Chapter 6 - Temporal-Difference Learning

Exercise 6.1

If V changes during the episode, then (6.6) only holds approximately; what would the difference be between the two sides? Let Vt denote the array of state values used at time t in the TD error (6.5) and in the TD update (6.2). Redo the derivation above to determine the additional amount that must be added to the sum of TD errors in order to equal the Monte Carlo error.

$G_t - V(S_t) = R_{t+1} + \gamma G_{t+1} - V_t(S_t) + \gamma V_t(S_{t+1}) - \gamma V_t(S_{t+1})$

$= \delta_t + \gamma(G_{t+1} - V_t(S_{t+1}))$

$= \delta_t + \gamma(R_{t+2} + \gamma G_{t+2} - (V_{t+1}(S_{t+1}) + (V_0(S_{t+1}) - V_{t+1}(S_{t+1}))))$

$= \delta_t + \gamma(R_{t+2} + \gamma G_{t+2} - V_{t+1}(S_{t+1}) - V_0(S_{t+1}) + V_{t+1}(S_{t+1}))$

$= \delta_t + \gamma(R_{t+2} + \gamma G_{t+2} - V_{t+1}(S_{t+1}) - V_0(S_{t+1}) + V_{t+1}(S_{t+1}) + \gamma V_{t+1}(S_{t+2}) + \gamma V_0(S_{t+2}) - \gamma V_{t+1}(S_{t+2}) - (\gamma V_{t_1}(S_{t+2}) + (\gamma V_0(S_{t+2}) - \gamma V_{t+1}(S_{t+2}))))$

$= \delta_t + \gamma(R_{t+2} + \gamma V_{t+1}(S_{t+2}) - V_{t+1}(S_{t+1}) - V_0(S_{t+1}) + V_{t+1}(S_{t+1}) + \gamma V_0(S_{t+2}) - \gamma V_{t+1}(S_{t+2}) + \gamma G_{t+2} - (\gamma V_{t_1}(S_{t+2}) + (\gamma V_0(S_{t+2}) - \gamma V_{t+1}(S_{t+2}))))$

Define $d_t = -V_0(S_t) + V_t(S_t) + \gamma V_0(S_{t+1}) - \gamma V_t(S_{t+1})$

$= \delta_t + \gamma(\delta_{t+1} + d_t + \gamma G_{t+2} - (\gamma V_{t_1}(S_{t+2}) + (\gamma V_0(S_{t+2}) - \gamma V_{t+1}(S_{t+2}))))$

$= \delta_t + \gamma\delta_{t+1} + \gamma d_t + \gamma^2 G_{t+2} - \gamma(\gamma V_{t_1}(S_{t+2}) + (\gamma V_0(S_{t+2}) - \gamma V_{t+1}(S_{t+2})))$

$= \delta_t + \gamma\delta_{t+1} + \gamma d_t + \gamma^2\delta_{t+2+\gamma^2 d_{t+1}} + \ldots + \gamma^{T-t-1}\delta_{T-1} + \gamma^{T-t-1}d_{T-1} + \gamma^{T-t}(0-0)$

$= \sum_{k=t}^{T-1} \gamma^{k-t}\delta_k + \sum_{k=t+1}^{T-1} \gamma^{k-t} d_k$

Exercise 6.2

This is an exercise to help develop your intuition about why TD methods are often more efficient than Monte Carlo methods. Consider the driving home example and how it is addressed by TD and Monte Carlo methods. Can you imagine a scenario in which a TD update would be better on average than a Monte Carlo update? Give an example scenario—a description of past experience and a current state—in which you would expect the TD update to be better. Here's a hint: Suppose you have lots of experience driving home from work. Then you move to a new building and a new parking lot (but you still enter the highway at the same place). Now you are starting to learn predictions for the new building. Can you see why TD updates are likely to be much better, at least initially, in this case? Might the same sort of thing happen in the original scenario?

In the case given by the hint, our value predictions from entering a highway to exiting logically should remain the same since we are entering it from the same place. When we start to learn predictions from the new building using TD Learning it should keep our value estimates for the highway the same while changing our value estimates from the parking lot to the highway, rather than treating the whole as a different problem. If we use MC Methods we don't take advantage of the value predictions we already have for the highway, and would be subject to a greater variance, variables that affect time after the highway would 'ebb' into the parking lot time and vice versa.

In the original scenario TD Learning would benefit learning speed by being able to isolate variances to the sub problems. If we think of the sub problems as time steps, our agent has more 'temporal' awareness of where variances are coming from with regard to the overall problem.

Exercise 6.3

From the results shown in the left graph of the random walk example it appears that the first episode results in a change in only V (A). What does this tell you about what happened on the first episode? Why was only the estimate for this one state changed? By exactly how much was it changed?

On the first episode the agent ended up going to the left. Only the estimate for V (A) changes because the updates are done on each step. Rewards are 0 and all values are the same so change would only occur on a step that incurs reward on the first run.

Exercise 6.4

The specific results shown in the right graph of the random walk example are dependent on the value of the step-size parameter, α. Do you think the conclusions about which algorithm is better would be a↵ected if a wider range of ↵ values were used? Is there a di↵erent, fixed value of α at which either algorithm would have performed significantly better than shown? Why or why not?

Assuming that MC used the same range of alpha values, it is possible that there is a different value that either algorithm could have performed better. Perhaps between the ranges of .1 and .05, TD-Learning could have performed better. We see a pattern, however, of a "sweet spot" for the alpha values so we know .05 is near optimal.

The conclusions of which algorithm is better is probably not affected by this since MC and TD-Learning's alpha values are used in a near identical way. Even though TD-learning only applies its alpha value to the immediate reward, the subsequent updates are compounding the alpha's multiplication the same way it would be done with a MC method.

Exercise 6.5

In the right graph of the random walk example, the RMS error of the TD method seems to go down and then up again, particularly at high ↵'s. What could have caused this? Do you think this always occurs, or might it be a function of how the approximate value function was initialized?

It could be a result of the initializations being close to optimal for V(C), it initially goes down as it updates those farther away from C which would be updated first, but then once it starts updating C it goes down because any changes to C would be making the error larger.

I don't believe this should always occur since we can expect fluctuations but an overall pattern of the error going down the back up is unusual, especially for high values of alpha, which indicates that it is due to a pattern in the updates.

Exercise 6.6

In Example 6.2 we stated that the true values for the random walk example are 1/6 , 2/6 , 3/6 , 4/6 , and 5/6, for states A through E. Describe at least two different ways that these could have been computed. Which would you guess we actually used? Why?

One way would be to use pure probability and mathematics. Let $P(x, y)$ denote the probability of an agent at node x ending up at node y. x and y ∈ [s,A,B,C,D,E,e]

$P(A, s) = 0.5 + 0.5 * P(B, s)$

$= 0.5 + 0.5(0.5 * P(C, s) + 0.5 * P(A, s)$

$= 0.5 + 0.25 * P(C, s) + 0.25 * P(A, s)$

Thanks to symmetry we know $P(C, s) = 0.5$

$= 0.5 + 0.25 * 0.5 + 0.25 * P(A, s)$

$0.75 * P(A, s) = 0.625$

$P(A, s) = 0.8333... = 5/6$

$P(E, s) = 1 - P(A, s) = 1/6$

This matches the result! We can find the rest of the probabilities in a similar fashion.

Another way would be to calculate a 'tree' of probabilities by iterating over all possible paths. The tree would have the node being calculated for, and with 0.5 probability weighting for each of the two branches, it leads to its neighbor nodes. The probability would be a sum of the paths to its leaf nodes that result in leading to the right side. The deeper the tree, the more accurate the calculation. This tree has infinite depth so one would calculate it to a certain depth depending on how accurate of an answer is desired

I would guess that the mathematical way was used because it takes less time to do and yields optimally accurate results.

Exercise 6.7

Design an off policy version of the TD(0) update that can be used with arbitrary target policy π and covering behavior policy b, using at each step t the importance sampling ratio pt:t (5.3).

$V(S) < -V(S) + p_{t:t}[R + \gamma V(S') - V(S)]$

Exercise 6.8

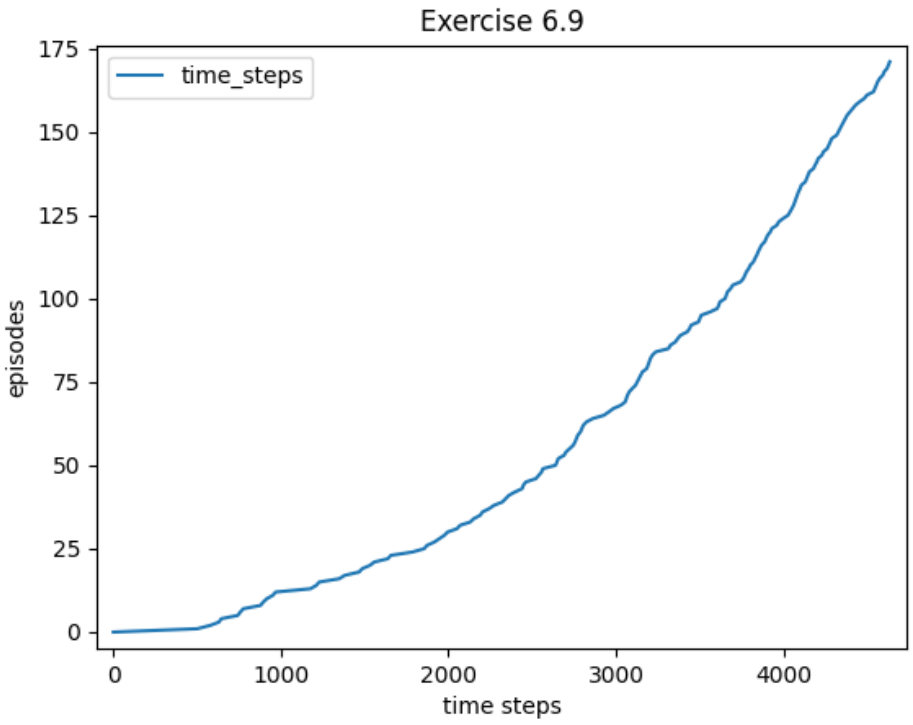
$G_t - Q(S_t, A_t) = R_{t+1} + \gamma G_{t+1} - Q(S_t, A_t) + \gamma Q(S_{t+1}, A_{t+1}) - \gamma Q(S_{t+1}, A_{t+1})$

$= \delta_t + \gamma(G_{t+1} + \gamma Q(S_{t+1}, A_{t+1}))$

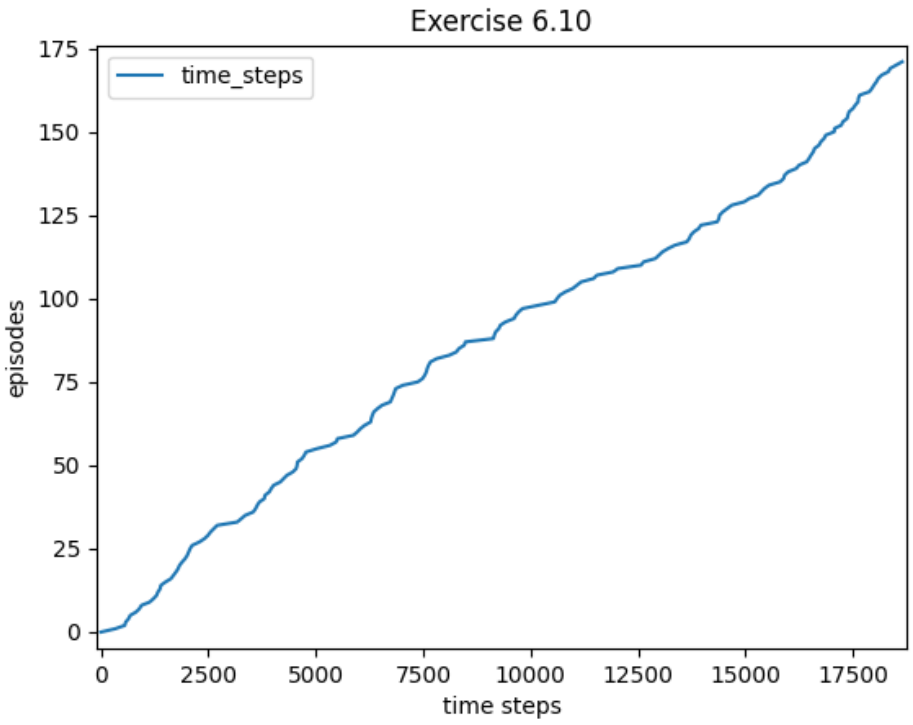
$= \delta_t + \gamma\delta_{t+1} + \gamma^2(G_{t+2} - Q(S_{t+2}, A_{t+2}))$

$= \sum_{k=t}^{T-1} \gamma^{k-t}\delta_k$

Exercise 6.9



Exercise 6.10



Exercise 6.11

Why is Q-learning considered an off-policy control method?

It is considered an off-policy control method because it's learning the value function for policy π^* that is completely greedy from the actions of policy pi which is only epsilon-greedy. It does this by performing the actions of an epsilon greedy policy, but basing its updates to Q with the next state-action pair being based on a completely greedy policy, taking advantage of the structure of a TD Q-based update rule to ensure that all action-states are updated while finding the values for the optimal policy in order to gaurantee convergence to the optimal policy. Very clever.

Exercise 6.12

Suppose action selection is greedy. Is Q-learning then exactly the same algorithm as Sarsa? Will they make exactly the same action selections and weight updates?

Yes, then Q-learning would be the same as fully greedy Sarsa. The key difference is that action selection does not match the update rule.

Exercise 6.13

What are the update equations for Double Expected Sarsa with an ϵ-greedy target policy?

$Q_1(S,A) < -Q_1(S,A) + \alpha(R + \gamma E[Q_2(S,A)])$

Where $E[Q_2(S,A)]$ is based on the ϵ-greedy target policy.

Exercise 6.14

Describe how the task of Jack's Car Rental (Example 4.2) could be reformulated in terms of afterstates. Why, in terms of this specific task, would such a reformulation be likely to speed convergence?

Instead of basing it on action values as we did, we could base our values on the states that result from choosing actions. Then, we would choose actions based on the values of the resulting states since we know what immediate states occur- before the stochastic randomness occurs to them. This would speed up learning because state-action pairs that lead to the same initial state would be treated the same- as they should.