1.10 N이 2의 거듭제곱일 때, 다음 점화식을 풀어라.

$$C_N = 2C_{N/2} + N^2, \quad N \geq 2,\ C_1 = 0.$$

$N = 2^n$이라 하면,

$$C_{2^n} = 2C_{2^{n-1}} + 2^{2n}$$

$$\frac{C_{2^n}}{2^n} = \frac{C_{2^{n-1}}}{2^{n-1}} + 2^n$$

$$= \frac{C_{2^{n-2}}}{2^{n-2}} + 2^{n-1} + 2^n$$

$$= \frac{C_{2^{n-3}}}{2^{n-3}} + 2^{n-2} + 2^{n-1} + 2^n$$

$$\vdots$$

$$= C_1 + \cdots + 2^{n-2} + 2^{n-1} + 2^n$$

$$\frac{C_N}{N} = N + \frac{N}{2} + \frac{N}{4} + \cdots$$

$$= 2N$$

$$C_N \approx 2N^2$$

$$C_N = O(N^2)$$