

Digital Signal Processing

Lecture 11 – digital image processing I

상명대학교
컴퓨터과학과
강상욱 교수

Digital Image Definition

An image is a two-dimensional function $f(x,y)$, where x and y are the **spatial** (plane) coordinates, and the amplitude of f at any pair of coordinates (x,y) is called the intensity of the image at that level.

If x,y and the **amplitude** values of f are **finite** and **discrete quantities**, we call the image a **digital image**. A digital image is composed of a finite number of elements called **pixels**, each of which has a particular location and value.

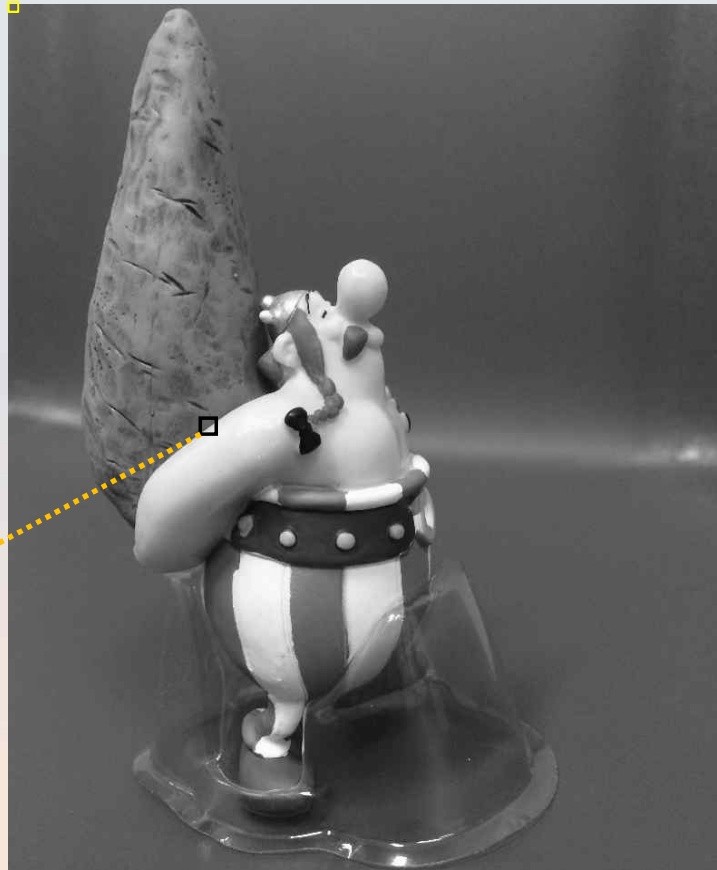
Digital Image Representation

Pixel intensity value
 $f(l, l) = 103$
 Pixel location

rows columns
 $f(645:650, 1323:1328) =$

83	82	82	82	82	82
82	82	82	81	81	81
82	82	81	81	80	80
82	82	81	80	80	79
80	79	78	77	77	77
80	79	78	78	77	77

3



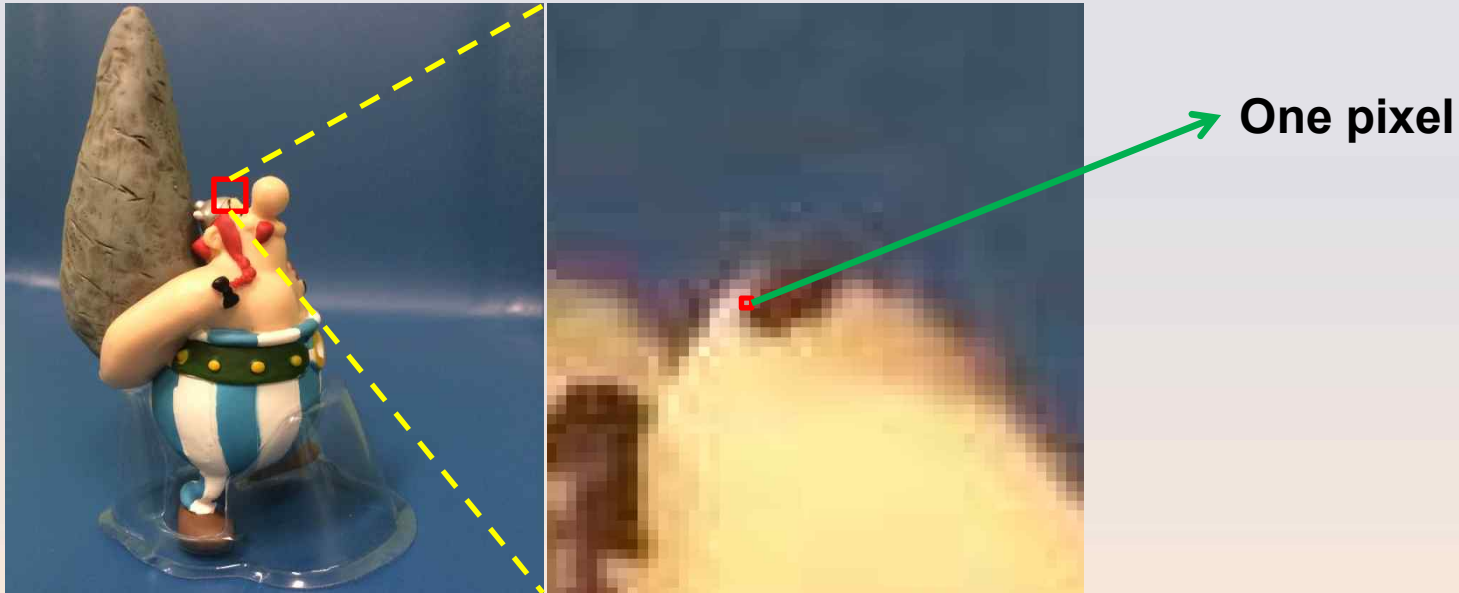
$f(2724, 2336) = 88$

Consider the following image (2724x2336 pixels) to be 2D function or a **matrix** with **r**ows and **c**olumns

In **8-bit** representation
 Pixel intensity values change between **0 (Black)** and **255 (White)**

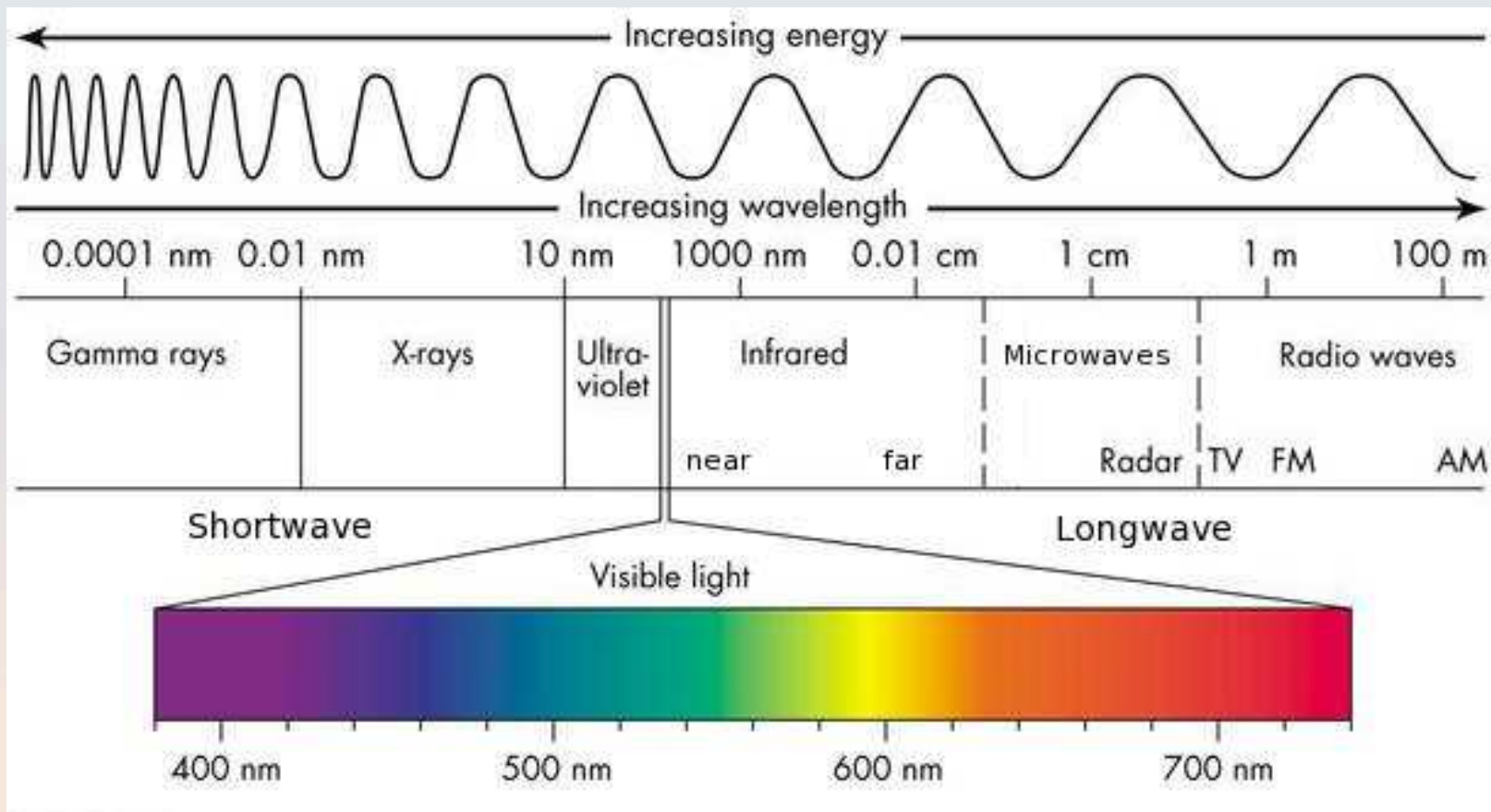
Digitization

Remember *digitization* implies that a digital image is an *approximation* of a real scene



Sources of Digital Images

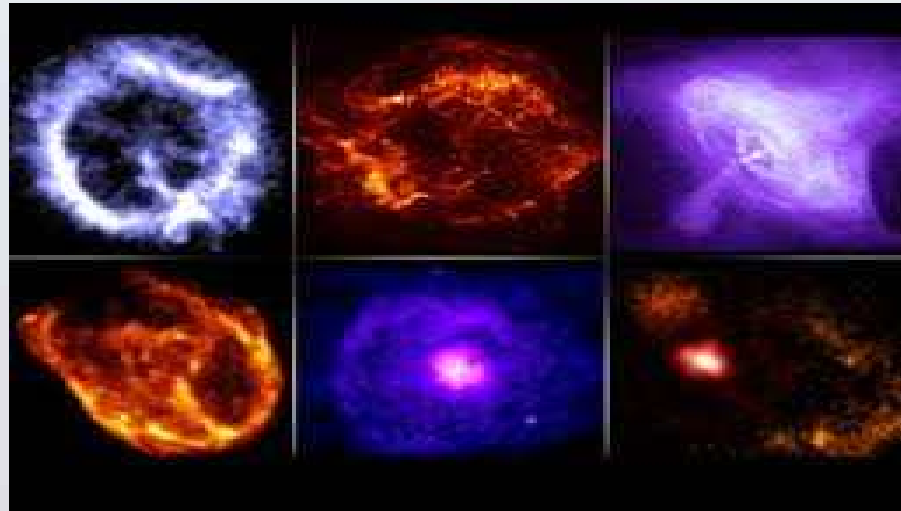
The principal source for the images is the **electromagnetic (EM) energy spectrum**.



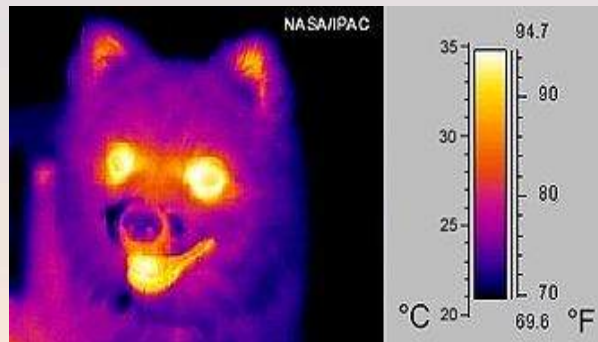
Examples



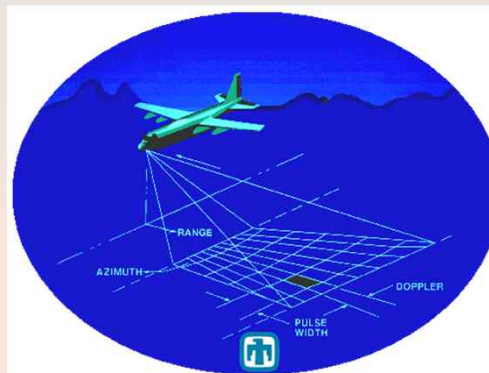
**Gamma-Ray imaging of
A starburst galaxy about 12
million light-years away**



**X-ray images from the space
The Chandra X-Ray Observatory**

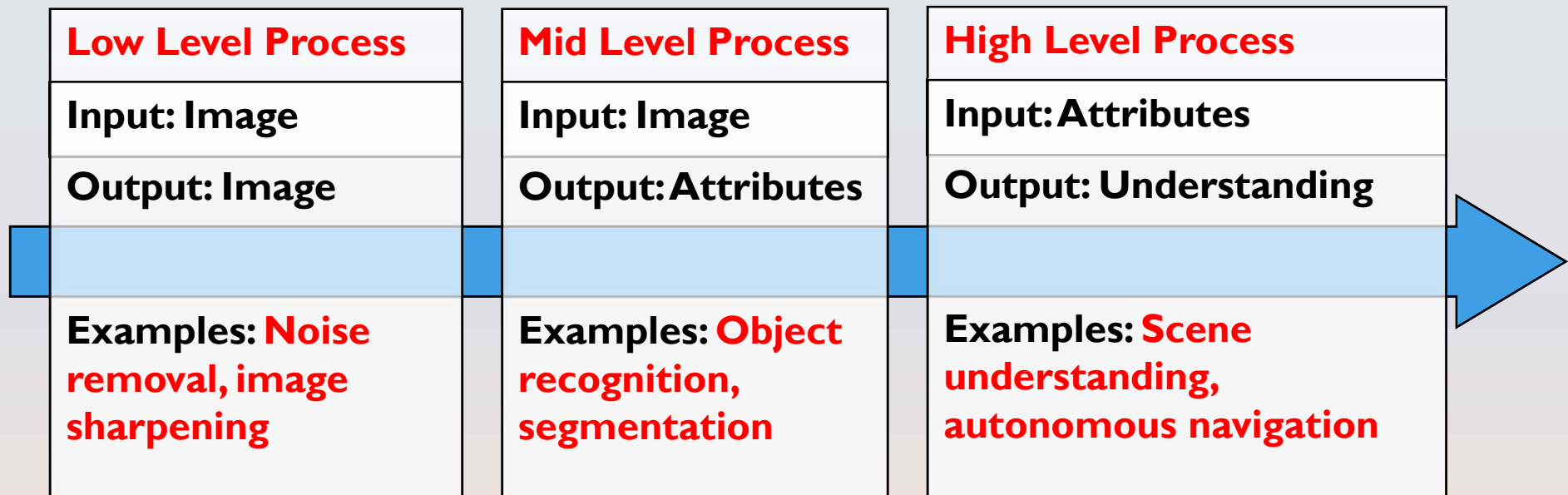


Infrared thermal image



**Synthetic Aperture Radar
System**

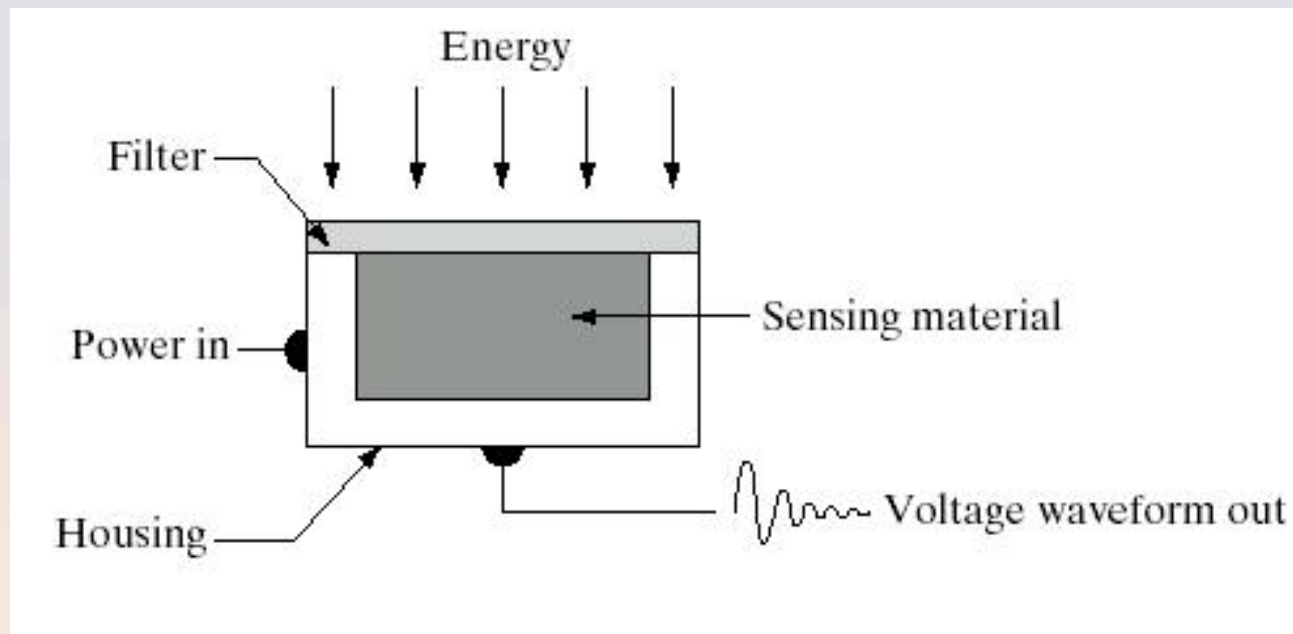
Digital Image Processing Level



Acquisition of Images

The images are generated by the combination of an *illumination source* and the reflection or absorption of energy from that source by the elements of the *scene* being imaged.

Imaging sensors are used to transform the *illumination energy* into digital images.

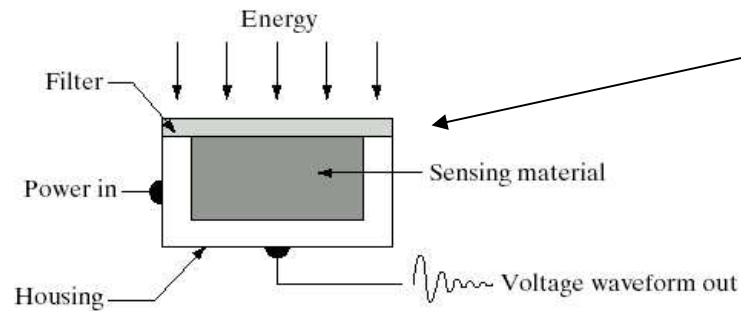


Types of Image Sensors

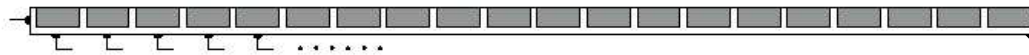
a
b
c

FIGURE 2.12

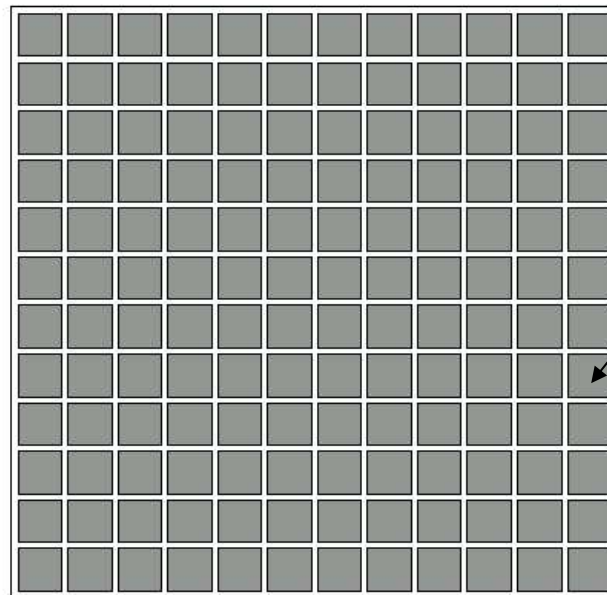
(a) Single imaging sensor.
(b) Line sensor.
(c) Array sensor.



Single Sensor



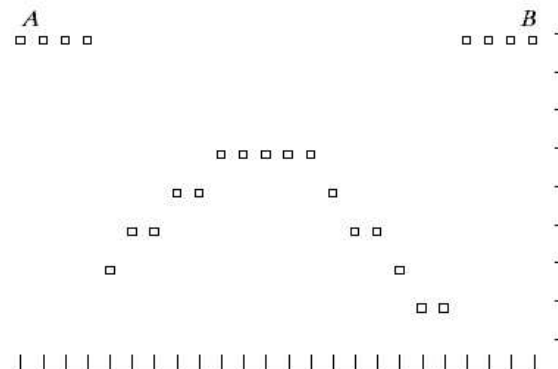
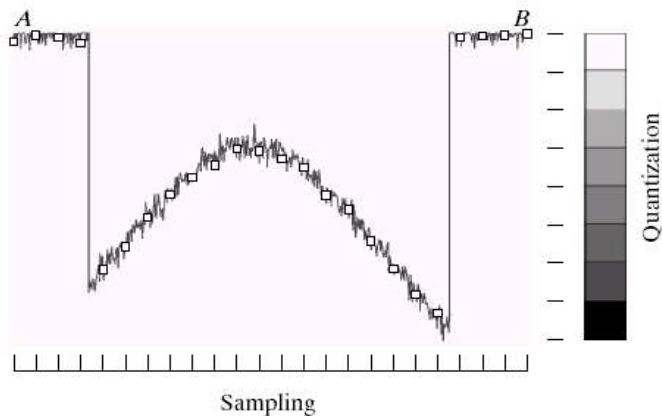
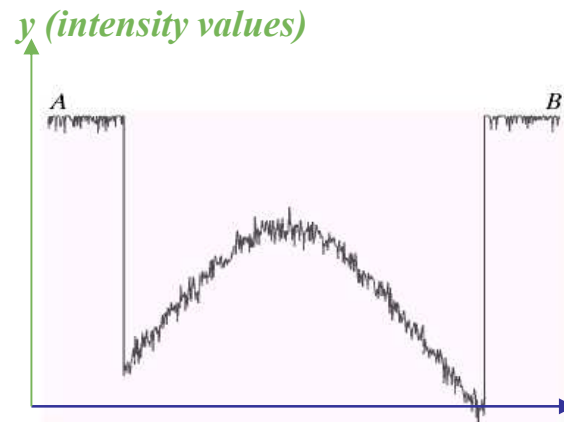
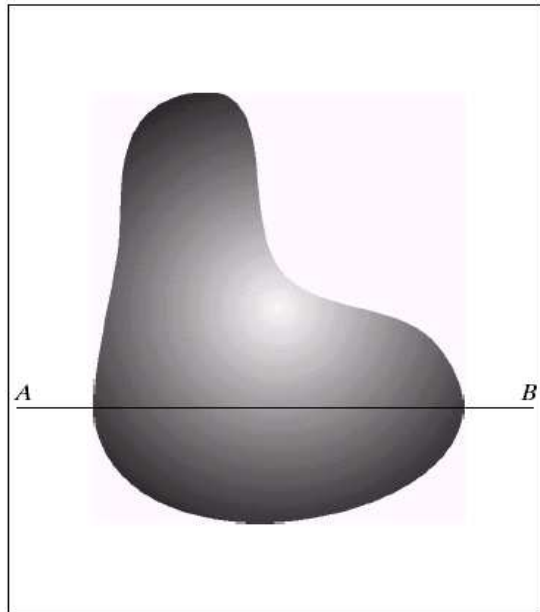
Line Sensor



Array Sensor

CCD is popular.
(Charge
Coupled
Device)

Sampling & Quantization



a	b
c	d

Generating a digital image.

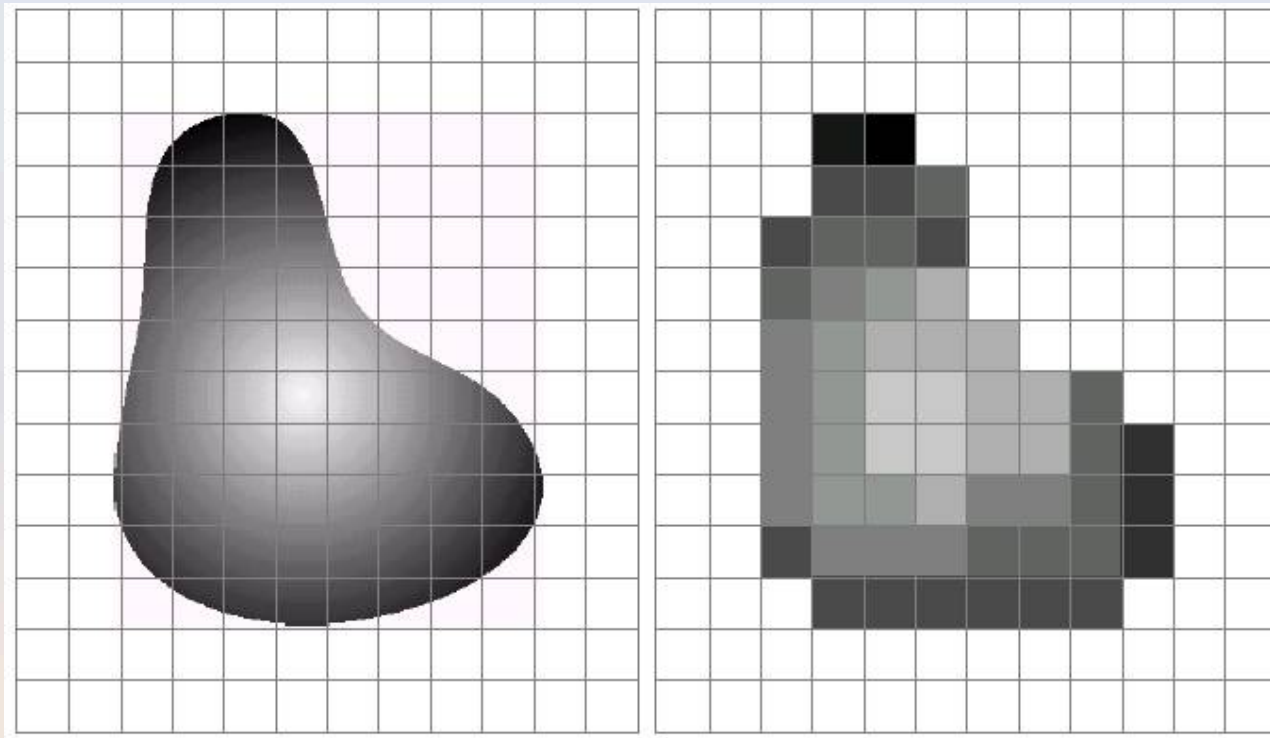
(a) Continuous image.

(b) A scaling line from A to B in the continuous image.

(c) sampling and quantization.

(d) Digital scan line.

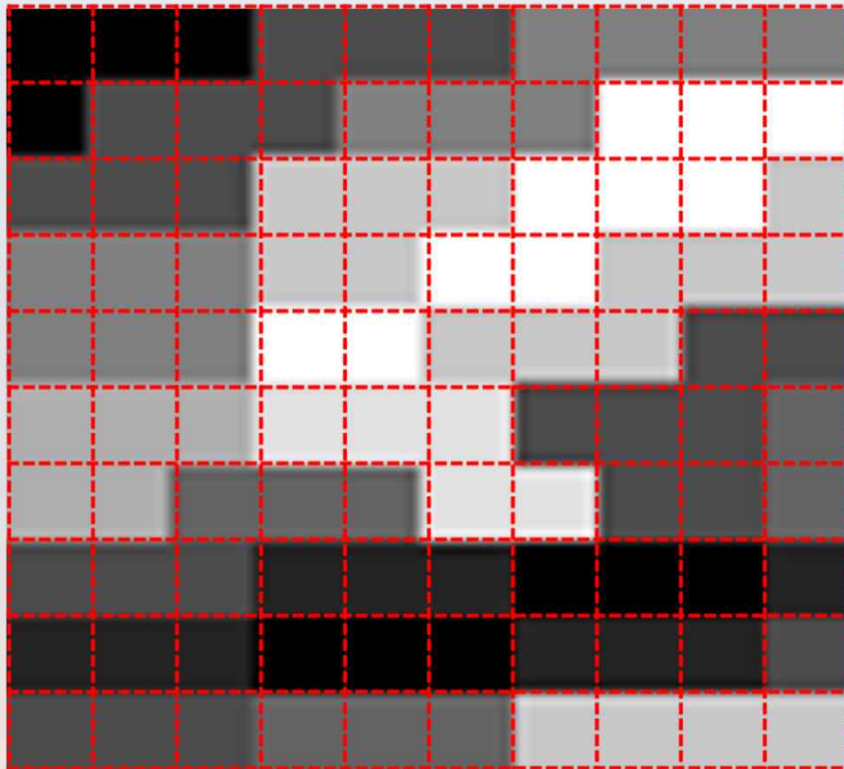
Sampling & Quantization



a **b**

- (a) Continuous image projected onto a sensor array.
- (b) Result of image sampling and quantization.

Image as a Signal



0	0	0	75	75	75	128	128	128	128
0	75	75	75	128	128	128	255	255	255
75	75	75	200	200	200	255	255	255	200
128	128	128	200	200	255	255	200	200	200
128	128	128	255	255	200	200	200	75	75
175	175	175	225	225	225	75	75	75	100
175	175	100	100	100	225	225	75	75	100
75	75	75	35	35	35	0	0	0	35
35	35	35	0	0	0	35	35	35	75
75	75	75	100	100	100	200	200	200	200

Sampling Rates – The same pixel size



1024



512



256



128



Sampling Rates – The same picture size



1024



512



256



128



64



32

Quantization Precision



8-bit



7-bit



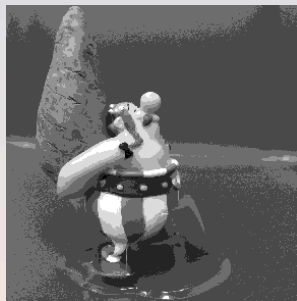
6-bit



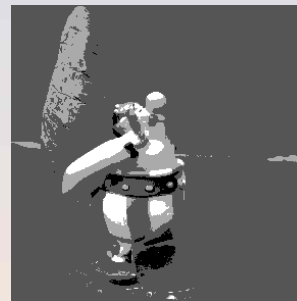
5-bit



4-bit



3-bit



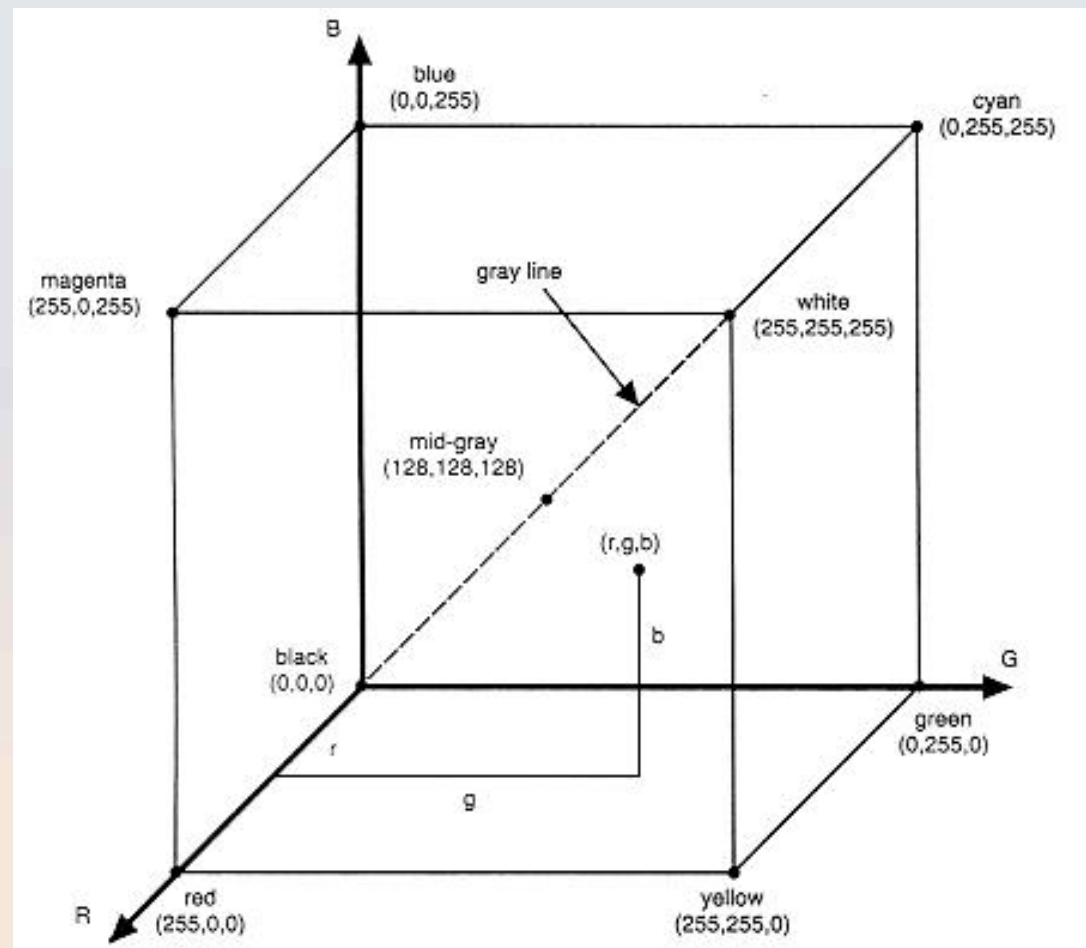
2-bit



1-bit

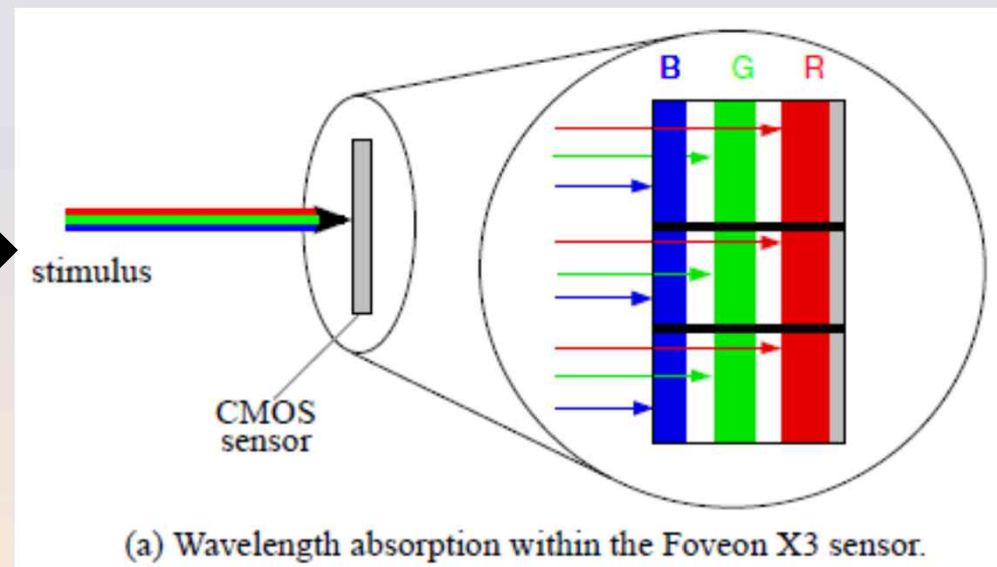
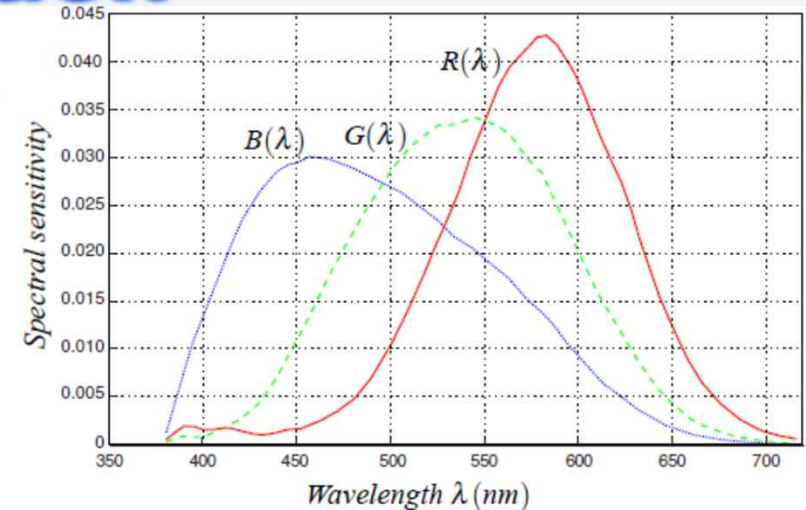
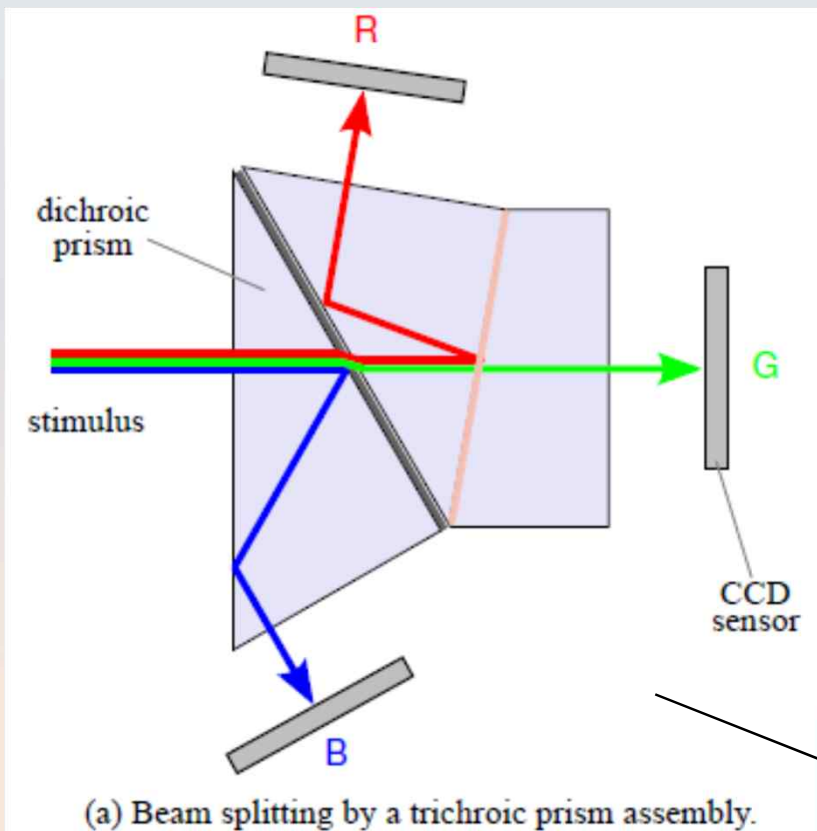
Color Image Representation

- Hardware generally delivers or displays color via RGB model (red, green, and blue).
- Gray has the same amount of r, g, b components.



Color Image Acquisition

- Three-CCD sensor
 - High manufacturing cost

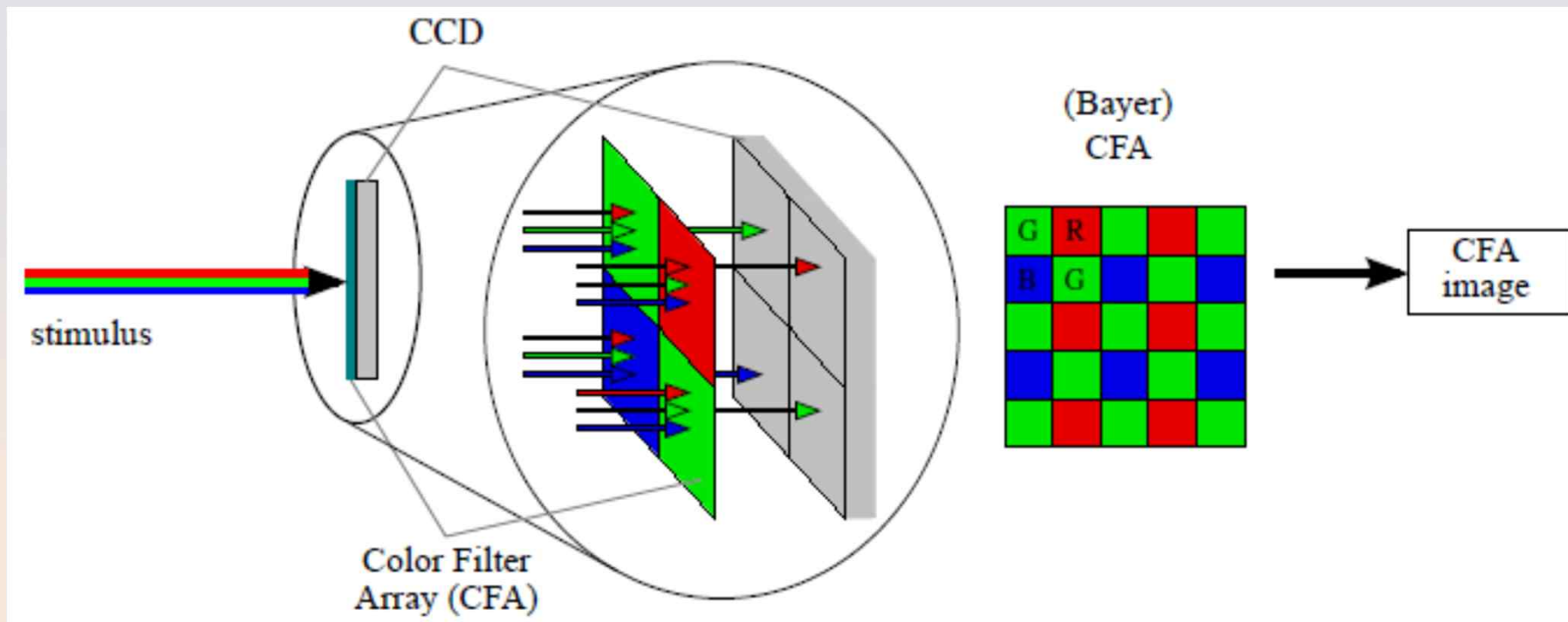


The spectral sensitivity differs from human's one.

Color Image Acquisition

■ One CCD sensor

- Color filter array (CFA) is a mosaic of spectrally selective color filters.
- There are many CFA patterns, Bayer's CFA is most widespread.
- Bayer's observation :The human eye has a greater resolving power for green light.



Demosaicing

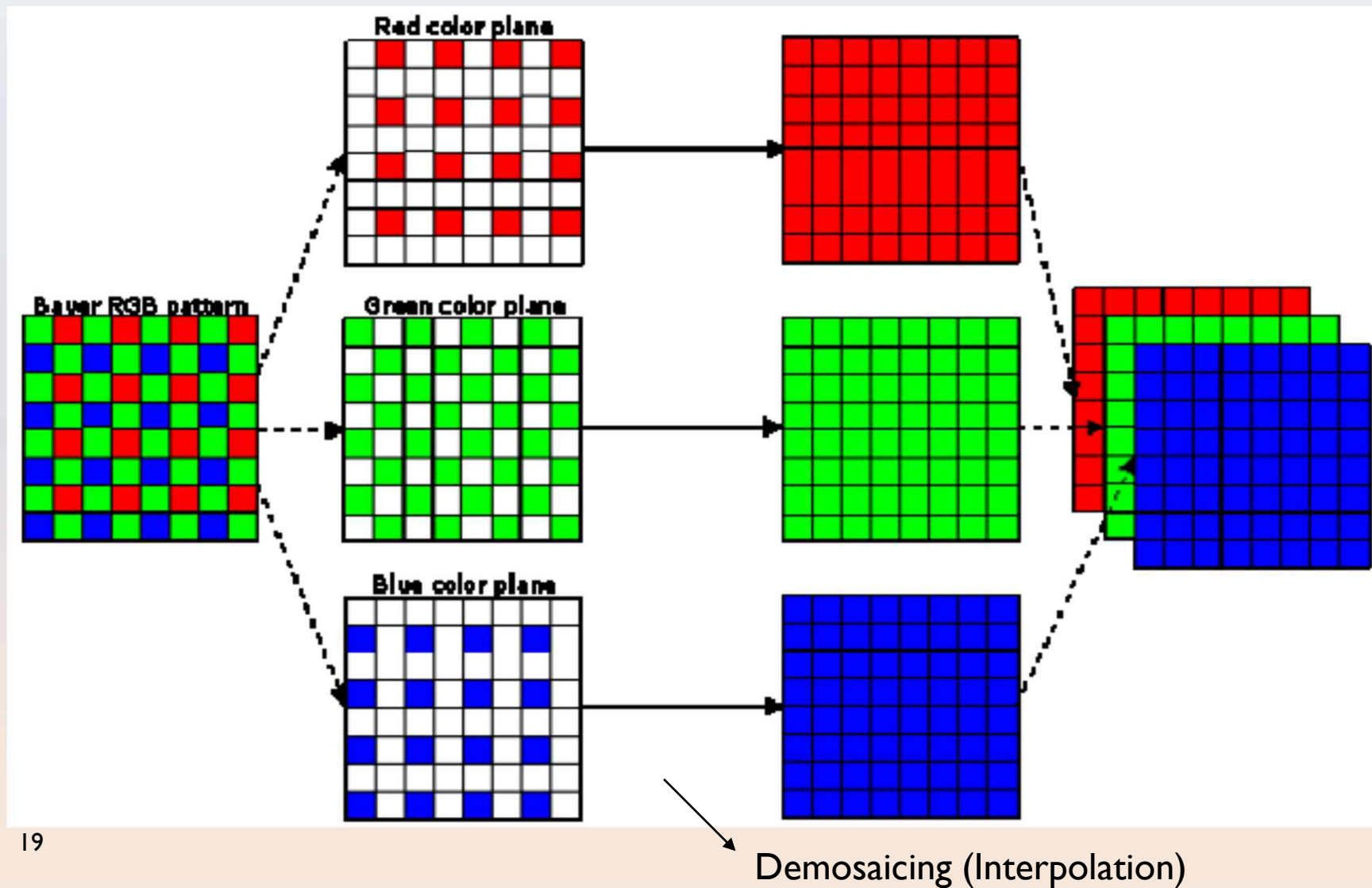
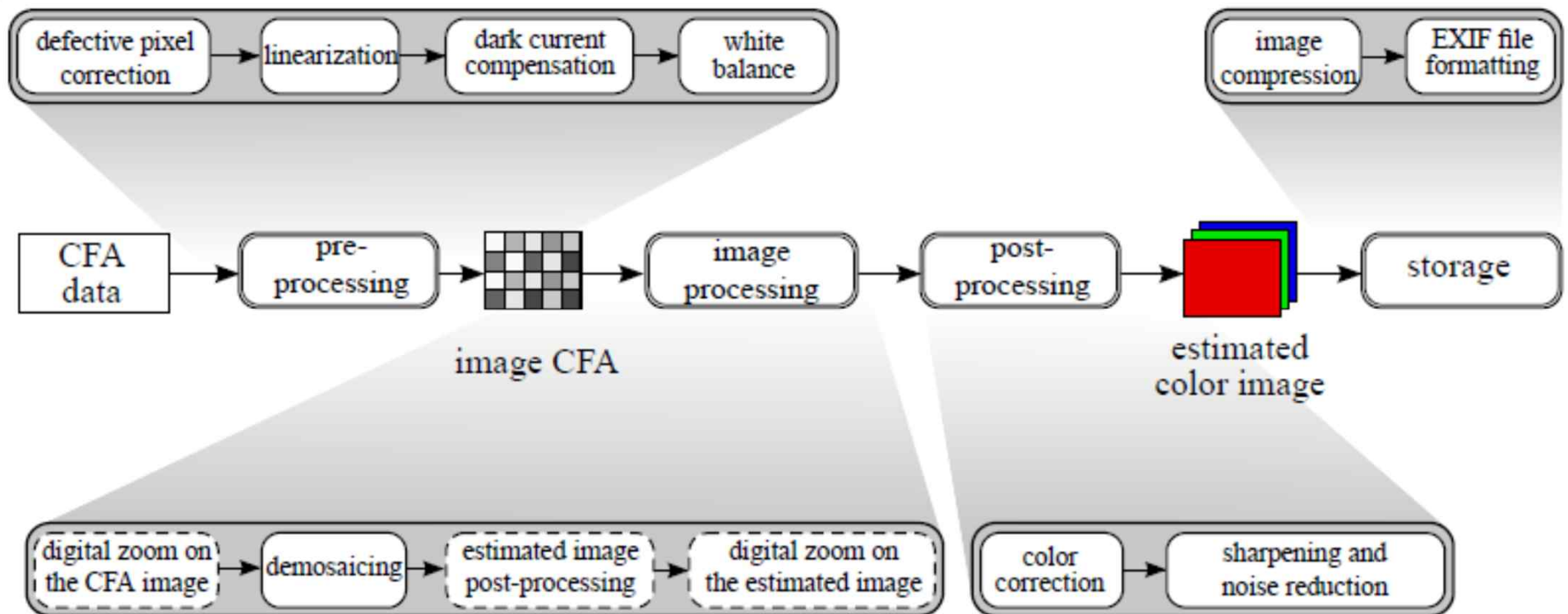


Image Acquisition - Pipeline



Lab

- ▣ Image read & write
- ▣ Image color manipulation
- ▣ Image display



IMAGE FILTERING

Mask (Kernel)

$w(-1,-1)$	$w(-1,0)$	$w(-1,1)$
$w(0,-1)$	$w(0,0)$	$w(0,1)$
$w(1,-1)$	$w(1,0)$	$w(1,1)$

**Mask coefficients showing
coordinate arrangement**

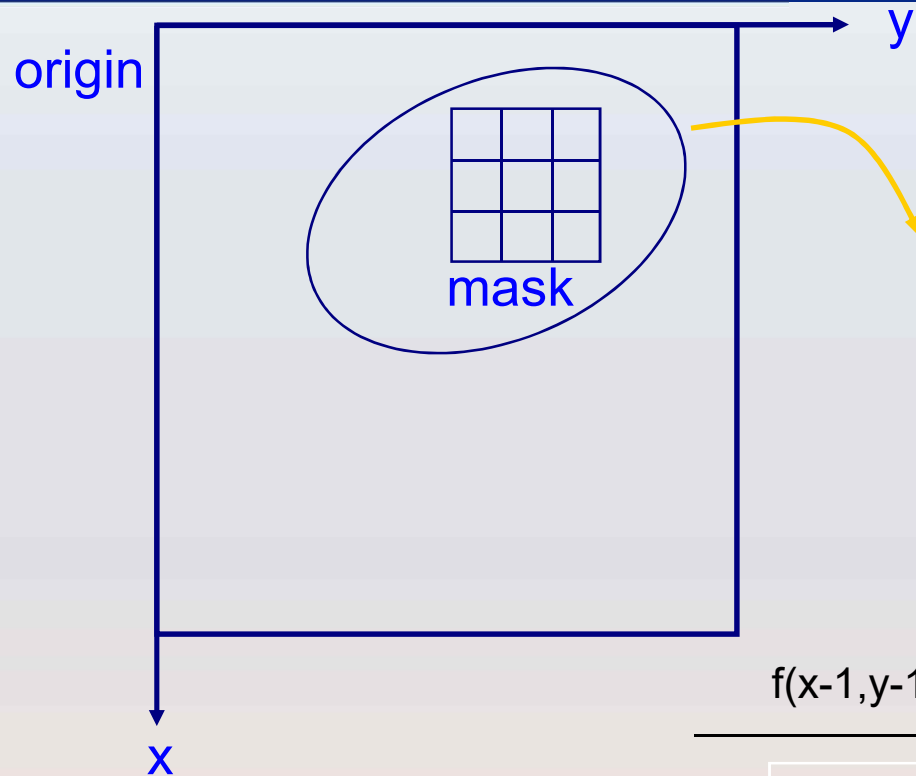
Spatial Filtering

Convolution

$$g(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b \omega(s, t) f(x-s, y-t)$$

$$g = \omega * f$$

Spatial Filtering



$W(-1,-1)$	$W(-1,0)$	$W(-1,1)$
$W(0,-1)$	$W(0,0)$	$W(0,1)$
$W(1,-1)$	$W(1,0)$	$W(1,1)$

Mask coefficients

$f(x-1,y-1)$	$f(x-1,y)$	$f(x-1,y+1)$
$f(x,y-1)$	$f(x,y)$	$f(x,y+1)$
$f(x+1,y-1)$	$f(x+1,y)$	$f(x+1,y+1)$

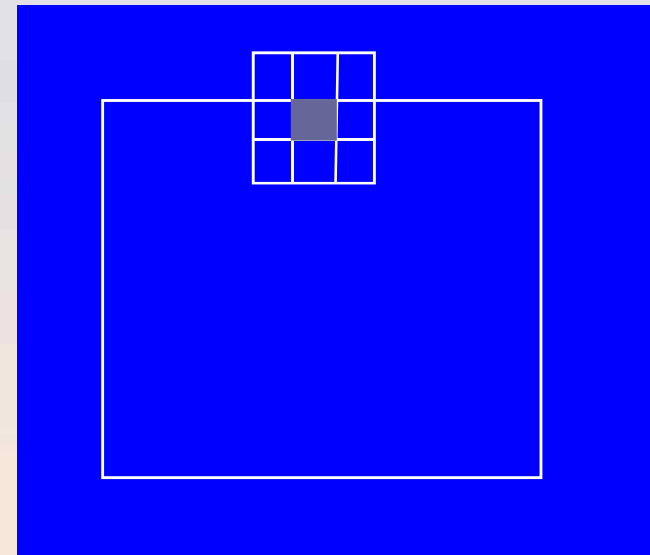
Image section under mask

$$g(x, y) =$$

$$\sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$

What happens at the borders?

- ▣ The mask falls outside the edge.
- ▣ Solutions?
 - ▣ Ignore the edges
 - ▣ The resultant image is smaller than the original
 - ▣ With 3x3 kernel, the size is reduced by 2, both horizontally and vertically.
 - ▣ Pad with zeros
 - ▣ Introducing unwanted artifacts



Values Outside the Range

- Linear filtering might bring the intensity outside the display range.

- Solutions?

- Clip values

$$y = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 255 \\ 255 & \text{if } x > 255 \end{cases}$$

- Scaling transformation

$$y = 255 \frac{x - g_L}{g_H - g_L}$$

Diagram illustrating the scaling transformation formula:

- The term g_L in the numerator is labeled "New min" (green text in a red box).
- The term $g_H - g_L$ in the denominator is labeled "New max" (green text in a red box).

Transform values in $[g_L, g_H]$ to $[0, 255]$

Spatial Filtering – step by step

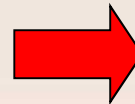
1	1	1
-1	2	1
-1	-1	1

Convolution
kernel, ω

Input Image, f

2	2	2	3
2	1	3	3
2	2	1	2
1	3	2	2

1	1	1		
-1	4	2	2	3
-1	-2	1	3	3
	2	2	1	2
	1	3	2	2



5			

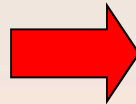
Output
Image, g

Spatial Filtering – step by step

1	1	1
-1	2	1
-1	-1	1

2	2	2	3
2	1	3	3
2	2	1	2
1	3	2	2

1	1	1	
-2	4	2	3
-2	-1	3	3
2	2	1	2
1	3	2	2



5	4		

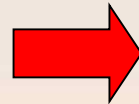
**Output I
mage, g**

Spatial Filtering – step by step

1	1	1
-1	2	1
-1	-1	1

2	2	2	3
2	1	3	3
2	2	1	2
1	3	2	2

1	1	1	
-2	4	2	3
-2	-1	3	3
2	2	1	2
1	3	2	2



5	4		

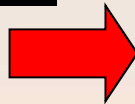
**Output I
mage, g**

Spatial Filtering – step by step

1	1	1
-1	2	1
-1	-1	1

2	2	2	3
2	1	3	3
2	2	1	2
1	3	2	2

		1	1	1
2	2	-2	6	1
2	1	-3	-3	1
2	2	1	2	
1	3	2	2	

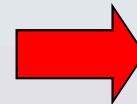


5	4	4	-2

**Output I
mage, g**

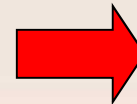
Spatial Filtering – step by step

1	2	2	2	3
-1	4	1	3	3
-1	-2	2	1	2
	1	3	2	2



5	4	4	-2
9			

2	2	2	3
-2	2	3	3
-2	-2	1	2
1	3	2	2



5	4	4	-2
9	6		

Spatial Filtering – step by step

5	4	4	-2
9	6	14	5
11	7	6	5
9	12	8	5

Final output Image, g

Low Pass Filter - Averaging

- One simple example is smoothing using a 3x3 mask.

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \quad \frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

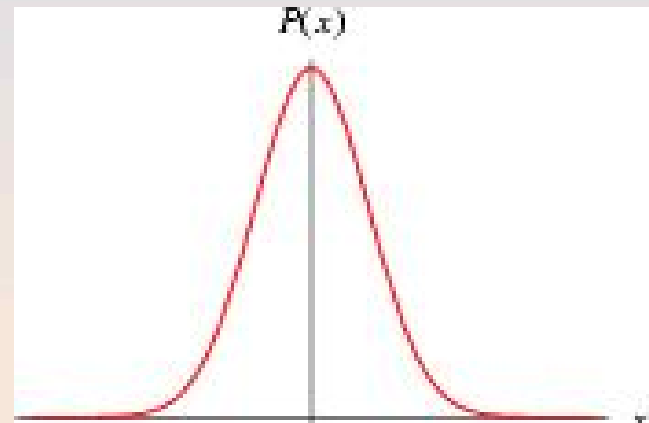
Box filter

Weighted average filter

Low Pass Filter - Gaussian

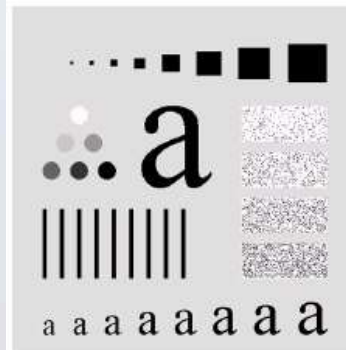
- ▣ Mostly for noise reduction/removal and smoothing
 - ▣ 3x3 averaging filter to blur edges
 - ▣ Gaussian filter,
 - ▣ based on Gaussian probability distribution function
 - ▣ a popular filter for smoothing
 - ▣ more later when we discuss image restoration

In 1D: $P(x) = e^{-x^2/2\sigma^2}$

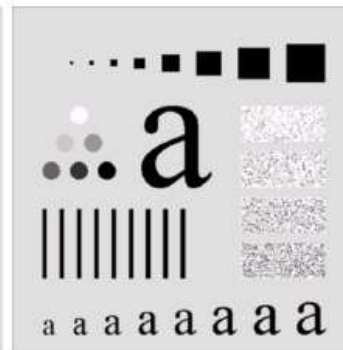


LPF – Different Kernel Size

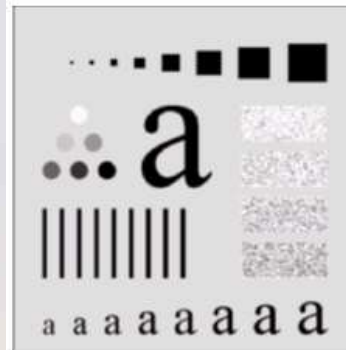
Original



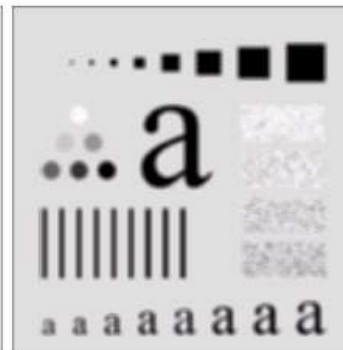
3x3



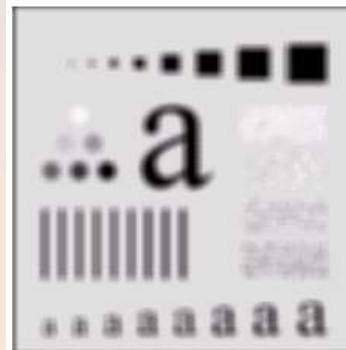
5x5



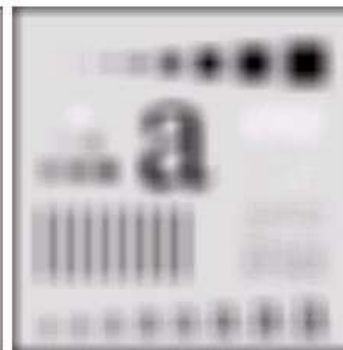
9x9



15x15

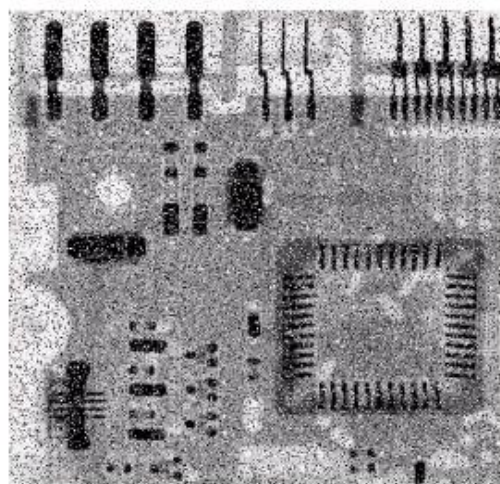


35x35

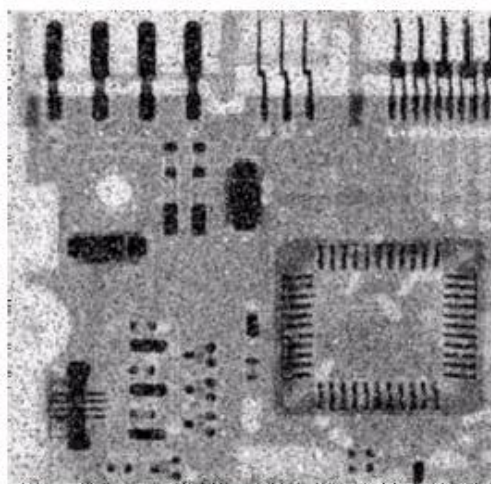


Median Filter

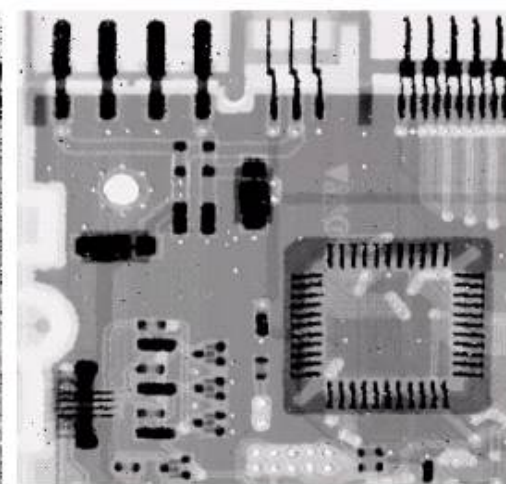
$a_1 < a_2 < \dots < a_n$ then median is $a_{(1+n)/2}$



Corrupted by salt and pepper noise



Averaging filter



Median filter

Laplacian Filter - I

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

$$\begin{aligned}\frac{\partial^2 f}{\partial x^2} &= f(x+1) - f(x) - (f(x) - f(x-1)) \\ &= f(x+1) + f(x-1) - 2f(x)\end{aligned}$$

Laplacian Filter - 2

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

The 2D Laplacian

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

Laplacian Filter - 3

The 2D Laplacian

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

0	1	0	1	1	1
1	-4	1	1	-8	1
0	1	0	1	1	1

0	-1	0	-1	-1	-1
-1	4	-1	-1	8	-1
0	-1	0	-1	-1	-1

Laplacian Filter - 4

Sobel Filter - I

The Gradient

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\nabla f = \text{mag}(\nabla \mathbf{f}) = \left[G_x^2 + G_y^2 \right]^{1/2} = \sqrt{\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2}$$

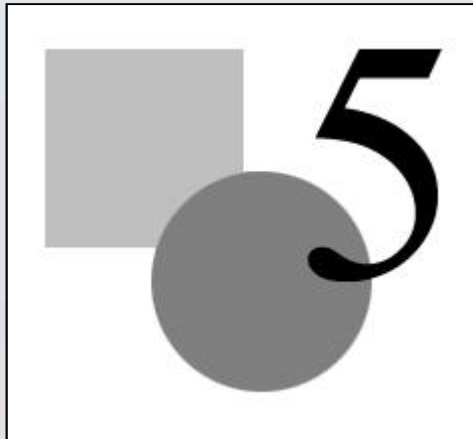
Sobel Filter - 2

$$\nabla f \approx |G_x| + |G_y|$$

$$\begin{aligned} \nabla f \approx |G_x| + |G_y| = & |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| \\ & + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)| \end{aligned}$$

-1	-2	-1	-1	0	1
0	0	0	-2	0	2
1	2	1	-1	0	1

Sobel Filter - 3



Horizontal

Vertical

Comparison (Laplacian, Gradient)

- ▣ Gradient

- ▣ The value is proportional to the degree of value change.

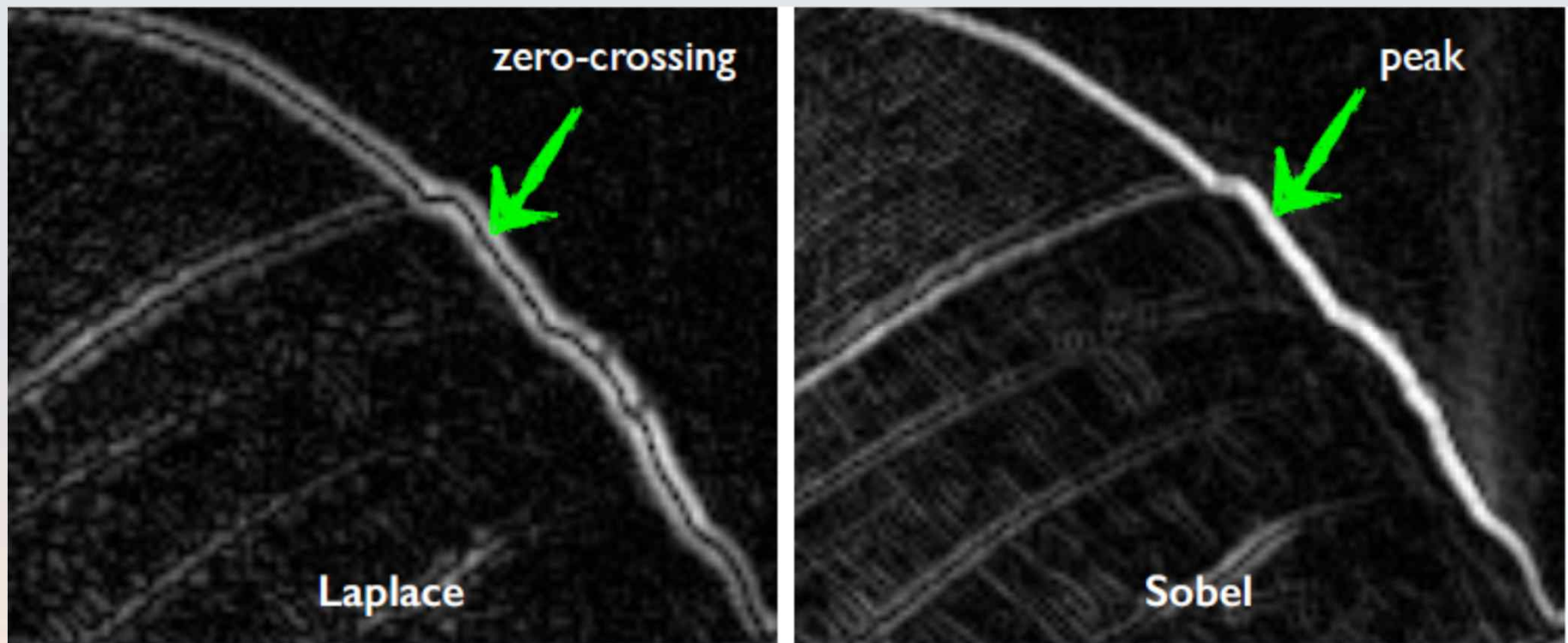
- ▣ Laplacian

- ▣

Comparison (Laplacian, Gradient)

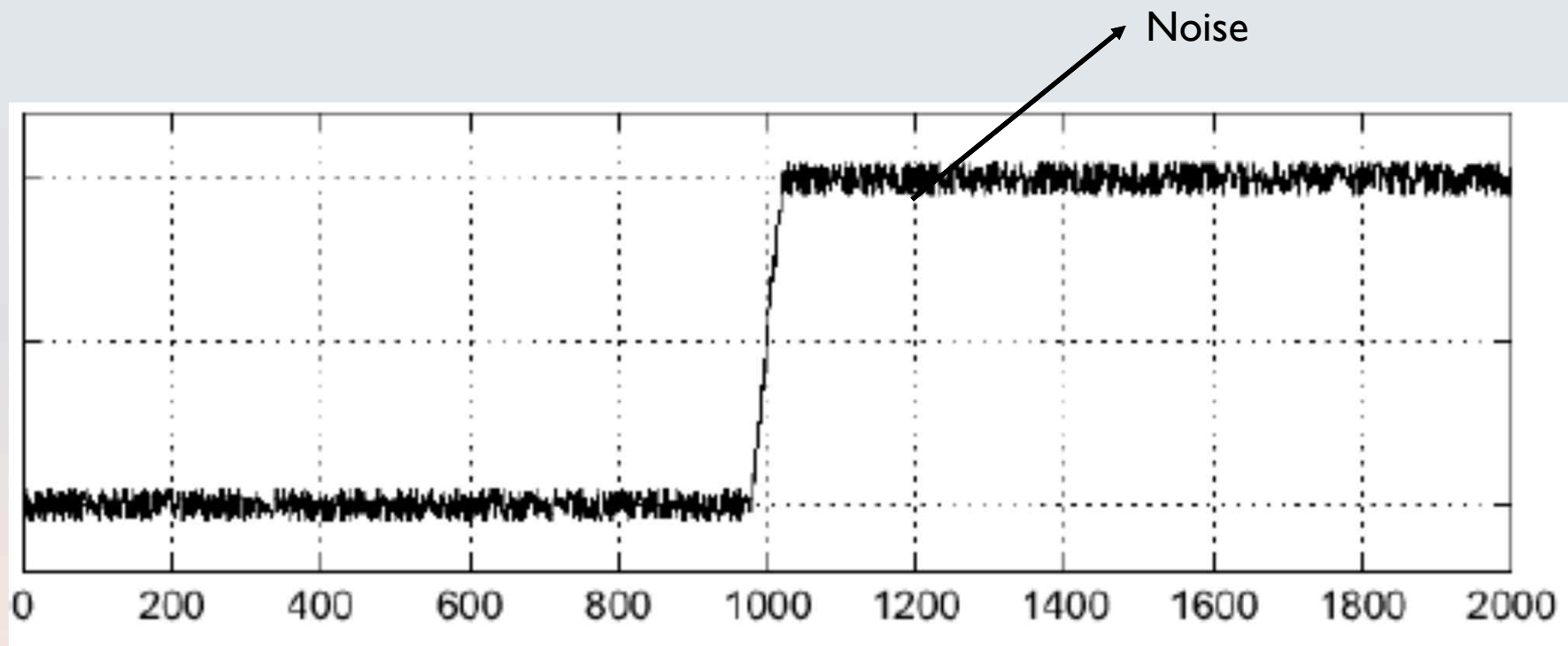


Comparison (Laplacian, Gradient)



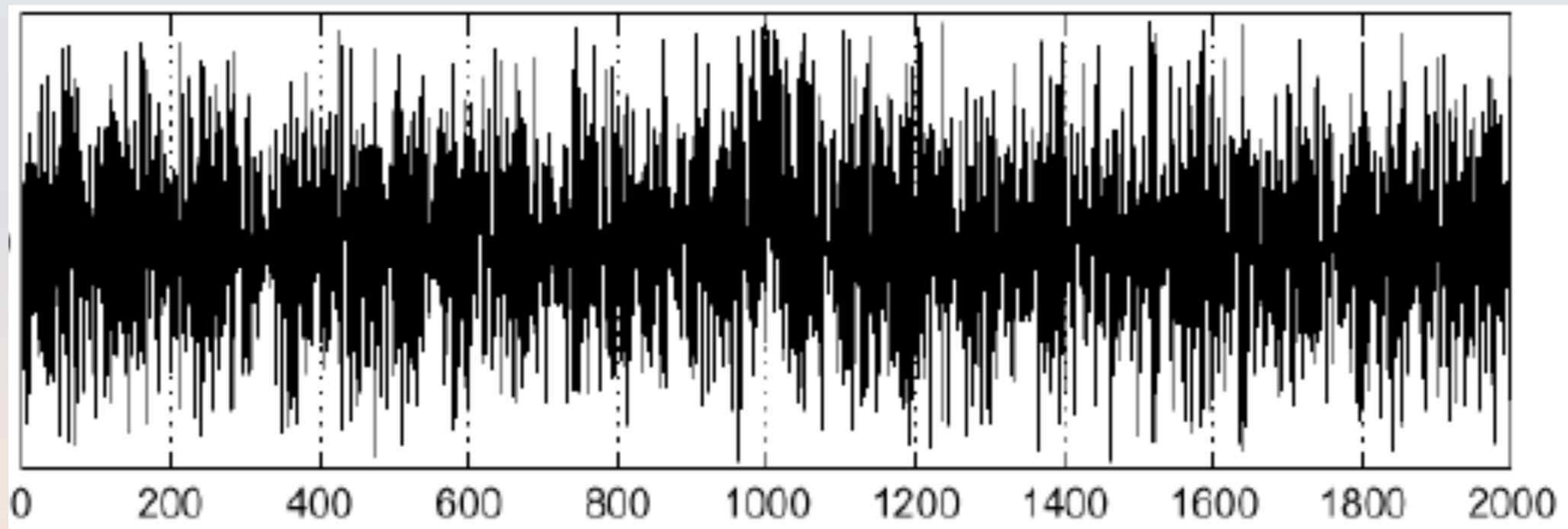
Noise & Filtering

- ▣ How do you find the edge from this signal?



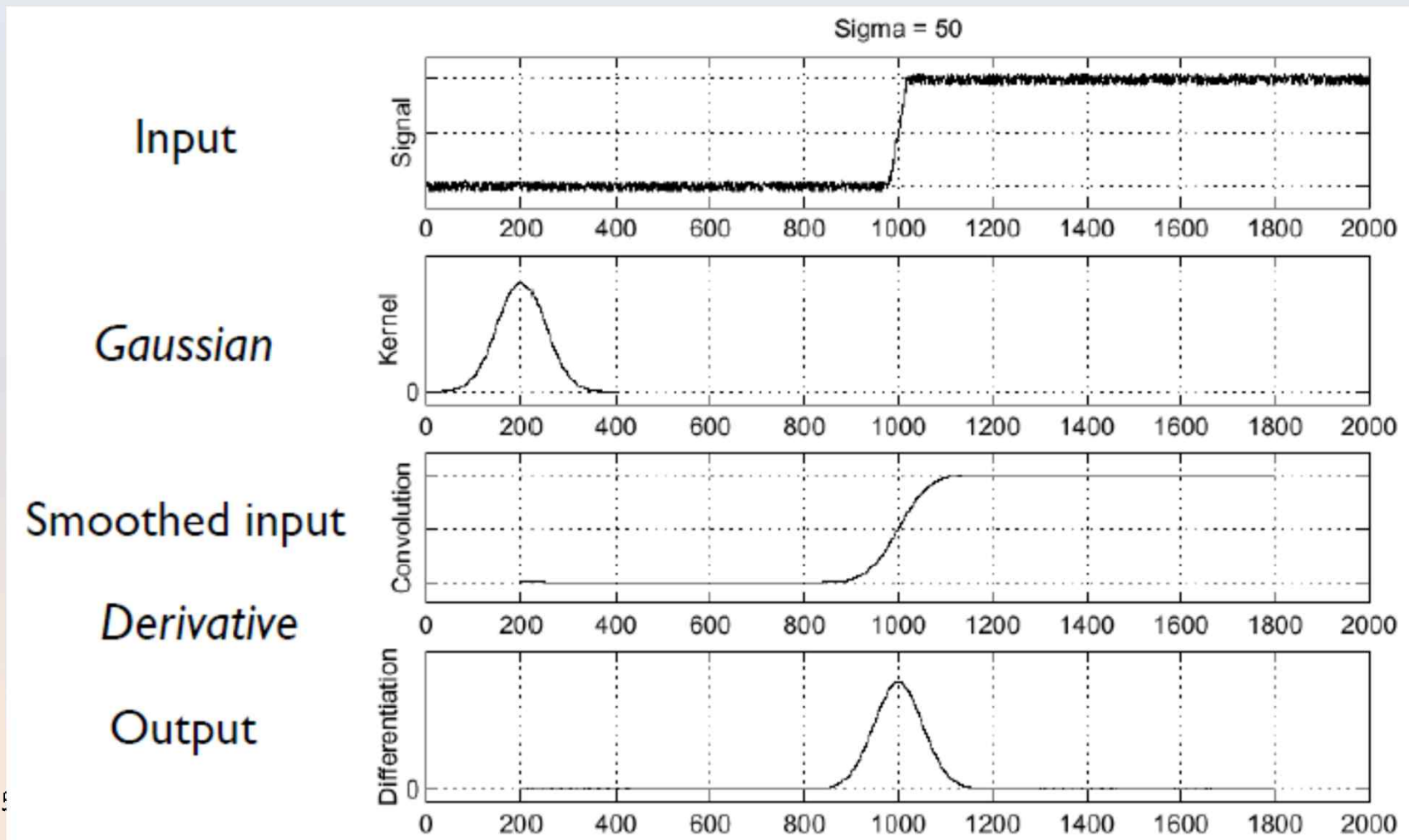
Noise & Filtering

- Use a derivative filter
 - Sensitive to noise

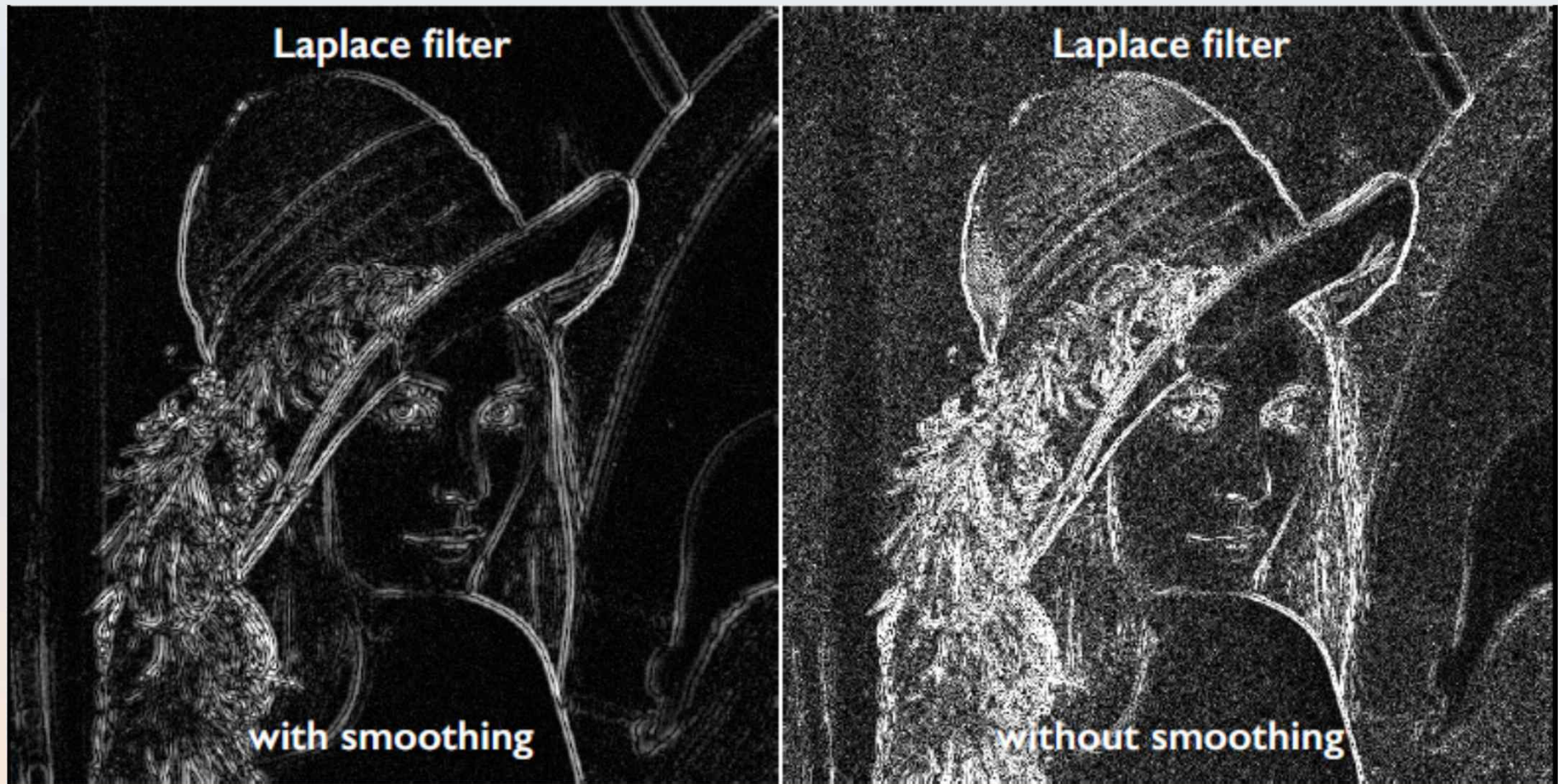


Noise & Filtering

- Don't forget to smooth before running derivative filters



Noise & Filtering



LAB

- ▣ Smoothing filter (low pass filter)
 - ▣ Box
 - ▣ Gaussian
- ▣ Edge detection filter (high pass filter)
 - ▣ Sobel
 - ▣ Laplacian
- ▣ More filters