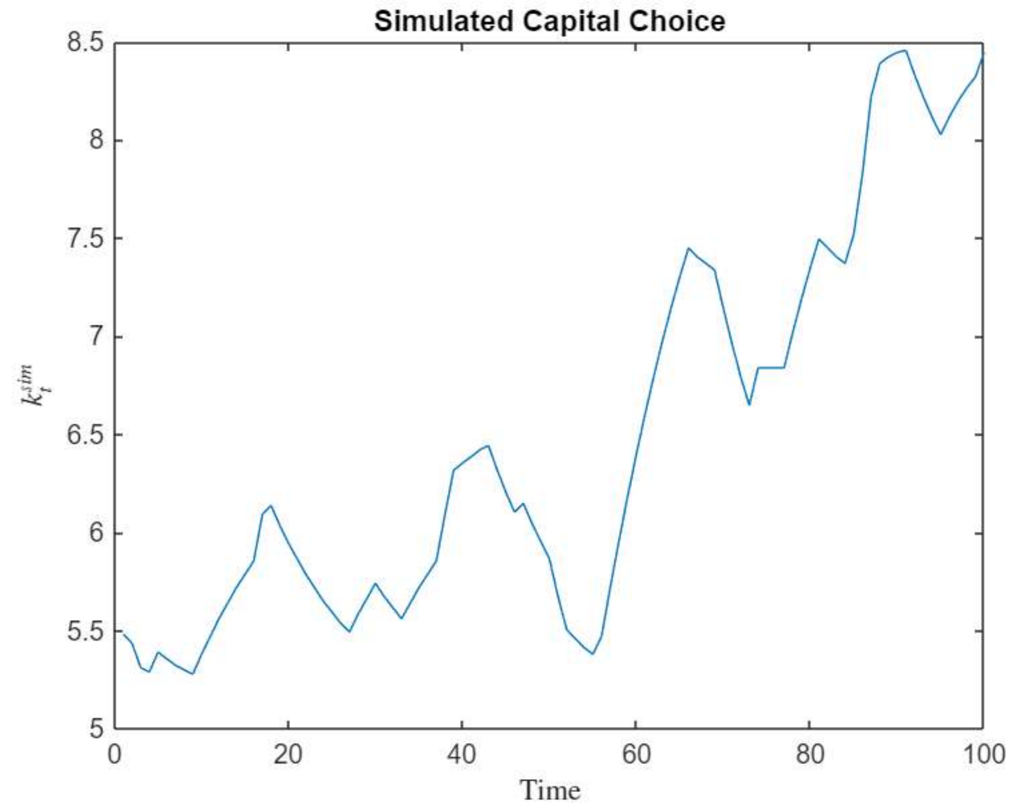
Capital Policy Function
k_{t+1}
k_t
8.5
8
7.5
7
6.5
6
5.5
5
5
5.5
6
6.5
7
7.5
8
8.5

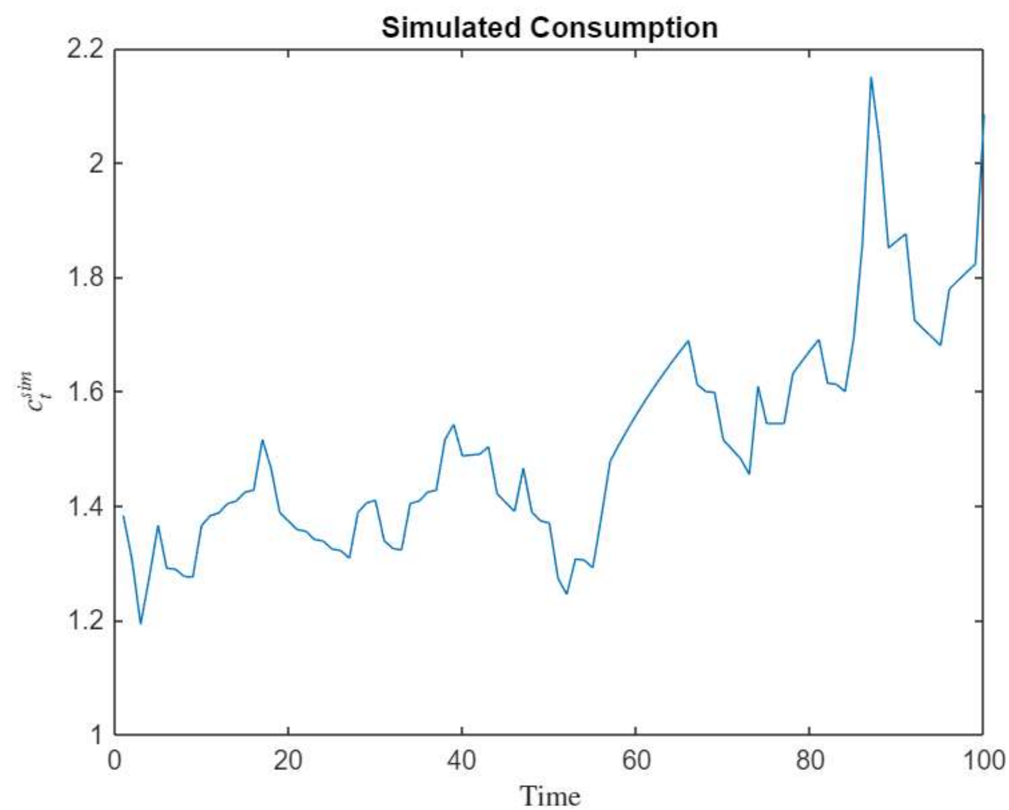

Consumption Policy Function
c_t
k_t
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
5
5.5
6
6.5
7
7.5
8
8.5

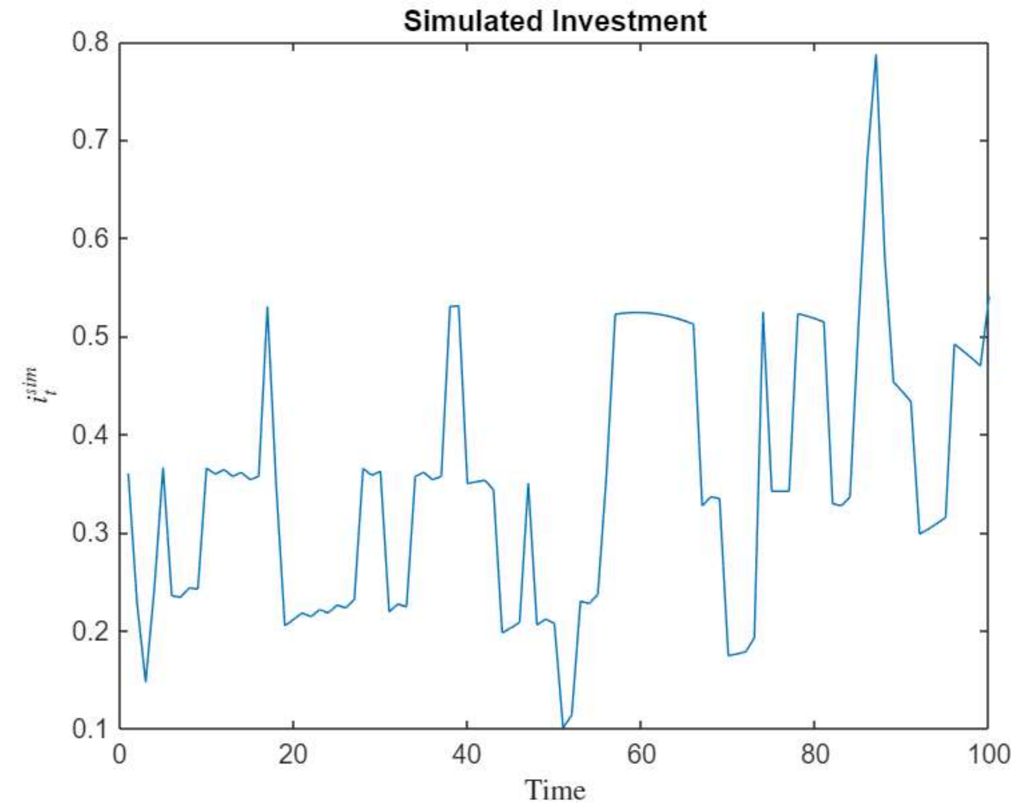

Investment Policy Function
i_t
k_t
1
0.8
0.6
0.4
0.2
0
-0.2
5
5.5
6
6.5
7
7.5
8
8.5

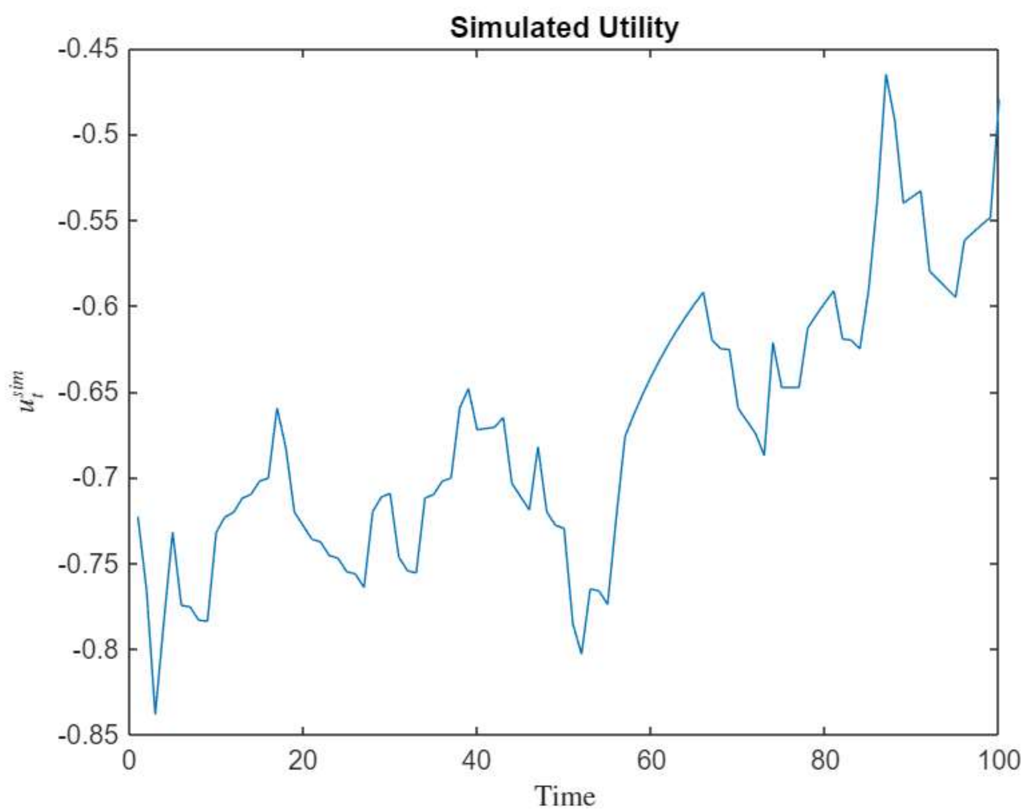

Value Function
$v_t(k_t, A_t)$
k_t
-14
-15
-16
-17
-18
-19
-20
5
5.5
6
6.5
7
7.5
8
8.5

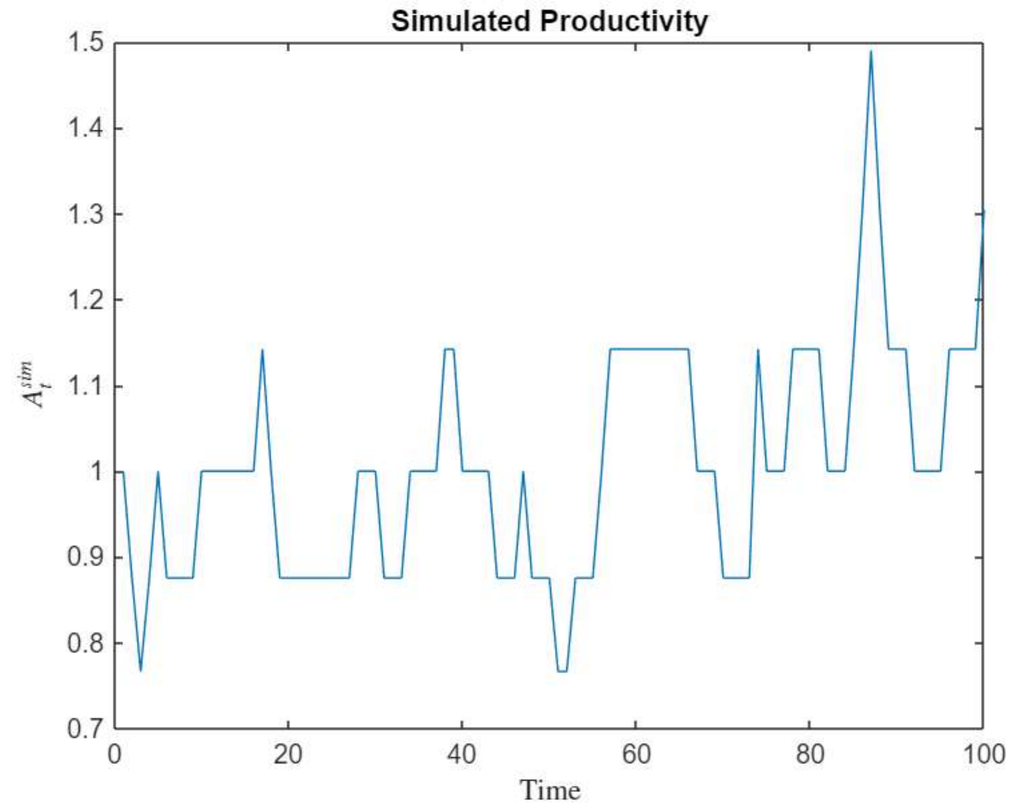

Simulated Output
y_t^{sim}
Time
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
0
20
40
60
80
100


Simulated Capital Choice
8.5
8
7.5
7
6.5
6
5.5
5
0
20
40
60
80
100
Time
k_t^{sim}


Simulated Consumption
2.2
2
1.8
1.6
1.4
1.2
1
0
20
40
60
80
100
Time
c_t^{sim}


Simulated Investment
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
i_t^{sim}
0
20
40
60
80
100
Time


Simulated Utility
-0.45
-0.5
-0.55
-0.6
-0.65
-0.7
-0.75
-0.8
-0.85
u_t^{sim}
0
20
40
60
80
100
Time


Simulated Productivity
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
A_t^{sim}
0
20
40
60
80
100
Time