

Fundamentals of Programming for Artificial Intelligence

**Session 09
Numpy**

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Numpy

- NumPy is a Python library used for working with arrays
- One of the most important foundational packages for numerical computing in Python.
- Most computational packages providing scientific functionality use NumPy's array objects as the lingua franca for data exchange.
- Providing fast array-oriented operations and flexible broadcasting capabilities

Array

- NumPy is used to work with arrays. The array object in NumPy is called **ndarray**
- Create a NumPy **ndarray** object by using the **array()** function

```
import numpy as np  
  
arr = np.array([1, 2, 3, 4, 5])  
  
print(arr) # [1 2 3 4 5]  
print(type(arr)) # <class 'numpy.ndarray'>
```

Array

```
import numpy as np

# from list
print(np.array([1, 2, 3])) # [1 2 3]

# from tuple
print(np.array((1, 2, 3))) # [1 2 3]
print(np.array((1, 2, 3))[2]) # 3

# from set
print(np.array({1, 2, 3})) # {1, 2, 3}
print(np.array({1, 2, 3})[2])

# IndexError: too many indices for array:
# array is 0-dimensional,
# but 1 were indexed
```

Dimensions in Arrays

- A dimension in arrays is one level of array depth (nested arrays)
- 0-D array or Scalars, are the elements in an array
- 1-D array has 0-D arrays as its elements, called uni-dimensional
- 2-D array has 1-D arrays as its elements, called 2-d matrix
- 3-D array has 2-D matrices as its elements

Dimensions in Arrays

```
import numpy as np

arr2d_1 = np.array([[1, 2, 3], [4, 5, 6]])
arr3d_2 = np.array([1, 2, 3, 4], ndmin=3)

print(arr2d_1) # [[1 2 3] [4 5 6]]
print(arr3d_2) # [[[1 2 3 4]]]
print(arr2d_1.ndim, arr3d_2.ndim) # 2 3
```

Shape & Type of ndarray

- Attribute: `ndarray.shape` – Tuple of array dimensions

```
a = np.array([[1, 2], [3, 4], [5, 6]])  
print(a) # [[1 2]  
# [3 4]  
# [5 6]]  
print(a.shape) # (3, 2)
```

- Attribute: `ndarray.dtype` – Data-type of the array's elements

```
a = np.array([0.5, 1, 2])  
print(a.dtype) # float64
```

Create Array

| Function | Description |
|------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>array</code> | Convert input data (list, tuple, array, or other sequence type) to an ndarray either by inferring a dtype or explicitly specifying a dtype; copies the input data by default |
| <code>asarray</code> | Convert input to ndarray, but do not copy if the input is already an ndarray |
| <code>arange</code> | Like the built-in <code>range</code> but returns an ndarray instead of a list |
| <code>ones,</code> <code>ones_like</code> | Produce an array of all 1s with the given shape and dtype; <code>ones_like</code> takes another array and produces a ones array of the same shape and dtype |
| <code>zeros,</code> <code>zeros_like</code> | Like <code>ones</code> and <code>ones_like</code> but producing arrays of 0s instead |
| <code>empty,</code> <code>empty_like</code> | Create new arrays by allocating new memory, but do not populate with any values like <code>ones</code> and <code>zeros</code> |
| <code>full,</code> <code>full_like</code> | Produce an array of the given shape and dtype with all values set to the indicated “fill value” <code>full_like</code> takes another array and produces a filled array of the same shape and dtype |
| <code>eye, identity</code> | Create a square $N \times N$ identity matrix (1s on the diagonal and 0s elsewhere) |

Example:

```
print(np.zeros(5)) # [0. 0. 0. 0. 0.]  
print(np.ones((2, 3))) # [[1. 1. 1.]  
# [1. 1. 1.]]
```

dtype

| Type | Type code | Description |
|-----------------------------------------|-----------------|------------------------------------------------------------------------------------------------------------------------|
| int8, uint8 | i1, u1 | Signed and unsigned 8-bit (1 byte) integer types |
| int16, uint16 | i2, u2 | Signed and unsigned 16-bit integer types |
| int32, uint32 | i4, u4 | Signed and unsigned 32-bit integer types |
| int64, uint64 | i8, u8 | Signed and unsigned 64-bit integer types |
| float16 | f2 | Half-precision floating point |
| float32 | f4 or f | Standard single-precision floating point; compatible with C float |
| float64 | f8 or d | Standard double-precision floating point; compatible with C double and Python float object |
| float128 | f16 or g | Extended-precision floating point |
| complex64, complex128, complex256 | c8, c16, c32 | Complex numbers represented by two 32, 64, or 128 floats, respectively |
| bool | ? | Boolean type storing True and False values |
| object | 0 | Python object type; a value can be any Python object |
| string_ | S | Fixed-length ASCII string type (1 byte per character); for example, to create a string dtype with length 10, use 'S10' |
| unicode_ | U | Fixed-length Unicode type (number of bytes platform specific); same specification semantics as string_ (e.g., 'U10') |

Type & Type cast

```
[3] #type cast
arr = np.array([1.5,2,3,4.5,5], dtype=np.float64) #dtype = float64
int_arr = arr.astype(np.int64) #dtype = int64
int_arr

array([1, 2, 3, 4, 5])
```

numpy.arange()

- **arange([start,] stop[, step,][, dtype])**

Parameters :

```
start : [optional] start of interval range. By default start = 0
stop  : end of interval range
step   : [optional] step size of interval. By default step size = 1,
For any output out, this is the distance between two adjacent values, out[i+1] - out[i].
dtype  : type of output array
```

Return:

Array of evenly spaced values.

Length of array being generated = `Ceil((stop - start) / step)`

numpy.arange()

```
a1 = np.arange(10)  
print(a1, "dim = %d" % a1.ndim)  
# [0 1 2 3 4 5 6 7 8 9] dim = 1  
  
a2 = np.arange(2, 10, 3)  
print(a2, "dim = %d" % a2.ndim)  
# [2 5 8] dim = 1
```

numpy.random.randn

- **random.randn(d0, d1, ..., dn):** Return a sample (or samples) from the “standard normal” distribution.

If positive int_like arguments are provided, `randn` generates an array of shape `(d0, d1, ..., dn)`, filled with random floats sampled from a univariate “normal” (Gaussian) distribution of mean 0 and variance 1. A single float randomly sampled from the distribution is returned if no argument is provided.

Parameters: `d0, d1, ..., dn : int, optional`

The dimensions of the returned array, must be non-negative. If no argument is given a single Python float is returned.

Returns: `Z : ndarray or float`

A `(d0, d1, ..., dn)`-shaped array of floating-point samples from the standard normal distribution, or a single such float if no parameters were supplied.

numpy.random.randn

```
a1 = np.random.randn()  
print(type(a1), a1) # <class 'float'> 1.3338705849916304  
  
a2 = np.random.randn(1)  
print(type(a2), a2) # <class 'numpy.ndarray'> [1.80212745]  
  
a3 = np.random.randn(2, 3)  
print(type(a3), a3.shape) # <class 'numpy.ndarray'> (2, 3)  
print(a3) # [[-0.2903103   0.03688071 -0.1302092 ]  
# [-1.27259517  0.69088078  1.98388177]]
```

Numerical operations on arrays

- Elementwise operations:

- With scalars:

```
a = np.array([1, 2, 3, 4])  
print(a + 1) # [2 3 4 5]  
print(3 - a) # [ 2  1  0 -1]  
print(a*2) # [2 4 6 8]  
print(2**a) # [ 2  4  8 16]  
print(a//2) # [0 1 1 2]
```

Numerical operations on arrays

- Elementwise operations: With array (same dimension):

```
a = np.array([1, 2, 3, 4])  
  
b = np.array([5, 1, 7, 4])  
  
print(a + b) # [ 6  3 10  8]  
print(a - b) # [-4  1 -4  0]  
print(a * b) # [ 5  2 21 16]  
print(a % b) # [1 0 3 0]  
print(2**a - b) # [-3  3  1 12]
```

Numerical operations on arrays

- Matrix multiplication

```
a = np.array([1, 2, 3, 4])
b = np.array([1, 2, 2, 1])
print(a * b) # element-wise multiplication
# [1 4 6 4]
print(a.dot(b)) # matrix multiplication
# 15
print(a.shape, b.shape) # (4,) (4,)
```

Numerical operations on arrays

Comparisons:

```
a = np.array([1, 2, 3, 4])
b = np.array([4, 2, 2, 4])
print(a == b) # [False  True False  True]
print(a > b) # [False False  True False]
```

Array-wise Comparisons:

```
a = np.array([1, 2, 3, 4])
b = np.array([4, 2, 2, 4])
print(np.array_equal(a, b)) # False
print(np.array_equal(a, a)) # True
```

Numerical operations on arrays

- Logical operations

```
a = np.array([1, 0, 3, 0])
b = np.array([4, 0, 2, 4])
print(np.logical_or(a, b))
# [ True False True True]
print(np.logical_and(a, b))
# [ True False True False]
```

List vs numpy array in Math operation

```
[ ] #Multidimensional array object  
my_np_data = np.random.randn(2,3)  
print(my_np_data)
```

```
[[ -0.08221029  0.36287271  0.44168154]  
 [ 0.639073    0.52044355 -1.09013561]]
```

```
[ ] my_list = [[-0.14660949,  0.10938535 ,  0.73807359],  
 [ 0.23479429, -0.18461008 , -1.22907498]]
```

```
[ ] my_np_data * 10
```

```
array([[ -0.82210292,   3.62872709,   4.41681535],  
       [  6.39073001,   5.20443547, -10.90135605]])
```

```
[ ] my_list * 2
```

```
[[ -0.14660949,  0.10938535 ,  0.73807359],  
 [ 0.23479429, -0.18461008 , -1.22907498],  
 [-0.14660949,  0.10938535 ,  0.73807359],  
 [ 0.23479429, -0.18461008 , -1.22907498]]
```

Indexing & Slicing

```
# One dimensions

arr = np.arange(10) # array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])
arr[5] # 5

arr[5:8] # array([5, 6, 7])

arr[5:8] = 12 # array([ 0,  1,  2,  3,  4, 12, 12, 12,  8,  9])

# Two dimensions

arr2d = np.array([[1, 2, 3], [4, 5, 6], [7, 8, 9]])
arr2d[2] #array([7, 8, 9])

arr2d[0][2] # 3

arr2d[0,2] # 3

arr2d[:2,1:] # array([[2, 3],[5, 6]])
```

Indexing & Slicing

```
# Three dimensions

a = np.array([[1, 2, 3], [4, 5, 6], [7, 8, 9], [10, 11, 12]])

print(a.shape) # (2, 2, 3)

temp = a[0] # [[1 2 3]
             #   [4 5 6]]

temp[:] = 0

# Explain the result if using the code line "temp = 0"

print(a) # [[[ 0  0  0]
           #   [ 0  0  0]]
           #   [[ 7  8  9]
           #   [10 11 12]]]
```

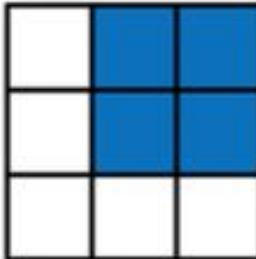
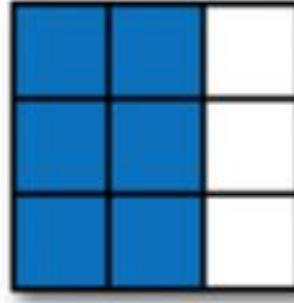
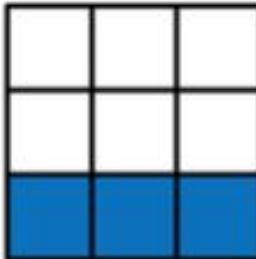
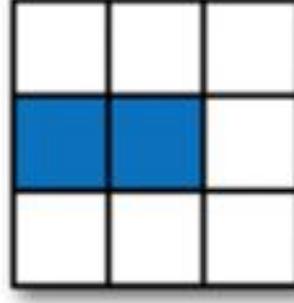
Indexing & Slicing

```
temp = 0
```

```
# Explain the result if using the code line "temp = 0"
```

```
print(a) # [[[ 1  2  3]
|           #   [ 4  5  6]]
|           #   [[ 7  8  9]
|           #   [10 11 12]]]
```

Indexing & Slicing

| Expression | Shape | | |
|------------------------------------------------------------------------------------------------------------|--------|--------------------------------------------------------------------------------------|----------------------------------|
|  <code>arr[:2, 1:]</code> | (2, 2) |  | <code>arr[:, :2]</code> (3, 2) |
|  <code>arr[2]</code> | (3,) |  | <code>arr[1, :2]</code> (2,) |
| <code>arr[2, :]</code> | (3,) | | <code>arr[1:2, :2]</code> (1, 2) |
| <code>arr[2:, :]</code> | (1, 3) | | |

Indexing & Slicing

```
>>> a[0, 3:5]
```

```
array([3, 4])
```

```
>>> a[4:, 4:]
```

```
array([[44, 55],  
       [54, 55]])
```

```
>>> a[:, 2]
```

```
a([2, 12, 22, 32, 42, 52])
```

```
>>> a[2::2, ::2]
```

```
array([[20, 22, 24],  
       [40, 42, 44]])
```

| | | | | | |
|----|----|----|----|----|----|
| 0 | 1 | 2 | 3 | 4 | 5 |
| 10 | 11 | 12 | 13 | 14 | 15 |
| 20 | 21 | 22 | 23 | 24 | 25 |
| 30 | 31 | 32 | 33 | 34 | 35 |
| 40 | 41 | 42 | 43 | 44 | 45 |
| 50 | 51 | 52 | 53 | 54 | 55 |

Copies & Views in ndarray

```
# View of ndarray via assignment operator  
  
a = np.array([[1, 2, 3], [4, 5, 6]])  
  
b = a[0] # creates a view  
  
print(b) # [1 2 3]  
  
a[0][0] = 0  
  
print(b) # [0 2 3]
```

Copies & Views in ndarray

```
# Copy of ndarray

a = np.array([[1, 2, 3], [4, 5, 6]])

b = a[0].copy() # creates a copy

print(b) # [1 2 3]

a[0][0] = 0

print(a[0]) # [0 2 3]

print(b) # [1 2 3]
```

Boolean indexing

- Numpy arrays support a feature called **conditional selection** to generate a new array of boolean values that state whether each element within the array satisfies a particular if statement.

```
arr = np.array([0.69, 0.94, 0.66, 0.73, 0.83])
index = arr > 0.7 # [False True False True  True]
print(arr[index]) # [0.94 0.73 0.83]
```

Boolean indexing

```
names = np.array(['Bob', 'Will', 'Joe', 'Bob', 'Will'])
data = np.random.randn(5, 3)
print(data)

[[ 1.64303 -0.91901438 -0.46691434]
 # [ 0.5523564 -1.29891734 -0.76211524]
 # [ 0.3985253  0.60424701  1.68980142]
 # [ 0.82260643  1.74365049 -0.01960414]
 # [ 0.94043782 -1.19145099  0.50533676]]

print(data[names == 'Bob'])

[[ 1.64303 -0.91901438 -0.46691434]
 # [ 0.82260643  1.74365049 -0.01960414]]

print(data[names == 'Bob', 1:])

#[[-0.91901438 -0.46691434]
# [ 1.74365049 -0.01960414]]

print(data[names == 'Bob'][1:]) # [[ 0.82260643  1.74365049 -0.01960414]]
```

Boolean indexing

```
data = np.random.randn(5, 3)
names = np.array(["Bob", "Will", "Joe", "Bob", "Will"])
print(data)

#[ [-1.54658869  0.08718386  0.3361064 ]
#  [-0.76355848  0.41091788  0.36225458]
#  [-0.66877597  1.33625116  1.15840969]
#  [ 0.3875389   -0.18037251  0.34739942]
#  [ 0.21145186  0.45118169  1.20429492]]

print(data[names != "Bob"])

#[ [-0.76355848  0.41091788  0.36225458]
#  [-0.66877597  1.33625116  1.15840969]
#  [ 0.21145186  0.45118169  1.20429492]]

print(~(names != "Bob")) # [ True False False  True False]
print(data[~(names != "Bob")])

#[ [-1.54658869  0.08718386  0.3361064 ]
#  [ 0.3875389   -0.18037251  0.34739942]]
```

Fancy Index

- Besides using indexing & slicing, NumPy provides you with a convenient way to index an array called fancy indexing.
- Fancy indexing allows you to index a numpy array using the following:
 - Another numpy array
 - A Python list
 - A sequence of integers

Fancy Index

```
a = np.arange(1, 10)
print(a) # [1 2 3 4 5 6 7 8 9]
indices = np.array([2, 3, 4])
print(a[indices]) # [3 4 5]
```

Fancy Index

```
[ ] arr = np.array([[ 0,  1,  2,  3],  
                  [ 4,  5,  6,  7],  
                  [ 8,  9, 10, 11],  
                  [12, 13, 14, 15],  
                  [16, 17, 18, 19],  
                  [20, 21, 22, 23],  
                  [24, 25, 26, 27],  
                  [28, 29, 30, 31]])
```

```
arr[[1,4,7]]
```

```
array([[ 4,  5,  6,  7],  
       [16, 17, 18, 19],  
       [28, 29, 30, 31]])
```

```
[ ] arr[[1,4, 7], [0, 0, 1]]
```

```
array([ 4, 16, 29])
```

```
[ ] arr[[1, 5, 7, 2]][:, [0, 3, 1, 2]]
```

```
array([[ 4,  7,  5,  6],  
       [20, 23, 21, 22],  
       [28, 31, 29, 30],  
       [ 8, 11,  9, 10]])
```

Reshape

- Reshaping means changing the shape of an array

```
arr = np.array([1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12])
newArr = arr.reshape(4, 3)
print(newArr)
# [[ 1  2  3]
#  [ 4  5  6]
#  [ 7  8  9]
#  [10 11 12]]
```

Reshape

- Reshaping returns the view of the original array

```
arr = np.array([1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12])
newArr = arr.reshape(4, 3)
print(newArr)
# [[ 1  2  3]
#  [ 4  5  6]
#  [ 7  8  9]
#  [10 11 12]]
arr[1] = 10
print(newArr)
# [[ 1 10  3]
#  [ 4  5  6]
#  [ 7  8  9]
#  [10 11 12]]
```

Reshape: Unknown Dimension

- Pass -1 as the value, and NumPy will calculate this number for you

```
arr = np.array([1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12])
newArr = arr.reshape(2, 3, -1)
print(newArr)
# [[[ 1  2]
#   [ 3  4]
#   [ 5  6]]
#   [[ 7  8]
#   [ 9 10]
#   [11 12]]]
```

Exercise

- Ex1: Given an integer matrix of NxM, set negative values in the matrix to 0
- Ex2: Given an integer matrix of NxM, set negative values in even rows to 0

Exercise

```
def setZero4Negative_Index(arr):  
    shape = arr.shape  
    for row in range(shape[0]):  
        for col in range(shape[1]):  
            if (arr[row][col] < 0):  
                arr[row][col] = 0
```

```
arr1 = np.array([[1, 2, -3], [-1, -2, 3]])  
setZero4Negative_Index(arr1)  
print(arr1)
```

Exercise

```
def setZero4Negative(arr):  
    arr[arr < 0] = 0  
  
arr2 = np.array([[1, 2, -3], [-1, -2, 3]])  
setZero4Negative(arr2)  
print(arr2)
```

Exercise

```
def setZero4Negative_Index(arr):
    shape = arr.shape
    for row in range(shape[0]):
        for col in range(shape[1]):
            if (row % 2 == 0):
                if (arr[row][col] < 0):
                    arr[row][col] = 0

arr1 = np.array([[1, 2, -3], [-1, -2, 3], [-3, -2, 4]])
setZero4Negative_Index(arr1)
print(arr1)
```

Exercise

```
def setZero4Negative(arr):
    index = arr < 0
    for i in range(len(arr)): # range(arr.shape[0])
        if (i % 2 == 0): arr[i][index[i]] = 0

arr2 = np.array([[1, 2, -3], [-1, -2, 3], [-3, -2, 4]])
setZero4Negative(arr2)
print(arr2)
```

Exercise

```
arr3 = np.array([[1, 2, -3], [-1, -2, 3], [-3, -2, 4]])  
arrEvenRow = arr3[::2]  
arrEvenRow[arrEvenRow < 0] = 0  
print(arr3)
```

Exercise

Ex3: Create a checkboard matrix NxN as follows:

```
[[0 1 0 1 0 1 0 1]
 [1 0 1 0 1 0 1 0]
 [0 1 0 1 0 1 0 1]
 [1 0 1 0 1 0 1 0]
 [0 1 0 1 0 1 0 1]
 [1 0 1 0 1 0 1 0]
 [0 1 0 1 0 1 0 1]
 [1 0 1 0 1 0 1 0]]
```

Exercise

```
def createMatrix(n: int):
    arr = np.zeros((n, n), dtype = np.int64)
    mark = 0
    for row in range(n):
        for col in range(n):
            arr[row][col] = mark
            mark = not(mark)
        mark = not(mark)
    return arr
```

Exercise

```
def createMatrixVer2(n: int):  
    arr = np.zeros((n, n), dtype = np.int64)  
    arr[1::2, ::2] = 1  
    arr[::-2, 1::2] = 1  
    return arr
```

Exercise

Ex4: Create a matrix NxM with 1 in borders and 0 for inside:

```
[[1 1 1 1 1]
 [1 0 0 0 1]
 [1 0 0 0 1]
 [1 0 0 0 1]
 [1 1 1 1 1]]
```

Exercise

```
def createMatrix(n: int, m: int):
    arr = np.zeros((n, m), dtype = np.int64)
    for row in range(n):
        for col in range(n):
            if (row == 0 or row == n-1):
                arr[row][col] = 1
            if (col == 0 or col == m - 1):
                arr[row][col] = 1
    return arr
```

Exercise

```
def createMatrixVer2(n: int, m: int):  
    arr = np.zeros((n, m), dtype = np.int64)  
    arr[0, :] = 1  
    arr[n-1, :] = 1  
    arr[:, 0] = 1  
    arr[:, m-1] = 1  
    return arr
```

Exercise

```
def createMatrixVer3(n: int, m: int):  
    arr = np.ones((n, m), dtype = np.int64)  
    arr[1:-1, 1:-1] = 0  
    return arr
```

Exercise

Ex5: Create a NxN matrix as follows:

N = 4

```
[[100    1    2    3]
 [  4 100    6    7]
 [  8    9 100   11]
 [ 12   13   14 100]]
```

Exercise

```
def createMatrix(n: int):
    content = list(range(n*n))
    arr = np.array(content)
    arr = arr.reshape(n, -1)
    for row in range(n):
        for col in range(n):
            if (col == row):
                arr[row][col] = 100
    return arr
```

Exercise

```
def createMatrixVer2(n: int):  
    arr = np.arange(n*n)  
    arr[0::5] = 100  
    return arr.reshape(n, -1)
```

Exercise

Ex6: Write a function to multiply two matrices NxN without using np.dot()

Exercise

```
def matrix_multiply(a, b):
    n = len(a)
    c = np.array([[0 for i in range(n)] for j in range(n)])
    for i in range(n):
        for j in range(n):
            for k in range(n):
                c[i][j] += a[i][k] * b[k][j]
    return c
```

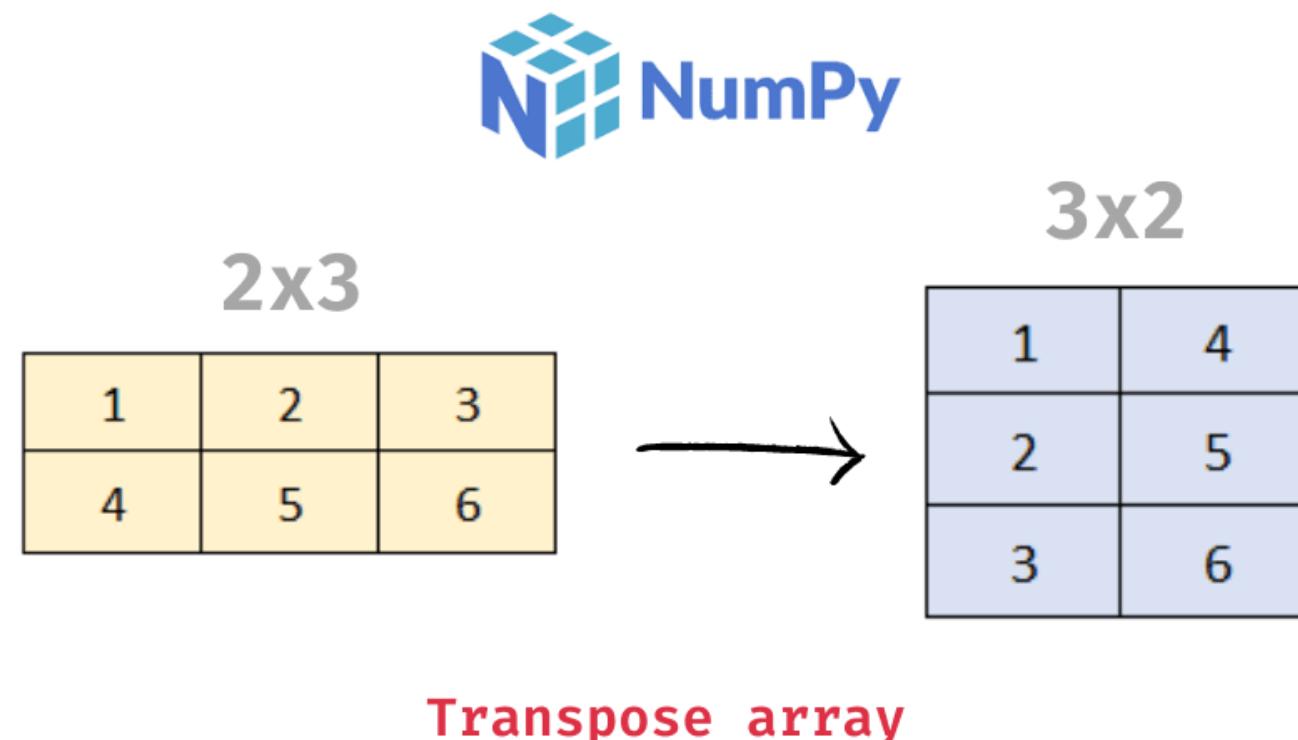
Exercise

```
def matrixMul(a, b):  
    return a@b
```

Transposing Arrays

fit@hcmus

- In Python, numpy.`transpose()` is used to get the permute or reserve the dimension of the input array



`numpy.transpose(a, axes=None)`

[source]

Returns an array with axes transposed.

For a 1-D array, this returns an unchanged view of the original array, as a transposed vector is simply the same vector. To convert a 1-D array into a 2-D column vector, an additional dimension must be added, e.g., `np.atleast2d(a).T` achieves this, as does `a[:, np.newaxis]`. For a 2-D array, this is the standard matrix transpose. For an n-D array, if axes are given, their order indicates how the axes are permuted (see Examples). If axes are not provided, then `transpose(a).shape == a.shape[::-1]`.

Transposing Arrays

fit@hcmus

`numpy.transpose(a, axes=None)`

Returns an array with axes transposed.

Parameters: `a : array_like`

Input array.

`axes : tuple or list of ints, optional`

If specified, it must be a tuple or list which contains a permutation of $[0, 1, \dots, N-1]$ where N is the number of axes of a . The i 'th axis of the returned array will correspond to the axis numbered `axes[i]` of the input. If not specified, defaults to `range(a.ndim)[::-1]`, which reverses the order of the axes.

Returns: `p : ndarray`

a with its axes permuted. A view is returned whenever possible.

Transposing Arrays

fit@hcmus

```
# 1-D array  
a = np.array([1, 2, 3, 4])  
print(a.transpose()) # [1 2 3 4]
```

```
#2-D array  
a = np.array([[1, 2], [3, 4]])  
print(a.transpose())  
# [[1 3]  
#  [2 4]]
```

Transposing Arrays

fit@hcmus

- The result of the following code:

```
a = np.arange(8)  
a = a.reshape((4, 2))  
print(a.T)
```

- The result of the following code:

```
a = np.arange(8)  
a = a.reshape((4, 2))  
print(a.T)  
# [[0 2 4 6]  
#  [1 3 5 7]]
```

Transposing Arrays

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- The result of the following code:

```
a = np.arange(6).reshape((3, 2))  
print(a.dot(a.T))
```

$$\begin{bmatrix} [0 1] \\ [2 3] \\ [4 5] \end{bmatrix} \times \begin{bmatrix} [0 2 4] \\ [1 3 5] \end{bmatrix} = \begin{bmatrix} [1 3 5] \\ [3 13 23] \\ [5 23 41] \end{bmatrix}$$

Transposing Arrays

fit@hcmus

- 3-d Array

```
a = np.arange(2*2*3).reshape((2,2,3))

print(a)

# [[[ 0  1  2]
#   [ 3  4  5]
#   [ 6  7  8]
#   [ 9 10 11]]]

print(a.T)

# [[[ 0  6]
#   [ 3  9]
#   [ 1  7]
#   [ 4 10]]
#   [[ 2  8]
#   [ 5 11]]]
```

Transposing Arrays

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- 3-d Array

```
a = np.arange(2*2*3).reshape((2,2,3))

print(a.transpose())

# equivalent to

print(a.transpose(2, 1, 0))

# [[[ 0  6]
#   [ 3  9]]
#   [[ 1  7]
#   [ 4 10]]
#   [[ 2  8]
#   [ 5 11]]]
```

- 3-d Array

```
a = np.arange(12).reshape((2, 2, 3))

print(a.transpose(1, 0, 2))

# [[[ 0  1  2]
#   [ 6  7  8]]
#
# [[ 3  4  5]
#   [ 9 10 11]]]
```

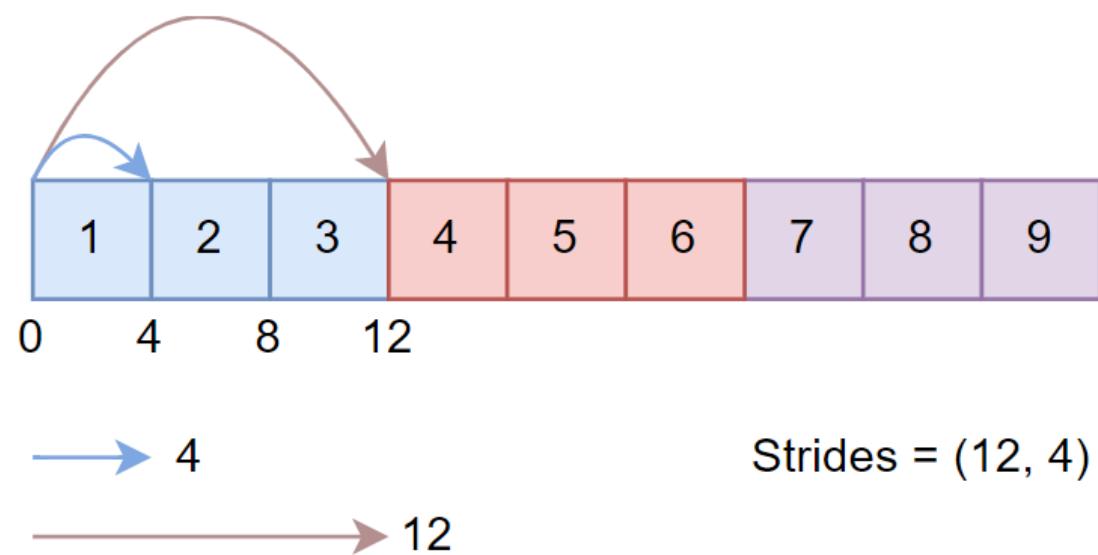
Strides in ndarray

fit@hcmus

- A stride is a tuple of integer numbers, each of which indicates the bytes for a particular dimension
- NumPy uses strides to tell how many bytes to jump in the data buffer

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |

Array



Data Buffer

- When we transpose an array, its stride is also changed via its corresponding axis.

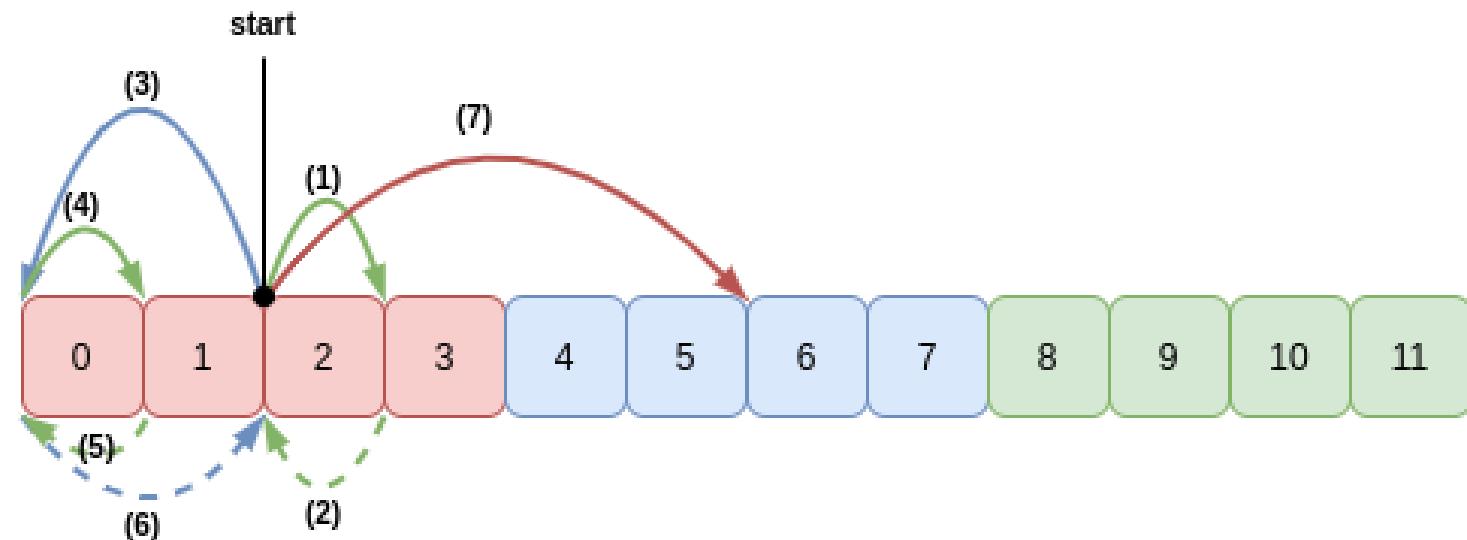
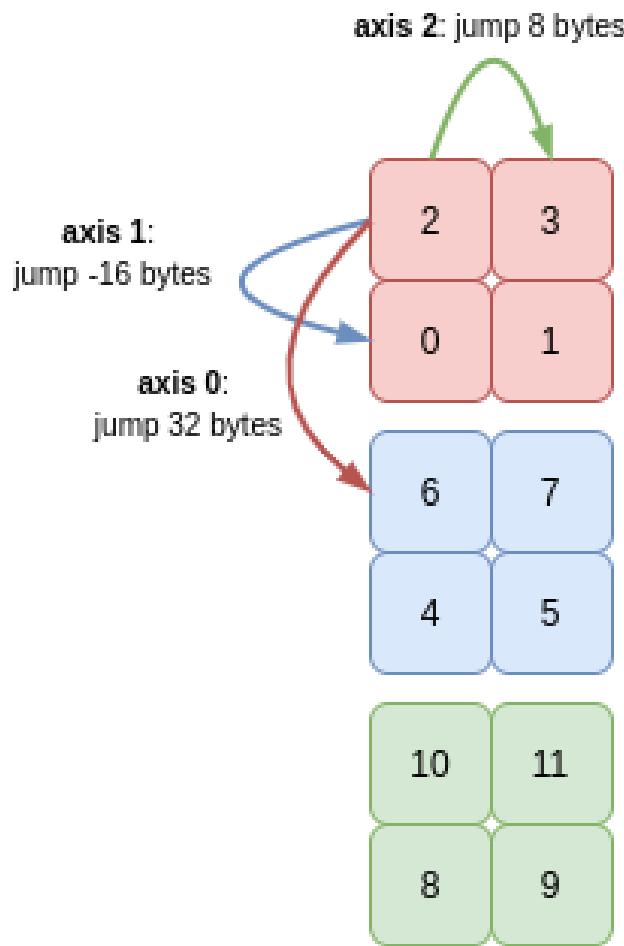
```
a = np.arange(9).reshape(3,3)
print(a.strides) # (12, 4)

b = a.transpose()
print(b.strides) # (4, 12)
```

Strides in ndarray

fit@hcmus

- In 3-d Array:



- What is the output of the following code

```
a = np.arange(16).reshape((2, 2, 4))  
b = a.transpose(1, 0, 2)  
print(b)
```

Transposing Arrays

fit@hcmus

- What is the output of the following code

```
a = np.arange(16).reshape((2, 2, 4))  
b = a.transpose(1, 0, 2)  
print(b)
```

```
[[[ 0  1  2  3]  
 [ 4  5  6  7]]
```

```
[[ 8  9 10 11]  
 [12 13 14 15]]]
```

```
[[[ 0  1  2  3]  
 [ 8  9 10 11]]
```

```
[[ 4  5  6  7]  
 [12 13 14 15]]]
```

- Given an array as follows:

```
array([ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11])
```

- Let convert to

```
array([ 0, 4, 8, 1, 5, 9, 2, 6, 10, 3, 7, 11])
```

```
import numpy as np
arr = np.array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11])

matrix = arr.reshape(3, 4)

# Transpose the matrix
transposed_matrix = matrix.T

# Flatten the transposed matrix back into a 1D array
result_arr = transposed_matrix.flatten()

print(result_arr)
```

- **numpy.swapaxes()** function interchange two axes of an array

```
numpy.swapaxes(a, axis1, axis2) #
```

[source]

Interchange two axes of an array.

Parameters: a : *array_like*

Input array.

axis1 : *int*

First axis.

axis2 : *int*

Second axis.

Returns: a_swapped : *ndarray*

For NumPy >= 1.10.0, if *a* is an ndarray, then a view of *a* is returned; otherwise a new array is created. For earlier NumPy versions a view of *a* is returned only if the order of the axes is changed, otherwise the input array is returned.

- `numpy.swapaxes()` is quite similar to transpose

```
a = np.arange(9).reshape(3,3)
print(a.transpose())
print(a.swapaxes(0, 1))
# [[0 3 6]
#  [1 4 7]
#  [2 5 8]]
```

- What is the output of the following code:

```
a = np.arange(12).reshape((3,2,2))  
print(a.swapaxes(1, 2))
```

- What is the output of the following code:

```
a = np.arange(12).reshape((3,2,2))  
print(a.swapaxes(1, 2))
```

[[[0 2]
 [1 3]]]

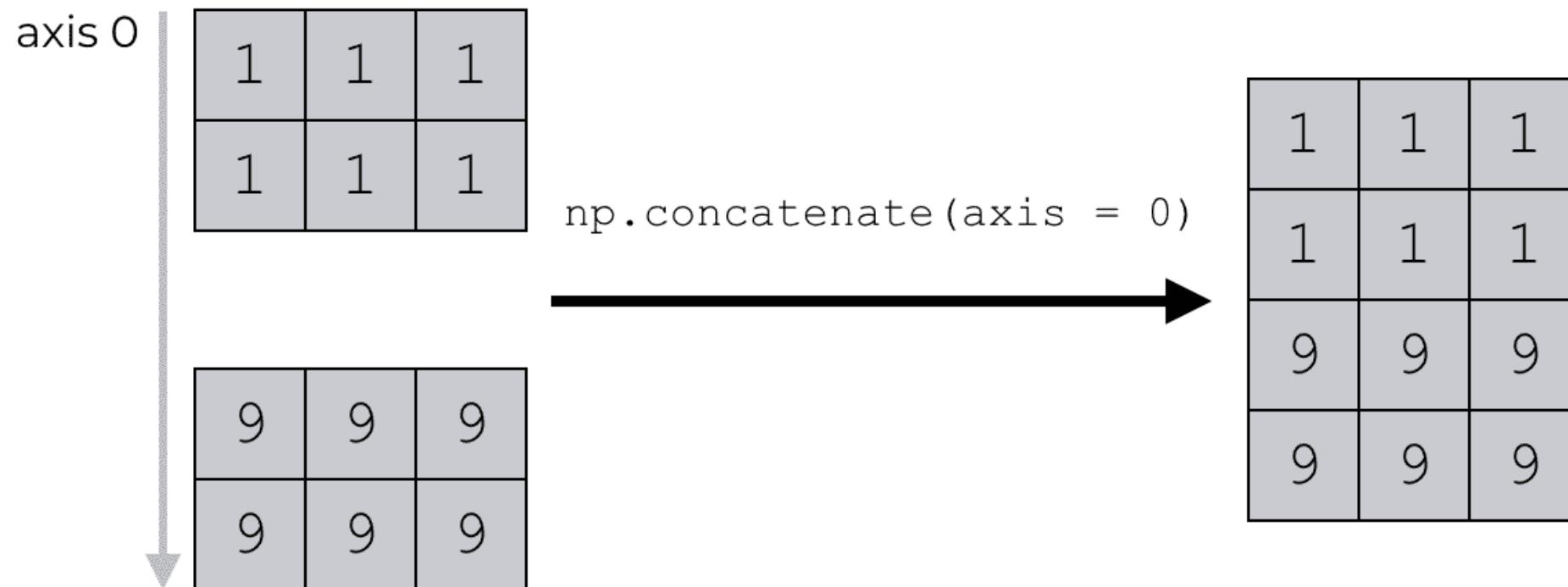
[[4 6]
 [5 7]]]

[[8 10]
 [9 11]]]

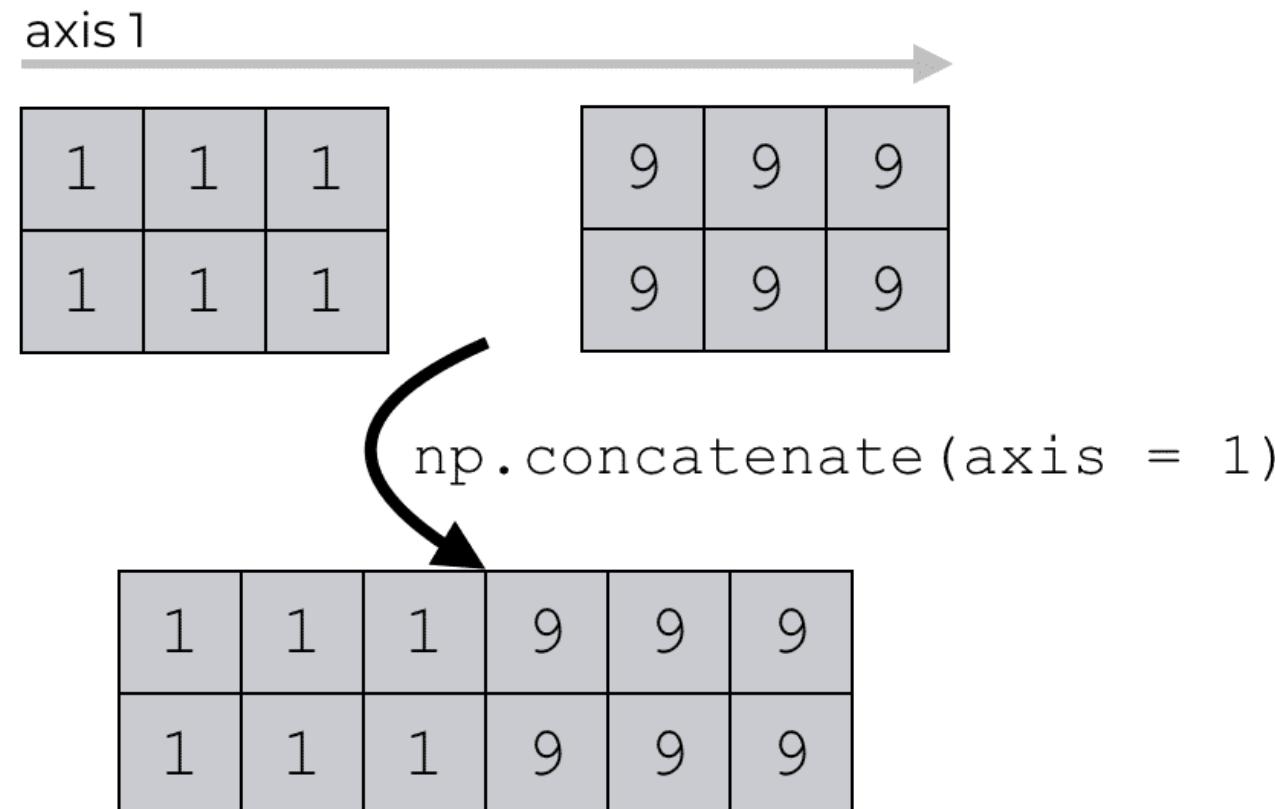
- Concatenation refers to joining.
- This function is used to join two or more arrays of the same shape along a specified axis

```
numpy.concatenate((a1, a2, ...), axis)
```

Setting axis=0 concatenates along the row axis



Setting axis=1 concatenates along the column axis



Concatenation

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1D array

| | | | |
|---|---|---|----|
| 7 | 2 | 9 | 10 |
|---|---|---|----|

axis 0 →

shape: (4,)

2D array

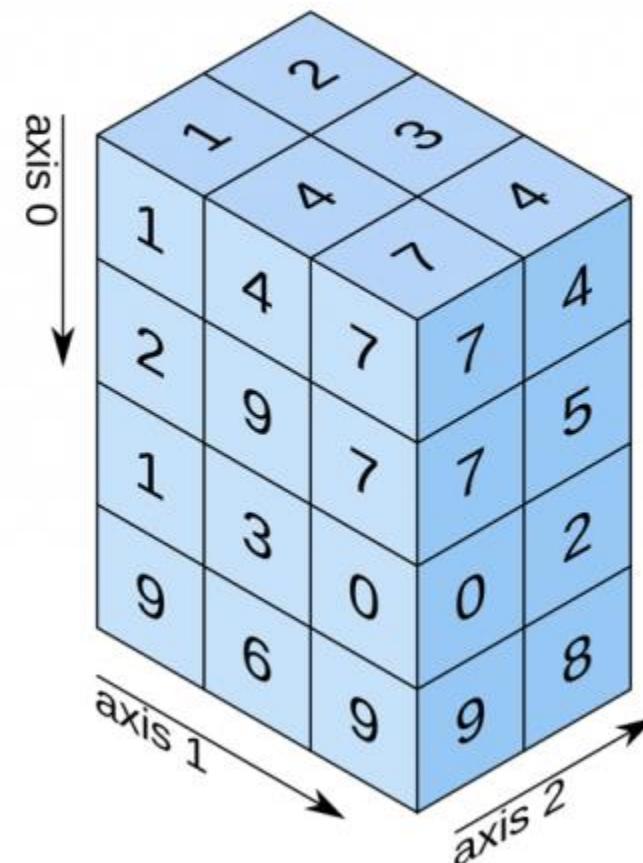
axis 0 ↓

| | | |
|-----|-----|-----|
| 5.2 | 3.0 | 4.5 |
| 9.1 | 0.1 | 0.3 |

axis 1 →

shape: (2, 3)

3D array



shape: (4, 3, 2)

Concatenation

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```
a = np.arange(12).reshape((3,2,2))
b = a + 2
c = np.concatenate([a, b], axis = 2)
print(c)
```

```
[[[ 0  1  2  3]
  [ 2  3  4  5]]
 [[ 4  5  6  7]
  [ 6  7  8  9]]
 [[ 8  9 10 11]
  [10 11 12 13]]]
```

Concatenation

```
[[[ 0  1]  
 [ 2  3]  
 [ 2  3]  
 [ 4  5]]
```

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```
a = np.arange(12).reshape((3,2,2))  
b = a + 2  
c = np.concatenate([a, b], axis = 1)  
print(c)
```

```
[[ 4  5]  
 [ 6  7]  
 [ 6  7]  
 [ 8  9]]
```

```
[[ 8  9]  
 [10 11]  
 [10 11]  
 [12 13]]]
```

```
arr = np.arange(3) # array([0, 1, 2])  
  
arr.repeat(3) # array([0, 0, 0, 1, 1, 1, 2, 2, 2])  
  
arr.repeat([2, 3, 4]) # array([0, 0, 1, 1, 1, 2, 2, 2, 2])
```

```
a = np.arange(4).reshape((1,2,2))
print(a)
# [[[0 1]
#   [2 3]]]
b = a.repeat(2)
print(b) # [0 0 1 1 2 2 3 3]
```

np.repeat()

fit@hcmus

```
arr = np.arange(4).reshape(2, 2)
arr
```

```
array([[0, 1],
       [2, 3]])
```

```
arr.repeat(2, axis=0)
```

```
array([[0, 1],
       [0, 1],
       [2, 3],
       [2, 3]])
```

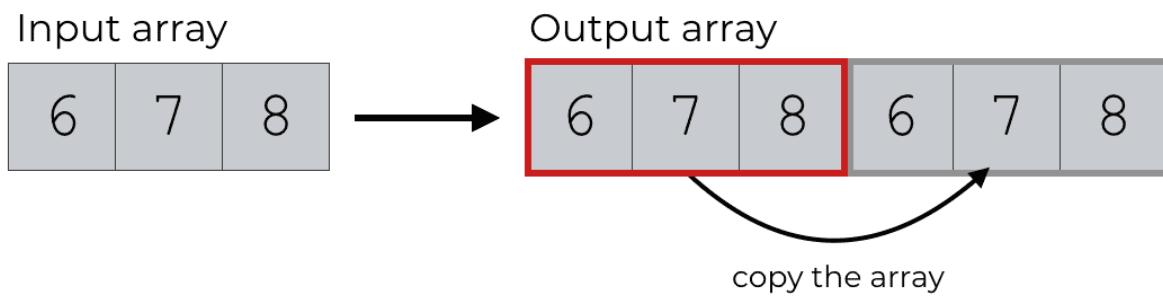
```
arr.repeat([2, 3], axis=0)
```

```
array([[0, 1],
       [0, 1],
       [2, 3],
       [2, 3],
       [2, 3]])
```

```
arr.repeat([2, 3], axis=1) #?
```

- numpy.tile() function constructs a new array by repeating array

np.tile CREATES A NEW ARRAY THAT CONTAINS
SEVERAL COPIES OF THE INPUT ARRAY



```
a = np.array([1, 2, 3, 4, 5])
b = np.tile(a, 2)
c = a.repeat(2)
print(b) # [1 2 3 4 5 1 2 3 4 5]
print(c) # [1 1 2 2 3 3 4 4 5 5]
```

numpy.tile()

fit@hcmus

```
[ ] np.tile(arr, 2)
```

```
array([[0, 1, 0, 1],  
       [2, 3, 2, 3]])
```

```
[ ] np.tile(arr, (2, 1))
```

```
array([[0, 1],  
       [2, 3],  
       [0, 1],  
       [2, 3]])
```

```
[ ] np.tile(arr, (2, 2)) #?
```

numpy.tile()

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```
[ ] np.tile(arr, 2)
```

```
array([[0, 1, 0, 1],  
       [2, 3, 2, 3]])
```

```
[ ] np.tile(arr, (2, 1))
```

```
array([[0, 1],  
       [2, 3],  
       [0, 1],  
       [2, 3]])
```

```
[[0 1 0 1]]
```

```
[2 3 2 3]
```

```
[0 1 0 1]
```

```
[2 3 2 3]]
```

```
[ ] np.tile(arr, (2, 2)) #?
```

Universal functions

fit@hcmus

▶ #Universal functions which perform element-wise operations on ndarrays

```
arr = np.random.randn(10)
```

```
np.square(arr)
```

```
⇒ array([3.16274687e+00, 2.29088562e+00, 6.14659767e-06, 1.11834661e-01,
          7.97263403e-02, 1.15980131e+00, 7.93531615e-01, 5.14354487e-01,
          2.02893154e+00, 2.65375327e-01])
```

```
[ ] np.exp(arr)
```

```
array([1.00000000e+00, 2.71828183e+00, 7.38905610e+00, 2.00855369e+01,
       5.45981500e+01, 1.48413159e+02, 4.03428793e+02, 1.09663316e+03,
       2.98095799e+03, 8.10308393e+03])
```

```
[ ] x = np.random.randn(8) #array([-0.0119,  1.0048,  1.3272, -0.9193, -1.5491,  0.0222,  0.7584,-0.6605])
y = np.random.randn(8) #array([ 0.8626, -0.01 ,  0.05 ,  0.6702,  0.853 , -0.9559, -0.0235,-2.3042])
np.maximum(x,y) # array([ 0.8626,  1.0048,  1.3272,  0.6702,  0.853 ,  0.0222,  0.7584,-0.6605])
```

Universal functions

| Function | Description |
|----------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------|
| <code>abs, fabs</code> | Compute the absolute value element-wise for integer, floating-point, or complex values |
| <code>sqrt</code> | Compute the square root of each element (equivalent to <code>arr ** 0.5</code>) |
| <code>square</code> | Compute the square of each element (equivalent to <code>arr ** 2</code>) |
| <code>exp</code> | Compute the exponent e^x of each element |
| <code>log, log10, log2, log1p</code> | Natural logarithm (base e), log base 10, log base 2, and $\log(1 + x)$, respectively |
| <code>sign</code> | Compute the sign of each element: 1 (positive), 0 (zero), or -1 (negative) |
| <code>ceil</code> | Compute the ceiling of each element (i.e., the smallest integer greater than or equal to that number) |
| <code>floor</code> | Compute the floor of each element (i.e., the largest integer less than or equal to each element) |
| <code>rint</code> | Round elements to the nearest integer, preserving the <code>dtype</code> |
| <code>modf</code> | Return fractional and integral parts of array as a separate array |
| <code>isnan</code> | Return boolean array indicating whether each value is NaN (Not a Number) |
| <code>isfinite, isinf</code> | Return boolean array indicating whether each element is finite (<code>non-inf, non-NaN</code>) or infinite, respectively |
| <code>cos, cosh, sin, sinh, tan, tanh</code> | Regular and hyperbolic trigonometric functions |
| <code>arccos, arccosh, arcsin, arcsinh, arctan, arctanh</code> | Inverse trigonometric functions |
| <code>logical_not</code> | Compute truth value of <code>not x</code> element-wise (equivalent to <code>~arr</code>). |

| Function | Description |
|------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| add | Add corresponding elements in arrays |
| subtract | Subtract elements in second array from first array |
| multiply | Multiply array elements |
| divide, floor_divide | Divide or floor divide (truncating the remainder) |
| power | Raise elements in first array to powers indicated in second array |
| maximum, fmax | Element-wise maximum; fmax ignores NaN |
| minimum, fmin | Element-wise minimum; fmin ignores NaN |
| mod | Element-wise modulus (remainder of division) |
| copysign | Copy sign of values in second argument to values in first argument |
| greater, greater_equal, less, less_equal, equal, not_equal | Perform element-wise comparison, yielding boolean array (equivalent to infix operators <code>></code> , <code>>=</code> , <code><</code> , <code><=</code> , <code>==</code> , <code>!=</code>) |
| logical_and, logical_or, logical_xor | Compute element-wise truth value of logical operation (equivalent to infix operators <code>&</code> , <code>^</code>) |

Conditional logic as Array operations

fit@hcmus

```
[ ] #Conditional logis as Array Operations
xarr = np.array([1.1, 1.2, 1.3, 1.4, 1.5])
yarr = np.array([2.1, 2.2, 2.3, 2.4, 2.5])
cond = np.array([True, False, True, True, False])

result = [(x if c else y) for x, y, c in zip(xarr, yarr, cond)]
result
```

```
[1.1, 2.2, 1.3, 1.4, 2.5]
```

```
[ ] result = np.where(cond, xarr, yarr)
result
```

```
array([1.1, 2.2, 1.3, 1.4, 2.5])
```

- Create a random $N \times M$ matrix. Set all positive values x to $2x$, and -2 for negative values

Exercise 02

fit@hcmus

```
# Parameters for the matrix dimensions
N = 4 # Number of rows
M = 5 # Number of columns

# Create a random NxM matrix with values from -10 to 10
matrix = np.random.randn(N, M) * 10

# Apply the condition:
# Double positive values, set negative values to -2
modified_matrix = np.where(matrix > 0, 2 * matrix, -2)

# Print the modified matrix
print("Modified Matrix:")
print(modified_matrix)
```

Mathematical & Statistical methods

fit@hcmus

```
[ ] #Mathematical and Statistical methods
```

```
arr = np.random.randn(5, 4)
```

```
arr
```

```
array([[-0.54926229,  0.18278111, -0.83024367,  0.69999463],  
      [-0.1905132 , -0.34356363,  1.0181388 ,  0.24849718],  
      [ 0.46647855,  0.33590169,  0.44325881,  0.20612658],  
      [ 0.2589447 ,  0.29961133, -0.24995255,  0.49049097],  
      [-0.46239346, -0.30404133,  1.20203899, -1.21725287]])
```

```
[ ] arr.mean()
```

```
arr.sum()
```

```
1.7050403500621718
```

```
[ ] arr.mean(axis=1)
```

```
array([-0.12418255,  0.18313979,  0.36294141,  0.19977361, -0.19541217])
```

```
[ ] arr.mean(axis=0)
```

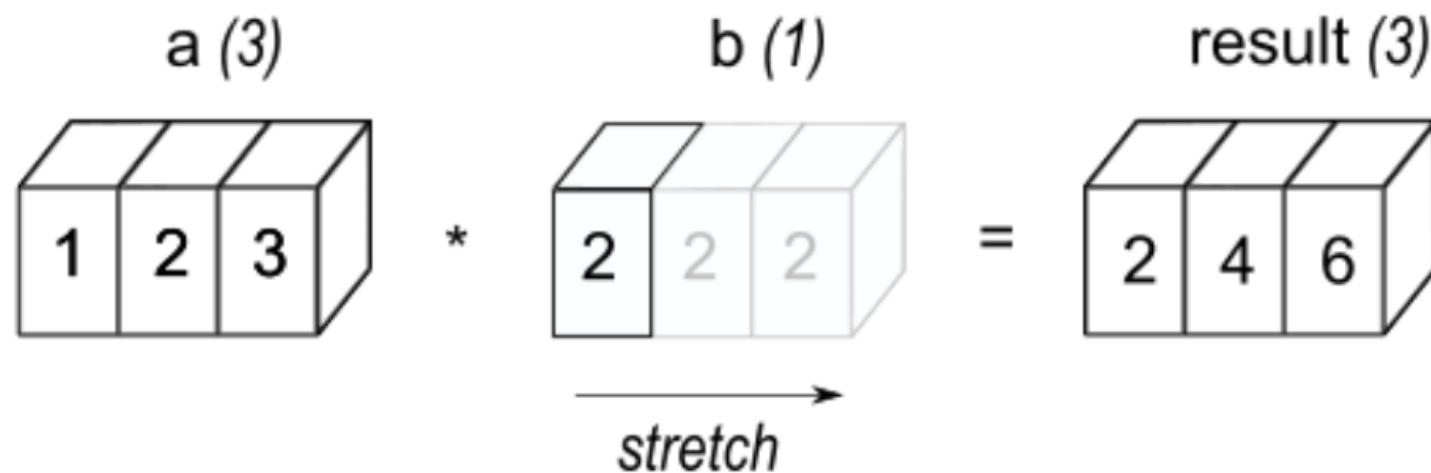
```
array([-0.09534914,  0.03413784,  0.31664808,  0.0855713 ])
```

| Method | Description |
|----------------|--------------------------------------------------------------------------------------------------------------------|
| sum | Sum of all the elements in the array or along an axis; zero-length arrays have sum 0 |
| mean | Arithmetic mean; zero-length arrays have NaN mean |
| std, var | Standard deviation and variance, respectively, with optional degrees of freedom adjustment (default denominator n) |
| min, max | Minimum and maximum |
| argmin, argmax | Indices of minimum and maximum elements, respectively |
| cumsum | Cumulative sum of elements starting from 0 |
| cumprod | Cumulative product of elements starting from 1 |

```
[ ] #File with numpy  
  
arr = np.arange(10)  
np.save('some_array', arr)
```

```
[ ] np.load('some_array.npy')  
  
array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])
```

```
a = np.array([1, 2, 3])  
print(a *2)
```



```
a = np.arange(12).reshape(4, 3) # shape = (4, 3)
b = a.mean(0)

print(b) # [4.5 5.5 6.5], shape = (3,)
print(a - b) # shape = (4, 3)
# [[-4.5 -4.5 -4.5]
#  [-1.5 -1.5 -1.5]
#  [ 1.5  1.5  1.5]
#  [ 4.5  4.5  4.5]]
```

```
[ ] arr = np.arange(5) #array([0, 1, 2, 3, 4])  
arr*4
```

```
array([ 0,  4,  8, 12, 16])
```

```
[ ] arr = np.random.randn(4, 3)  
arr.mean(0)
```

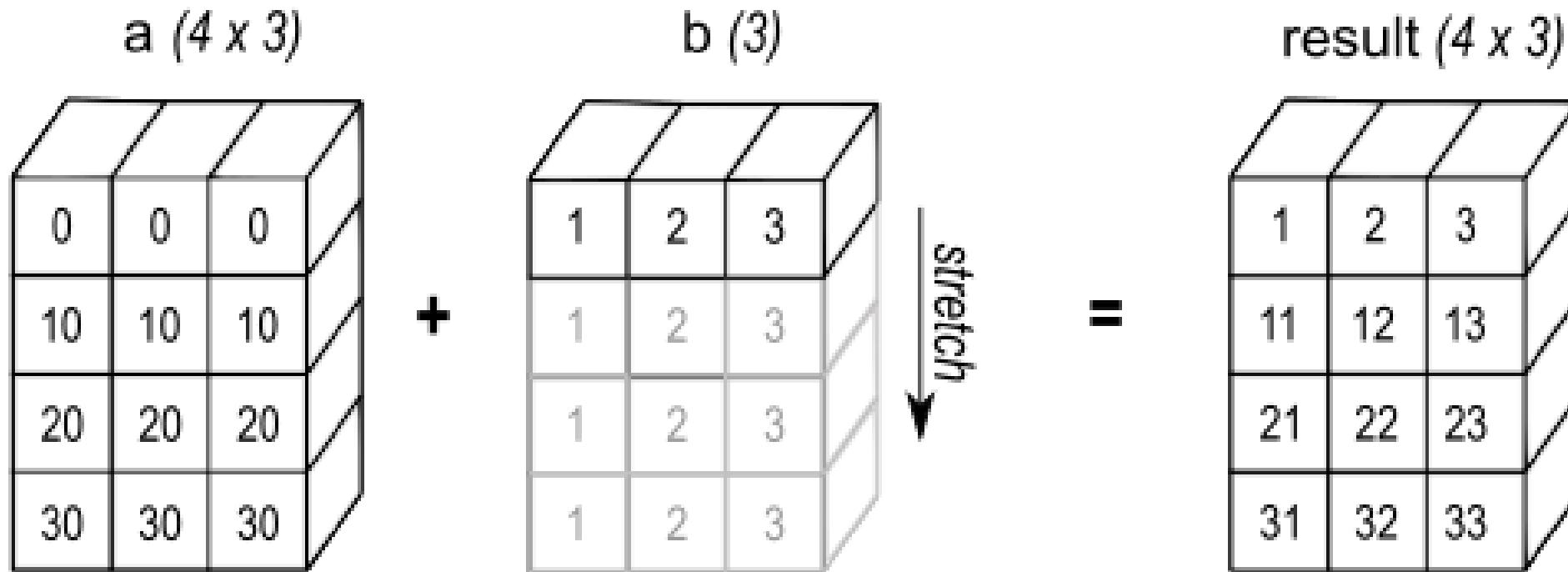
```
array([0.48837945, 0.52888814, 0.08470346])
```

```
[ ] de = arr - arr.mean(0)  
de
```

```
array([[ 0.04160849,  0.38942007,  0.77749224],  
      [ 0.49715569, -1.44295791, -0.95987177],  
      [ 0.39977265,  0.2456237 ,  1.06945136],  
      [-0.93853683,  0.80791414, -0.88707184]])
```

Broadcasting

fit@hcmus



Broadcasting

```
[ ] arr.mean(1)  
array([ 0.77016395, -0.26790098,  0.93893959,  0.02809217])
```

```
[ ] de = arr - arr.mean(1) #?
```

```
-----  
ValueError                                     Traceback (most recent call last)  
<ipython-input-24-952255e5b562> in <module>()  
----> 1 de = arr - arr.mean(1) #?  
      2
```

```
ValueError: operands could not be broadcast together with shapes (4,3) (4,)
```

SEARCH STACK OVERFLOW

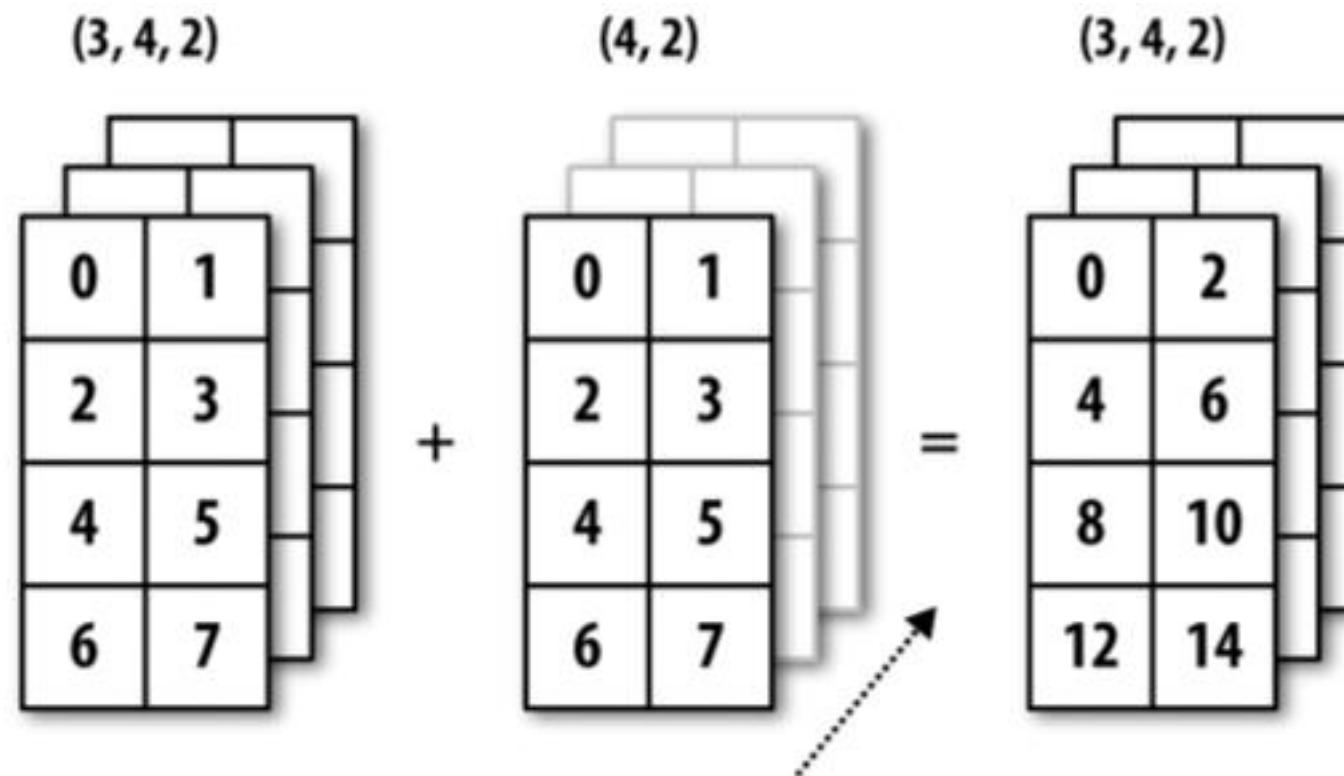
$$\begin{matrix} (4,3) \\ \begin{array}{|c|c|c|} \hline 0 & 0 & 0 \\ \hline 1 & 1 & 1 \\ \hline 2 & 2 & 2 \\ \hline 3 & 3 & 3 \\ \hline \end{array} \end{matrix} + \begin{matrix} (4,1) \\ \begin{array}{|c|} \hline 1 \\ \hline 2 \\ \hline 3 \\ \hline 4 \\ \hline \end{array} \end{matrix} = \begin{matrix} (4,3) \\ \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 3 & 3 & 3 \\ \hline 5 & 5 & 5 \\ \hline 7 & 7 & 7 \\ \hline \end{array} \end{matrix}$$

.....→

```
a = np.arange(12).reshape(4, 3)
b = np.array([1, 2, 3, 4]).reshape(4, 1)
print(a + b)
# [[ 1  2  3]
#  [ 5  6  7]
#  [ 9 10 11]
# [13 14 15]]
```

```
a = np.arange(12).reshape(4, 3)
b = np.array([1, 2, 3, 4])
#print(a - b) # error
print(a - b[:, np.newaxis])
# [[-1  0  1]
# [ 1  2  3]
# [ 3  4  5]
# [ 5  6  7]]
```

- Broadcasting in NumPy follows a strict set of rules to determine the interaction between the two arrays
 - Rule 1: If the two arrays differ in their number of dimensions, the shape of the one with fewer dimensions is padded with ones on its leading (left) side. (e.g: $(4,3)$ and $(4,)$ $\Rightarrow (4,3)$ and $(1, 4)$)
 - Rule 2: If the shape of the two arrays does not match in any dimension, the array with shape equal to 1 in that dimension is stretched to match the other shape. (e.g: $(4, 3)$ and $(1, 4)$ $\Rightarrow (4, 3)$ and $(4, 4)$)
 - Rule 3: If in any dimension the sizes disagree and neither is equal to 1, an error is raised.



- Create the following array with n inputted from keyboard.
- Ex: n = 10

```
[ [0, 1, 2, 3, 4, 1, 1, 1, 1, 1],  
  [5, 6, 7, 8, 9, 1, 1, 1, 1, 1]]
```

```
import numpy as np

def create_matrix(n: int) -> np.ndarray:
    arr = np.arange(n).reshape(2, -1)
    ones = np.ones(n, dtype= int).reshape(2, -1)
    result = np.concatenate([arr, ones], axis = 1)
    return result

print(create_matrix(10))
```

- Given `a = np.array([1, 2, 3])`
- Create a following array without using iteration and array initialization:
[1, 1, 1, 2, 2, 2, 3, 3, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3]

```
# Given array
a = np.array([1, 2, 3])

# Repeat each element three times
part1 = np.repeat(a, 3)
# Tile the entire array three times
part2 = np.tile(a, 3)
# Concatenate the two parts
result = np.concatenate((part1, part2))

# Print the result
print(result)
```

- What is the output of the code fragment

```
import numpy as np  
  
arr = np.arange(9).reshape(3,3)  
print(arr[::-1])
```

- What is the output of the code fragment

```
import numpy as np  
  
arr = np.arange(9).reshape(3,3)  
print(arr[::-1])
```

[[6 7 8]
 [3 4 5]
 [0 1 2]]

- Given a even number n from keyboards, create a $2 * ((N * N) // 2)$ array as follows:
- Ex: $n = 4$

```
[ [ 0 -8 1 -9 2 -10 3 -11 ]  
  [ 4 -12 5 -13 6 -14 7 -15 ] ]
```

Exercise 06

```
import numpy as np

def create_interleaved_array(n):
    if n % 2 != 0: return None
    positive_numbers = np.arange(n * n // 2)
    negative_numbers = -1*np.arange(n * n // 2, n*n)

    result_array = np.empty(n * n, dtype=int)
    result_array[::2] = positive_numbers
    result_array[1::2] = negative_numbers
    # Reshape the result into a 2x(N*N//2) array
    final_array = result_array.reshape(2, n * n // 2)
    return final_array

print(create_interleaved_array(4))
```

THANK YOU
for YOUR ATTENTION