

# Chapter 5

## 문맥 자유 문법과 파싱 알고리즘 - **Part II**

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- **Abstract Syntax Trees**
- Ambiguity
- Extended Notations
  - EBNF and Syntax Diagrams
- Chomsky's Classification

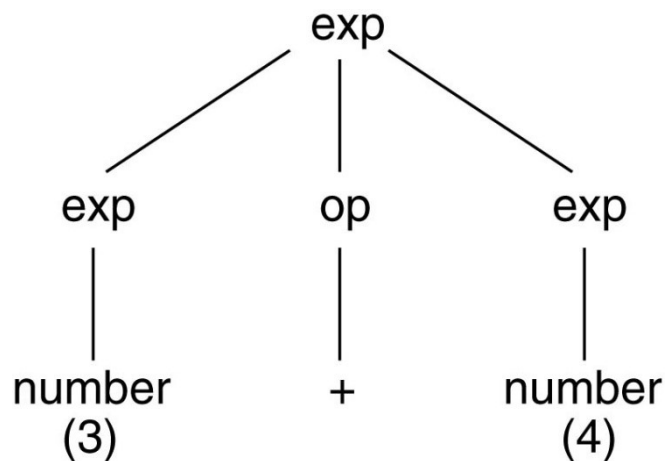
# Abstract Syntax Tree (AST)

## ■ parse tree는 지나치게 많은 정보를 표현

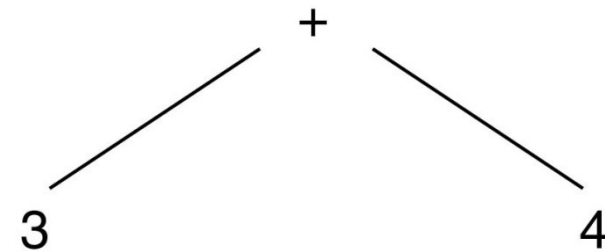
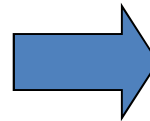
- Tree 탐색에 시간이 많이 걸림
- Tree 와 같은 동적 구조를 구현하기 위해 많은 메모리가 필요

## ■ AST(추상 구문 트리)

- contains all the information **needed for translation**
- in a more efficient form than parse trees.

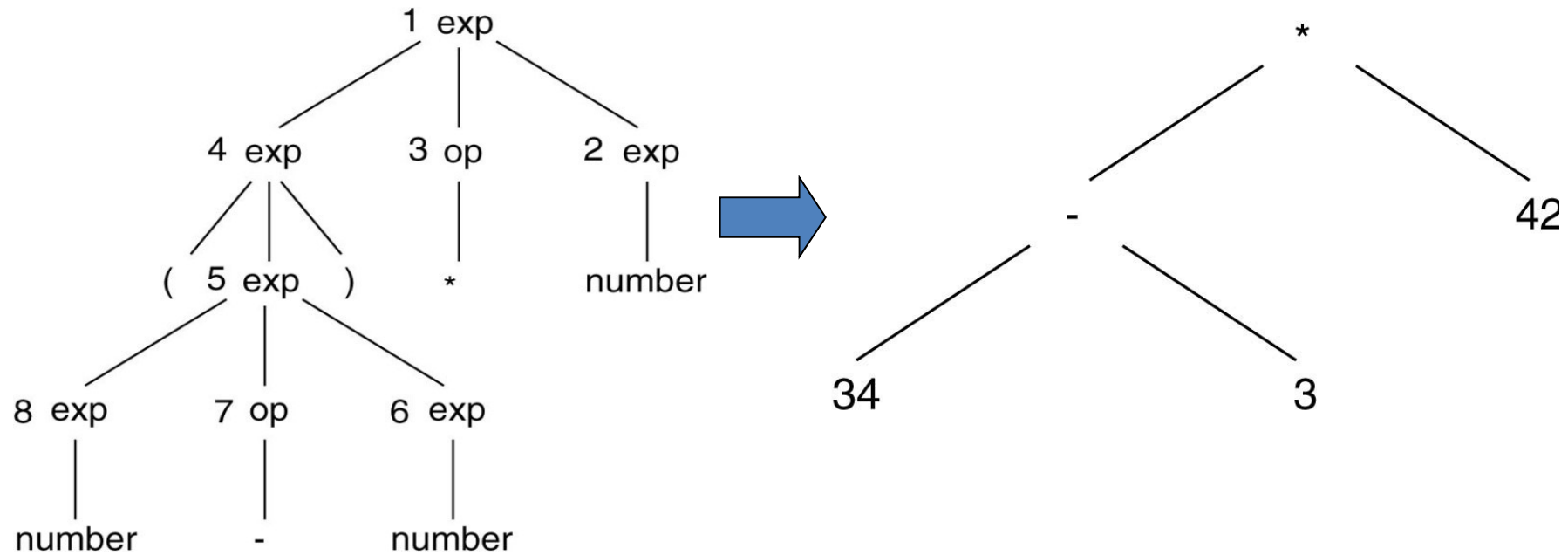


*Concrete (Ordinary) Syntax Tree*



*Abstract Syntax Tree*

## AST for $(34 - 3) * 42$



단점 : 원래 *source code sequence*로 되돌릴 수 없음  
(reverse compile이 불가능)

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# Ambiguous grammar(1/2)

## ■ 예: Grammar 의 생성 규칙이 다음과 같을 때

$E \rightarrow E + E \mid E - E \mid E * E \mid E / E \mid ( E )$

$E \rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$

입력  $3 + 4 * 5$  에 대해 구문 분석을 하시오.

$E \Rightarrow E + E$

$\Rightarrow 3 + E$

$\Rightarrow 3 + E * E$

$\Rightarrow 3 + 4 * E$

$\Rightarrow 3 + 4 * 5$

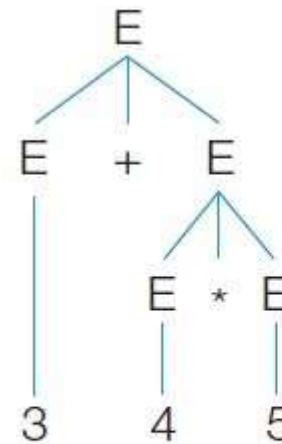
$E \Rightarrow E * E$

$\Rightarrow E + E * E$

$\Rightarrow 3 + E * E$

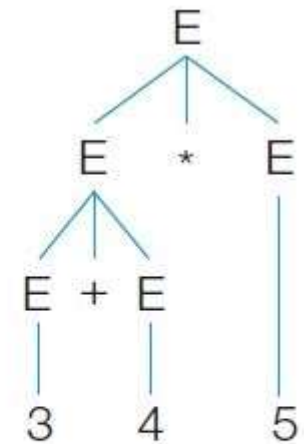
$\Rightarrow 3 + 4 * E$

$\Rightarrow 3 + 4 * 5$



(a)

$$3 + (4 * 5) = 23$$



(b)

$$(3 + 4) * 5 = 35$$

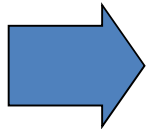
# Ambiguous grammar(2/2)

- 하나의 입력에 대해 두 개 이상의 parse tree를 생성할 수 있으면, 문법  $G$ 는 모호하다(ambiguous).
  - 전혀 다른 machine code 생성 → 똑같은 코딩인데 실행 결과가 다름
- 모호한 문법을 해결하는 방법
  - 문법 변환 : 모호한 문법 → **모호하지 않은 동등한 문법**
    - [예] 산술 연산 : 우선 순위와 결합 법칙을 생성 규칙에 반영
  - 모호하지 않은 문법으로 변환할 수 없는 경우
    - 적용 가능한 생성 규칙 중 **일정한 기준에 따라** 하나를 선택
    - [예] Dangling-else : else는 가장 가까이 있는 if와 짝이 된다!



# 문법 변환 : Precedence and Associative rule

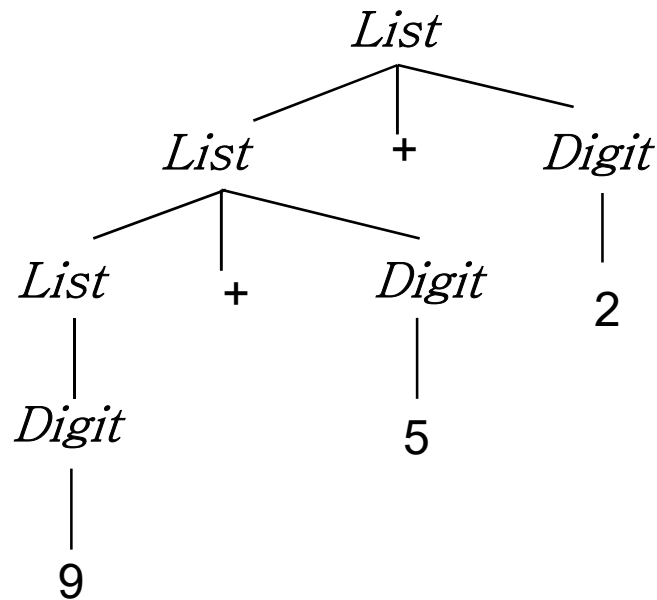
- 우선 순위가 같은 연산기호들은 한데 묶음  
    ➔ 하나의 rule에 함께 포함
- 우선 순위가 낮을수록 parse tree의 root에 가깝게 배치!
- 좌 결합법칙을 갖는 연산기호는 *left-recursive* rule로!
- 우 결합법칙을 갖는 연산기호는 *right-recursive* rule로!


$$\begin{aligned} E &\rightarrow E + T \mid E - T \mid T \\ T &\rightarrow T * F \mid T / F \mid F \\ F &\rightarrow (E) \mid 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \end{aligned}$$

# 생성 규칙에서 좌 결합법칙 표현하기

$$\begin{aligned} List &\rightarrow List + Digit \mid Digit \\ Digit &\rightarrow 0 \mid 1 \mid 2 \mid \dots \mid 9 \end{aligned}$$

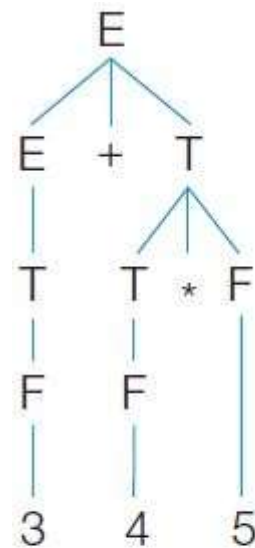
**Left Associative Rule** (좌결합 법칙 = 좌순환 규칙)



9+5+2 = (9+5)+2

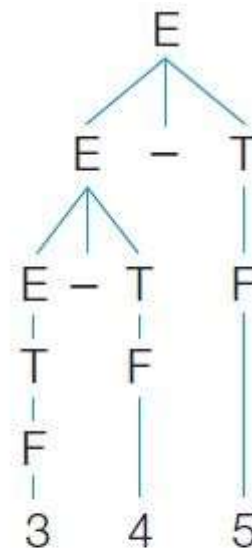
A diagram showing a box containing the equation 9+5+2 = (9+5)+2. Two arrows point from the box to the left, towards the parse tree, indicating that the tree structure corresponds to this left-associative evaluation.

## 예 10: 아래 입력에 대해 구문 분석시오.

$$\begin{aligned} E &\rightarrow E + T \mid E - T \mid T \\ T &\rightarrow T * F \mid T / F \mid F \\ F &\rightarrow (E) \mid 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \end{aligned}$$


(a)

3+4\*5



(b)

3-4-5

## Quiz #4

$E \rightarrow E + T \mid E - T \mid T$

$T \rightarrow T * F \mid T / F \mid F$

$F \rightarrow P \wedge F \mid P$

$P \rightarrow -P \mid H$

$H \rightarrow ( E ) \mid 0 \mid 1 \mid \dots \mid 9$

// ^는 거듭 제곱:  $2 \wedge 3 \rightarrow 2^3$

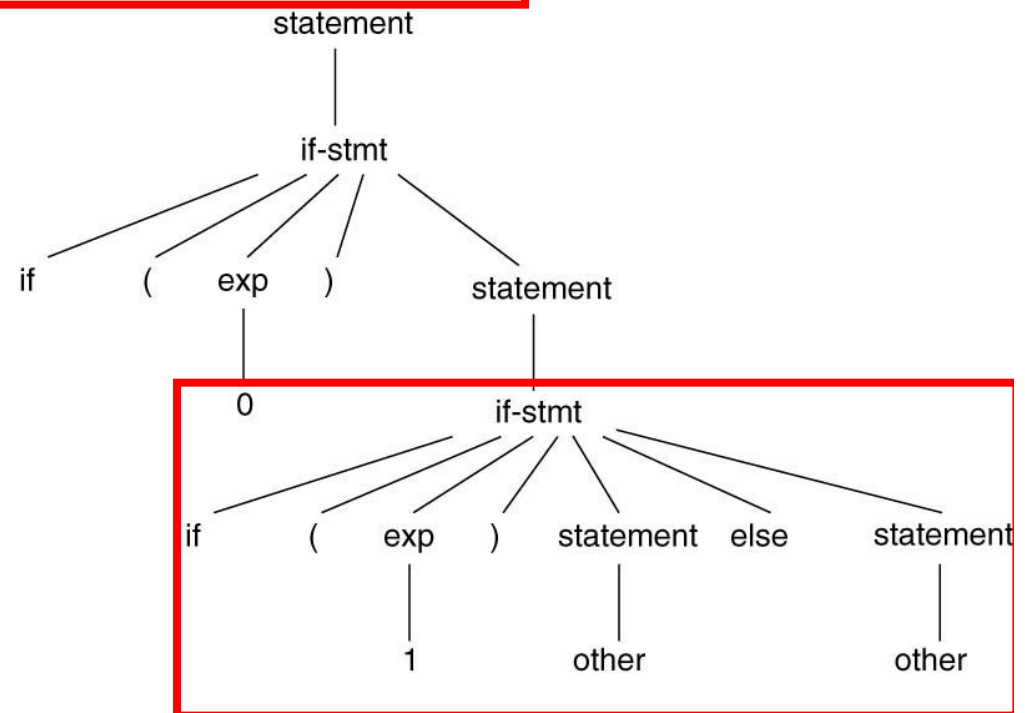
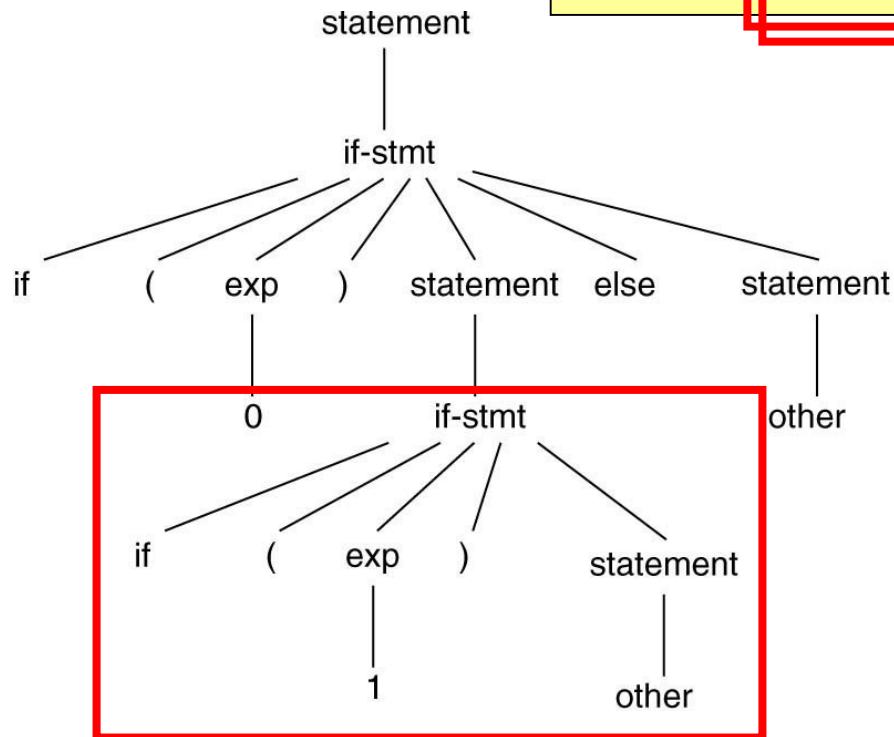
// - 는 단항 연산자. 마이너스 부호.

- a. 연산기호간 우선 순위는?
- b. 각 연산기호는 어떤 결합 법칙이 적용되는가?
- c. 입력  **$-2 + 3 * ( 4 \wedge 5 )$**  에 대해 구문 분석하시오.

# The Dangling Else Problem

$statement \rightarrow if\text{-}stmt \mid other$   
 $if\text{-}stmt \rightarrow if \ ( \ exp \ ) \ statement$   
 $\quad \quad \quad \mid if \ ( \ exp \ ) \ statement \ else \ statement$   
 $exp \rightarrow 0 \mid 1$

**if (0) if (1) other else other**



# Most closely nested rule

---

## ■ `else` 와 가장 가까이에 있는 `if` 와 짝을 이룸

- 2번째 parse tree 로 해석

```
if (x != 0)
    if (y == 1/x) ok = TRUE;
    else x = 1/x;
```

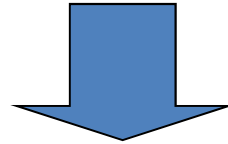
## ■ 첫 번째 `if` 와 짝을 이루려면 괄호를 사용

```
if (x != 0)
    { if (y == 1/x) ok = TRUE; }
else x = 1/x;
```

## a bracketing keyword for the *if* statement

```
if x/=0 then
  if y=1/x then ok := true;
  else z := 1/x;
  end if;
end if;
```

```
if x/=0 then
  if y=1/x then ok := true;
  end if;
  else z := 1/x;
end if;
```



```
statement → if-stmt | other  
if-stmt → if ( exp ) statement endif  
          | if ( exp ) statement else statement endif  
exp → 0 | 1
```

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## EBNF (*Extended* BNF) : Repetition { }(1/2)

### ■ Generic rules

- Left recursive :  $A \rightarrow A \alpha \mid \beta$   
→  $A \rightarrow \beta \{\alpha\}$
- Right recursive :  $A \rightarrow \alpha A \mid \beta$   
→  $A \rightarrow \{\alpha\} \beta$

### ■ EBNF로 변환하면 문제는 없을까?

- 문장 구조의 특징이 사라짐
  - *Parse tree*가 어떻게 만들어졌는지 알기 어려움

## EBNF : *Repetition { }* (2/2)

---

*stmt-sequence*  $\rightarrow$  *stmt-sequence* ; *stmt* | *stmt*

➡ **stmt-sequence**  $\rightarrow$  **stmt { ; stmt }** (*left* recursive)

*stmt-sequence*  $\rightarrow$  *stmt* ; *stmt-sequence* | *stmt*

➡ **stmt-sequence**  $\rightarrow$  **{ stmt ; } stmt** (*right* recursive)

*exp*  $\rightarrow$  *exp* addop *term* | *term*

➡ **exp**  $\rightarrow$  **term {addop term}**

*exp*  $\rightarrow$  *term* addop *exp* | *term*

➡ **exp**  $\rightarrow$  **{term addop} term**

## EBNF : *Optional* [ ] (1/2)

---

### ■ Rules for *if-statement* with optional *else-parts*

$$\begin{aligned} \textit{if-stmt} \rightarrow & \textit{if} \ ( \ \textit{exp} \ ) \ \textit{statement} \\ & | \ \textit{if} \ ( \ \textit{exp} \ ) \ \textit{statement} \ \textit{else} \ \textit{statement} \end{aligned}$$

$$\textit{if-stmt} \rightarrow \textit{if} \ (\textit{exp}) \ \textit{statement} \ [ \ \textit{else} \ \textit{statement} \ ]$$

## EBNF : *Optional* [ ] (2/2)

순환이 그대로  
살아 있음!!!

### ■ 우선순환 규칙을 [ ]로 표기

$stmt\_sequence \rightarrow stmt ; stmt\_sequence \mid stmt$

➡  $stmt\_sequence \rightarrow stmt [ ; stmt\_sequence ]$

### ♣ 참고 : 반복 기호 { } 사용

$stmt\_sequence \rightarrow \{ stmt ; \} stmt$

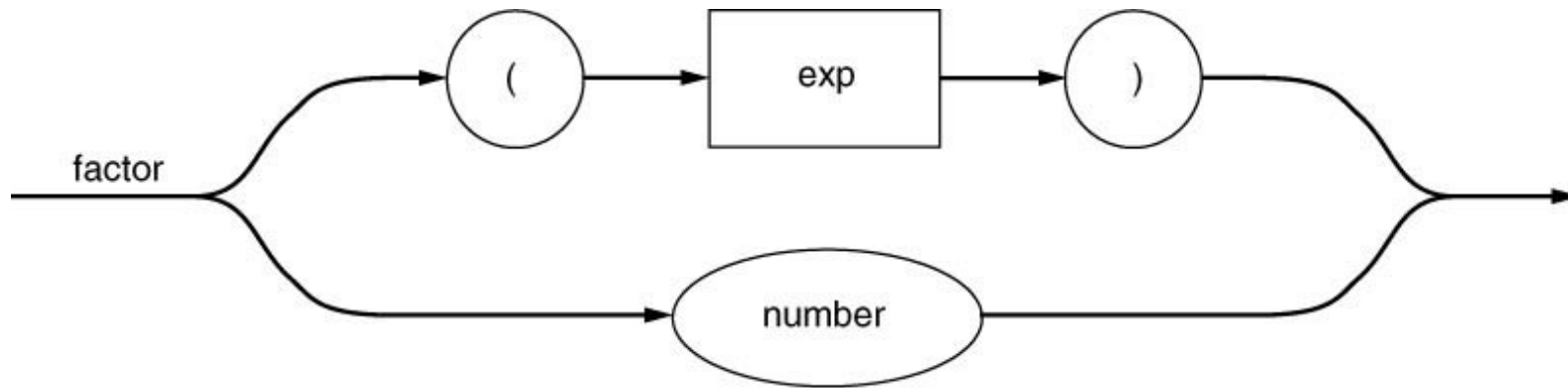
### ■ 우결합 형태를 [ ]로 표기

$exp \rightarrow term \ addop \ exp \mid term$

➡  $exp \rightarrow term [ \ addop \ exp ]$

# Syntax Diagram (1/2)

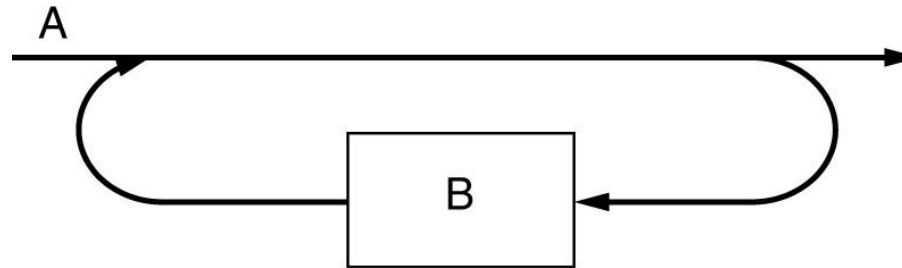
$factor \rightarrow ( exp ) \mid number$



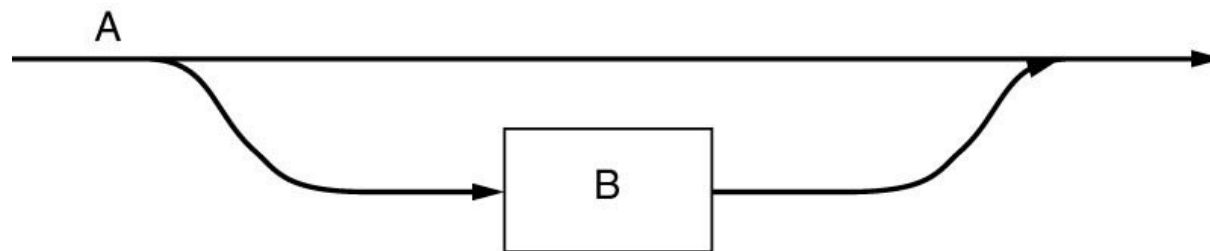
원, 타원  $\rightarrow$  terminal  
사각형  $\rightarrow$  nonterminal

## Syntax Diagram (2/2)

$A \rightarrow \{ B \}$

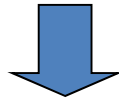


$A \rightarrow [ B ]$

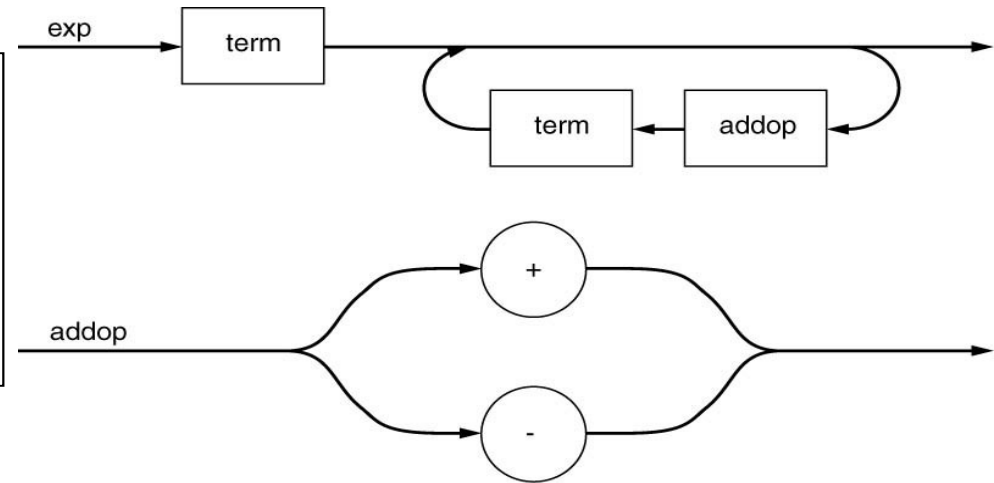


## 예 11: Syntax diagram (1)

$exp \rightarrow exp \text{ addop } term \mid term$   
 $addop \rightarrow + \mid -$   
 $term \rightarrow term \text{ mulop } factor \mid factor$   
 $mulop \rightarrow *$   
 $factor \rightarrow ( exp ) \mid number$

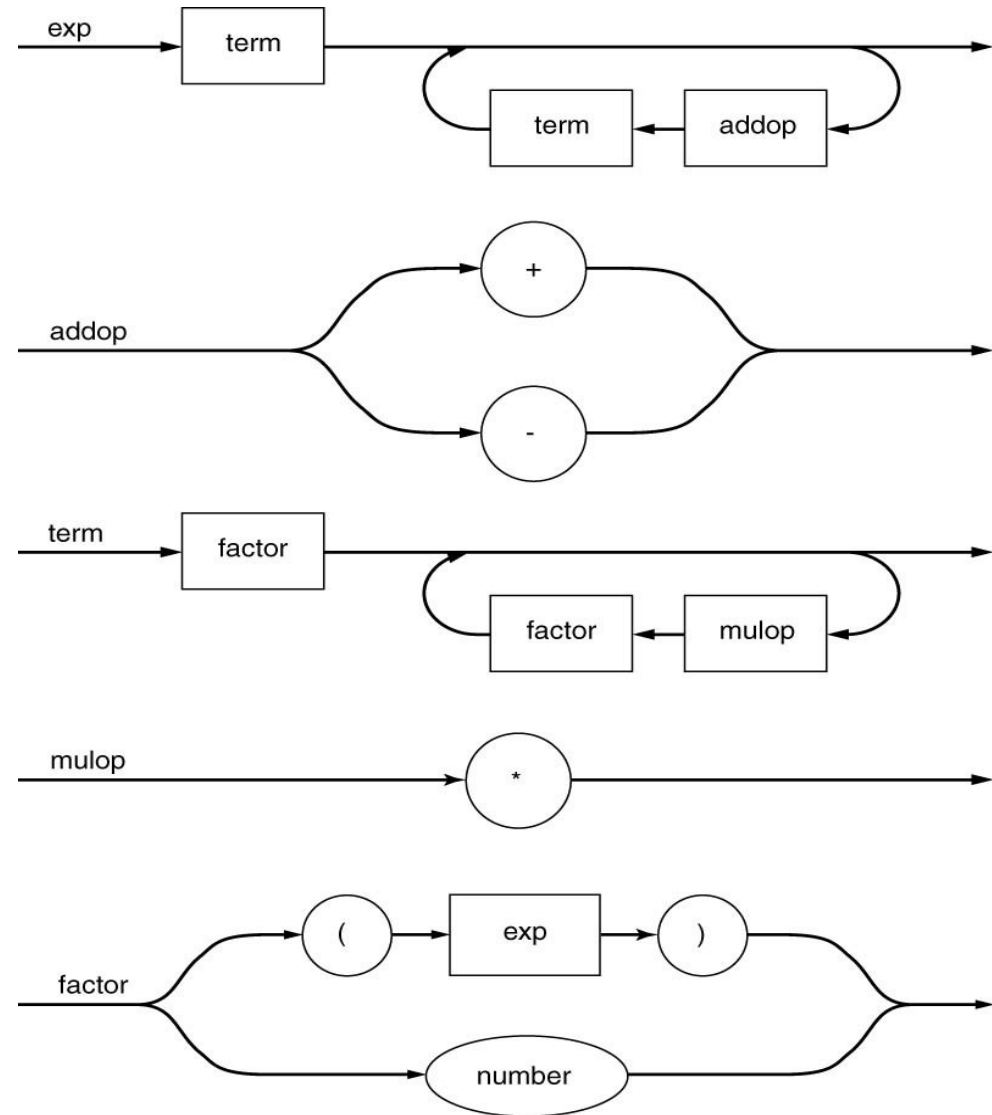


$exp \rightarrow term \{ addop term \}$   
 $addop \rightarrow + \mid -$   
 $term \rightarrow factor \{ mulop factor \}$   
 $mulop \rightarrow *$   
 $factor \rightarrow ( exp ) \mid number$



## 예 11: Syntax diagram (2)

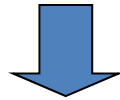
$exp \rightarrow term \{ addop \ term \}$   
 $addop \rightarrow + \mid -$   
 $term \rightarrow factor \{ mulop \ factor \}$   
 $mulop \rightarrow *$   
 $factor \rightarrow ( \ exp \ ) \mid number$



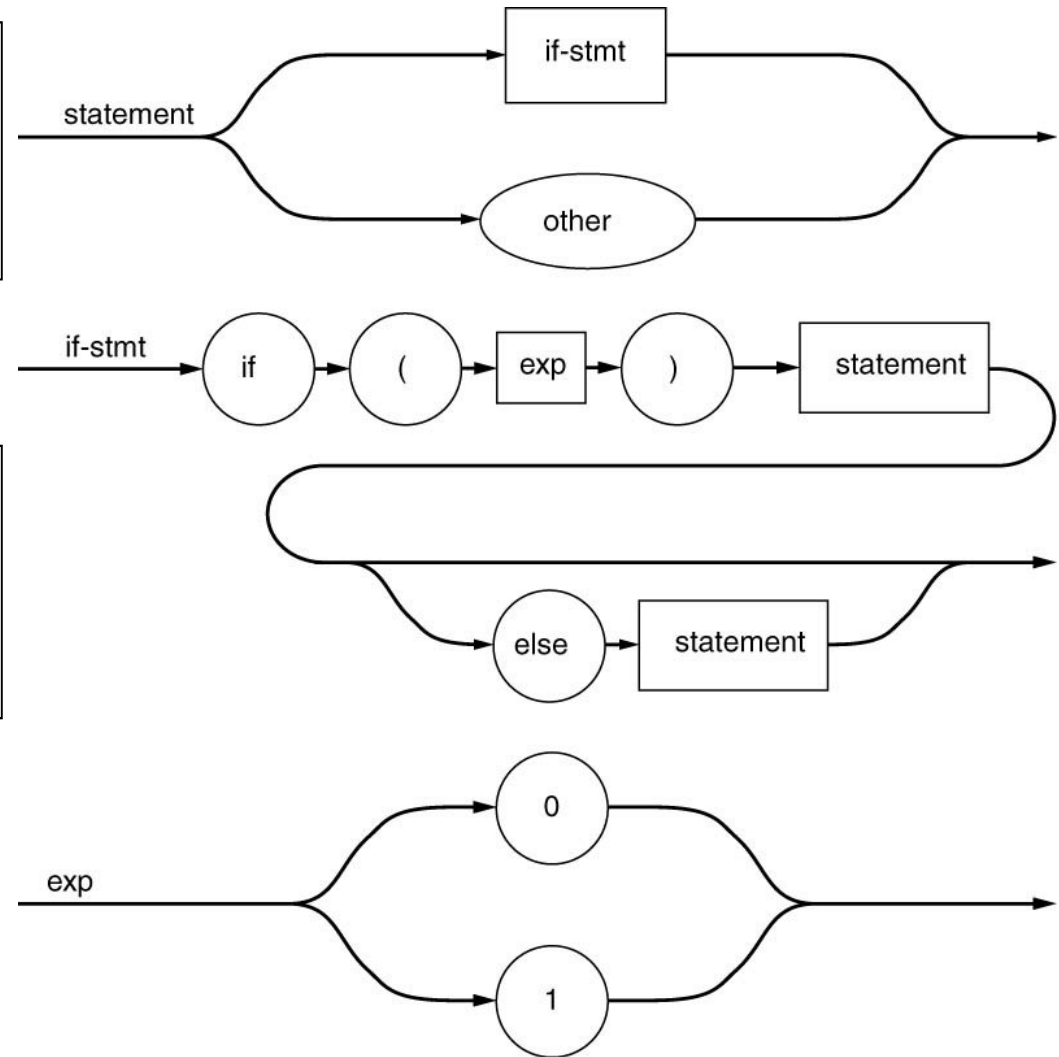


## 예 11: Syntax diagram (3)

$statement \rightarrow if\text{-}stmt / other$   
 $if\text{-}stmt \rightarrow if ( exp ) statement$   
 $\quad \quad | if ( exp ) statement else statement$   
 $exp \rightarrow 0 | 1$



$statement \rightarrow if\text{-}stmt / other$   
 $if\text{-}stmt \rightarrow if ( exp ) statement$   
 $\quad \quad \quad [else statement]$   
 $exp \rightarrow 0 | 1$



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# A formal definition of CFG

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## ■ 4 *tuple notation* $G = (T, N, P, S)$ , 단 $S \in N$

- A set ***T*** of *Terminals (input symbols)*
- A set ***N*** of *Nonterminals* ( $T \cap N = \emptyset$ )
- A set ***P*** of *productions or grammar rules*
  - 생성 규칙의 왼쪽에는 non-terminal 한 개만 있다.  
$$A \rightarrow \alpha, \alpha \in (T \cup N)^*$$
- A start symbol ***S***.  $S \in N$

# Classes of Grammars

0. An Unrestricted Grammar : no *restrictions* on the rewriting rules

$$\text{예} : SaB \rightarrow cS$$

1. A Context-Sensitive Grammar(CSG)

$$\alpha A \gamma \rightarrow \alpha \beta \gamma$$

$$\text{예} : SaB \rightarrow caB$$

2. A Context-Free Grammar(CFG)

$$A \rightarrow \alpha$$

$$\text{예} : A \rightarrow aABb$$

3. Regular Grammar

$$A \rightarrow aB \text{ 또는 } A \rightarrow a$$

$$A \rightarrow Ba \text{ 또는 } A \rightarrow a$$

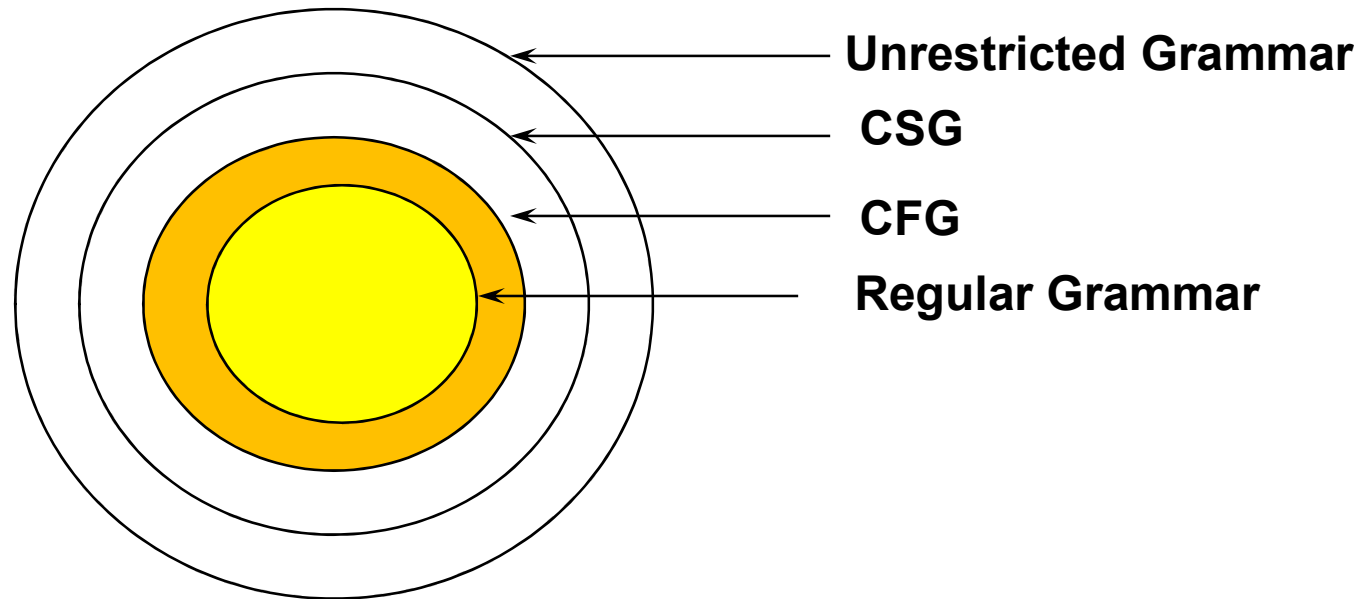
$\alpha, \beta, \gamma$  : 임의의 문자열  
 $A, B, S$  : 비단말 기호  
 $a, b, c$  : 단말기호

Right linear regular grammar

Left linear regular grammar

# Classes of Grammars (*by N. Chomsky*)

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**Type 0 : Unrestricted**  
**Type 1 : Context-Sensitive**  
**Type 2 : Context-Free**  
**Type 3 : Regular Grammar**

# 정규 표현은 정규 문법

정규 표현 (*Regular Expression*)

$$\begin{aligned} \text{digit} &= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \\ \text{number} &= \text{digit} \text{ digit}^* \end{aligned}$$

정규 문법 (*Regular Grammar*)

$$\begin{aligned} \text{digit} &\rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \\ \text{number} &\rightarrow \text{number} \text{ digit} \mid \text{digit} \end{aligned}$$
$$A \rightarrow A a \mid a$$

## 예 12: CSG, not CFG

P :

1.  $S \rightarrow aSBC$
2.  $S \rightarrow \varepsilon$
3.  $aB \rightarrow ab$
4.  $bB \rightarrow bb$
5.  $C \rightarrow c$
6.  $CB \rightarrow CX$
7.  $CX \rightarrow BX$
8.  $BX \rightarrow BC$

$S \Rightarrow a\mathbf{S}BC \Rightarrow aa\mathbf{S}BCBC \Rightarrow aaB\mathbf{C}BC$   
 $\Rightarrow aaB\mathbf{C}XC \Rightarrow aaB\mathbf{B}XC \Rightarrow aa\mathbf{B}BCC$   
 $\Rightarrow aa\mathbf{b}BCC \Rightarrow aabbC\mathbf{C} \Rightarrow aabb\mathbf{C}c$   
 $\Rightarrow aabbcc$

$$L(G) = \{\varepsilon, abc, aabbcc, \dots\}$$
$$= \{a^n b^n c^n, n \geq 0\}$$

## 예 13: 문법 종류

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- |                            |                             |
|----------------------------|-----------------------------|
| 1. $aSb \rightarrow aAcBb$ | (Type 1. Context-Sensitive) |
| 2. $B \rightarrow aA$      | (Type 3. Right Linear)      |
| 3. $a \rightarrow ABC$     | (Type 0. Unrestricted)      |
| 4. $S \rightarrow aBc$     | (Type 2. Context-Free)      |
| 5. $Ab \rightarrow b$      | (Type 1. Context-Sensitive) |
| 6. $AB \rightarrow BA$     | (Type 0. Unrestricted)      |