This is the final simplified Hartree term, H_{Hartree}, reduced to two momenta in the summation with the interaction potential $V(0)$ representing the static screened Coulomb interaction at zero momentum transfer.
Starting with the Fock term in $\hat{H}_{\text{int}}^{\text{HF}}$:

$$\hat{H}_{\text{int}}^{\text{HF}} = -\frac{1}{A} \sum_{\substack{\tau_\alpha,\tau_\beta \\ l_\alpha,l_\beta}} \sum_{\boldsymbol{k}_\alpha,\boldsymbol{k}_\beta,\boldsymbol{k}_\gamma,\boldsymbol{k}_\delta} \langle b^\dagger_{\boldsymbol{k}_\alpha,l_\alpha,\tau_\alpha} b_{\boldsymbol{k}_\gamma,l_\beta,\tau_\beta} \rangle b^\dagger_{\boldsymbol{k}_\beta,l_\beta,\tau_\beta} b_{\boldsymbol{k}_\delta,l_\alpha,\tau_\alpha} V(\boldsymbol{k}_\alpha - \boldsymbol{k}_\delta) \delta_{\boldsymbol{k}_\alpha+\boldsymbol{k}_\beta,\boldsymbol{k}_\gamma+\boldsymbol{k}_\delta}$$

By using the property that the expectation value is nonzero only when $k_\alpha = k_\gamma$:

$$\langle b^\dagger_{\boldsymbol{k}_\alpha,l_\alpha,\tau_\alpha} b_{\boldsymbol{k}_\gamma,l_\beta,\tau_\beta} \rangle = \langle b^\dagger_{\boldsymbol{k}_\alpha,l_\alpha,\tau_\alpha} b_{\boldsymbol{k}_\gamma,l_\beta,\tau_\beta} \rangle \delta_{\boldsymbol{k}_\alpha,\boldsymbol{k}_\gamma}$$

The Fock term becomes:

$$\hat{H}_{\text{Fock}} = -\frac{1}{A} \sum_{\substack{\tau_\alpha,\tau_\beta \\ l_\alpha,l_\beta}} \sum_{\boldsymbol{k}_\alpha,\boldsymbol{k}_\beta,\boldsymbol{k}_\delta} \langle b^\dagger_{\boldsymbol{k}_\alpha,l_\alpha,\tau_\alpha} b_{\boldsymbol{k}_\alpha,l_\beta,\tau_\beta} \rangle b^\dagger_{\boldsymbol{k}_\beta,l_\beta,\tau_\beta} b_{\boldsymbol{k}_\delta,l_\alpha,\tau_\alpha} V(\boldsymbol{k}_\alpha - \boldsymbol{k}_\delta) \delta_{\boldsymbol{k}_\alpha+\boldsymbol{k}_\beta,\boldsymbol{k}_\alpha+\boldsymbol{k}_\delta}$$

Simplifying further using momentum conservation:

$$\hat{H}_{\text{Fock}} = -\frac{1}{A} \sum_{\substack{\tau_\alpha,\tau_\beta \\ l_\alpha,l_\beta}} \sum_{\boldsymbol{k}_\alpha,\boldsymbol{k}_\beta} \langle b^\dagger_{\boldsymbol{k}_\alpha,l_\alpha,\tau_\alpha} b_{\boldsymbol{k}_\alpha,l_\beta,\tau_\beta} \rangle b^\dagger_{\boldsymbol{k}_\beta,l_\beta,\tau_\beta} b_{\boldsymbol{k}_\beta,l_\alpha,\tau_\alpha} V(\boldsymbol{k}_\alpha - \boldsymbol{k}_\beta)$$

This is the final simplified Fock term, H_{Fock}, reduced to two momenta in the summation with the interaction potential $V(\boldsymbol{k}_\alpha - \boldsymbol{k}_\beta)$ representing the screened Coulomb interaction between the relevant momentum states.
Starting with the given Hartree term H_{Hartree} and Fock term H_{Fock}:
Hartree term H_{Hartree}:

$$H_{\text{Hartree}} = \frac{1}{A} \sum_{\substack{\tau_\alpha,\tau_\beta \\ l_\alpha,l_\beta}} \sum_{\boldsymbol{k}_\alpha,\boldsymbol{k}_\beta} \langle b^\dagger_{\boldsymbol{k}_\alpha,l_\alpha,\tau_\alpha} b_{\boldsymbol{k}_\alpha,l_\alpha,\tau_\alpha} \rangle b^\dagger_{\boldsymbol{k}_\beta,l_\beta,\tau_\beta} b_{\boldsymbol{k}_\beta,l_\beta,\tau_\beta} V(0)$$

Fock term H_{Fock}:

$$H_{\text{Fock}} = -\frac{1}{A} \sum_{\substack{\tau_\alpha,\tau_\beta \\ l_\alpha,l_\beta}} \sum_{\boldsymbol{k}_\alpha,\boldsymbol{k}_\beta} \langle b^\dagger_{\boldsymbol{k}_\alpha,l_\alpha,\tau_\alpha} b_{\boldsymbol{k}_\alpha,l_\beta,\tau_\beta} \rangle b^\dagger_{\boldsymbol{k}_\beta,l_\beta,\tau_\beta} b_{\boldsymbol{k}_\beta,l_\alpha,\tau_\alpha} V(\boldsymbol{k}_\alpha - \boldsymbol{k}_\beta)$$

Since the quadratic operators in the Fock term already match those in the Hartree term, there is no need for additional relabeling. The final combined sum of the Hartree and Fock terms is:

$$H_{\text{combined}} = H_{\text{Hartree}} + H_{\text{Fock}}$$

$$= \frac{1}{A} \sum_{\substack{\tau_\alpha,\tau_\beta \\ l_\alpha,l_\beta}} \sum_{\boldsymbol{k}_\alpha,\boldsymbol{k}_\beta} \Bigg(\langle b^\dagger_{\boldsymbol{k}_\alpha,l_\alpha,\tau_\alpha} b_{\boldsymbol{k}_\alpha,l_\alpha,\tau_\alpha} \rangle b^\dagger_{\boldsymbol{k}_\beta,l_\beta,\tau_\beta} b_{\boldsymbol{k}_\beta,l_\beta,\tau_\beta} V(0)$$

$$- \langle b^\dagger_{\boldsymbol{k}_\alpha,l_\alpha,\tau_\alpha} b_{\boldsymbol{k}_\alpha,l_\beta,\tau_\beta} \rangle b^\dagger_{\boldsymbol{k}_\beta,l_\beta,\tau_\beta} b_{\boldsymbol{k}_\beta,l_\alpha,\tau_\alpha} V(\boldsymbol{k}_\alpha - \boldsymbol{k}_\beta) \Bigg)$$

This expression represents the final combined sum of the Hartree and Fock terms, simplified and ready to be used in further calculations.