Linear Algebra

Vector Part 2:
Vector Extension Concept

Automotive Intelligence Lab.

Contents

Vector set

Vector Set

Definition of vector set

- A collection of vectors.

Notation of vector set

- Vector set is indicated as $\boldsymbol{S}$ or $\boldsymbol{V}$, represented in capital italics letters.

- Mathematical representation of a vector set : $\boldsymbol{V} = \{\boldsymbol{v_1}, \dots, \boldsymbol{v_n}\}$.

Characteristics of vector set

- Vector set can contain a finite or an infinite number of vectors.

- Vector set can also be empty which is indicated as $\boldsymbol{V} = \boxed{\{\ \}}$

Linear weighted combination

$V = \{v_1, v_2, \cdots v_n\}$

$\Rightarrow \lambda_1 v_1 + \lambda_2 v_2 + \cdots + \lambda_n v_n$

Linear Weighted Combination

What is linear weighted combination?

- A way of mixing information from multiple variables, with some variables contributing more than others.

- It is also sometimes called **linear mixture** or **weighted combination** when linear part is assumed.

- Weight can also be expressed as **coefficient**.

- Linear weighted combination simply means **scalar-vector multiplication** and **addition** as Eq 1.
  - It is assumed that all vectors $\boldsymbol{v}_i$ have the same ______; otherwise, the addition is invalid.
  - The λ_i can be any real number, including zero.
  - Subtraction can be handled by setting a λ_i to be negative.

$$\boxed{\mathbf{w}} = \lambda_1 \mathbf{v_1} + \lambda_2 \mathbf{v_2} + \ \dots + \ \lambda_n \mathbf{v_n}$$

Eq 1. Standard form of linear weighted combination

$$w = \begin{bmatrix} | & | & | \\ v_1 & v_2 & v_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}$$

Subtraction of Linear Weighted Combination

- **In Eq 2., λ_3 shows an example of subtraction.**

$$\begin{array}{ccc} 4 & -8 & -3 \\ 5 & 0 & -9 \\ 1 & -8 & -6 \end{array}$$

$$\lambda_1 = 1\,, \lambda_2 = 2\,, \lambda_3 = -3\,,\ \mathbf{v_1} = \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} -4 \\ 0 \\ -4 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

$$\mathbf{w} = \lambda_1 \mathbf{v_1} + \lambda_2 \mathbf{v_2} + \lambda_3 \mathbf{v_3} = \begin{bmatrix} \boxed{} \end{bmatrix}$$

Eq 2. Example of linear weighted combination

Code Exercise of Linear Weighted Combination using Matlab

Code Exercise (03_01)

- Linear weighted combination.

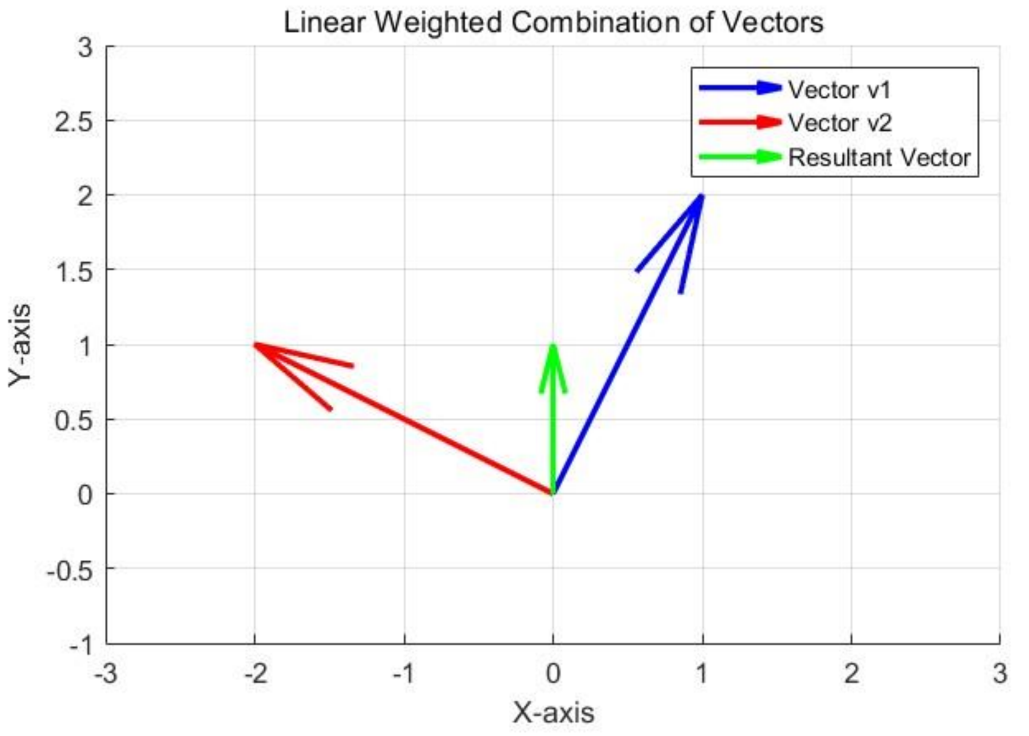
```matlab
% Define vectors
v1 = [1; 2]; % example vector 1
v2 = [-2; 1]; % example vector 2


% Define weights
alpha = 0.4;
beta = 0.2;


% Compute the linear weighted combination
resultant_vector = alpha * v1 + beta * v2;


% Visualize vectors
figure;
hold on;
% Plot vectors
quiver(0, 0, v1(1), v1(2), 'Color', 'b', 'LineWidth', 2, 'MaxHeadSize', 1,
'AutoScale', 'off');
quiver(0, 0, v2(1), v2(2), 'Color', 'r', 'LineWidth', 2, 'MaxHeadSize', 1,
'AutoScale', 'off');
quiver(0, 0, resultant_vector(1), resultant_vector(2), 'Color', 'g',
'LineWidth',2, 'MaxHeadSize', 1, 'AutoScale', 'off');

% Set axis equal, xlim, and ylim
axis equal;
xlim([-3, 3]);
ylim([-1, 3]);


% Add labels
xlabel('X-axis');
ylabel('Y-axis');


% Add legend
legend('Vector v1', 'Vector v2', 'Resultant Vector');


% Set title
title('Linear Weighted Combination of Vectors');


% Show grid
grid on;
hold off;
```

Source code

Visualization Result of Linear Weighted Combination using Matlab

Code Exercise (03_01)

- Linear weighted combination.



Source code result

Applications of Linear Weighted Combination

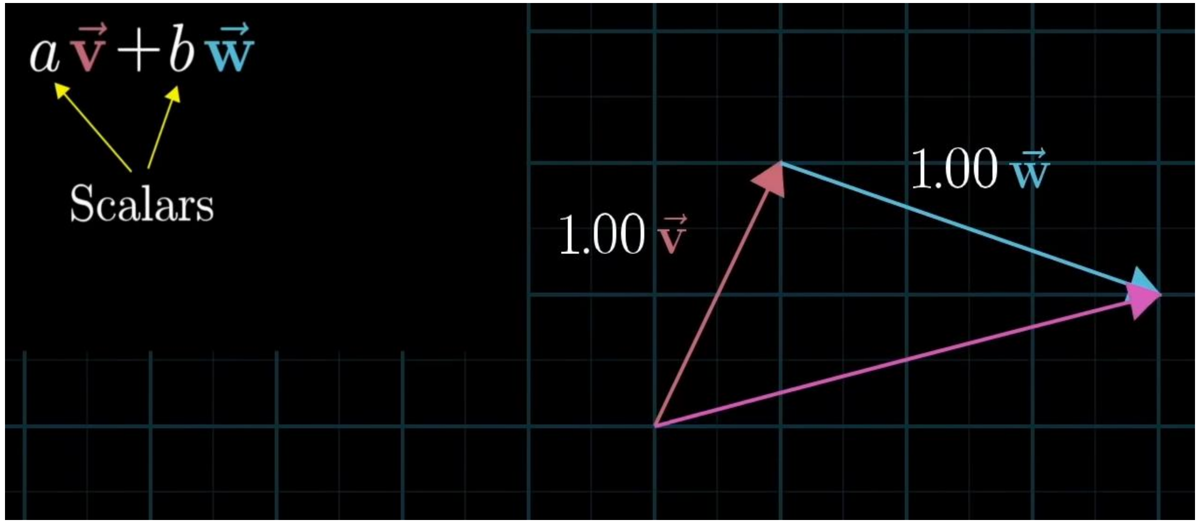
- **The predicted data from a statistical model**
  - The linear weighted combination of regressors (predictor variables) and coefficients (scalars).
  - Regressors and coefficients are computed via the least squares algorithm.

- **Dimension-reduction procedures**
  - The linear weighted combination of the data channels and weights.
  - The weights selected to maximize the variance of the component.
    - such as PCA (Principal Components Analysis).

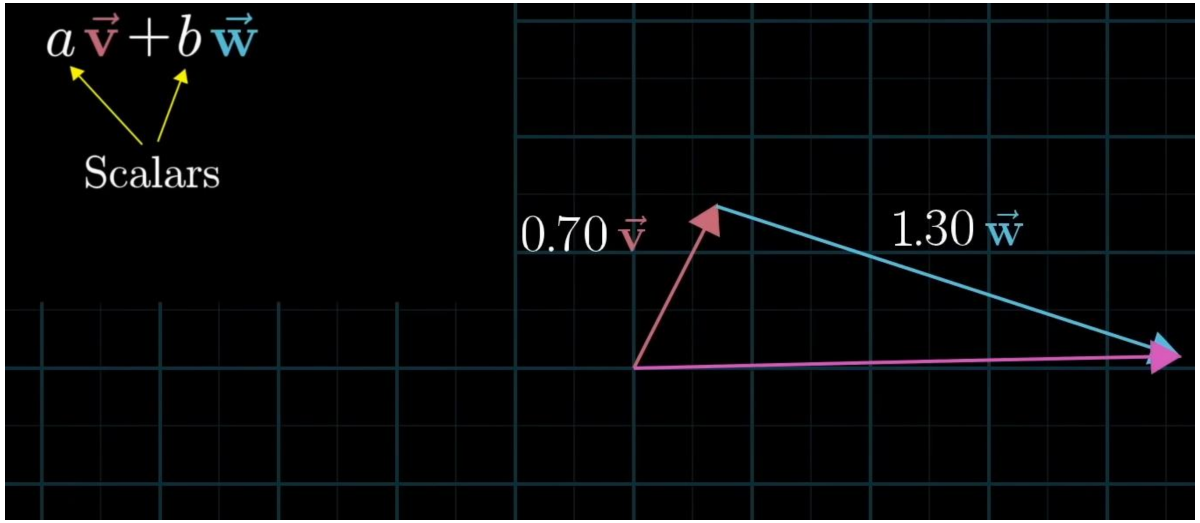
- **Artificial neural networks**
  - Two operations: Linear weighted combination of input data and nonlinear transformation.
  - The weights are learned by minimizing a cost function.
    - Cost function: difference between the model prediction and the real-world target variable.

Visual Materials

- **Geometric representation of linear weighted combination.**
  - Linear combination (2:28 ~ 3:46)
  - https://youtu.be/k7RM-ot2NWY?si=Cc5H_I_8cmfEoDsK&t=148



Visualization of linear combination v and w (when scala a is 1.0, and scala b is 1.0)



Visualization of linear combination v and w (when scala a is 0.7, and scala b is 1.3)

Linear independence

Definition of Linear Independence

Linearly dependent

- At least one vector in vector set can be expressed as a linear weighted combination of other vectors in that set.
  - Infinite number of such combinations, two of which are $\boldsymbol{s_1} = 0.5 * \boldsymbol{s_2}$ and $\boxed{S_2 = 2 * S_1}$

$$\boldsymbol{S} = \left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \end{bmatrix} \right\}$$

Vector set S

Linearly independent

- No vector in vector set can be expressed as a linear weighted combination of other vectors in that set.
  - No possible scalar λ for which $\boldsymbol{v_1} = \lambda * \boldsymbol{v_2}$.

$$\boldsymbol{V} = \left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 7 \end{bmatrix} \right\}$$

Vector set V

Linear Independence in Complex Vector

How about below vector set T?

- The sum of the first three vectors equals twice the fourth vector → **Linearly dependent**

Determine whether linearly independent

- It is hard to figure out just from visual inspection.
  - In Ch.5, we will learn **matrix rank** for determine independence of vector set.
    - Create a matrix from the vector set.
    - Compute the rank of the matrix rank.
    - Compare the rank to the smaller of the number of rows or columns.

$$\boldsymbol{T} = \left\{ \begin{bmatrix} 8 \\ -4 \\ 14 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 14 \\ 2 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 13 \\ 2 \\ 9 \\ 8 \end{bmatrix} \right\}$$

$$2 * \boldsymbol{t_4} = \boldsymbol{t_1} + \boldsymbol{t_2} + \boldsymbol{t_3}$$

$$\boldsymbol{t_4} = \frac{1}{2}(\boldsymbol{t_1} + \boldsymbol{t_2} + \boldsymbol{t_3})$$

Example of linearly dependent

Property of Linear Independence

Independent sets

- Independence is a property of a **set** of vectors, not individual vector within a set.
  - A set of vectors can be $\boxed{\text{linearly independent}}$ or $\boxed{\text{linearly dependent}}$
- In case of below vector set $\boldsymbol{V}$, is it linearly independent?

dependent

- If then, each of v_1, v_2, v_3, v_4 is linearly independent?

$$\boldsymbol{V} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ 12 \\ 7 \end{bmatrix} \right\} \quad when\ \boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \boldsymbol{v}_2 = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix}, \boldsymbol{v}_3 = \begin{bmatrix} 3 \\ 6 \\ 9 \\ 6 \end{bmatrix}, \boldsymbol{v}_4 = \begin{bmatrix} 4 \\ 8 \\ 12 \\ 7 \end{bmatrix}$$

Vector set **V**

The Math of Linear Independence

Formal mathematical definition of Linear dependence

- Define some linear weighted combination of the vectors in the set to produce the zeros vector.
  - If there are some λs that make the Eq. 1. true, set of vectors is linearly $\boxed{\text{dependent}}$
  - Conversely, If no possible way to linearly combine the vectors to produce zeros vector, set of vectors is linearly $\boxed{\text{independent}}$ (알면).

Why do we care about the zeros vector regarding the question of Linear dependence?

- how about expressing equation like Eq. 2.?

$$0 = \lambda_1 \boldsymbol{v}_1 + \lambda_2 \boldsymbol{v}_2 + \cdots + \lambda_n \boldsymbol{v}_n, \qquad \lambda \in \mathbb{R}$$

Eq 1. The mathematical definition of linear dependence

$$\lambda_1 \boldsymbol{v}_1 = \lambda_2 \boldsymbol{v}_2 + \cdots + \lambda_n \boldsymbol{v}_n, \qquad \lambda \in \mathbb{R}$$

Eq 2. Another definition of linear dependence

Express Zero Vector for Linear Independence

- **Equation with zeros vector on the left-hand side**
  - Setting the equation to zero helps reinforce the principle that the entire set is dependent or independent.
  - No individual vector has the privileged position of being “dependent vector”.
    - When independence, vector sets are purely egalitarian.

Definition and Constraint of Trivial Solution

(자명한).

Trivial solution

- Set all λ's to zero, and the equation reads $0 = 0$, regardless of the vectors in the set.
  - But, trivial solutions involving zeros are often ignored in linear algebra.

Mathematical definition of linear dependence with constraint

- Trivial solutions involving zeros are often ignored in linear algebra as mentioned before slide, so the constraint that at least one $\lambda \neq 0$ is added.

- This constraint can be incorporated into the equation by dividing through by one of the scalars.
  - $\boldsymbol{v}_1$ and λ_1 can refer to any vector/scalar pair in the set.

$$0 = \boldsymbol{v}_1 + \cdots + \frac{\lambda_n}{\lambda_1}\boldsymbol{v}_n, \qquad \lambda \in \mathbb{R}, \lambda_1 \neq 0$$

The mathematical definition of linear dependence with constraint

Independence and The Zeros Vector

Linear independence and the zeros vector

- Any vector set that includes the zeros vector is automatically a linearly dependent set.
  - Because any scalar multiple of the zeros vector is still the zeros vector

Linear dependence

- As long as $\lambda_0 \neq 0$, it has a **nontrivial solution**, and the set fits with the definition of linear dependence.

Nontrivial solution

- Solution to a homogeneous equation that is not the zero solution.
  - Any solution in which at least one variable has a nonzero value.

$$\lambda_0 0 = 0\boldsymbol{v}_1 + 0\boldsymbol{v}_2 + \cdots + 0\boldsymbol{v}_n$$

Equation of Linear dependence with zero vector

Question about Nonlinear Independence

- **About nonlinear independence**
  - Linear algebra is all about, well, linear operations.
  - If you can express one vector as a nonlinear combination of other vectors, then those vectors still form a linearly independent set.
- **Reason for the linearity constraint**
  - For express transformations as matrix multiplication.
- **Nonlinear systems can be well approximated using linear functions!**

Subspace and span

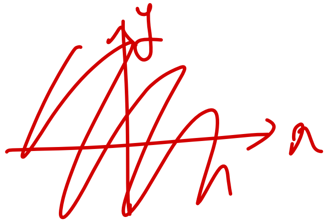
Concept of Subspace and Span

$V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $V_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Subspace : span을 해서 나온 공간들

- The space formed by infinitely linear combination of vectors within a vector set, where each vector is multiplied by different weights.

$= \lambda_1 V_1 + \lambda_2 V_2 + \cdots + \lambda_n V_n$



Span $\lambda_1 V_1 + \lambda_2 V_2 + \cdots + \lambda_n V_n$ (하는).

- The **mechanism** of combining all possible linear weighted combination.

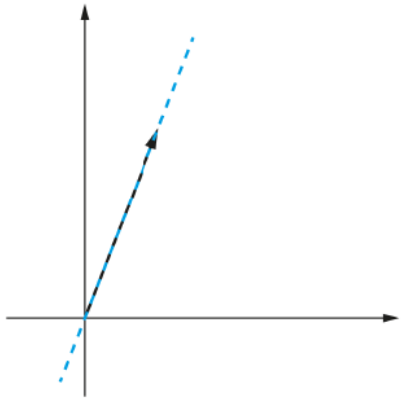
Difference between subspace and span

- Think of span as **verb** and subspace as **noun**.
  - A set of vectors spans, and the result of their spanning is a subspace.
- If using span as a noun, span and subspace refer to the same infinite vector set.

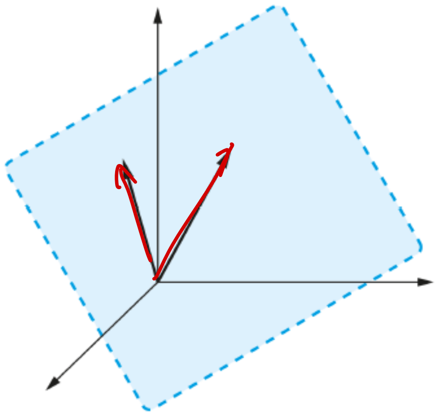
Subspace Spanned by Vector Set

- **Example of subspace of a set of linearly independent vector set refer to below figures**
  - The $\boxed{\text{blue}}$ color represents the subspace generated by the $\boxed{\text{black.}}$ colored vector.
  - $\boldsymbol{V_1}$ has one vector and its span is 1D subspace, $\boldsymbol{V_2}$ has two vectors and their span is 2D subspace, is there a pattern?
    - Dimensionality of the spanned subspace and the number of vectors in the set?

$$\boldsymbol{V_1} = \left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\} \qquad \boldsymbol{V_2} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} \right\}$$



Subspace of V_1



Subspace of V_2

Relation Between Dimensionality of Subspace and Number of Vectors

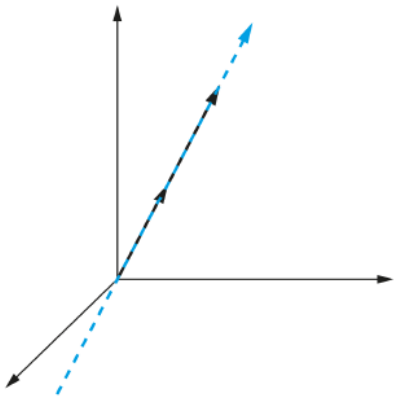
Example of subspace of a set of linearly dependent vector set refer to below figure

- Two vectors in $\mathbb{R}^3$, but subspace of $\boldsymbol{V_3}$ is still only a $\boxed{1D}$ subspace.
  - One vector in the set is already in the space of the other vector.

Relation between dimensionality of spanned subspace and the number of vectors in vector set

- Related to **linear independence.**

$$\boldsymbol{V_3} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \right\}$$

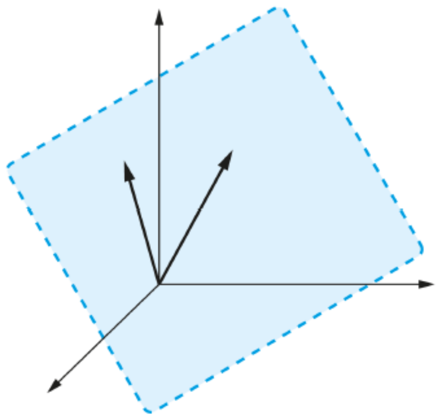


Subspace of V_3

Dimension of Subspace and Linear Independence

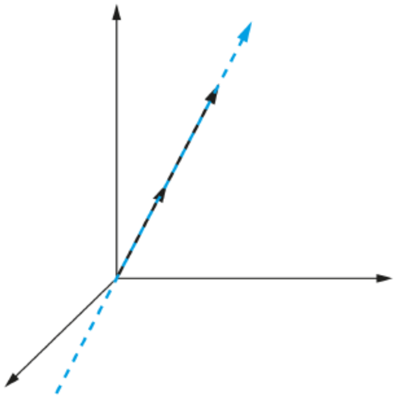
- **If the vectors in a vector set are linearly independent,**
  - the dimension of the subspace $\boxed{\text{equals}}$ the number of vectors in the set like $\boldsymbol{V}_2$.
    (벡터의 차원과 같은 것X).
- **If the vectors in a vector set are linearly dependent,**
  - the dimension of the subspace is $\boxed{\text{less than.}}$ the number of vectors in the set like $\boldsymbol{V}_3$.

$$\boldsymbol{V}_2 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} \right\}$$



Subspace of V_2

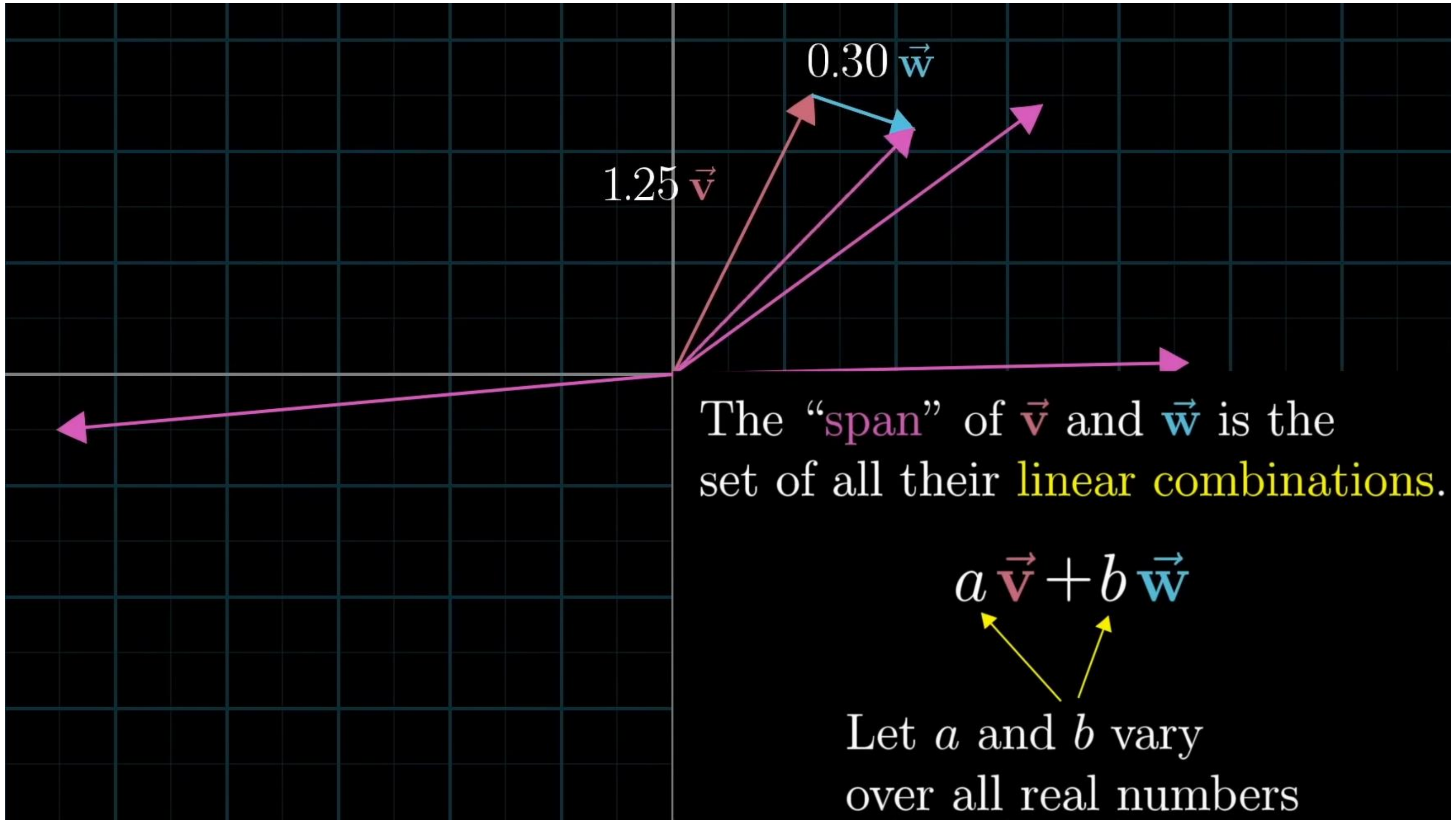
$$\boldsymbol{V}_3 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \right\}$$



Subspace of V_3

Visual Materials of Span

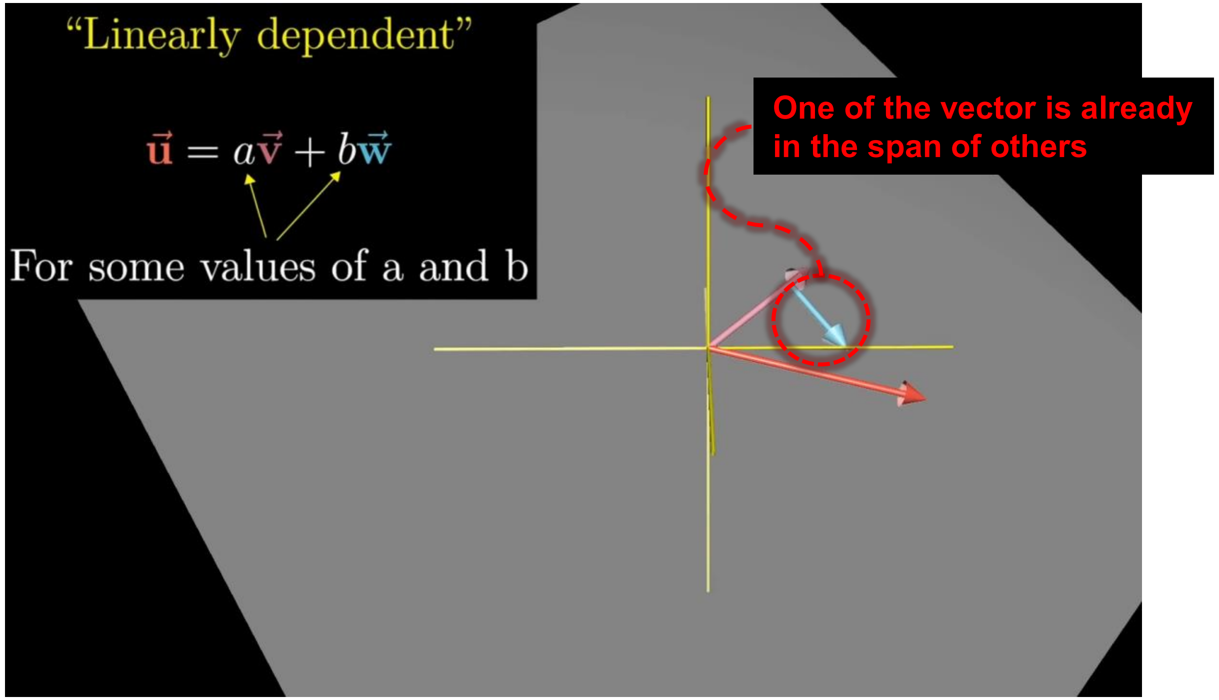
- **Geometric representation of span**
  - Span (3:47 ~ 7:54)
  - https://youtu.be/k7RM-ot2NWY?si=Zm_LRnsDlnohsn1y&t=227



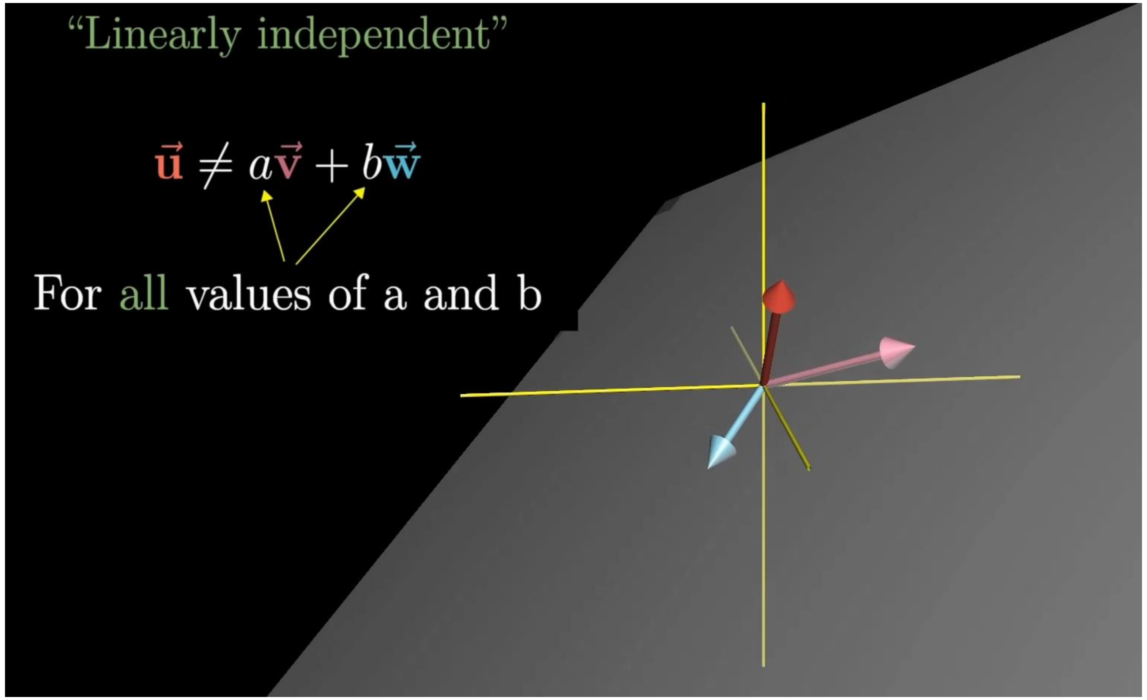
Visualization of span

Visual Materials of Linear Dependence

- **Geometric representation of linear dependence and linear independence**
  - Linear dependence (7:55 ~)
  - https://youtu.be/k7RM-ot2NWY?si=NX_fK1ppEP5aGmPU&t=475



Visualization of linear dependent



Visualization of linear independent

Code Exercise of Vector Span using Matlab

Code Exercise (03_02)

- Vector Span.

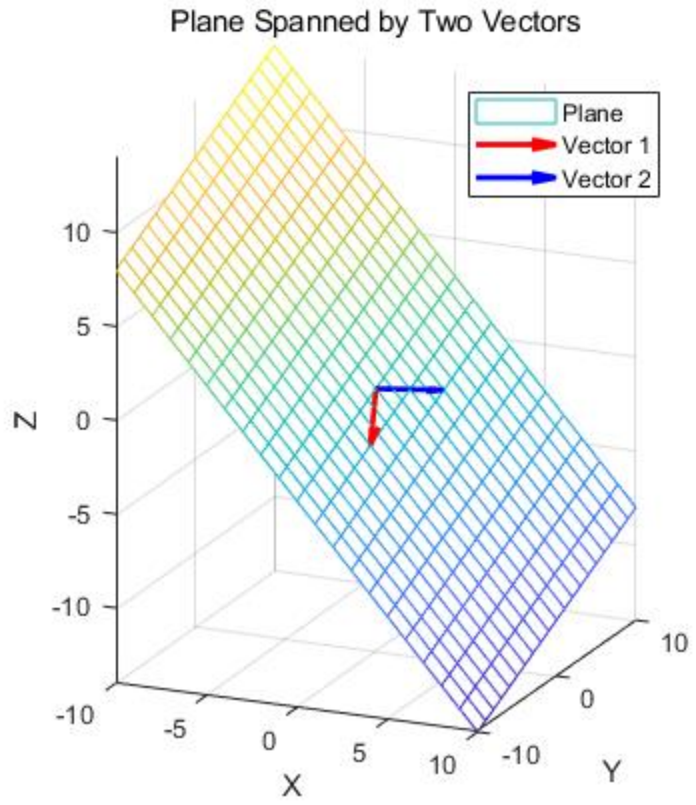
```
% Define two 3D vectors
v1 = [1; -3; -2];
v2 = [2; 4; -1];

% Calculate the normal vector (cross product of the two vectors)
normal = cross(v1, v2);

% Choose a point on the plane (for example, using v1)
point = v1;

% Equation of the plane: ax + by + cz = d
% where a, b, c are components of the normal vector, and d is the constant term
of the plane equation
a = normal(1);
b = normal(2);
c = normal(3);
d = -dot(normal, point);

% Code for visualization
[x, y] = meshgrid(-10:1:10, -10:1:10); % Create a grid to represent the plane
z = (-d - a*x - b*y) / c; % Calculate z values of the plane

% Draw the plane
figure;
mesh(x, y, z);
hold on;

% Draw the two vectors
quiver3(0, 0, 0, v1(1), v1(2), v1(3), 'r', 'LineWidth', 2, 'AutoScale', 'off',
'MaxHeadSize', 2);
quiver3(0, 0, 0, v2(1), v2(2), v2(3), 'b', 'LineWidth', 2, 'AutoScale', 'off',
'MaxHeadSize', 2);

xlabel('X'); ylabel('Y'); zlabel('Z');
title('Plane Spanned by Two Vectors');
legend('Plane', 'Vector 1', 'Vector 2');
axis equal;
grid on;
```

Source code

Visualization Result of Vector Span using Matlab

Code Exercise (03_02)

- Vector Span.



Source code result

Basis

Concept of Basis

- **In linear algebra, a basis is a set of $\boxed{\text{ruler}}$ that describes the information in the matrix.**

- **Most common basis set is the Cartesian axis.**
  - The Cartesian basis set comprises vectors that are mutually orthogonal and unit length.
    - It's why the Cartesian basis sets are so ubiquitous.
    - They are called the standard basis set.

$$\boldsymbol{S}_2 = \left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\} \qquad \boldsymbol{S}_3 = \left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$$

Basis sets for 2D and 3D Cartesian graphs

Determining Basis

Cartesian axis is not the only basis sets

- Basis set **S**, **T** both span the same subspace.
  - Subspace : all of R^2

$$\boldsymbol{S} = \left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\} \quad \Bigg| \quad \boldsymbol{T} = \left\{\begin{bmatrix}3\\1\end{bmatrix}, \begin{bmatrix}-3\\1\end{bmatrix}\right\}$$

$$\boldsymbol{s_1} = \begin{bmatrix}1\\0\end{bmatrix}, \boldsymbol{s_2} = \begin{bmatrix}0\\1\end{bmatrix} \quad \Bigg| \quad \boldsymbol{t_1} = \begin{bmatrix}3\\1\end{bmatrix}, \boldsymbol{t_2} = \begin{bmatrix}-3\\1\end{bmatrix}$$

Basis set of S and T

- Let's represent point p and q in terms of the basis S and T.
  - In basis $\boldsymbol{S}$, point p is (3,1), point q is (-6,2).
    - Because $p = 3\boldsymbol{s_1} + \boldsymbol{s_2}$, $q = -6\boldsymbol{s_1} + 2\boldsymbol{s_2}$
  - In basis $\boldsymbol{T}$, point p is (1,0), point q is (0,2).
    - Because $p = 1\boldsymbol{t_1} + 0\boldsymbol{t_2}$, $q = 0\boldsymbol{t_1} + 2\boldsymbol{t_2}$

- Data points p and q are the same regardless of the basis set, but $\boldsymbol{T}$ provide a **compact** and **orthogonal** description.

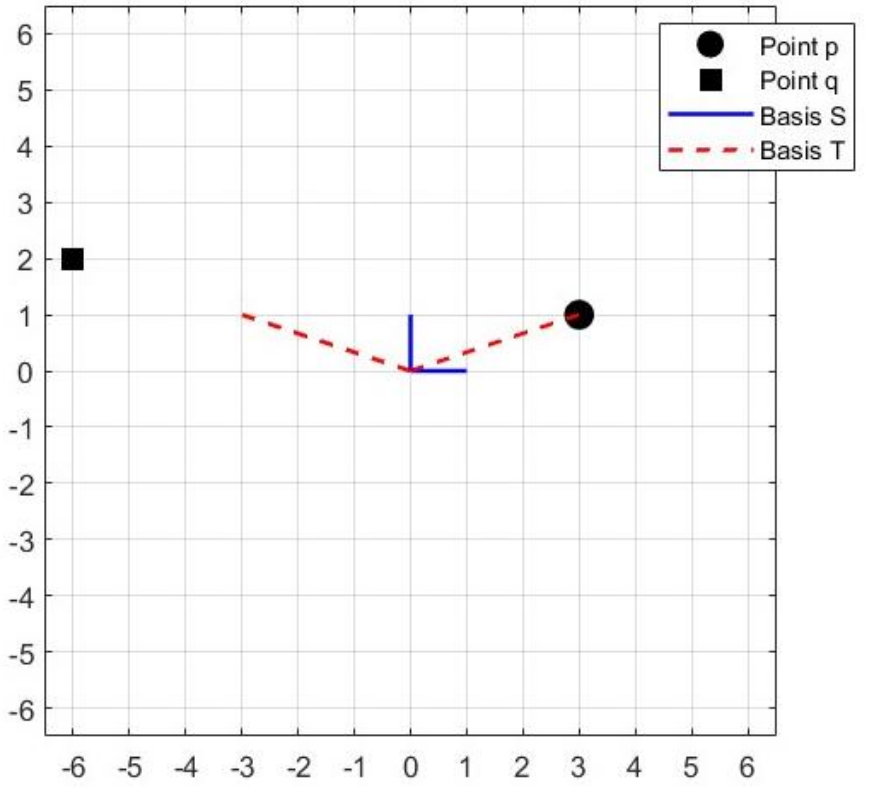


Figure 1. The same points can be described by different basis set

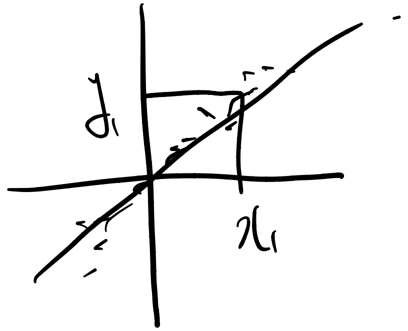
Standard Basis

Basis in vector space

- A set of vectors $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_n\}$ within a vector space $\boldsymbol{V}$ is called a basis of $\boldsymbol{V}$ if it satisfies the following two conditions.
  - The set $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_n\}$ is linearly independent
  - The set $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_n\}$ spans $\boldsymbol{V}$, meaning it generates the whole space $\boldsymbol{V}$.

Applications of Basis

- **Application of basis in data science and machine learning**
  - Many problems in applied linear algebra can be conceptualized as finding the best set of basis vectors to describe some subspace.
    - Dimension reduction
    - Feature extraction
    - Principal components analysis
    - Independent components analysis
    - Factor analysis
    - Singular value decomposition
    - Linear discriminant analysis
    - Image approximation
    - Data compression
    - …



- **All of those analyses are essentially ways of identifying optimal basis vectors for a specific problem.**

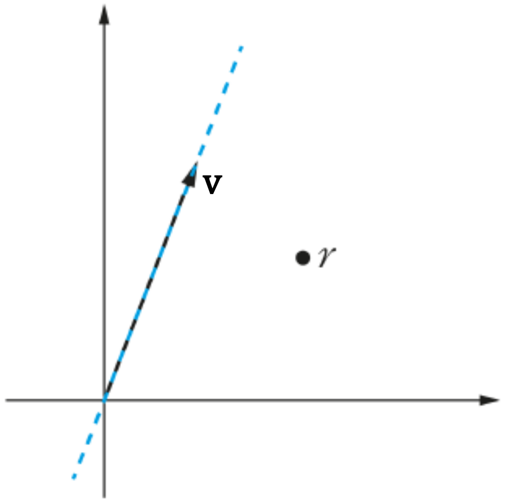
Definition of Basis

Formal definition of basis

- A basis is the combination of $\boxed{\text{span}}$ and $\boxed{\text{independence.}}$
  - 1. Spans a certain subspace.
  - 2. Independent set of vectors.

The basis needs to span a subspace for it to be used as a basis for that subspace

- A basis set can measure only what is contained its span.
  - A basis vector for $\boxed{\text{blue}}$ colored subspace cannot measure point r which is not in subspace of vector $\mathbf{v}$.



Basis and Subspace

Linearly Independency of Basis

Why a basis set must be linearly independent?

- All vectors in a subspace must have unique coordinates with respect to that basis.
- To ensure this uniqueness, the basis vectors must no be linearly combinable in any way.
  - For example, U is a valid set of vectors.
  - But if you want to describe point (3,1) , the coefficients for the linear weighted combination of the three vectors in U could be (3, 0, 1) or (0, 1.5, 1) or … a bajillion other possibilities.
  - It's so confusing, and so mathematicians decided that a vector must have unique coordinates within a basis set.

$$\boldsymbol{U} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \quad \boldsymbol{u_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \boldsymbol{u_2} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \boldsymbol{u_3} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

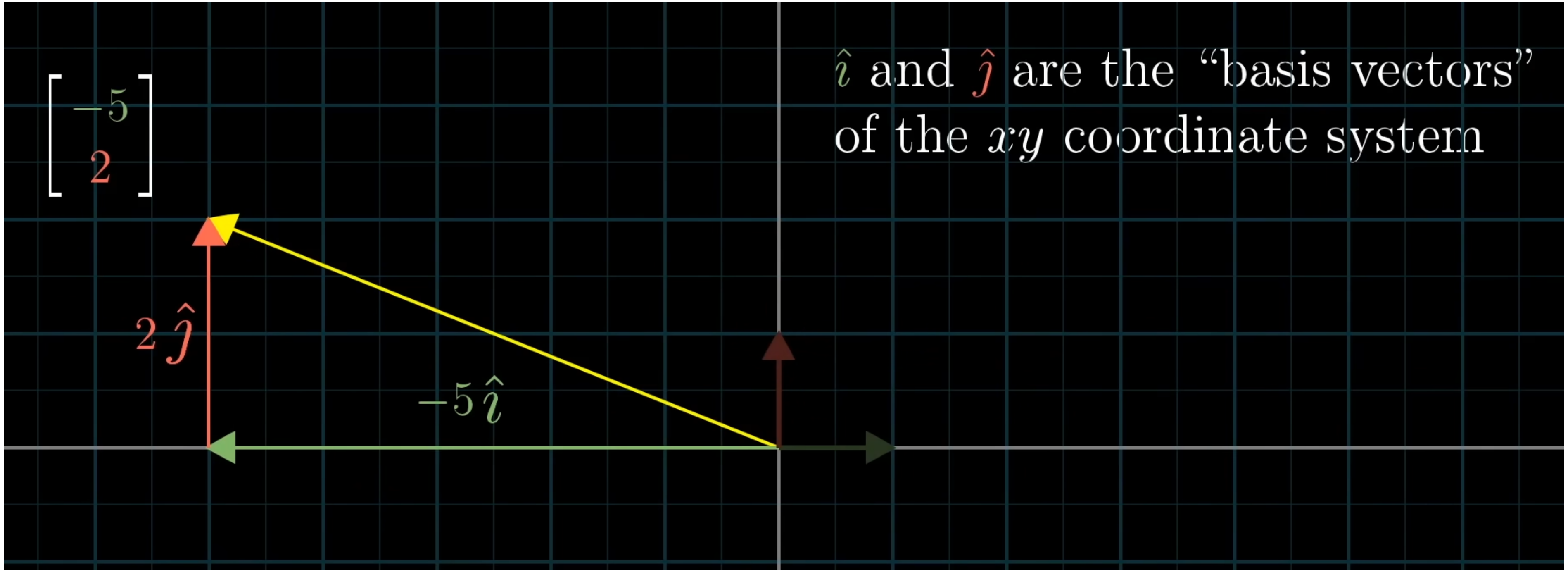
Vector set of U

- To be clear, any point can be described using an infinite numbers of basis sets. Within a basis set, a point is defined by exactly one linear weighted combination.

Visual Materials of Basis

Geometric representation of basis

- Basis (0:12 ~ 2:27)
- https://youtu.be/k7RM-ot2NWY?si=-MomPGwuSEOwvwev&t=12



Visualization of basis

Code Exercise of Basis using Matlab

Code Exercise (03_03)

▶ Basis

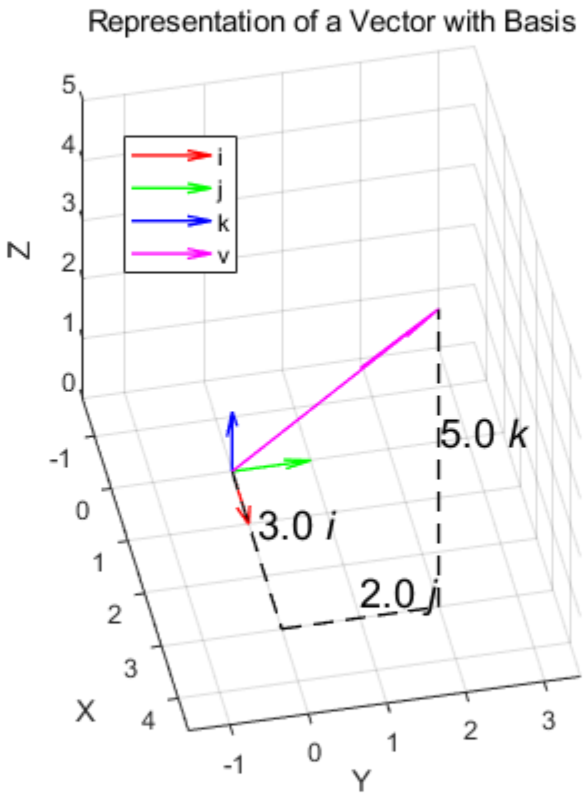
```
% Define basis vectors
i = [1; 0; 0];
j = [0; 1; 0];
k = [0; 0; 1];

% Define an arbitrary 3D vector v
v = [3; 2; 5];

% Visualization
figure;
hold on;
grid on;
axis equal;

% Draw the basis vectors
quiver3(0, 0, 0, i(1), i(2), i(3), 'r', 'LineWidth', 1, 'AutoScale', 'off',
'MaxHeadSize', 1, 'DisplayName', 'i');
quiver3(0, 0, 0, j(1), j(2), j(3), 'g', 'LineWidth', 1, 'AutoScale', 'off',
'MaxHeadSize', 1, 'DisplayName', 'j');
quiver3(0, 0, 0, k(1), k(2), k(3), 'b', 'LineWidth', 1, 'AutoScale', 'off',
'MaxHeadSize', 1, 'DisplayName', 'k');

% Draw the vector v
quiver3(0, 0, 0, v(1), v(2), v(3), 'm', 'LineWidth', 1, 'AutoScale', 'off',
'MaxHeadSize', 1, 'DisplayName', 'v');

% Visualize each multiplication of basis vector
plot3([0 v(1)], [0 0], [0 0], 'k--', 'LineWidth', 1);
plot3([v(1) v(1)], [0 v(2)], [0 0], 'k--', 'LineWidth', 1);
plot3([v(1) v(1)], [v(2) v(2)], [0 v(3)], 'k--', 'LineWidth', 1);

% Visualize the values of multiplication
text(v(1)/2, 0, 0, sprintf('%0.1f \\iti', v(1)), 'VerticalAlignment', 'bottom',
'FontSize', 15);
text(v(1), v(2)/2, 0, sprintf('%0.1f \\itj', v(2)), 'VerticalAlignment',
'bottom','FontSize', 15);
text(v(1), v(2), v(3)/2, sprintf('%0.1f \\itk', v(3)), 'VerticalAlignment',
'bottom', 'FontSize', 15);

xlabel('X');
ylabel('Y');
zlabel('Z');
title('Representation of a Vector with Basis');
legend('i','j','k','v','Location', 'northeastoutside');
view(45, 45); % Adjust the 3D view angle
hold off;
```

Source code

Visualization Result of Basis using Matlab

Code Exercise (03_03)

- Basis



Source code result

Summary

Summary

- **A vector set**
  - The set can contain either a finite or an infinite number of vectors.
- **Linear weighted combination**
  - Multiply scalar and add vectors in a set.
  - One of the single most important concepts in linear algebra.
- **Linear dependent vs linear independent**
  - If a vector can be expressed as a linear weighted combination, the set is linearly dependent.
  - If no such linearly weighted combination, the set is linearly independent.
- **A subspace**
  - The infinite set of all possible linearly weighted combination of vector set.
- **A basis**
  - If vector set $(\mathbf{1})$spans a certain subspace and $(\mathbf{2})$is linearly independent, it can be a basis for subspace.

Exercises (1)

1. **Rewrite the Original code for linear weighted combination, but put the scalars in array and the vectors as elements in an array**
   - you will have two arrays, one of the **scalars** and one of **vectors.**
2. **Then use a for loop to implement the linear weighted combination operation.**
   - Initialize the output vector using zeros().
3. **Display var** *linCombo2*
   - *linCombo1* & *linCombo2* display same result.

```
% Scalars
l1 = 1;
l2 = 2;
l3 = -3;

% Vectors
v1 = [4, 5, 1];
v2 = [-4, 0, -4];
v3 = [1, 3, 2];

% Linear weighted combination
linCombo1 = l1 * v1 + l2 * v2 + l3 * v3;
disp(linCombo1);
```

Original code

```
% Scalars
l1 = 1;
l2 = 2;
l3 = -3;

% Vectors
v1 = [4, 5, 1];
v2 = [-4, 0, -4];
v3 = [1, 3, 2];

% Scalars and vectors organized into arrays
scalars = 리스트
vectors = "

% Initialize the linear combination
linCombo2 =

% Implement linear weighted combination using a loop
for i = 1:length(scalars)
    linCombo2 =
end

% Confirm it's the same answer as above
disp(linCombo2);
```

Generated code

Exercises (2)

(1) 활용,

1. **Create a scalar list and vector list like the original code in Exercise (1), but length of scalar is different**
   - Scalar
     - length : 4
   - Vector
     - dimension : 3
     - length : 3
   - You can use any number in the list.

2. **Write code for linear weighted combination and execute code**
   - If code run successfully, write comment the result.
   - If code isn't run successfully, write comment the reason of error.

THANK YOU
FOR YOUR ATTENTION