ECON 310: Dynamic Macroeconomics

Spring 2025

Fulbright University Vietnam

Model: Deterministic Growth Model.

The social planner's problem is

$$\max_{\{c_t\}_{t=1}^{\infty},\{k_{t+1}\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \ ,$$

$$\text{s.t. } y_t = c_t + i_t \ ,$$

$$y_t = k_t^{\alpha} \ ,$$

$$k_{t+1} = (1-\delta)k_t + i_t \ ,$$

$$c_t > 0, k_t > 0 \ ,$$

where the constraints hold for for $t = 1, 2, ...,$. The first constraint is the aggregate resource constraint; the second is production; the third is the law of motion for capital. Consumption is restricted to be positive, meaning people should not starve. Capital is restricted to be positive so goods can be produced for consumption.

Recursive Formulation.

The recursive formulation, combining the constraints into one, is

$$V_t(k_t) = \max_{c_t, k_{t+1}} \frac{c_t^{1-\sigma}}{1-\sigma} + \beta V_{t+1}(k_{t+1}) \ ,$$

$$\text{s.t. } k_t^{\alpha} = c_t + k_{t+1} - (1-\delta)k_t \ \text{ for } \ t = 1, 2, \dots \ ,$$

$$c_t > 0, k_t > 0 \ .$$

The state space consists only of k_t while the choice space consists of c_t and k_{t+1}. Substituting the combined constraint into the utility function gives

$$V_t(k_t) = \max_{k_{t+1}} \frac{\left(k_t^{\alpha} + (1-\delta)k_t - k_{t+1}\right)^{1-\sigma}}{1-\sigma} + \beta V_{t+1}(k_{t+1}) \ ,$$

$$k_t > 0 \ ,$$

which reduces the choice space to k_{t+1}.

Set up directories and paths.

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The directory is C:/Users/xmgb/Dropbox/02_FUV/teaching/spring_2025/dynamic_macro/code/vfi_dgm_matla
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Set the parameters and generate the state space.

Calls: model.m.

From (5.3) in Adda and Cooper, the Euler Equation is

$$c_t^{-\sigma} = \beta c_{t+1}^{-\sigma}(\alpha k_{t+1}^{\alpha-1} + (1-\delta)) \ .$$

In the steady state, $c_t = c_{t+1} = c^{ss}$ so

$$1 = \beta(\alpha k_{t+1}^{\alpha-1} + (1-\delta)) \ ,$$

$$\frac{1}{\beta} = \alpha k_{t+1}^{\alpha-1} + (1-\delta) \ ,$$

$$k^{ss} = \left(\frac{\alpha}{\frac{1}{\beta} - 1 + \delta}\right)^{\frac{1}{1-\alpha}} \ ,$$

which we can construct our grid for k_t around.

Solve the model using Value Function Iteration.

Calls: solve.m and model.m.

We will solve the model using value function iteration using

$$V(k_t) = \frac{1}{1-\beta} \frac{\left(k_t^{\alpha} - (1-\delta)k_t\right)^{1-\sigma}}{1-\sigma} \ ,$$

which is the sum of an infinite geometric series of utility with discount factor β, as a guess.

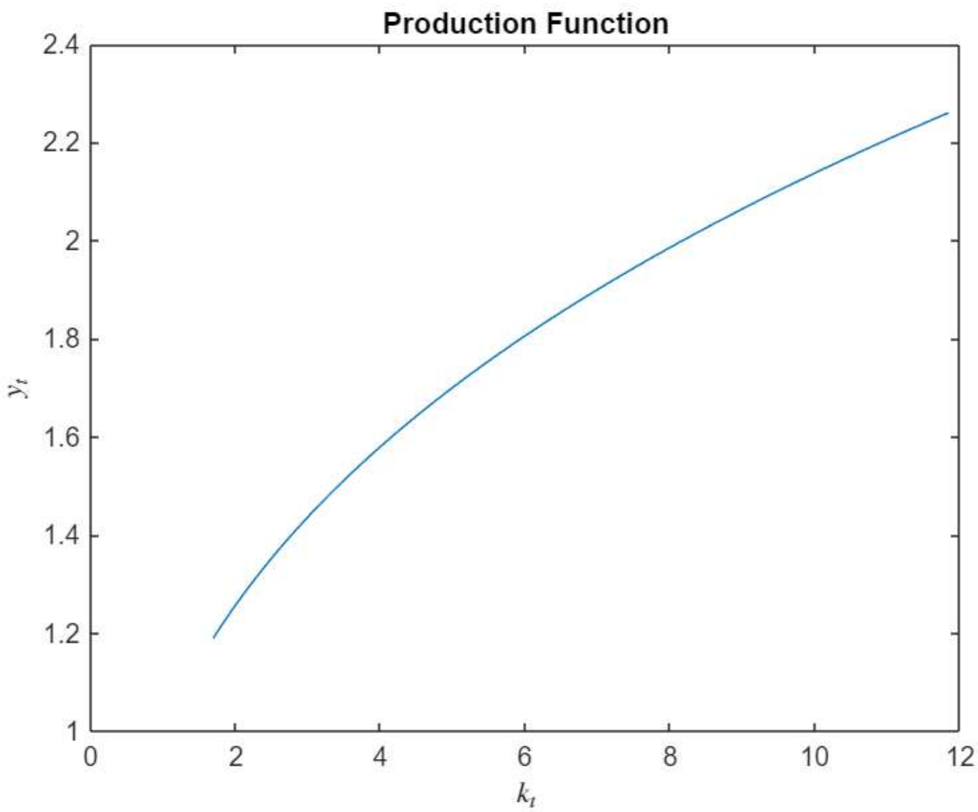
```
------------Beginning Value Function Iteration.------------

Iteration: 25.
Iteration: 50.
Iteration: 75.
Iteration: 100.
Iteration: 125.
Iteration: 150.
Iteration: 175.
Iteration: 200.
Iteration: 225.
Iteration: 250.
Iteration: 275.
Iteration: 300.
Iteration: 325.
Iteration: 350.
Iteration: 375.
Iteration: 400.
Iteration: 425.

Converged in 438 iterations

Model solved in 0.4219 seconds.
```

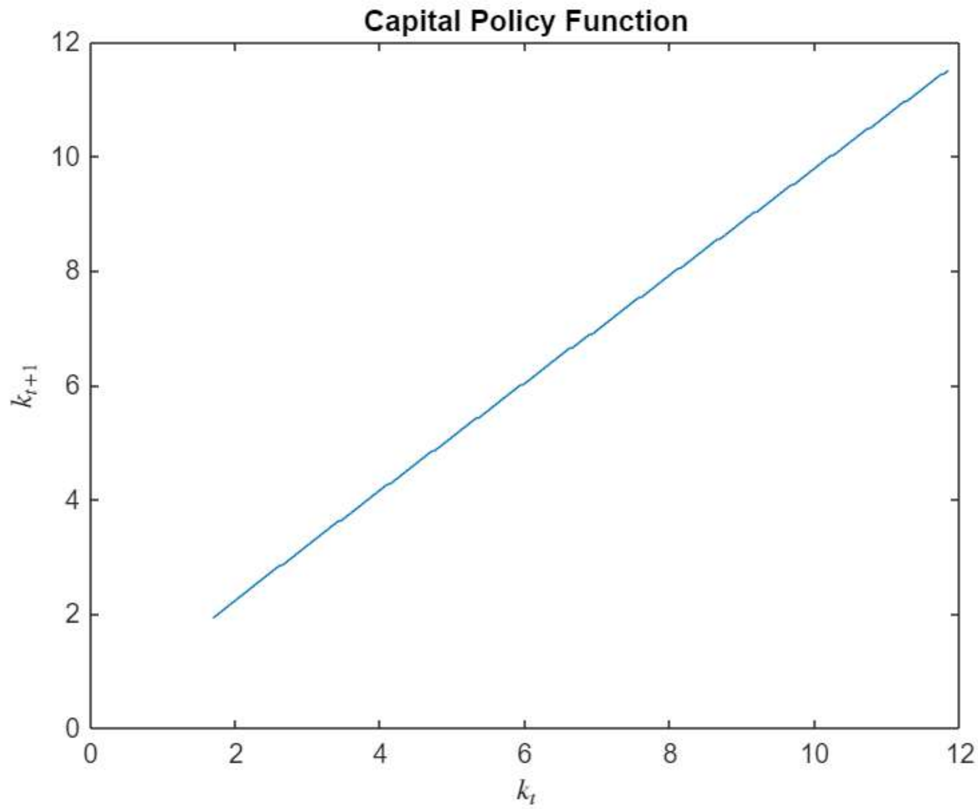
Simulate the model and plot the model functions and time series.

Calls: simulate.m and my_graph.m.

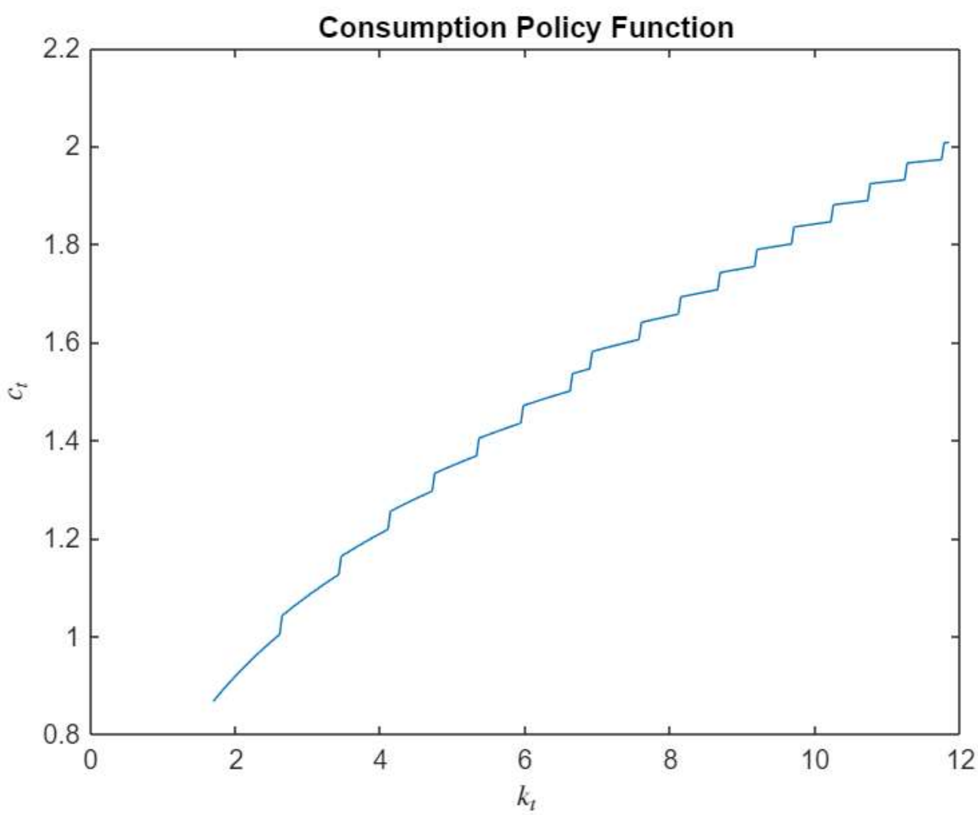
The production and value functions increase with k_t at a decreasing rate, as seen below, for most parameterizations of the model. The curvature of the former depends on α, while the curvatrure of the latter depends on σ. The policy functions for capital and consumption increase with k_t because y_t increases with k_t, leading to more resources for consumption and investment. The investment function is sensitive to the depreciation rate and may increase or decrease depending on how large it is. If δ is high, i_t is increasing in k_t because more investment is needed to maintain capital; otherwise, it is decreasing in k_t because less investment is needed to maintain capital when k_t is large.

When initial capital is below the steady state level, consumers accumulate capital which leads to growth in output and consumption. Investment increases or decreases depending on the current stock of capital and how fast it depreciates. Because consumption is increasing over time, so does utility received each period. The opposite happens if initial capital is above steady state. At some point, the economy reaches a steady state and there is no growth in the variables, indicated by the horizontal segment beyond time time period.

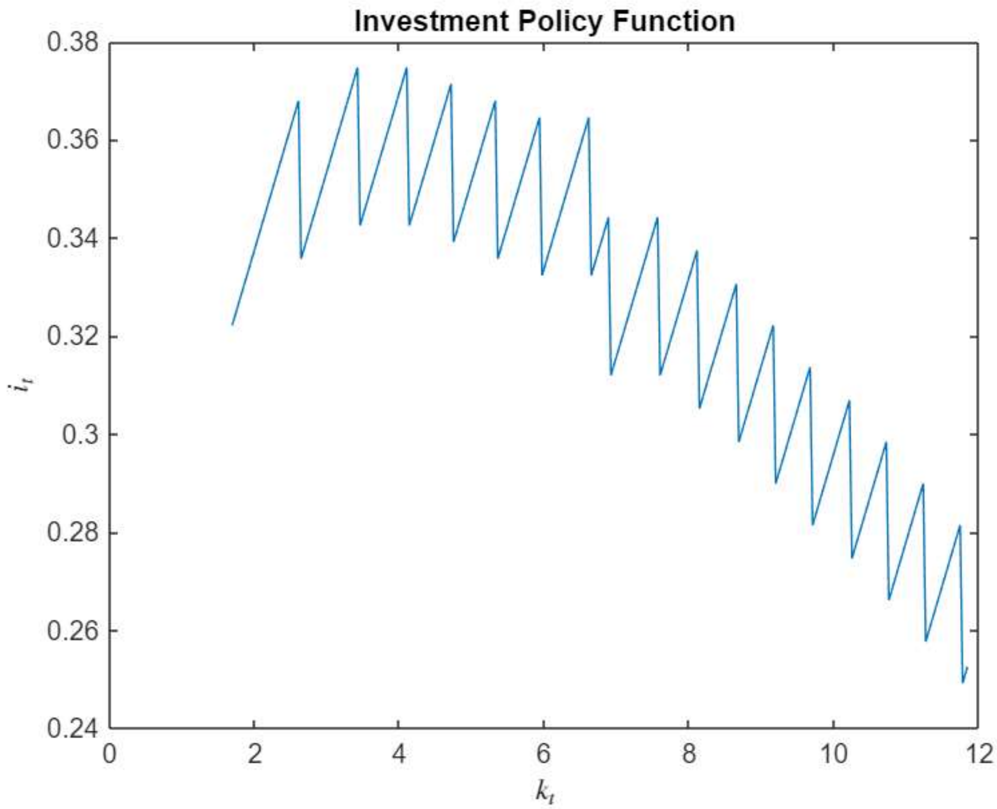


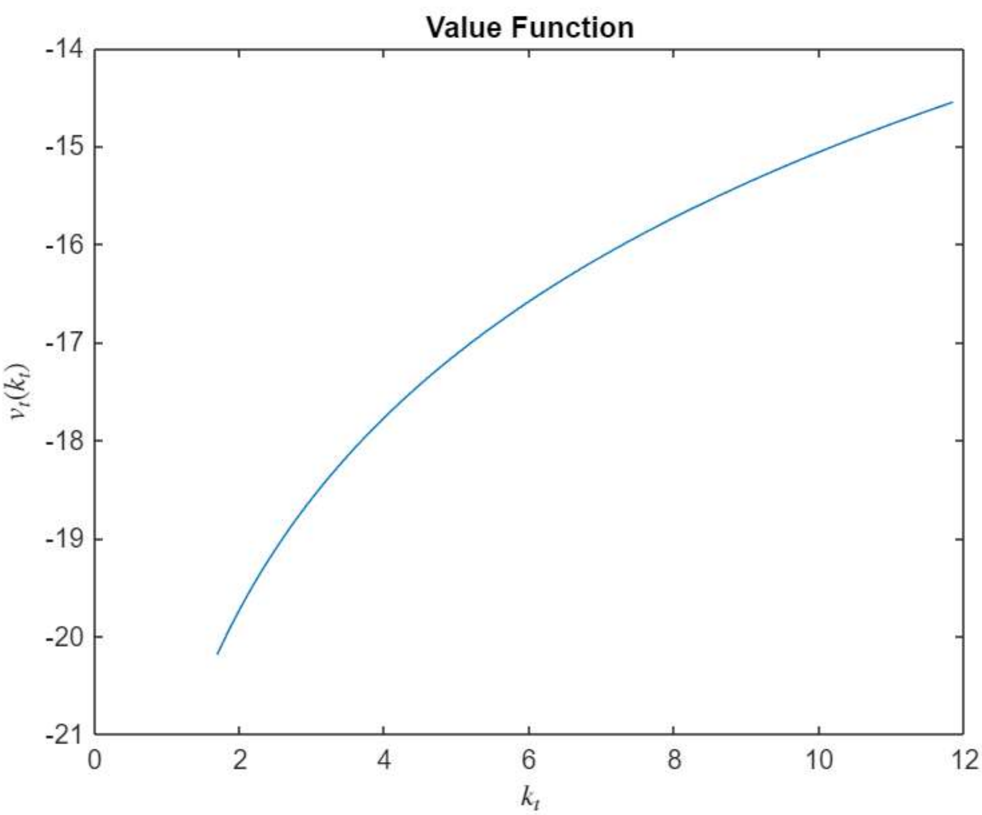


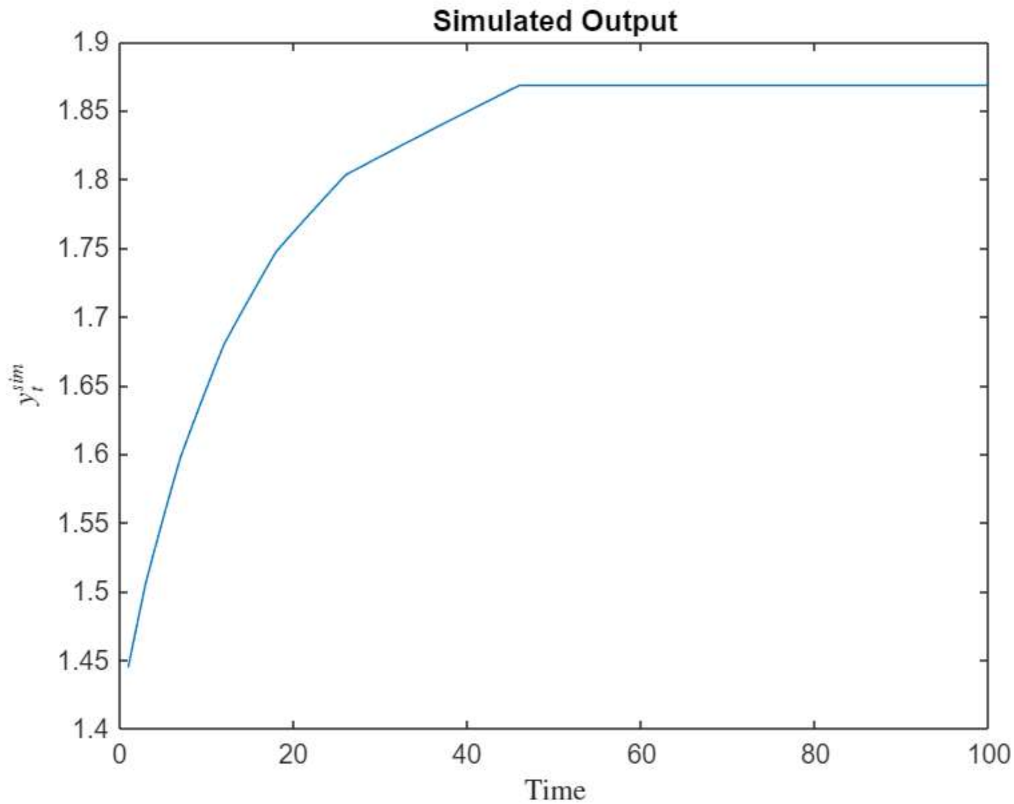
Capital Policy Function
k_{t+1}
k_t
0
2
4
6
8
10
12

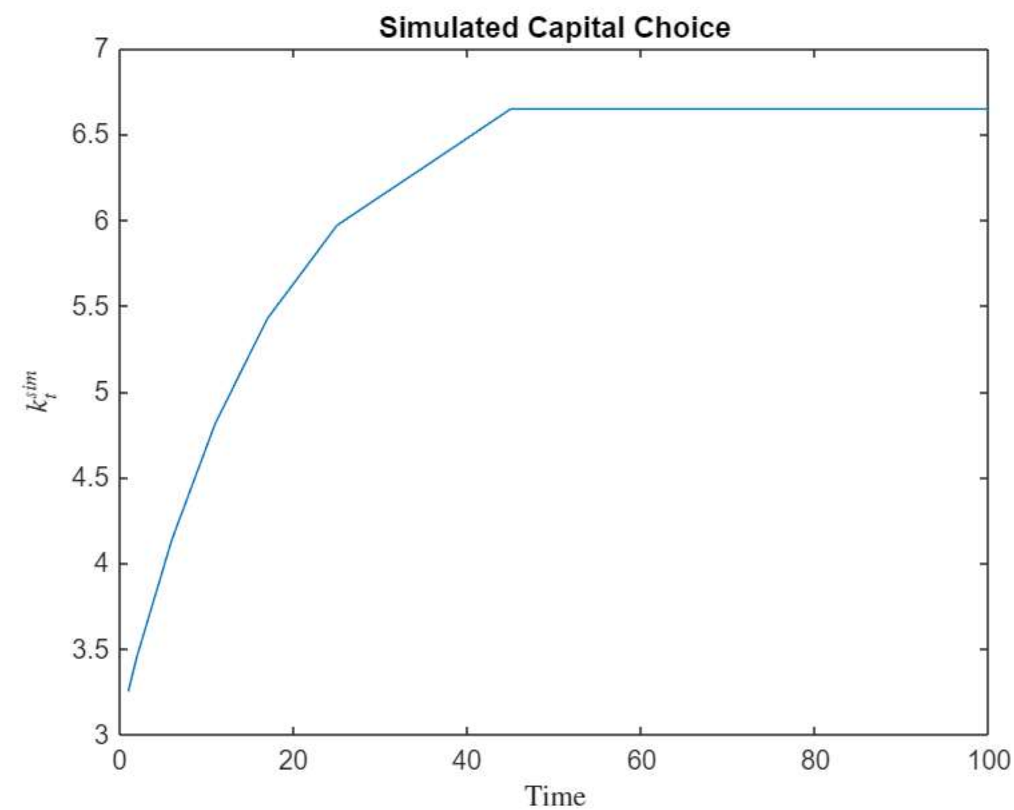


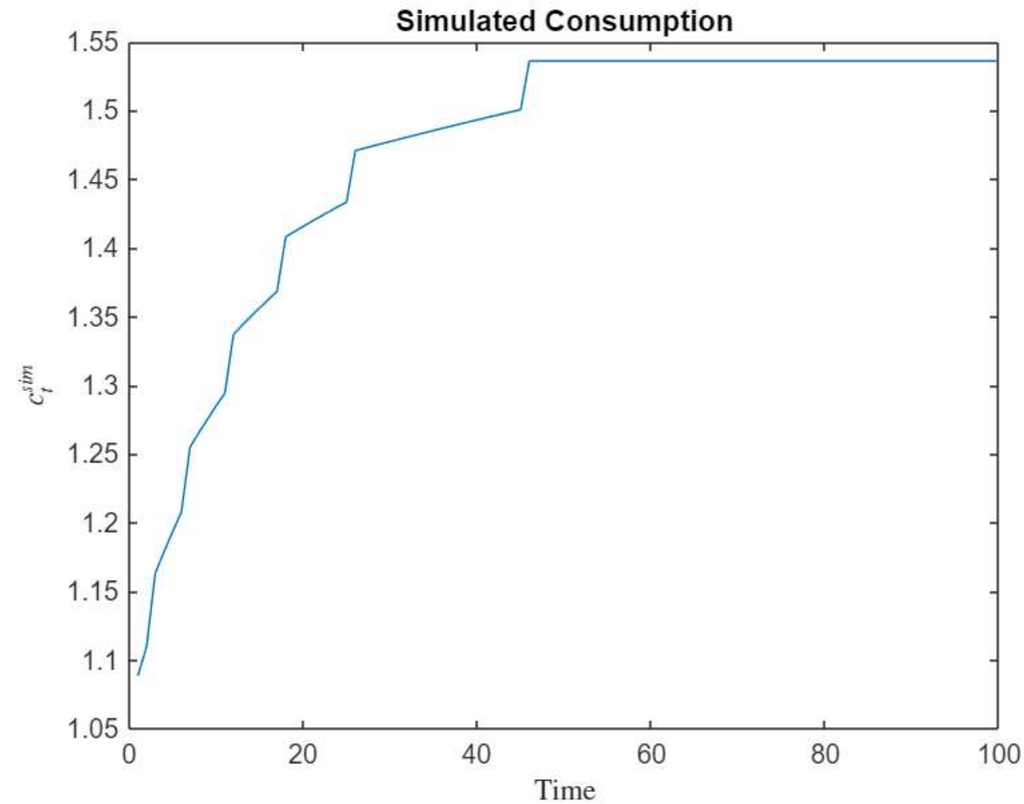
Consumption Policy Function
c_t
k_t
0.8
1
1.2
1.4
1.6
1.8
2
2.2
0
2
4
6
8
10
12

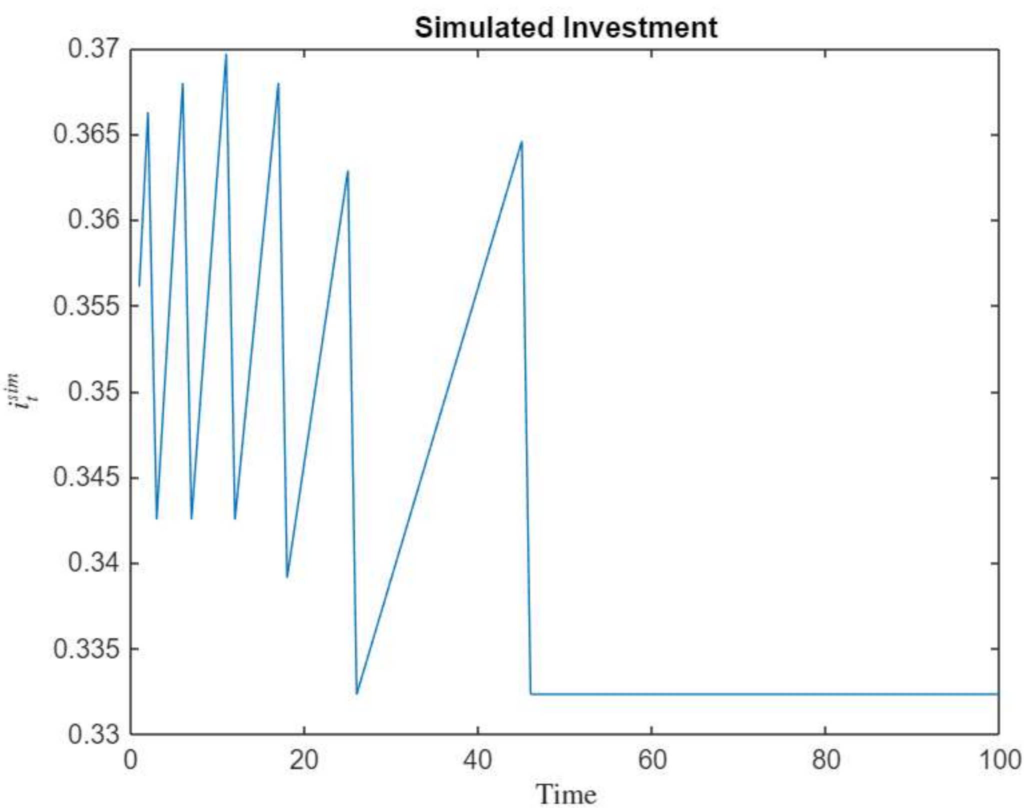

Investment Policy Function
0.38
0.36
0.34
0.32
0.3
0.28
0.26
0.24
i_t
0
2
4
6
8
10
12
k_t

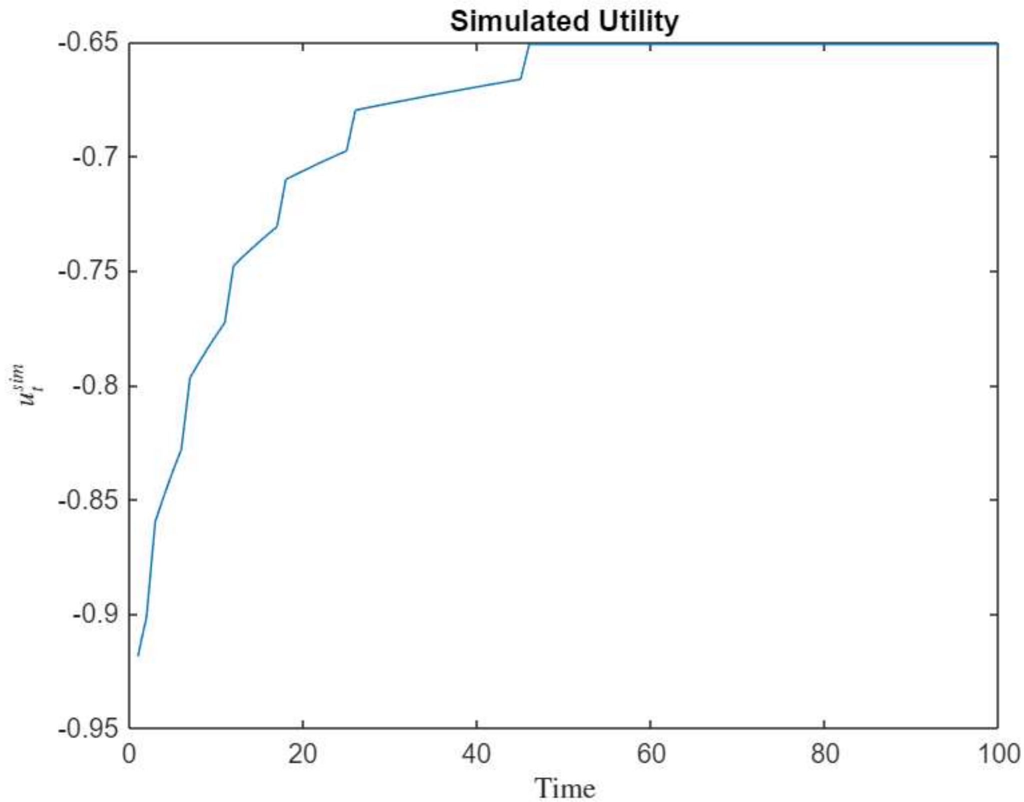

Value Function
-14
-15
-16
-17
-18
-19
-20
-21
$v_t(k_t)$
0
2
4
6
8
10
12
k_t


Simulated Output
1.9
1.85
1.8
1.75
1.7
1.65
1.6
1.55
1.5
1.45
1.4
y_t^{sim}
0
20
40
60
80
100
Time


Simulated Capital Choice
7
6.5
6
5.5
5
4.5
4
3.5
3
k_t^{sim}
0
20
40
60
80
100
Time


Simulated Consumption
1.55
1.5
1.45
1.4
1.35
1.3
1.25
1.2
1.15
1.1
1.05
c_t^{sim}
0
20
40
60
80
100
Time


Simulated Investment
0.37
0.365
0.36
0.355
0.35
0.345
0.34
0.335
0.33
i_t^{sim}
0
20
40
60
80
100
Time


Simulated Utility
-0.65
-0.7
-0.75
-0.8
-0.85
-0.9
-0.95
0
20
40
60
80
100
Time
u_t^{sim}