Linear Algebra

# Row Reduction and LU Decomposition: Part 1

Automotive Intelligence Lab.

HANYANG UNIVERSITY

# Contents

# Simultaneous Equations and Matrix

# Solution of Simultaneous Equations

- **Think about solution of simultaneous equations as Eq 1..**
- **To solve simultaneous equations, one** Variable **must be eliminated from either upper or lower equation.**
- **Let's multiply upper equation by 2 and subtract it from lower equation in Eq 1..**
  - Upper equation: $r_1$, lower equation: $r_2$
  - $r_2 \rightarrow r_2 - 2r_1$ as Eq 2..
  - In this process, we can know that $\boldsymbol{y} = \mathbf{1}$.
  - By substituting $y = 1$ into upper equation, we can know that $\boldsymbol{x} = -\mathbf{1}$.

$$\begin{cases} 2x + 3y = 1 \\ 4x + 7y = 3 \end{cases}$$

Eq 1. Example of simultaneous equations

$$4x + 7y - 2(2x + 3y) = 3 - 2 \times 1 = 1$$
$$\Rightarrow (4x - 4x) + (7y - 6y) = y = 1$$
$$2x + 3(1) = 1 \Rightarrow x = -1$$

Eq 2. Process of solving simultaneous equations

# Method to Solve Simultaneous Equations

- **We can know two methods to solve simultaneous equations.**
  - Multiplying both sides of an equation by number.
  - Combining two equations.

- **Additional technique to solve simultaneous equations.**
  - Swapping the order of two equations.

- **In summary, there are three skills for solving simultaneous equations.**
  1. Multiplying both sides of an equation by scalar number.
  2. Combining two equations.
  3. Swapping. the order of two equations.

# Representation of Simultaneous Equations using Matrix

- **Simultaneous equations in Eq 1. can be expressed as matrix in Eq 2..**

- **Eq 2. can be expressed as augmented matrix in Eq 3..**
  - In augmented matrix, it can be treated like regular $2 \times 3$ matrix.
  - Long vertical bars are just auxiliary lines for visual aid.

- **In conclusion, we can solve simultaneous equations.**
  - By treating each 'row' of this augmented matrix as if it were **single equation**.

$$\begin{cases} 2x + 3y = 1 \\ 4x + 7y = 3 \end{cases}$$

Eq 1. Example of simultaneous equations

$$\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Eq 2. Representation of simultaneous equations as matrix

$$\boldsymbol{A} = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} \quad \boldsymbol{b} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$[\boldsymbol{A}|\boldsymbol{b}] = \left[\begin{array}{cc|c} 2 & 3 & 1 \\ 4 & 7 & 3 \end{array}\right]$$

Eq 3. Representation of simultaneous equations as augmented matrix

# Why Represent Simultaneous Equations as Matrix?

## Isn't it more complicated?

- Attempting to solve simultaneous equations using $\boxed{\text{Computer}}$.

## To do this, calculations corresponding to "three methods" for finding solution described below must be able to be expressed on computer.

- Multiplying both sides of an equation by scalar number.
- Combining two equations.
- Swapping the order of two equations.

$\rightsquigarrow E \times [A|b]$

$Ax B \to$

## In another view…,

- When there are two matrices $\boldsymbol{A}$ and $\boldsymbol{B}$, the operation of multiplying $\boldsymbol{A}$ and $\boldsymbol{B}$ involves $\boldsymbol{A}$ performing the operation and $\boldsymbol{B}$ functioning as the operand object that receives operation
- Also in case of $[\boldsymbol{A}|\boldsymbol{b}]$ as mentioned in before page, we can consider operation matrix that three skills of simultaneous equations mentioned above.
  - By multiplying operation matrix front of $[\boldsymbol{A}|\boldsymbol{b}]$ matrix, operations can be performed on the rows of the addition matrix.
  - We called this operation matrix as "Elementary matrix".

# Elementary Matrix



$$\underset{m\times m.}{E}\times \underset{m\times n}{[A|b]} = \overset{m\times n.}{}$$

- **There are a total of three elementary row operations.**
  - 1. Row multiplication
  - 2. Row switching
  - 3. Row addition

- **If size of matrix to be multiplied later is $m \times n$,**
  - Size of elementary matrix should be $m \times m$.
  - Size of matrix may remain the same.

- **With some modification on identity matrix $\boldsymbol{I_m}$ of size $m \times m$ we can obtain.**
  - Change a single number in matrix $\boldsymbol{I_m}$ as below matrices.
  - Manipulate the order of rows in matrix $\boldsymbol{I_m}$.

$$\boldsymbol{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \boldsymbol{E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example case of change single number for row operations

# 1. Row Multiplication

- **Elementary matrix that performs row multiplication**
  - Matrix that changed number of one of $\boxed{\text{diagonal}}$ elements of identity matrix as Eq 1.
- **In matrix $\boldsymbol{E}$ in Eq 1.,**
  - The second diagonal component was modified and changed to constant $s$
    - Results in operation that takes constant multiple in second row.
  - If indicated with symbol: $r_2 \rightarrow sr_2$
  - If you perform matrix operation on random $3 \times 4$ matrix $\boldsymbol{A}$, following operations are Eq 2..

$$\begin{bmatrix} s & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \boldsymbol{E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Eq 1. Elementary matrix that performs row multiplication

$$\boldsymbol{EA} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ sa_{21} & sa_{22} & sa_{23} & sa_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

Eq 2. Example of row multiplication

# Inverse Operation of Row Multiplication

- **Inverse operation for row multiplication is to perform $\boxed{1/s}$ times again.**
  - Inverse operation for operation that multiples 2 rows by $s$ is as in Eq 1..

- **This means the form of inverse matrix of elementary matrix corresponding to row multiplication operation.**

$$\boldsymbol{E}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/s & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Eq 1. Example of inverse operation of row multiplication

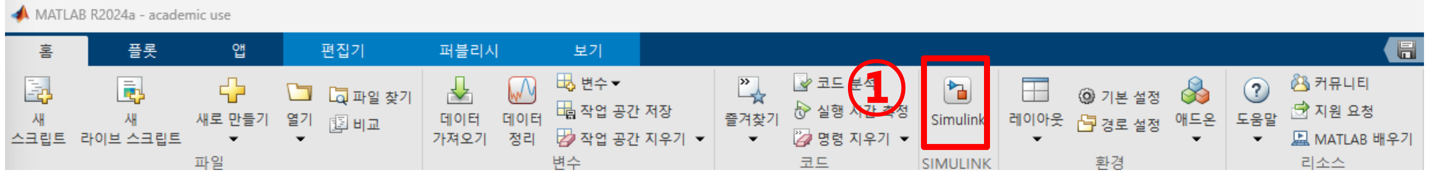
$$\boldsymbol{E}\boldsymbol{E}^{-1} = \boldsymbol{E}^{-1}\boldsymbol{E} = I$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/s & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/s & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

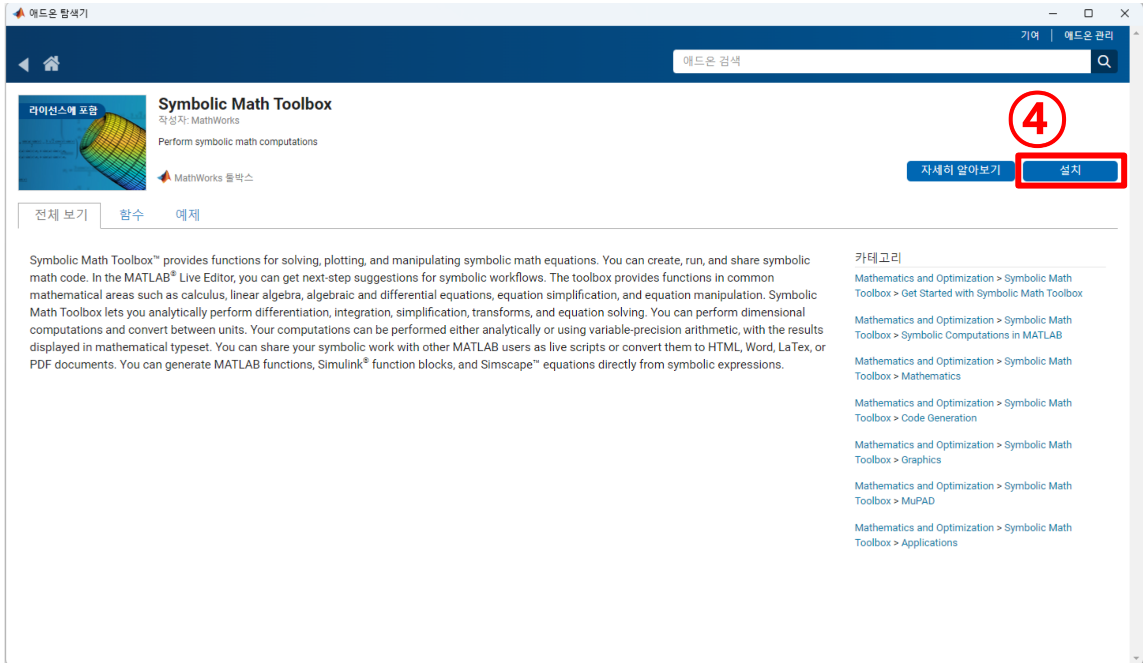
Eq 2. Inverse operation of row multiplication

# Preparation for 'Symbolic math toolbox'

- You need 'Symbolic Math Toolbox' to run the code in this lecture.
- Follow the procedure to install the toolbox.







# Code Exercise of Row Multiplication

## Code Exercise (10_01)

- You need ‘**Symbolic Math Toolbox**’ to run this code.

```
% Clear workspace, command window, and close all figures
clc; clear; close all;

% Include the symbolic toolbox
syms s;

% Define a 3x3 diagonal matrix E with diagonal entries 1, s, 1
E = diag([1 s 1]);

% Calculate the inverse of matrix E
E_inv = inv(E);

% Calculate the product of E and its inverse
product = E * E_inv;

% Display the matrix E, its inverse, and their product
disp('Matrix E:');
disp(E);
disp('Inverse of Matrix E:');
disp(E_inv);
disp('Product of E and E_inv (should be the identity matrix):');
disp(product);
```

MATLAB code of Row Multiplication

# 2. Row Switching

- **Matrix that changed** $\boxed{\text{row}}$ **of identity matrix**

- **If you want to perform operation that switch rows 3 and 2, it is as in Eq 1..**

- **Permutation matrix**
  - Among elementary matrices, matrix that performs row switching.
  - If indicated with symbol
    - $\boldsymbol{P}$: Permutation
    - Numbers of two rows to be replaced are conventionally written, such as $\boldsymbol{P}_{ij}$
      - To specify which two rows you want to change the order of.

$$\boldsymbol{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \boldsymbol{P}_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Eq 1. Elementary matrix that performs row switching

# Example of Row Switching

- **If you perform matrix operation on random $3 \times 4$ matrix $A$,**
  - Following operations are performed as Eq 1..

$$\boldsymbol{P}_{32}\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix}$$

Eq 1. Example of row switching

# Inverse Operation of Row Switching

- **Inverse matrix of elementary matrix (or permutation matrix) is** $\boxed{\text{itself}}$
  - This may seem pretty obvious.
  - All you have to do is swap lines 1 and 3 again.
    - To reverse the operation that swaps lines 1 and 3.

$$\boldsymbol{P}_{31} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{P}_{31}^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Example of inverse operation of row switching

# Code Exercise of Row Switching

## Code Exercise (10_02)

- You need '**Symbolic Math Toolbox**' to run this code.
- You can change the permutation matrix.
  - How about changing permutation matrix to $P_{31}$?

```
% Clear workspace, command window, and close all figures
clc; clear; close all;

% Include the symbolic toolbox
syms a11 a12 a13 a14 a21 a22 a23 a24 a31 a32 a33 a34;

% Define a 3x4 symobolic matrix A
A = [a11 a12 a13 a14; a21 a22 a23 a24; a31 a32 a33 a34];

% Defeine the permutation matrix P32
P32 = [1 0 0; 0 0 1; 0 1 0];

% Calculate the product of P32 and A
product = P32 * A;

% Display the matrix A, permutation matrix P32, and their product
disp('Matrix A:');
disp(A);
disp('Matrix P32');
disp(P32);
disp('Product of P32 and A');
disp(product);
```

MATLAB code of Row Switching

# 3. Row-Addition Matrix

## ■ Operation that $\boxed{\text{adds.}}$ different rows

- ▶ You must also perform process of replacing added result in certain row as Eq 1..
  - ● Converting row 2 to 2 rows plus $s$ times row 1.

$$\begin{bmatrix} 1 & 0 & 0 \\ s & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} + sa_{11} & a_{22} + sa_{12} & a_{23} + sa_{13} & a_{24} + sa_{14} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

Eq 1. Example of row-addition matrix

# Example of Row-Addition Matrix

- **Let consider matrix $E$ that performs the above operation.**
  - If matrix before the operation is called $A$,
    - It can be thought of as Eq 1..

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow E = \begin{bmatrix} 1 & 0 & 0 \\ s & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ s & 0 & 1 \end{bmatrix}$$

$$EA = \begin{bmatrix} 1 & 0 & 0 \\ s & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} + sa_{11} & a_{22} + sa_{12} & a_{23} + sa_{13} & a_{24} + sa_{14} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

Eq 1. Example of row-addition matrix

# Code Exercise of Row-Addition Matrix

## Code Exercise (10_03)

- You need '**Symbolic Math Toolbox**' to run this code.
- You can change the matrix $E$.
  - For example, Change the matrix $E$ to $\begin{bmatrix} 1 & s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

```
% Clear workspace, command window, and close all figures
clc; clear; close all;

% Include the symbolic toolbox
syms s a11 a12 a13 a14 a21 a22 a23 a24 a31 a32 a33 a34;

% Define a 3x4 symobolic matrix A
A = [a11 a12 a13 a14; a21 a22 a23 a24; a31 a32 a33 a34];

% Defeine the permutation matrix E
E = [1 0 0; s 1 0; 0 0 1];

% Calculate the product of E and A
product = E * A;

% Display the matrix A, E, and their product
disp('Matrix A:');
disp(A);
disp('Matrix E');
disp(E);
disp('Product of E and A');
disp(product);
```

MATLAB code of Row-Addition

# Meaning of Row-Addition Matrix

- **Let's consider how matrix $E$ performs row-addition operations.**
- **First, operation affects each row of output matrix.**
  - Operation performed using each row of matrix $E$.
- **In Eq 1.,**
  - Each row of matrix multiplied on left affects each row of output matrix.
  - Also indicates how much $\boxed{\text{weight}}$ to give to each row of matrix being operated on.

$$\begin{bmatrix} 1 & 0 & 0 \\ \boxed{s \quad 1 \quad 0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = \begin{bmatrix} - & r_1 & - \\ - & r_2 & - \\ - & r_3 & - \end{bmatrix}$$

**weighting**$= s$ **for row 1**

**weighting**$= 1$ **for row 2**

**weighting**$= 0$ **for row 3**

Eq 1. Affect of each row of matrix

# Result of Row-Addition Matrix



- Therefore, when row addition operation is performed on output matrix, Eq 1. occurs.

$$\boxed{\begin{array}{c} s \times (a_{11}\ \ a_{12}\ \ a_{13}\ \ a_{14}) \\ +1 \times (a_{21}\ \ a_{22}\ \ a_{23}\ \ a_{24}) \\ \rightarrow \boldsymbol{r_2} \text{ of output matrix} \end{array}}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ s & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = \begin{bmatrix} - & r_1 & - \\ - & r_2 & - \\ - & r_3 & - \end{bmatrix}$$

Eq 1. Affect of each row of matrix

# Result of Row-Addition Matrix

- **Using row addition operation**
  - ▶ You can erase specific element to **0** as Eq 1..

- **You can use elementary matrix as follows to substitute $\boldsymbol{row\,2} = \boldsymbol{row\,2} - \boldsymbol{row\,1}$.**
  - ▶ To make the first element in row 2 of matrix $\boldsymbol{A}$ $0$.

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & -1 & -2 & 2 \\ 0 & 3 & 2 & 1 \end{bmatrix}$$

$$\boldsymbol{EA} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & -1 & -2 & 2 \\ 0 & 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -5 & -2 \\ 0 & 3 & 2 & 1 \end{bmatrix}$$

Eq 1. Example of row-addition matrix

# Inverse Operation of Row-Addition Matrix

- $-s$ **Multiplying and adding again.**

$$\boldsymbol{E} = \begin{bmatrix} 1 & 0 & 0 \\ s & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \boldsymbol{E}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -s & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{E}^{-1}\boldsymbol{E} = \boldsymbol{I}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -s & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ s & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example of inverse operation of row-addition matrix

# Code Exercise of Inverse of Row-Addition Matrix

## Code Exercise (10_04)

- You need '**Symbolic Math Toolbox**' to run this code.

```
% Clear workspace, command window, and close all figures
clc; clear; close all;

% Include the symbolic toolbox
syms s;

% Defeine the matrix E
E = [1 0 0; s 1 0; 0 0 1];

% Define the inverse matrix of E
E_inv = [1 0 0; -s 1 0; 0 0 1];

% Calculate the product of E_inv and E
product = E_inv * E;

% Display the matrix E ,its inverse and the product of them
disp('Matrix E:');
disp(E);
disp('Matrix E_inv');
disp(E_inv);
disp('Product of E_inv and E');
disp(product);
```

MATLAB code of Row-Addition

# Solving Simultaneous Equations using Elementary Matrix

- **Let's solve simultaneous equations**
  - Using elementary matrix and check results by implementing it directly in MATLAB.
- **Eq 1 can be represented in the form of a matrix, like Eq 2..**
- **To remove term related to $\boxed{x}$ in the second equation, perform Eq 3..**

$$\begin{cases} 2x + 3y = 1 \\ 4x + 7y = 3 \end{cases}$$

Eq 1. Example of simultaneous equations

$$[\boldsymbol{A}|\boldsymbol{b}] = \left[\begin{array}{cc|c} 2 & 3 & 1 \\ 4 & 7 & 3 \end{array}\right]$$

$$\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

Eq 2. Representation of simultaneous equations as augmented matrix

$$r_2 \rightarrow r_2 - 2r_1$$

Eq 3. Process of solving simultaneous equations

# Solving Simultaneous Equations using Augmented Matrix

- **Let's multiply augmented matrix.**

$$\boldsymbol{E}_1 = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$\Rightarrow \boldsymbol{E}_1[\boldsymbol{A}|\boldsymbol{b}] = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \left[\begin{array}{cc|c} 2 & 3 & 1 \\ 4 & 7 & 3 \end{array}\right]$$

$$= \left[\begin{array}{cc|c} 2 & 3 & 1 \\ 0 & 1 & 1 \end{array}\right]$$

Process of solving simultaneous equations using augmented matrix

# Solving Simultaneous Equations using Elementary Matrix

- **Let's perform following operation to remove second element 3 of first row as Eq 1..**
- **To do this, let's multiply elementary matrix as Eq 2..**

$$r_1 \rightarrow r_1 - 3r_2$$

Eq 1. Process of solving simultaneous equations

$$\boldsymbol{E}_2 = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$$

$E_2 E_1 [A|b]$

$$\Rightarrow \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \left[\begin{array}{cc|c} 2 & 3 & 1 \\ 0 & 1 & 1 \end{array}\right] = \left[\begin{array}{cc|c} 2 & 0 & -2 \\ 0 & 1 & 1 \end{array}\right]$$

Eq 2. Multiply elementary matrix

# Result of Simultaneous Equations

- Lastly, let's multiply the first row by $1/2$.
- To do this, let's multiply elementary matrix as Eq 1..
- Therefore, it can be confirmed through final result augmented matrix.
  - ▶ $x = \boxed{-1}, y = \boxed{1}$

$$\boldsymbol{E}_3 = \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix}$$

$E_3 E_2 E_1 [A|b]$

$$\Rightarrow \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix} \left[\begin{array}{cc|c} 2 & 0 & -2 \\ 0 & 1 & 1 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 1 \end{array}\right]$$

Eq 1. Multiply elementary matrix

# Summary of using Elementary Matrix

- **If you think about this process carefully,**
  - You can see that result can be obtained
    - By using elementary matrices $\boldsymbol{E}_1$, $\boldsymbol{E}_2$, and $\boldsymbol{E}_3$ in order as Eq 1..
- **With computer**
  - Represent operations and equations
  - Obtain solutions with simple coding as fig 1..

$E_3E_2E_1 = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}^{-1}.$

$$\boldsymbol{E}_3\boldsymbol{E}_2\boldsymbol{E}_1[\boldsymbol{A}|\boldsymbol{b}] = \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}\left[\begin{array}{cc|c} 2 & 3 & 1 \\ 4 & 7 & 3 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 1 \end{array}\right]$$

Eq 1. Result via elementary matrix

```
% Clear workspace, command window, and close all figures
clc; clear; close all;

% Matrix E1, E2, E3 and augmented A
E3 = [0.5 0; 0 1];
E2 = [1 -3; 0 1];
E1 = [1 0; -2 1];
augmented_A = [2 3 1; 4 7 3];

% Calculatge the multipllication
result = E3*E2*E1*augmented_A;

% Display the result
disp("result")
disp(result);
```

```
result
     1     0    -1
     0     1     1
```

Fig 1. Solving simultaneous equations using elementary matrix

# Summary

# Summary

## Simultaneous equations and matrix

- Expression of simultaneous equations with matrix
  - Augmented matrix
- Skill to solve
  - Row operation
    - Row multiplication
    - Row switching
    - Row addition

# THANK YOU
# FOR YOUR ATTENTION