

Linear Algebra

***Least Square-based
Sensor Data Processing***

Automotive Intelligence Lab.



Contents

- Lane model fitting for camera lane detector
- Ground model fitting for LiDAR measurements

Lane model fitting for camera lane detector

Various Camera based Perception Systems

■ StradVision



Cameras in Autonomous Vehicle



Least Square Example: Visual SLAM

■ Visual simultaneous localization and mapping (SLAM)

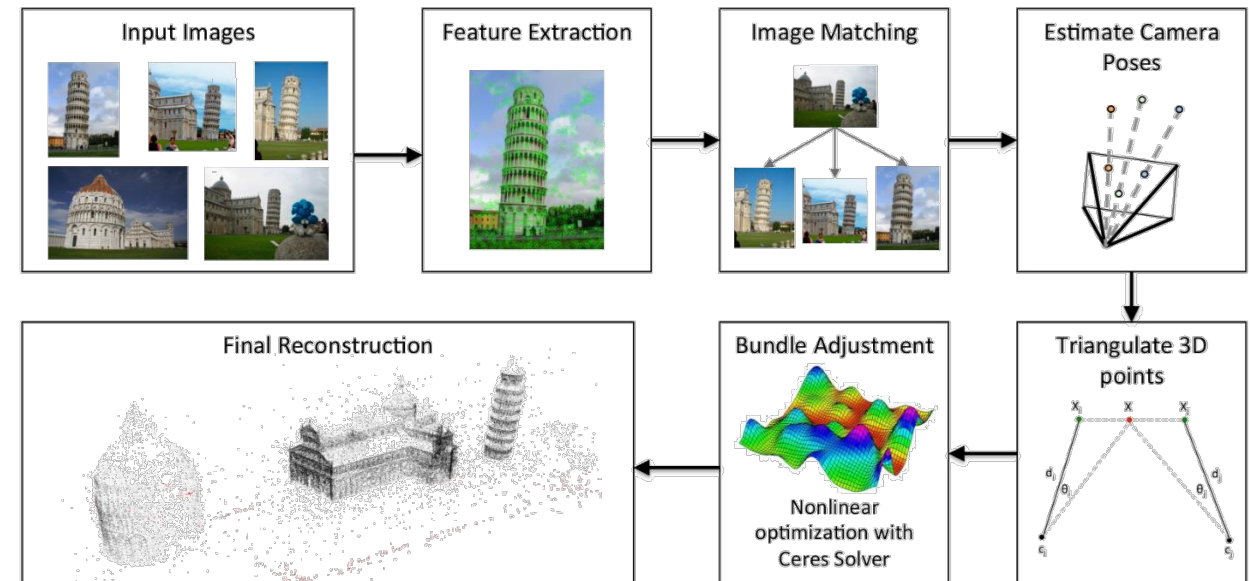
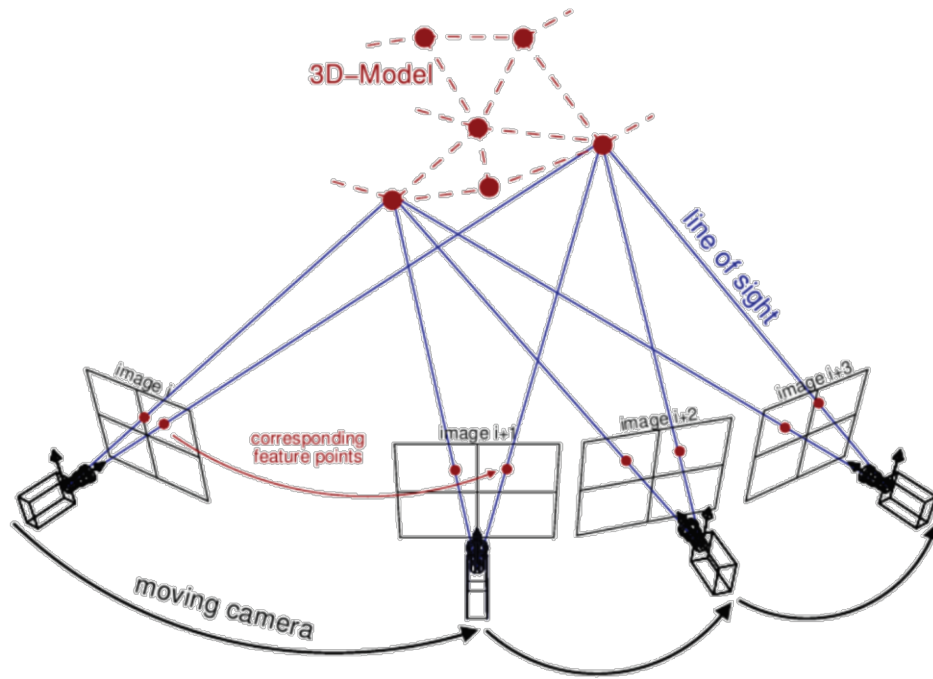
- ▶ SLAM means estimating pose of the sensor while creating a map of the environment
- ▶ Visual SLAM uses only vision(camera) sensor for SLAM



Least Square Example: Structure from Motion (SfM)

■ SfM with monocular camera

- ▶ From 2D-image to 3D-model with monocular camera
- ▶ A photogrammetric range imaging technique for estimating three-dimensional structures from two-dimensional image sequences that may be coupled with local motion signals.



Structure from Motion (SfM) Demo

■ SfM (Structure from Motion) demo video



Image Representation (1/2)

Image Acquisition Process

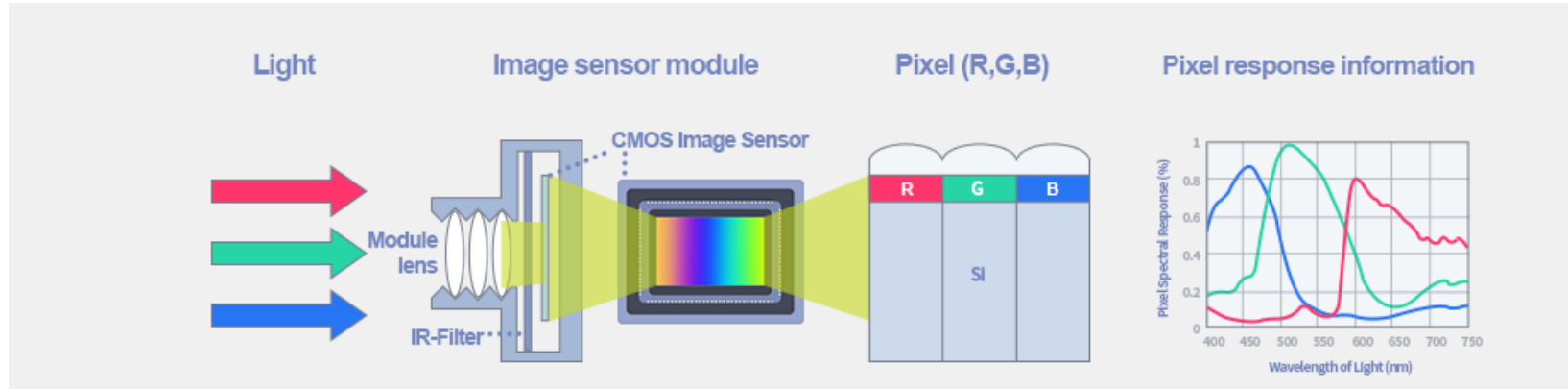


Image Representation

► Pixel

- The smallest unit that composes an image
- An image is an array of pixels

► Image coordinates

- Normally, the top left is (0,0)
- $f(x,y)$: pixel values at coordinates (x,y)

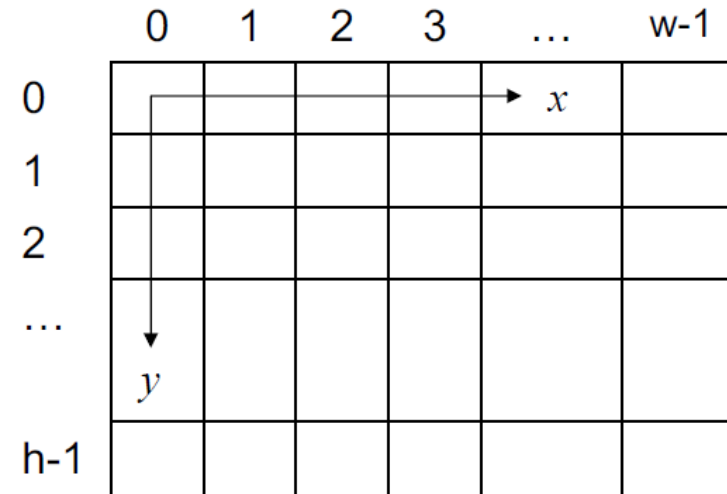
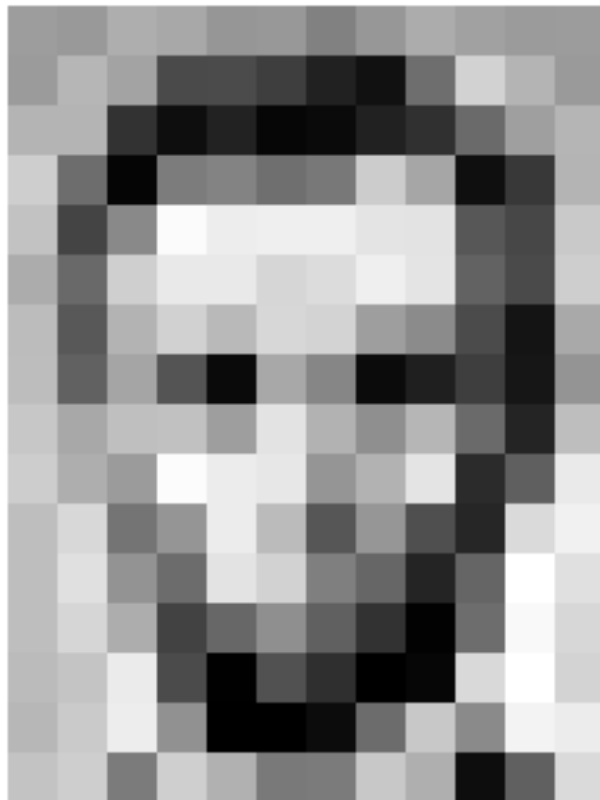


Image Representation (2/2)



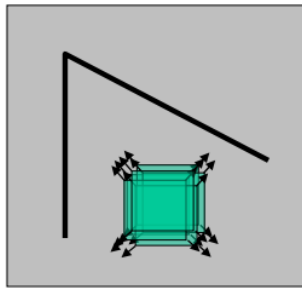
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155	182	163	74	75	62	33	17	110	210	180	154
180	180	50	14	34	6	10	33	48	106	159	181
206	109	5	124	131	111	120	204	166	15	56	180
194	68	137	251	237	239	239	228	227	87	71	201
172	105	207	233	233	214	220	239	228	98	74	206
188	88	179	209	185	215	211	158	139	75	20	169
189	97	165	84	10	168	134	11	31	62	22	148
199	168	191	193	158	227	178	143	182	106	36	190
205	174	155	252	236	231	149	178	228	43	95	234
190	216	116	149	236	187	86	150	79	38	218	241
190	224	147	108	227	210	127	102	36	101	255	224
190	214	173	66	103	143	96	50	2	109	249	215
187	196	235	75	1	81	47	0	6	217	255	211
183	202	237	145	0	0	12	108	200	138	243	236
195	206	123	207	177	121	123	200	175	13	96	218

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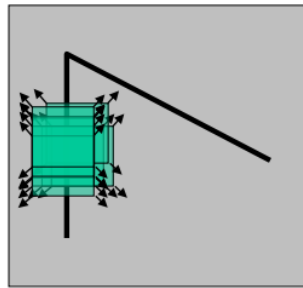
Image Feature Extraction

■ What is feature?

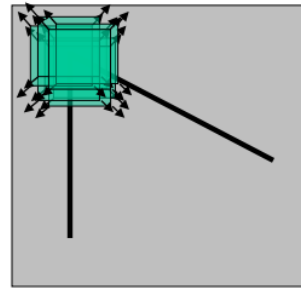
- ▶ Local, meaningful, detectable part of the image
- ▶ Edges
 - A part where pixel values change rapidly in one direction
 - Can determine the outline of an object
- ▶ Corners
 - A part where pixel values change rapidly in all direction



“flat” region:
no change in
all directions



“edge”:
no change along
the edge direction



“corner”:
significant change
in all directions

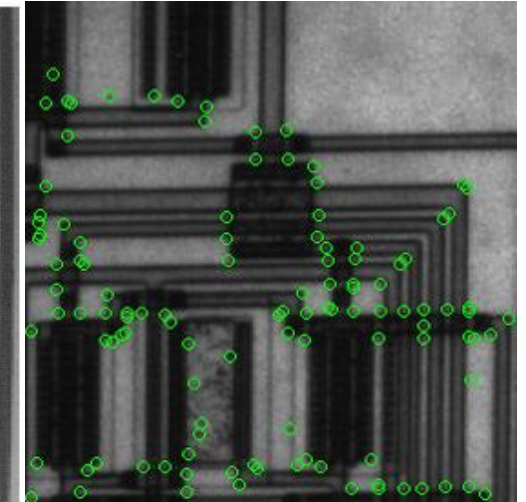
Original Image



Edge Detected



Example of edge detection

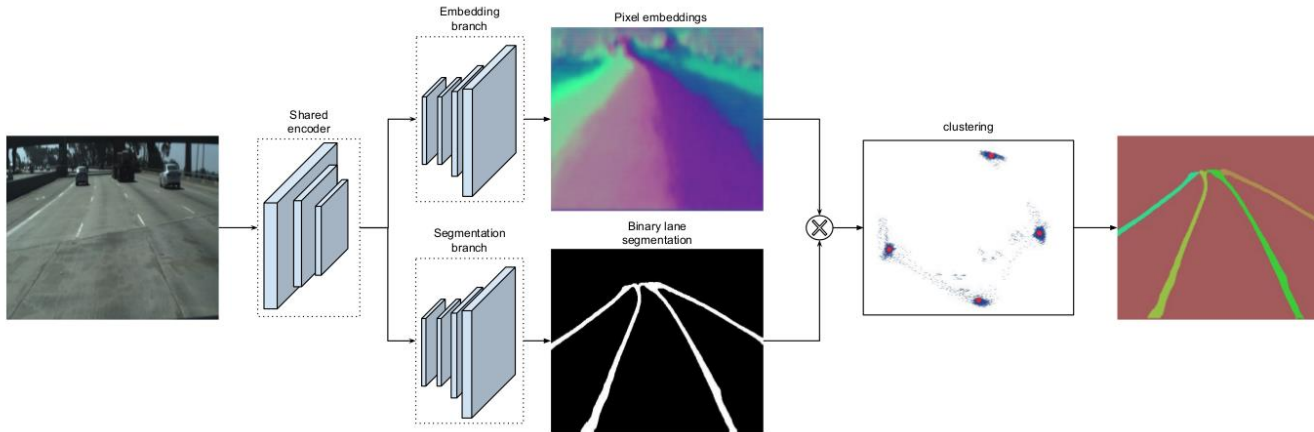


Example of Corner detection

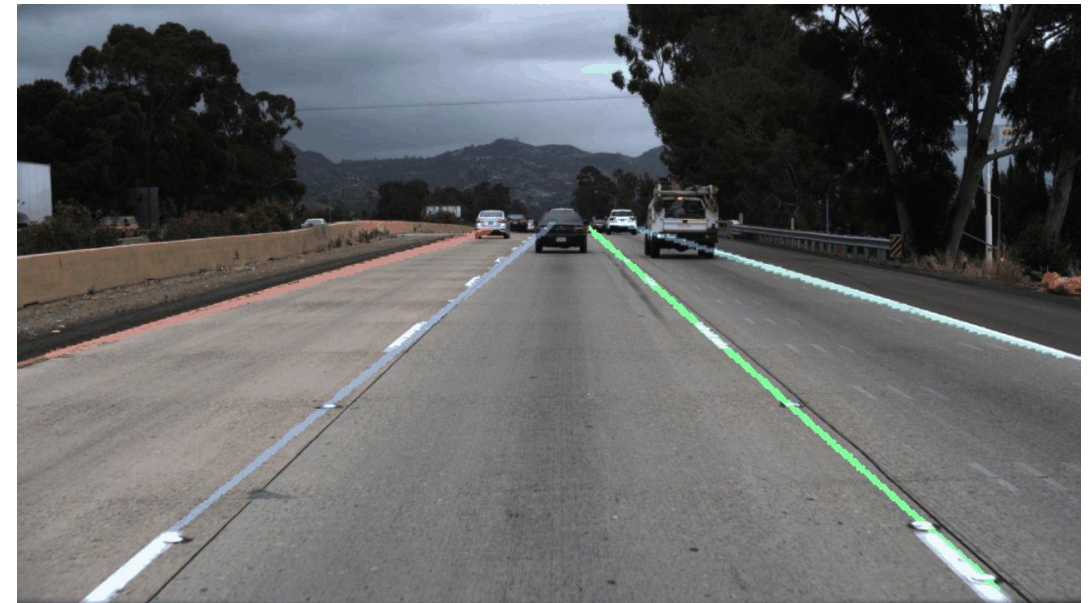
Machine Learning based Lane Detection

■ LaneNet: Real-time lane detection networks for autonomous driving

- ▶ Model takes driving scene image
- ▶ Shared layer extracts feature from image
- ▶ Two branch architecture
 - Embedding branch extracts instance information to give unique identification between lanes
 - Segmentation branch identifies lane segment



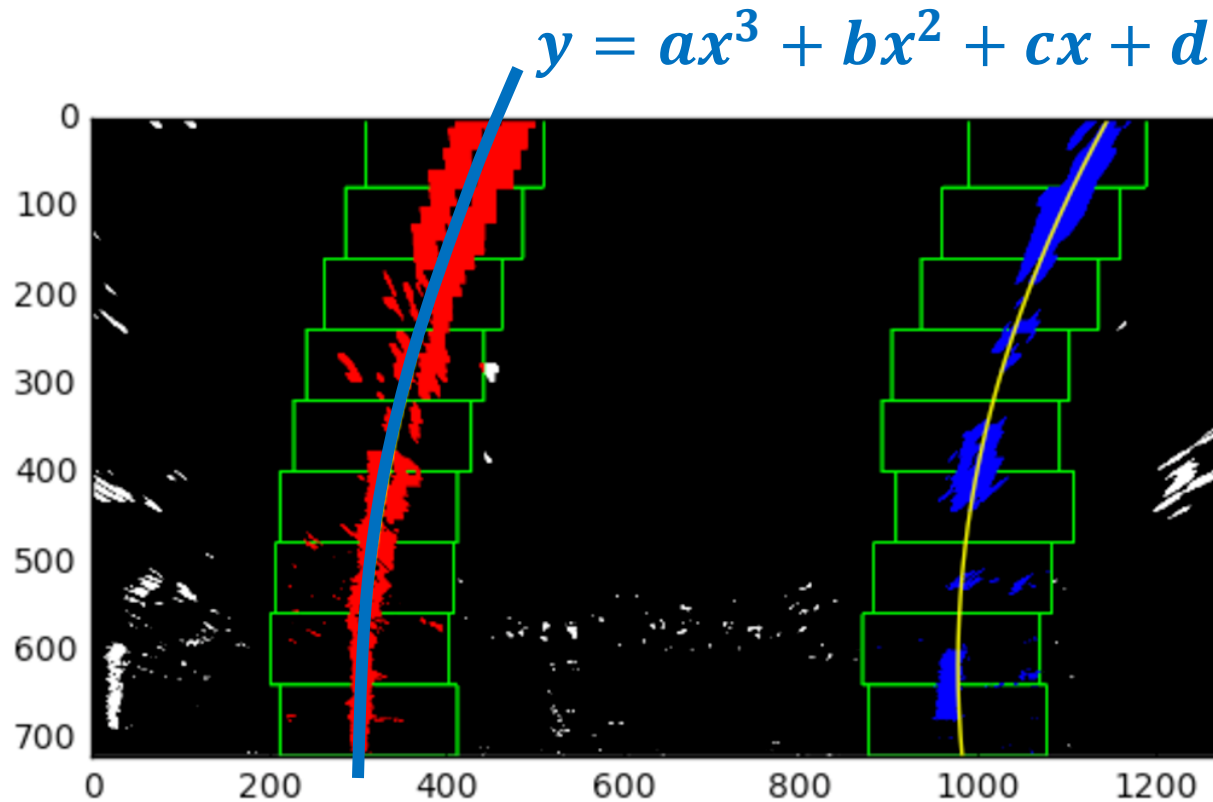
LaneNet Architecture



Lane detection demo

Lane Model Fitting with Least Square

- Find a polynomial equation from the detected pixel.



Meaning of Curve Equation

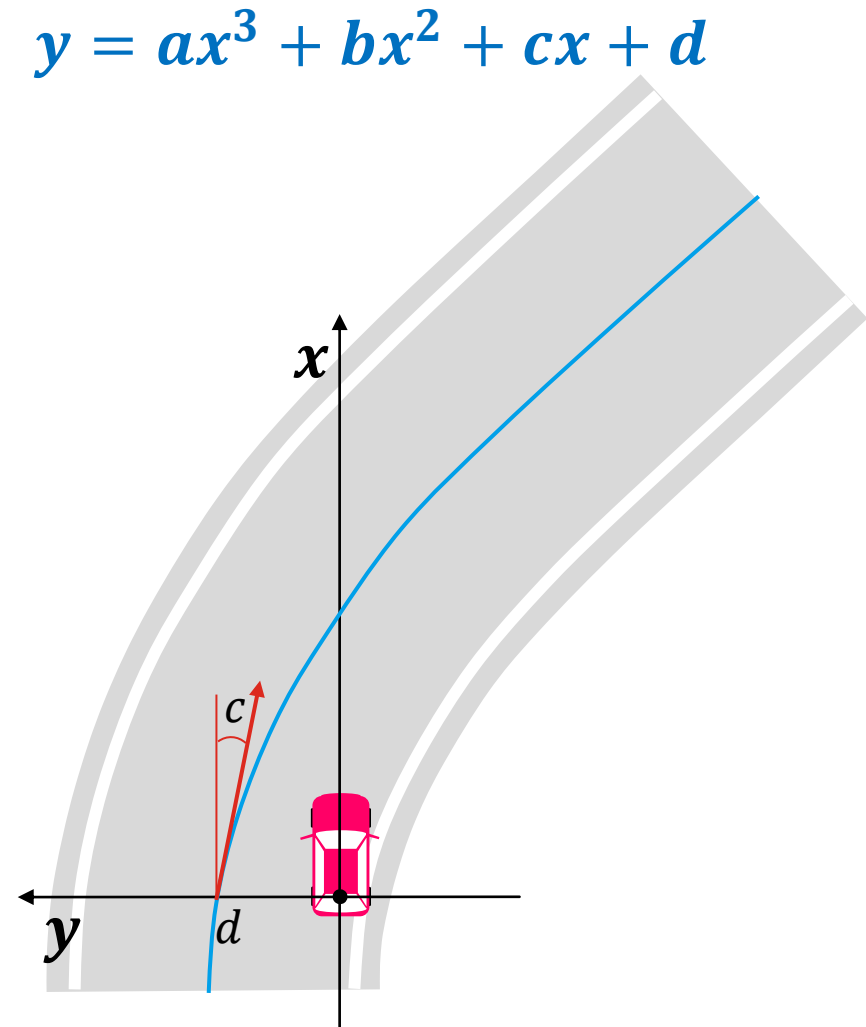
■ Lane Position

► $y|_{x=0} = a * 0^3 + b * 0^2 + c * 0 + d = d$

■ Heading Angle

► *Heading angle* = $\text{atan} \frac{dy}{dx} \Big|_{x=0} \approx \frac{dy}{dx} \Big|_{x=0}$

► $\frac{dy}{dx} \Big|_{x=0} = 3ax^2 + 2bx + c \Big|_{x=0} = c$



Meaning of Curve Equation

Curvature

$$\blacktriangleright R = \frac{1}{\kappa} = \frac{(1+y'^2)^{\frac{3}{2}}}{y''}, \quad y'' = \frac{d^2y}{dx^2} = 6ax + 2b$$

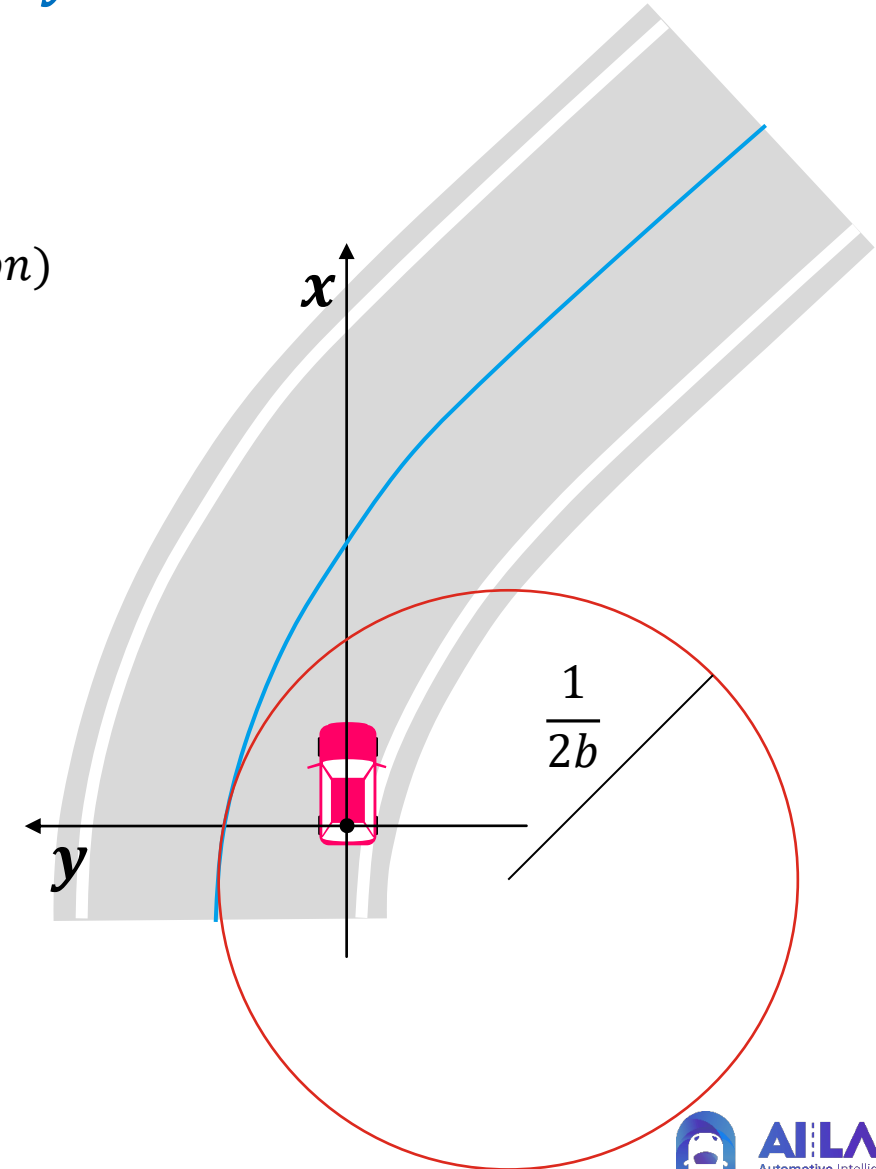
$$\blacktriangleright \kappa = \frac{y''}{(1+y'^2)^{\frac{3}{2}}} \bigg|_{x=0} = \frac{2b}{(1+c^2)^{\frac{3}{2}}} \approx 2b \quad (c \ll 1, \text{small angle estimation})$$

Curvature Derivative

$$\blacktriangleright \frac{d\kappa}{dx} = \frac{d}{dx} \frac{y''}{(1+y'^2)^{\frac{3}{2}}} = \frac{y'''(1+y'^2)^{\frac{3}{2}} - 3y''^2(1+y'^2)^{\frac{1}{2}}}{(1+y'^2)^3}$$

$$\blacktriangleright \kappa' \approx y'''|_{x=0} = 6a, \quad (\text{when } y' \ll 1 \text{ and } y'' \ll 1)$$

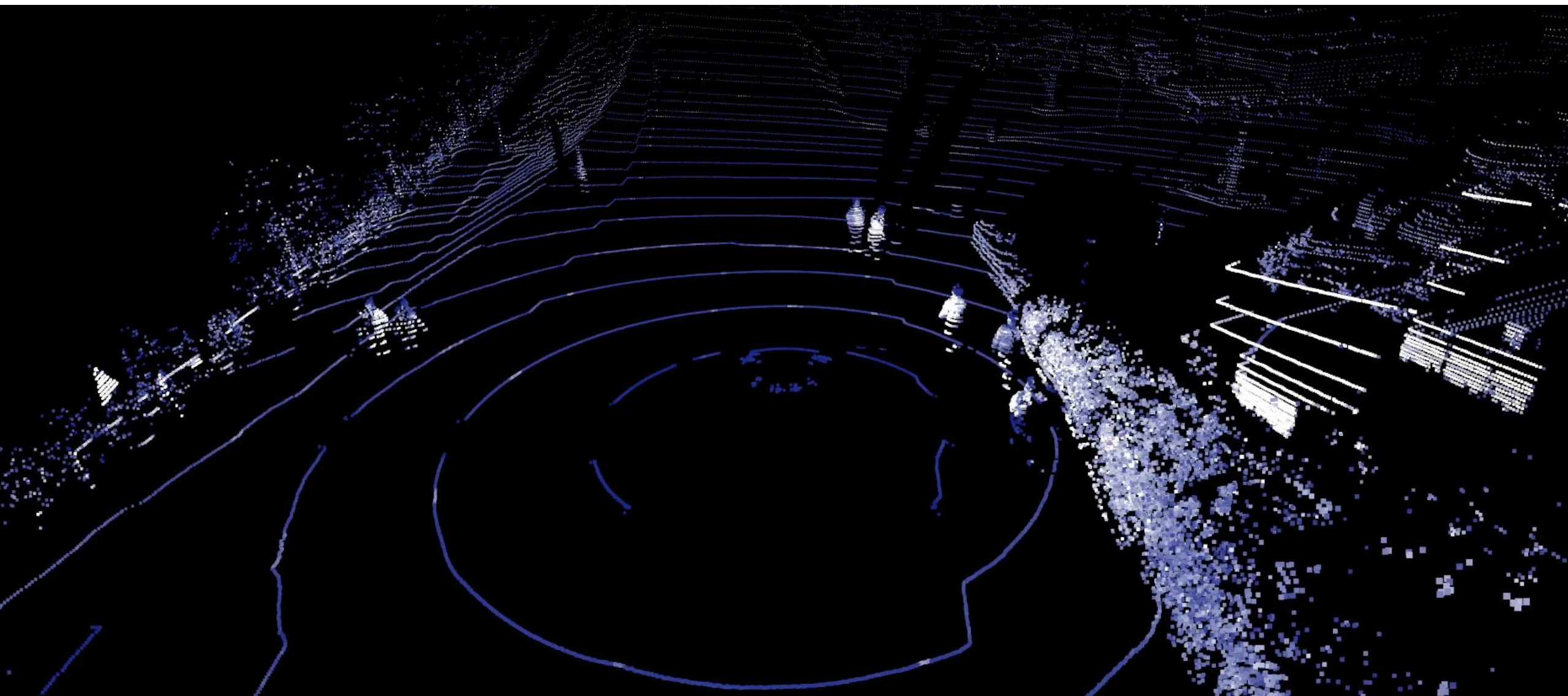
$$y = ax^3 + bx^2 + cx + d$$



<https://www.intmath.com/applications-differentiation/8-radius-curvature.php>

Ground model fitting for LiDAR measurements

LiDAR (Light Detection And Ranging)



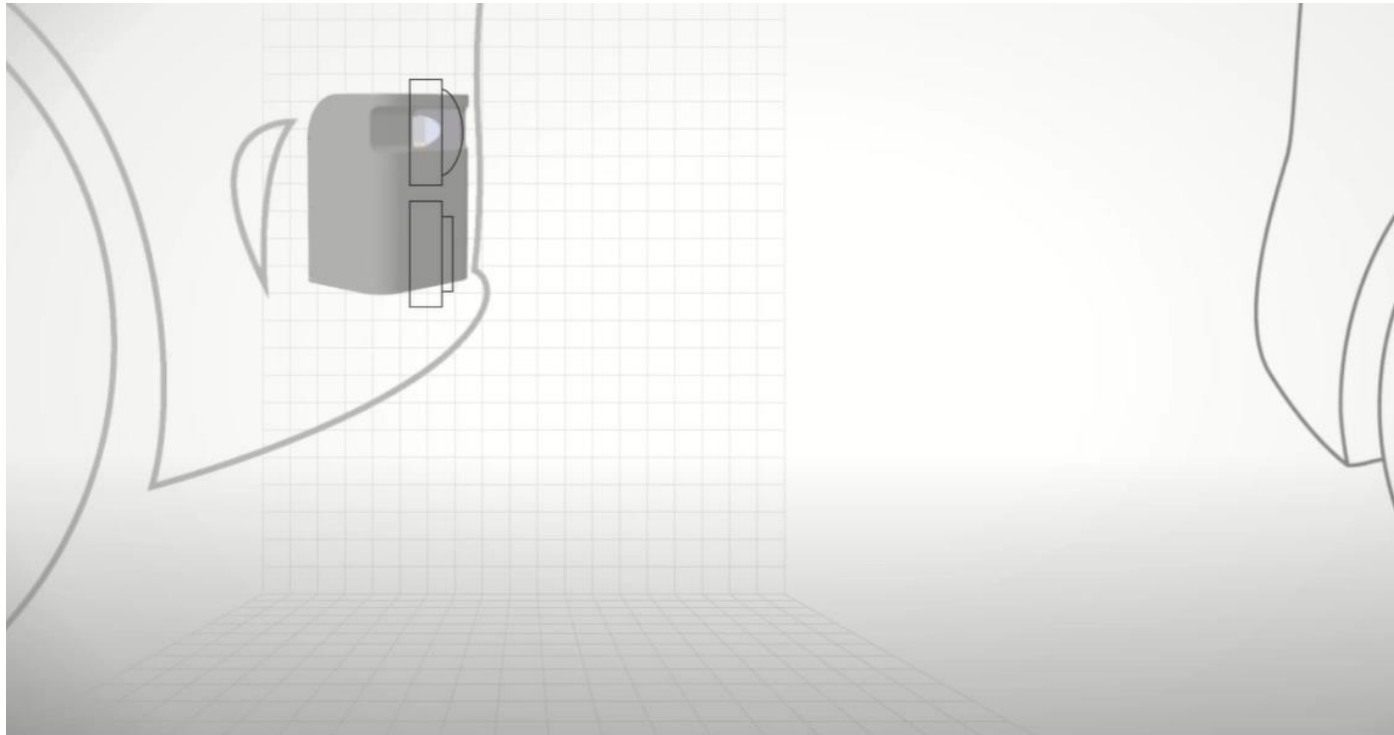
LiDARs on Autonomous Cars



ToF (Time of Flight)

■ Time of Flight (ToF)

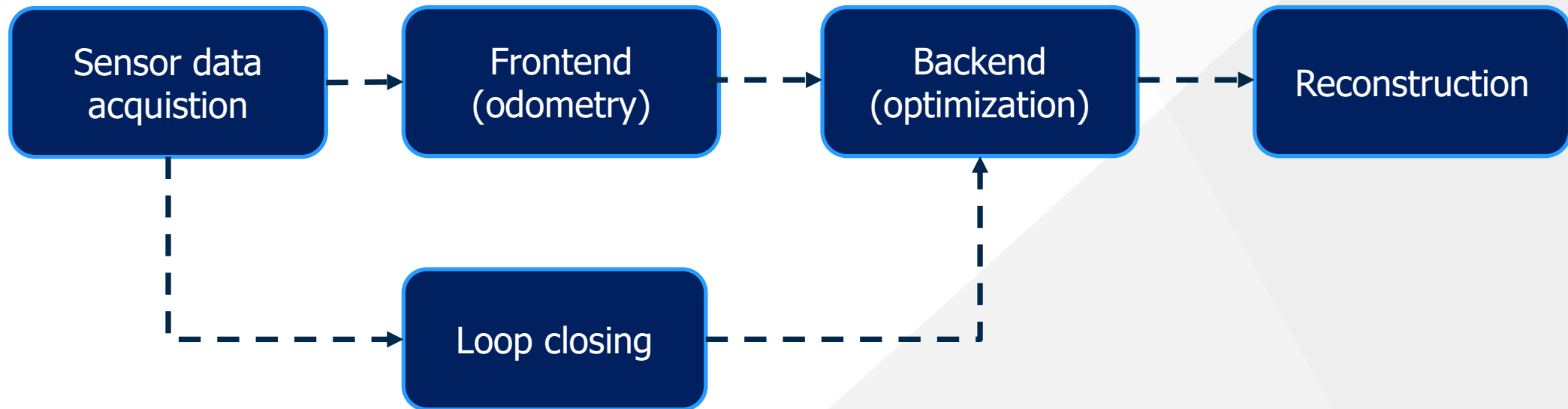
- ▶ Emitting a laser pulse on a surface
- ▶ Catching the echo pulse to the LiDAR pulse source with sensors
- ▶ Measuring the time laser travelled using TDC (time to digital converter) threshold



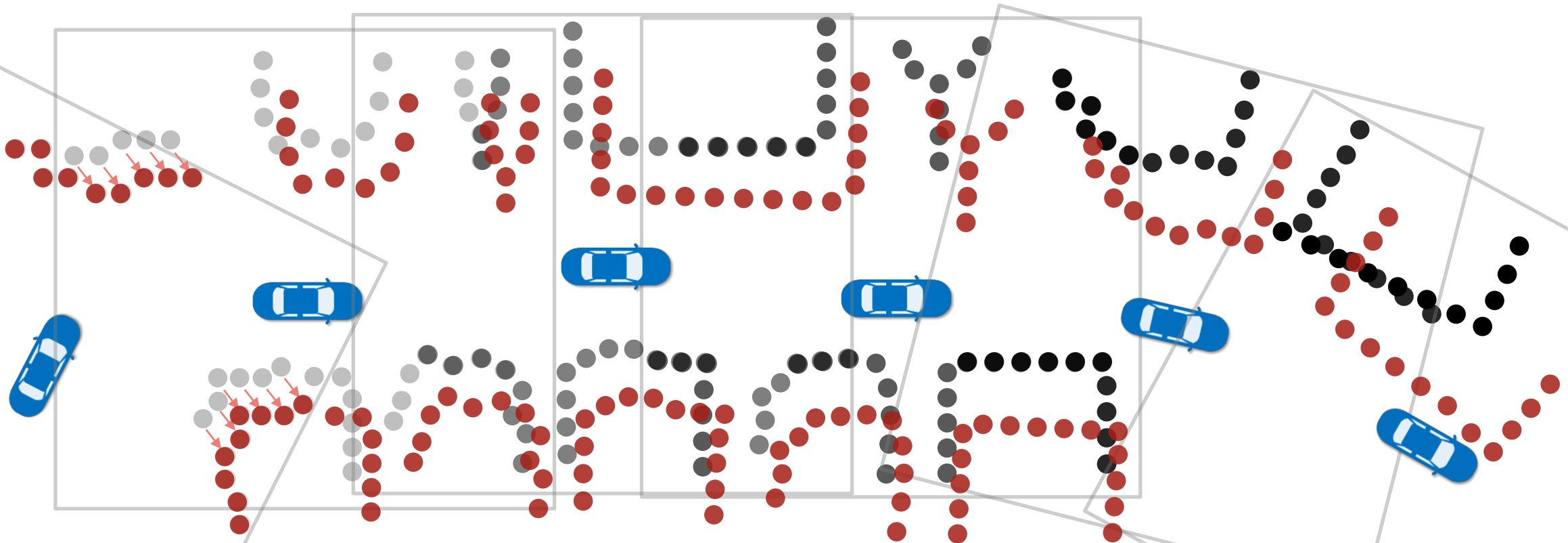
Application: Point Cloud-based SLAM

■ Optimization-based SLAM framework

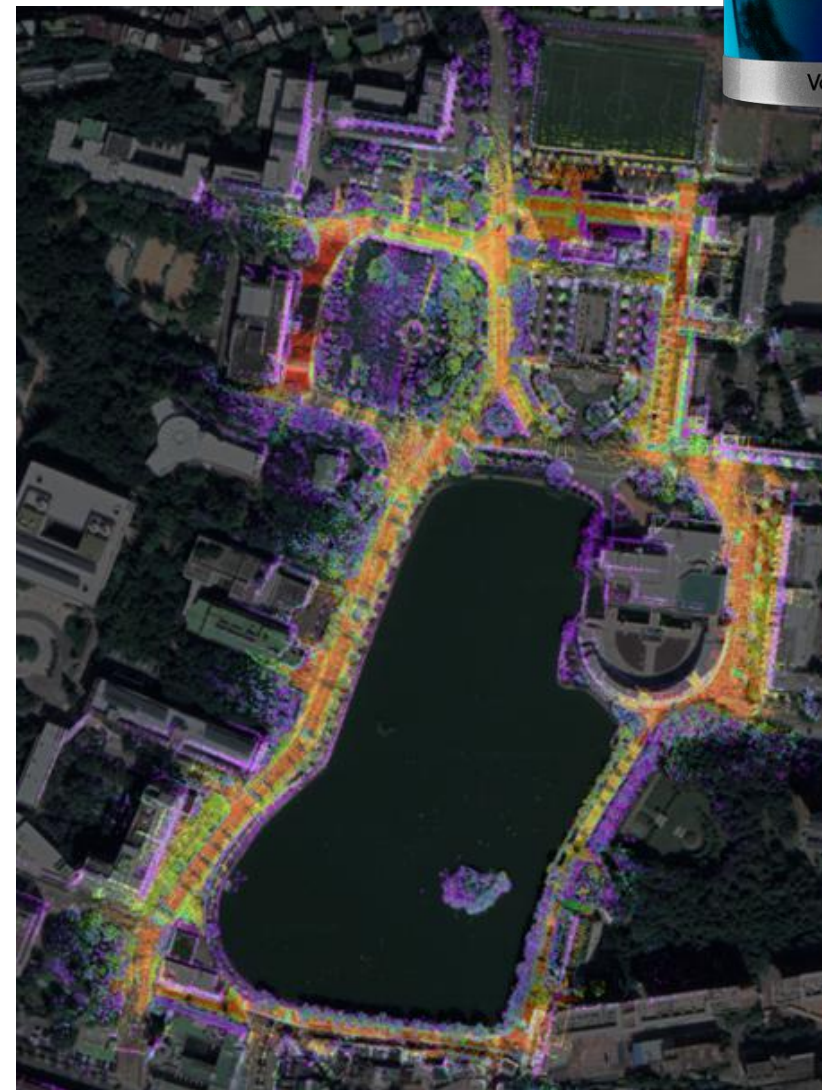
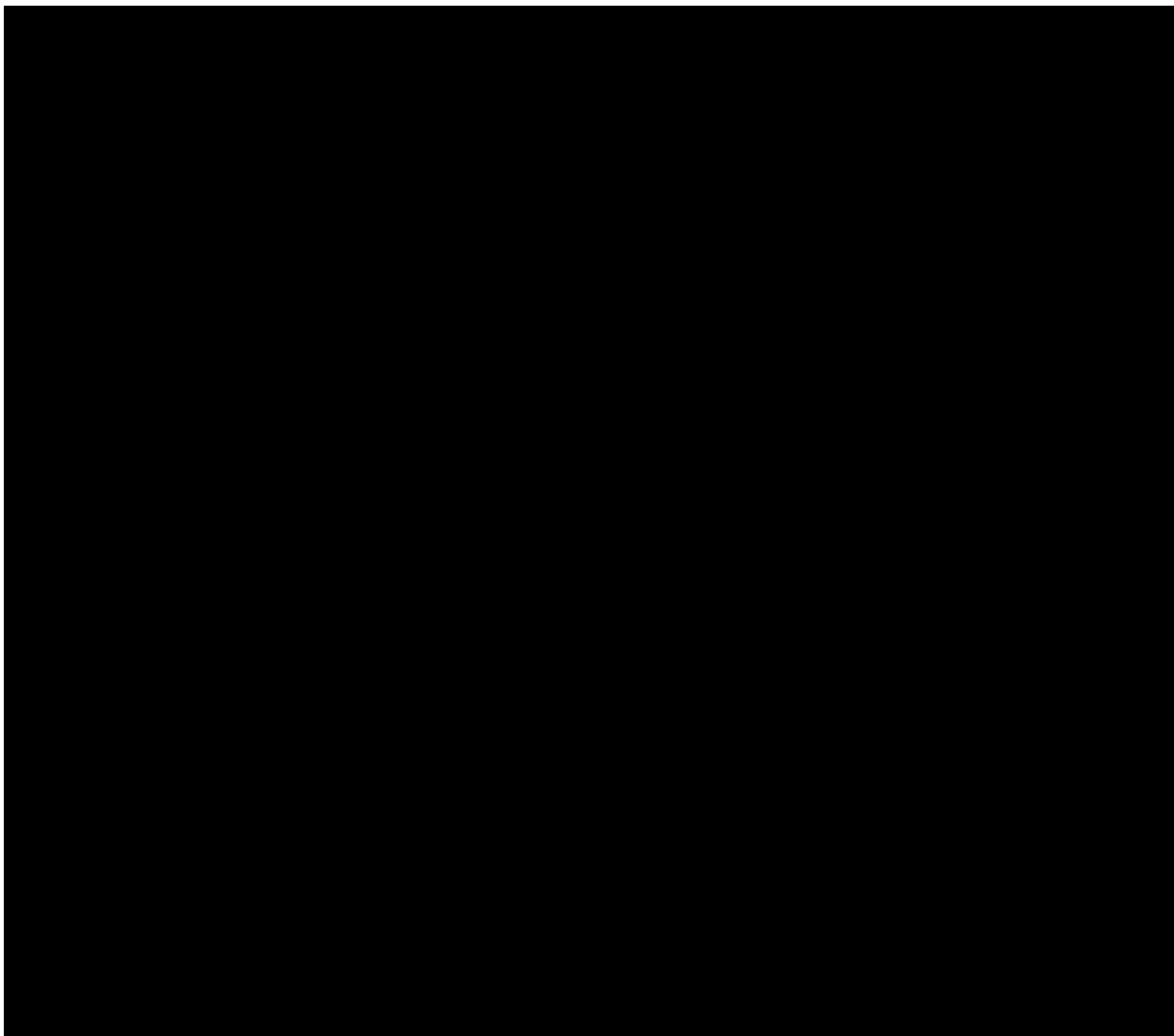
1. Sensor data acquisition
2. **Frontend**: Point cloud registration-based odometry
3. **Backend**: Filtering or Optimization
4. Loop Closing
5. Reconstruction



Application: Point Cloud SLAM for map generation



LiDAR-based SLAM for map generation (I)

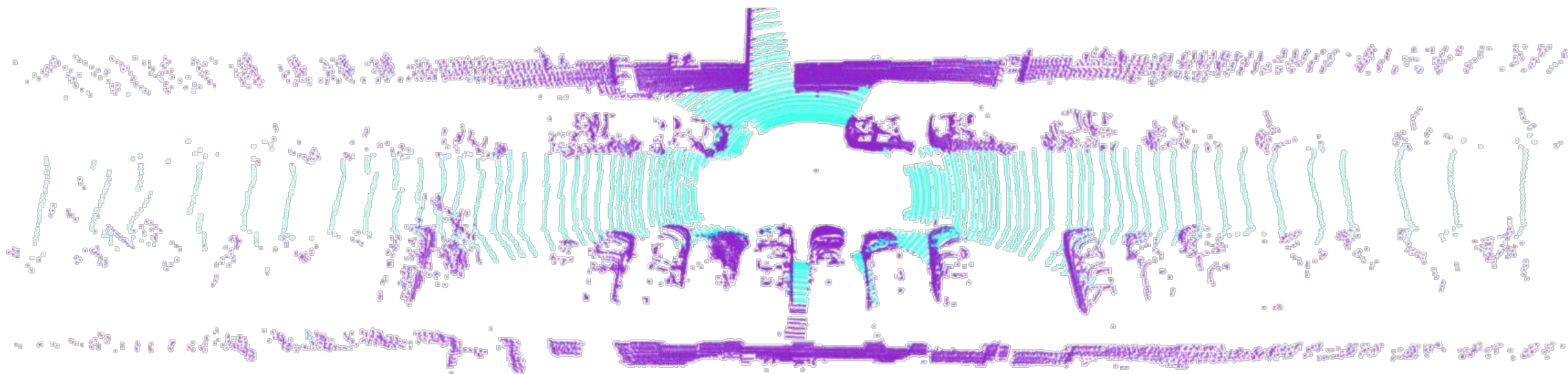


LiDAR-based SLAM for map generation (II)



Ground Model Fitting

- Identifying and removing ground points from the collected point cloud.
- It is needed to detect and analyze non-ground objects such as vehicles, pedestrians, and buildings within a 3D environment.
 - ▶ Least square, RANSAC(Random Sample Consensus), Height-based thresholding etc..





**THANK YOU
FOR YOUR ATTENTION**