

Chapter 6

상향식 파싱 알고리즘 - **Part II**

Table of Contents

- Overview of Bottom-Up Parsing
- Finite Automata of LR(0) items and LR(0) Parsing
- **SLR(1) Parsing**
- **General LR(1) and LALR(1) Parsing**

SLR(1) Parsing

■ *Simple* LR(1)

- DFA of sets of LR(0) items
- uses the next token in the input string
 - *consults* the input token before a shift
 - 해당 transition 존재 여부를 확인
 - ◆ 진짜 그런 기호가 입력에 있니?
- uses the **Follow** set of a nonterminal
 - reduction 실행 여부를 결정
 - ◆ 이렇게 reduction 하는 게 정말 맞는 거니?

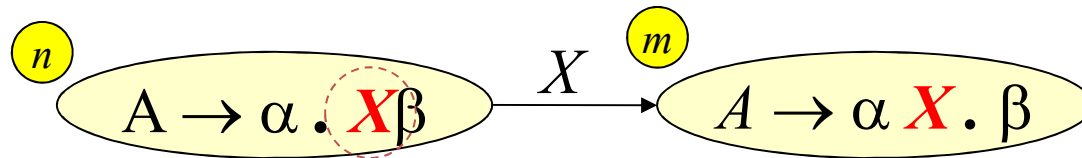
■ 이렇게만 해도 LR(0)에 비해 엄청난 power-up!!!

SLR(1) Parsing Algorithm (1/3)

■ 현재 상태 n 이

1. *shift* item $A \rightarrow \alpha .X\beta$ 을 갖고 있는 경우

- X 가 *terminal* 이고, 입력에서 읽어 온 token 도 X 이면,
 - *shift action* : input token X 를 stack에 push
 - $A \rightarrow \alpha X . \beta$ 의 item을 포함하는 상태 번호(m)를 push



Parsing Stack

\$ \dots n

\$ \dots n X m

Input

X \dots \$

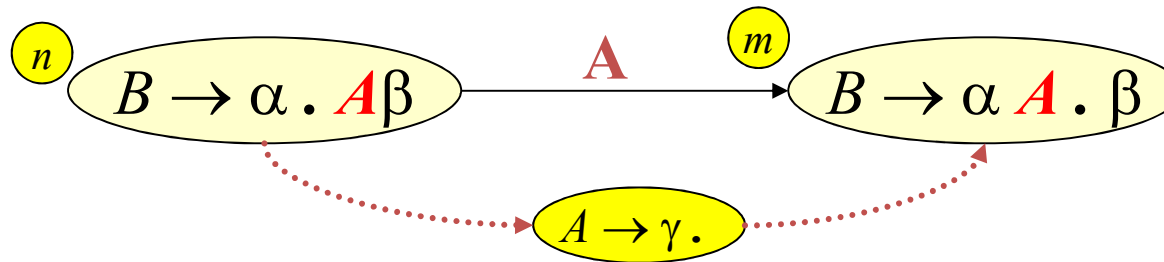
\dots \$

SLR(1) Parsing Algorithm (2/3)

■ 현재 상태 n 이

2. *complete* item $A \rightarrow \gamma \cdot$ 을 갖고 있는 경우

- 입력에서 읽어 온 token이 $\text{Follow}(A)$ 에 속하면,
- **reduce action:** reduction by $A \rightarrow \gamma$
 - stack에서 기호(γ)와 상태 번호를 함께 제거(*pop*)
 - stack에 nonterminal A 를 push
 - $B \rightarrow \alpha \cdot A \beta$ 에서 reduction 직후의 item $B \rightarrow \alpha A \cdot \beta$ 를 포함하는 상태 번호를 push



SLR(1) Parsing Algorithm (3/3)

■ 현재 상태 n 이

2. *complete* item $S' \rightarrow S.$ 을 갖고 있는 경우

- 입력 버퍼가 비어 있으면($LOOKAHEAD = '$'$)
 - ➔ Parsing은 여기서 멈춤
 - ➔ The input sentence is syntactically correct for a given grammar.

SLR(1) Grammar

■ A grammar is *SLR(1) grammar* if and only if the following two conditions are satisfied:

for any state s ,

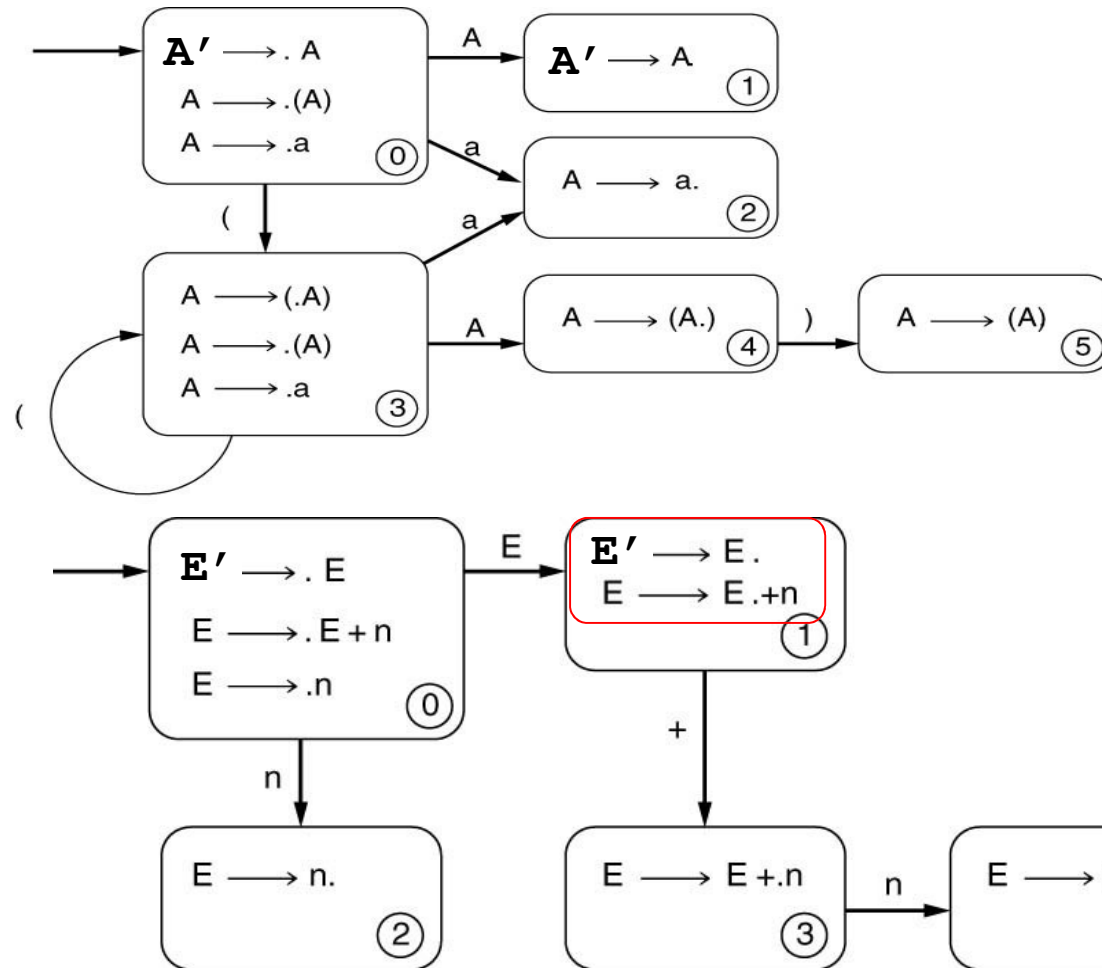
1. *no shift-reduce conflict*

- For any shift item $A \rightarrow \alpha.X\beta$ in s with X a *terminal*,
 - there is *no* complete item $B \rightarrow \gamma.$ in s with X in $\text{Follow}(B)$.

2. *no reduce-reduce conflict*

- For any two *complete* items $A \rightarrow \alpha.$ and $B \rightarrow \beta.$ in s ,
 - $\text{Follow}(A) \cap \text{Follow}(B)$ is *empty*.

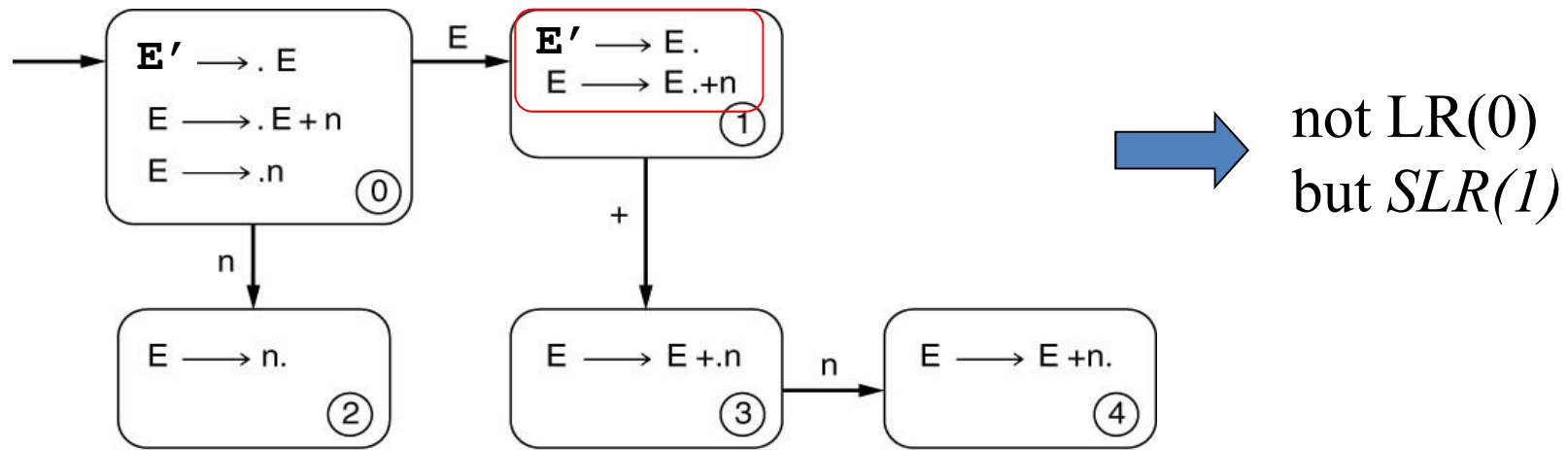
What's the difference?



➡ LR(0)

➡ not LR(0)
but *SLR(1)*

What's the difference?



Why?

shift item	$E \rightarrow E \cdot + n$
reduce item	$E' \rightarrow E \cdot$

SLR(1) Parsing Table (1/2)

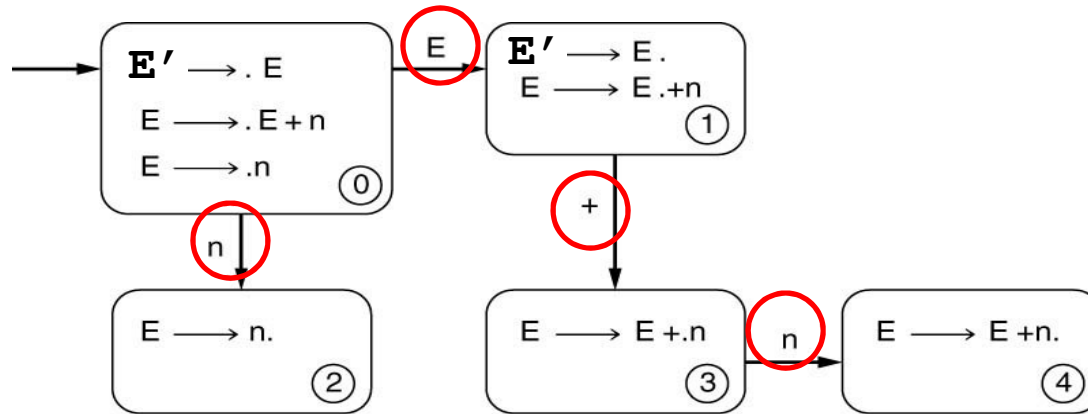
■ SLR(1) parsing table 구성

상태 \ 기호	ACTION 표	GOTO 표
	$V_T \cup \{\$ \}$	V_N
0	shift	상태 번호
1	reduce	
2	accept	
:	error	

■ 4 semantic actions

1. **Shift** : $\text{ACTION}[S_m, a_i] = \text{shift } S$
2. **Reduce** : $\text{ACTION}[S_m, a_i] = \text{reduce by } A \rightarrow \alpha$
3. **Accept** : $\text{ACTION}[S_m, a_i] = \text{accept}$
4. **Syntax error** : $\text{ACTION}[S_m, a_i] = \text{error (No action defined!)}$

SLR(1) Parsing Table for Example 10

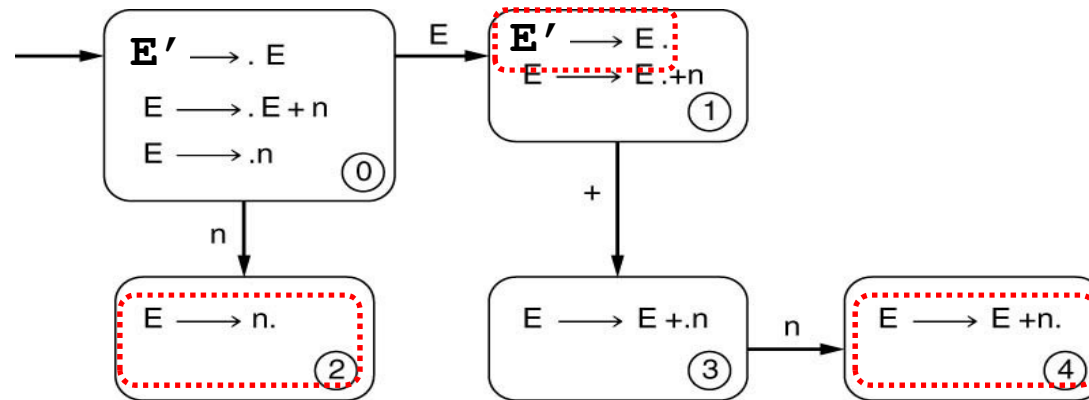


$E' \rightarrow E$
 $E \rightarrow E + n \mid n$

State	input			goto
	n	+	\$	E
0	s2			1
1		s3		
2				
3	s4			
4				

shift 와 *reduce* action이 같은 상태에
 함께 포함될 수 있음
shift action + 상태번호 \rightarrow s2

SLR(1) Parsing Table for Example 10



$E' \rightarrow E$

$E \rightarrow E + n \mid n$

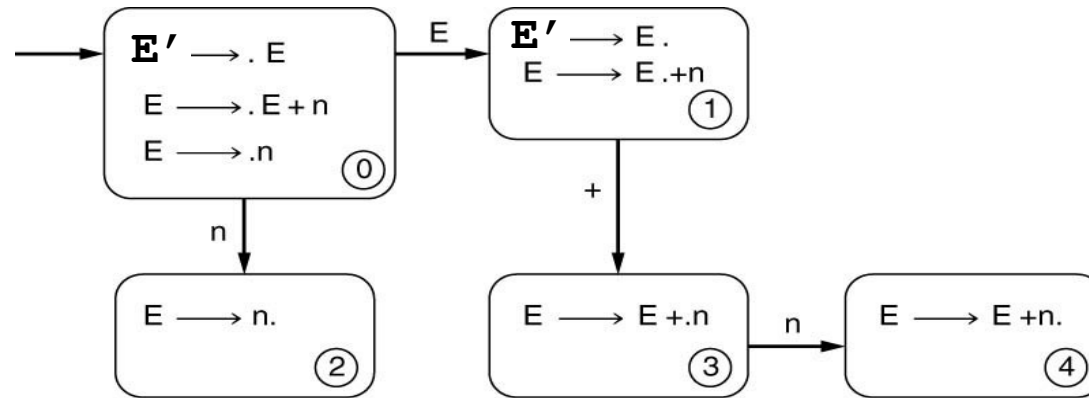
$\text{FOLLOW}(E') = \{\$ \}$

$\text{FOLLOW}(E) = \{+, \$ \}$

State	input			goto
	n	+	\$	E
0				1
1			accept	
2		$r(E \rightarrow n)$	$r(E \rightarrow n)$	
3				
4		$r(E \rightarrow E + n)$	$r(E \rightarrow E + n)$	

reduce action + rule

SLR(1) Parsing Table for Example 10



State	input			goto
	n	+	\$	E
0	s2			1
1		s3	accept	
2		r(E → n)	r(E → n)	
3	s4			
4		r(E → E+n)	r(E → E+n)	

What was changed?

State	Input			Goto
	n	+	\$	E
0	s2			1
1		s3	accept	
2		r (E→n)	r (E→n)	
3	s4			
4		r (E→E+n)	r (E→E+n)	

SLR(1)

LR(0)

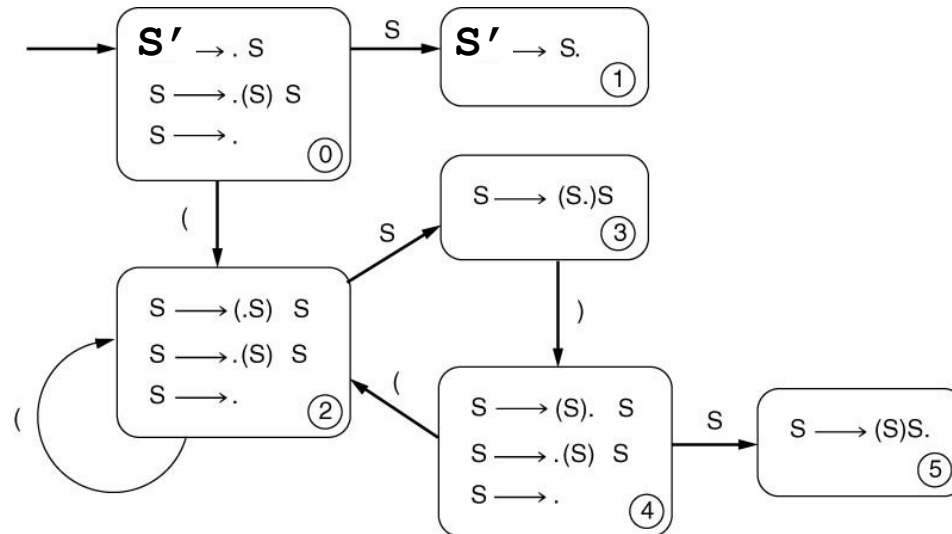
State	Action	Rule	Input			Goto
			(a)	A
0	shift		3	2		1
1	reduce	$A' \rightarrow A$				
2	reduce	$A \rightarrow a$				
3	shift		3	2		4
4	shift				5	
5	reduce	$A \rightarrow (A)$				

Parsing Actions for Example 10

State	Input			Goto
	n	+	\$	E
0	s2			1
1		s3	accept	
2		r (E→n)	r (E→n)	
3	s4			
4		r (E→E+n)	r (E→E+n)	

	<u>Parsing Stack</u>	<u>Input</u>	<u>Action</u>
1	\$0	n+n+n\$	shift 2
2	\$0n2	+n+n\$	reduce by E→n
3	\$0E1	+n+n\$	shift 3
4	\$0E1+3	n+n\$	shift 4
5	\$0E1+3n4	+n\$	reduce E→E+n
6	\$0E1	+n\$	shift 3
7	\$0E1+3	n\$	shift 4
8	\$0E1+3n4	\$	reduce by E→E+n
9	\$0E1	\$	accept

SLR(1) Parsing Table for Example 7



$S' \rightarrow S$
 $S \rightarrow (S) S \mid \epsilon$

$FOLLOW(S') = \{ \$ \}$
 $FOLLOW(S) = \{), \$ \}$

State	Input			Goto
	()	\$	
0	s2	r ($S \rightarrow \epsilon$)	r ($S \rightarrow \epsilon$)	1
1			accept	
2	s2	r ($S \rightarrow \epsilon$)	r ($S \rightarrow \epsilon$)	3
3		s4		
4	s2	r ($S \rightarrow \epsilon$)	r ($S \rightarrow \epsilon$)	5
5		r ($S \rightarrow (S) S$)	r ($S \rightarrow (S) S$)	

Parsing Actions for Example 7

State	Input			Goto
	()	\$	S
0	s2	$r(S \rightarrow \epsilon)$	$r(S \rightarrow \epsilon)$	1
1			accept	
2	s2	$r(S \rightarrow \epsilon)$	$r(S \rightarrow \epsilon)$	3
3		s4		
4	s2	$r(S \rightarrow \epsilon)$	$r(S \rightarrow \epsilon)$	5
5		$r(S \rightarrow (S) S)$	$r(S \rightarrow (S) S)$	

	<u>Parsing Stack</u>	<u>Input</u>	<u>Action</u>
1	\$0	() \$	shift 2
2	\$0 (2) () \$	reduce $S \rightarrow \epsilon$
3	\$0 (2S3) () \$	shift 4
4	\$0 (2S3) 4	() \$	shift 2
5	\$0 (2S3) 4 (2) \$	reduce $S \rightarrow \epsilon$
6	\$0 (2S3) 4 (2S3) \$	shift 4
7	\$0 (2S3) 4 (2S3) 4	\$	reduce $S \rightarrow \epsilon$
8	\$0 (2S3) 4 (2S3) 4S5	\$	reduce $S \rightarrow (S) S$
9	\$0 (2S3) 4S5	\$	reduce $S \rightarrow (S) S$
10	\$0S1	\$	accept

예 12 : SLR(1) parsing - C_0

■ C_0 : Canonical Collection of LR(0) items

■ [예] 1. $E \rightarrow E + T$ 2. $E \rightarrow T$ 3. $T \rightarrow T * F$ 4. $T \rightarrow F$ 5. $F \rightarrow (E)$ 6. $F \rightarrow id$

■ 확장 문법(augmented grammar)

0. $S' \rightarrow E$ 1. $E \rightarrow E + T$ 2. $E \rightarrow T$ 3. $T \rightarrow T * F$ 4. $T \rightarrow F$ 5. $F \rightarrow (E)$ 6. $F \rightarrow id$

$I_0 = \text{CLOSURE}([S' \rightarrow \bullet E])$
 $= \{[S' \rightarrow \bullet E], [E \rightarrow \bullet E + T], [E \rightarrow \bullet T], [T \rightarrow \bullet T * F], [T \rightarrow \bullet F], [F \rightarrow \bullet (E)], [F \rightarrow \bullet id]\}$

$\text{GOTO}(I_0, E) = I_1 = \{[S' \rightarrow E \bullet], [E \rightarrow E \bullet + T]\}$

$\text{GOTO}(I_0, T) = I_2 = \{[E \rightarrow T \bullet], [T \rightarrow T \bullet * F]\}$

$\text{GOTO}(I_0, F) = I_3 = \{[T \rightarrow F \bullet]\}$

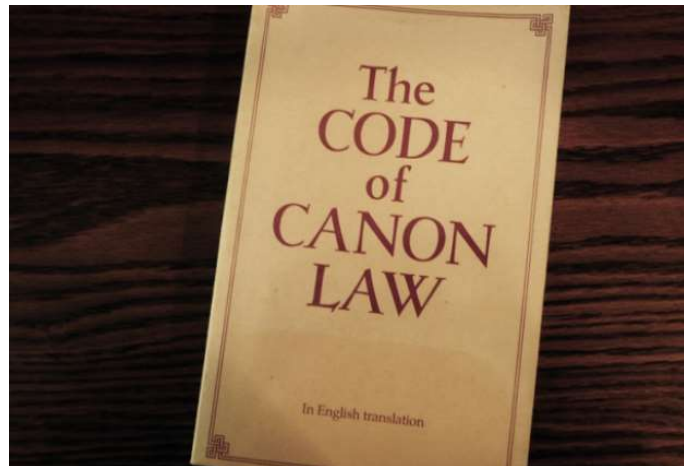
$\text{GOTO}(I_0, () = I_4 = \text{CLOSURE}([F \rightarrow (\bullet E)])$
 $= \{[F \rightarrow (\bullet E)], [E \rightarrow \bullet E + T], [E \rightarrow \bullet T], [T \rightarrow \bullet T * F], [T \rightarrow \bullet F], [F \rightarrow \bullet (E)], [F \rightarrow \bullet id]\}$

$\text{GOTO}(I_0, id) = I_5 = \{[F \rightarrow id \bullet]\}$

Canonical Collection of LR(0) items - C_0

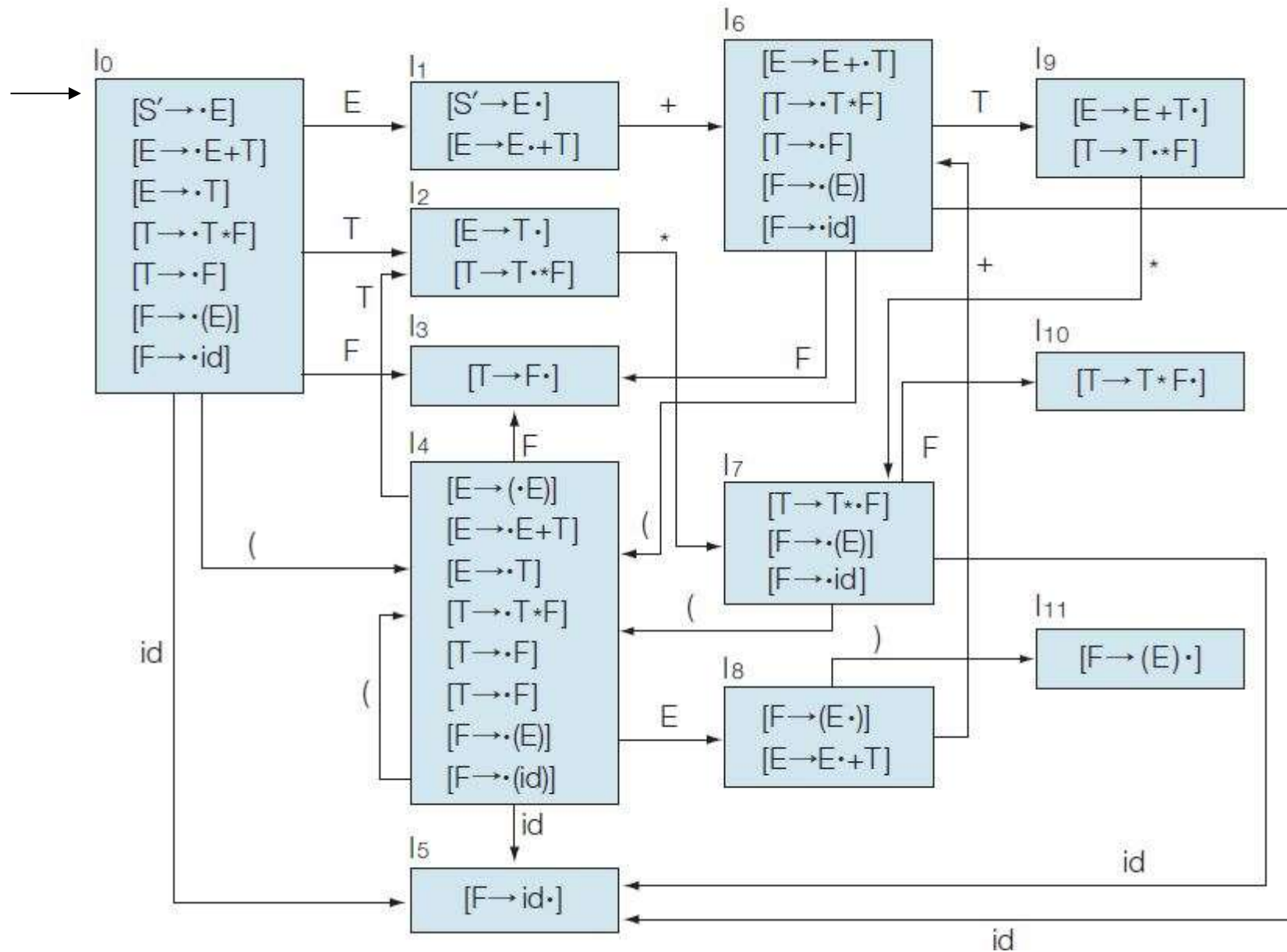
■ Canonical 은 무슨 뜻?

- **Canon law** : 교회의 권위에 의해 만들어진 일련의 조례 및 규정



- **Canonical Collection** → 표준 모음(*Standard Collection*)

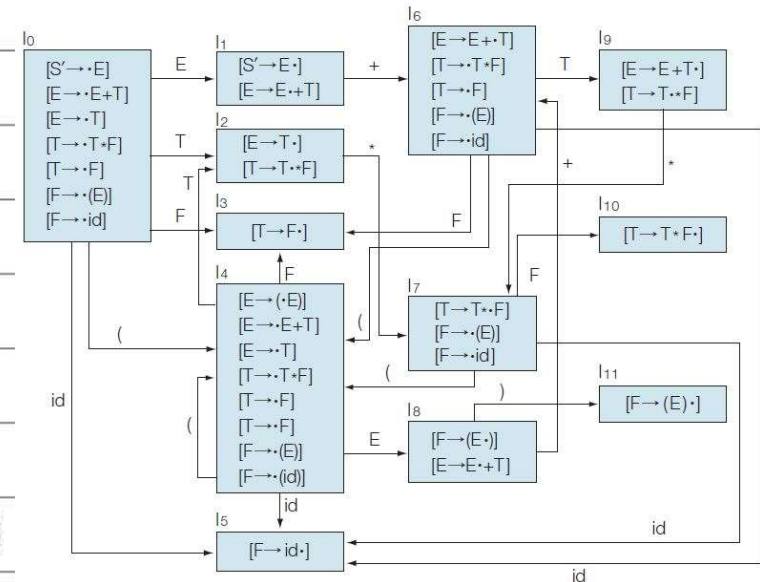
예 12 : SLR(1) parsing - DFA of LR(0) items



예 12 : SLR(1) parsing – parsing table

0. $S' \rightarrow E$
1. $E \rightarrow E + T$
2. $E \rightarrow T$
3. $T \rightarrow T * F$
4. $T \rightarrow F$
5. $F \rightarrow (E)$
6. $F \rightarrow id$

상태	구문 분석기 행동						GOTO 함수		
	id	+	*	()	\$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			



FOLLOW(E) = { \$, +,) }
FOLLOW(T) = { *, +,), \$ }
FOLLOW(F) = { *, +,), \$ }

예 12: SLR(1) parsing – syntax analysis (1/2)

표 6-3 SLR 파싱표

상태	구문 분석기 행동						GOTO 함수		
	id	+	*	()	\$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

단계	스택	입력 기호	구문 분석 내용
0	0	id * (id * id)\$	이동 5
1	0id5	* (id * id)\$	감축 6
2	0F	* (id * id)\$	GOTO 3
3	0F3	* (id * id)\$	감축 4
4	0T	* (id * id)\$	GOTO 2
5	0T2	* (id * id)\$	이동 7
6	0T2 * 7	(id * id)\$	이동 4
7	0T2 * 7(4	id * id)\$	이동 5
8	0T2 * 7(4id5	* id)\$	감축 6
9	0T2 * 7(4F	* id)\$	GOTO 3

예 12: SLR(1) parsing – syntax analysis (2/2)

표 6-3 SLR 파싱표

상태	구문 분석기 행동						GOTO 함수		
	id	+	*	()	\$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

단계	스택	입력 기호	구문 분석 내용
10	0T2 * 7(4F3	* id)\$	감축 4
11	0T2 * 7(4T	* id)\$	GOTO 2
12	0T2 * 7(4T2	* id)\$	이동 7
13	0T2 * 7(4T2 * 7	id)\$	이동 5
14	0T2 * 7(4T2 * 7id5)\$	감축 6
15	0T2 * 7(4T2 * 7F)\$	GOTO 10
16	0T2 * 7(4T2 * 7F10)\$	감축 3
17	0T2 * 7(4T)\$	GOTO 2
18	0T2 * 7(4T2)\$	감축 2
19	0T2 * 7(4E)\$	GOTO 8
20	0T2 * 7(4E8)\$	이동 11
21	0T2 * 7(4E8)11	\$	감축 5
22	0T2 * 7F	\$	GOTO 10
23	0T2 * 7F10	\$	감축 3
24	0T	\$	GOTO 2
25	0T2	\$	감축 2
26	0E	\$	GOTO 1
27	0E1	\$	수락

Disambiguating Rules for Parsing Conflicts

■ shift-reduce conflicts 해결 방법

- always *prefer the shift* over the reduce
 - incorporates the *most closely nested rule* for the dangling else ambiguity in **if**-statement
- 자연스러운 해결 방법
 - 문법에 모호함이 있더라도 이런 방식으로 해결 가능

■ reduce-reduce conflicts

- 대부분 문법 자체에 오류가 있는 경우 발생

예 13

■ Grammar with *dangling-else* ambiguity

$S \rightarrow I \mid \text{other}$

$I \rightarrow \text{if } S \mid \text{if } S \text{ else } S$

■ DFA를 구성하면 parsing conflict이 발생하는 상태 존재

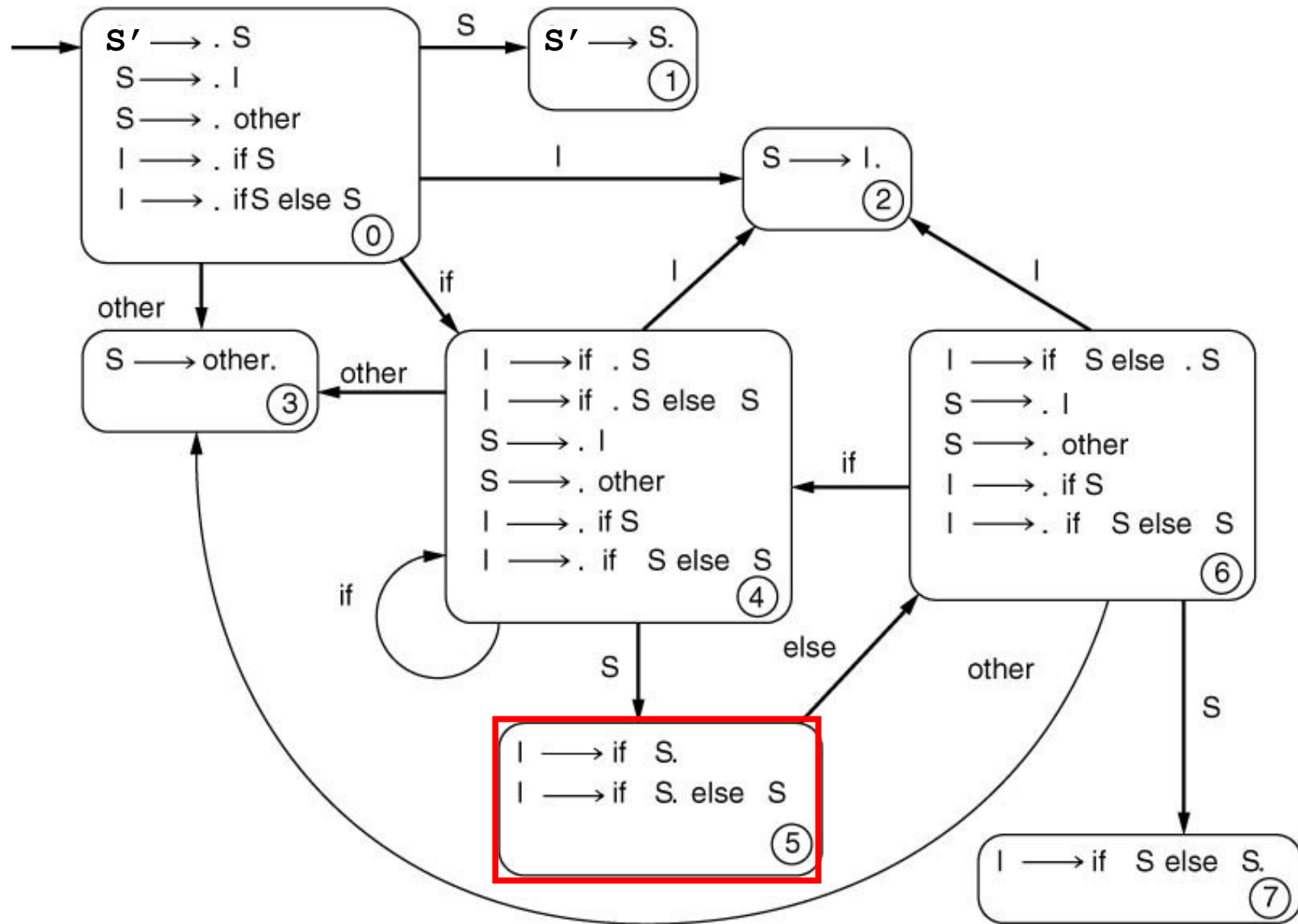
- *shift* item과 *reduce* item이 같은 상태에 포함됨

$I \rightarrow \text{if } S \cdot$ FOLLOW(I) = { **else**, \$ }

$I \rightarrow \text{if } S \cdot \text{else } S$ 다음 토큰이 **else** 일 때 *shift* action

■ *shift* action을 우선 적용 → resolve the shift-reduce conflict

- **else**는 가장 가까운 **if**와 짝을 이룸(*most closely nested rule*)



예 13: SLR(1) Parsing Table

State	Input				Goto	
	if	else	other	\$	<i>S</i>	<i>I</i>
0	s4		s3		1	2
1				accept		
2		r1		r1		
3		r2		r2		
4	r4		s3		5	2
5		s6		r3		
6	s4		s3		7	2
7		r4		r4		

(1) $S \rightarrow I$

(3) $I \rightarrow \text{if } S$

(2) $S \rightarrow \text{other}$

(4) $I \rightarrow \text{if } S \text{ else } S$

Limits of SLR(1) Parsing Power (1/3)

■ “phony” problem (장난 전화)

- 실제 입력에서 절대로 나타날 수 없는 상황 때문에 conflict가 발생
 - caused by the *weakness* of the SLR(1) method

■ SLR(1) parsing을 적용할 수 없는 예

$stmt \rightarrow call-stmt \mid assign-stmt$

$call-stmt \rightarrow \boxed{identifier}$

$assign-stmt \rightarrow var := exp$

$var \rightarrow var [exp] \mid \boxed{identifier}$

$exp \rightarrow var \mid number$

- 호출문 ($call-stmt$)과 할당문($assign-stmt$) 모두 **identifier**로 시작
 $:=$ 를 읽어오기 전까지는 호출문인지 할당문인지 알 수 없음

Limits of SLR(1) Parsing Power (2/3)

$S \rightarrow id \mid V := E$
 $V \rightarrow id$
 $E \rightarrow V \mid n$

시작 상태

$S' \rightarrow . S$
 $S \rightarrow . id$
 $S \rightarrow . V := E$
 $V \rightarrow . id$

id

$S \rightarrow id .$
 $V \rightarrow id .$

*reduce-reduce
conflict*

$S \rightarrow id .$

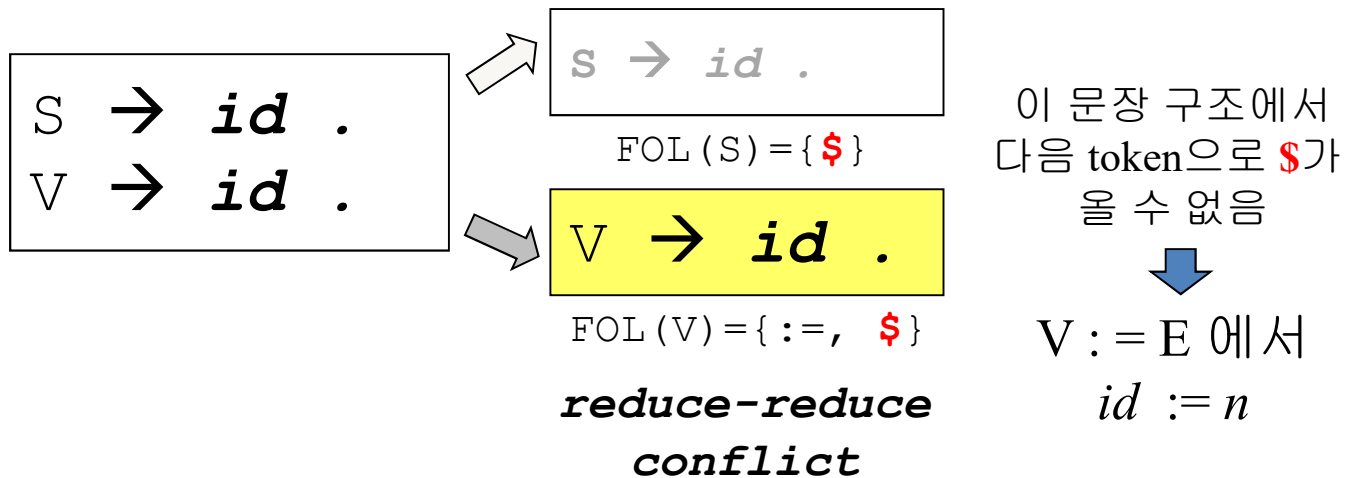
$FOL(S) = \{ \$ \}$

$V \rightarrow id .$

$FOL(V) = \{ :=, \$ \}$

Limits of SLR(1) Parsing Power (3/3)

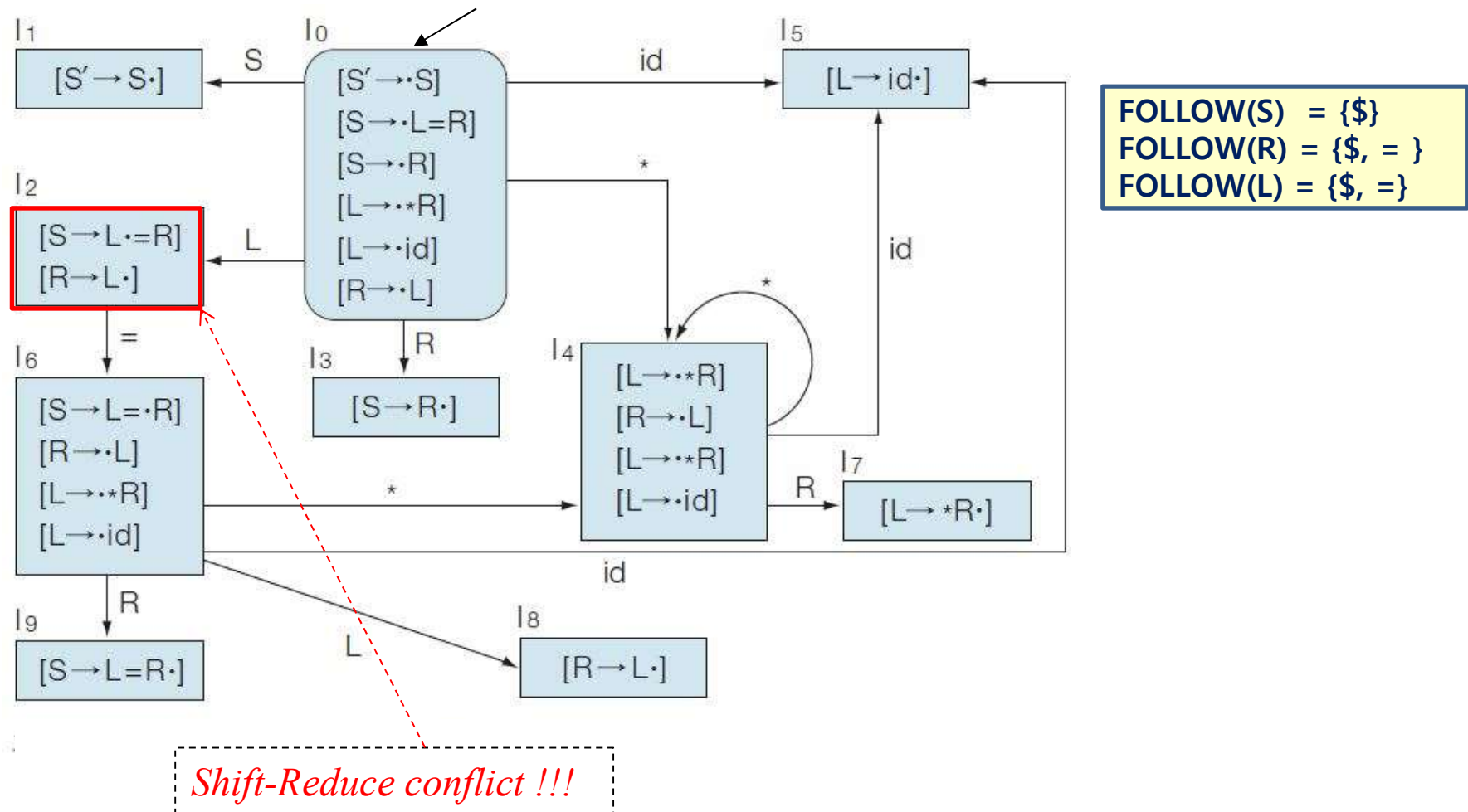
$S \rightarrow id \mid V := E$
 $V \rightarrow id$
 $E \rightarrow V \mid n$



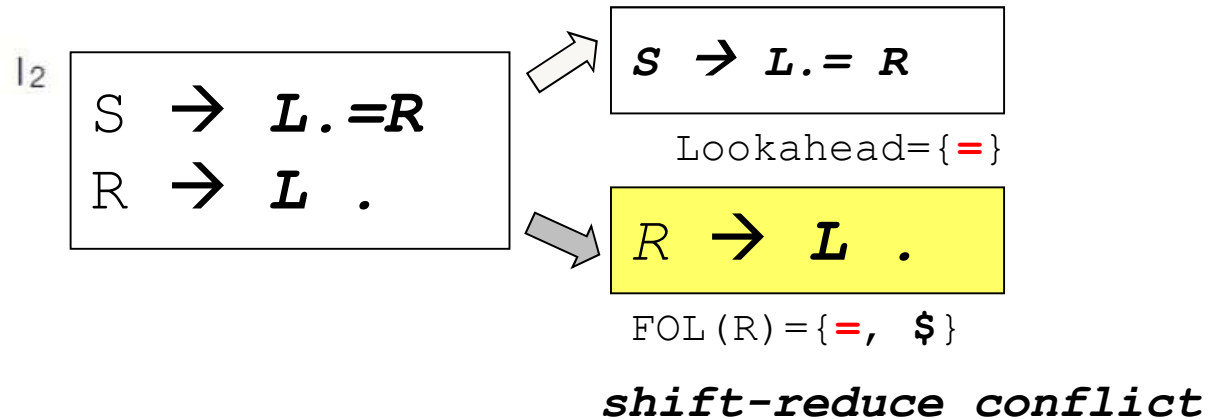
절대 올 수 없는
기호 때문에
parsing conflict
발생

예 14: Limits of SLR(1) Parsing Power (1/3)

- 문법 : 1. $S \rightarrow L = R$ 2. $S \rightarrow R$ 3. $L \rightarrow * R$ 4. $L \rightarrow id$ 5. $R \rightarrow L$
- 확장 문법 : 0. $S' \rightarrow S$ 1. $S \rightarrow L = R$ 2. $S \rightarrow R$ 3. $L \rightarrow * R$ 4. $L \rightarrow id$ 5. $R \rightarrow L$



예 14: Limits of SLR(1) Parsing Power (2/3)



$S \Rightarrow L=R \Rightarrow L=L \Rightarrow L=id \Rightarrow id=id$
 $S \Rightarrow R \Rightarrow L \Rightarrow *R = *L \Rightarrow *id$
 $S \Rightarrow R \Rightarrow L \Rightarrow id$

.....

R 다음에 =이 나타나는 *right sentential form*은 존재하지 않음.

예 14: Limits of SLR(1) Parsing Power (3/3)

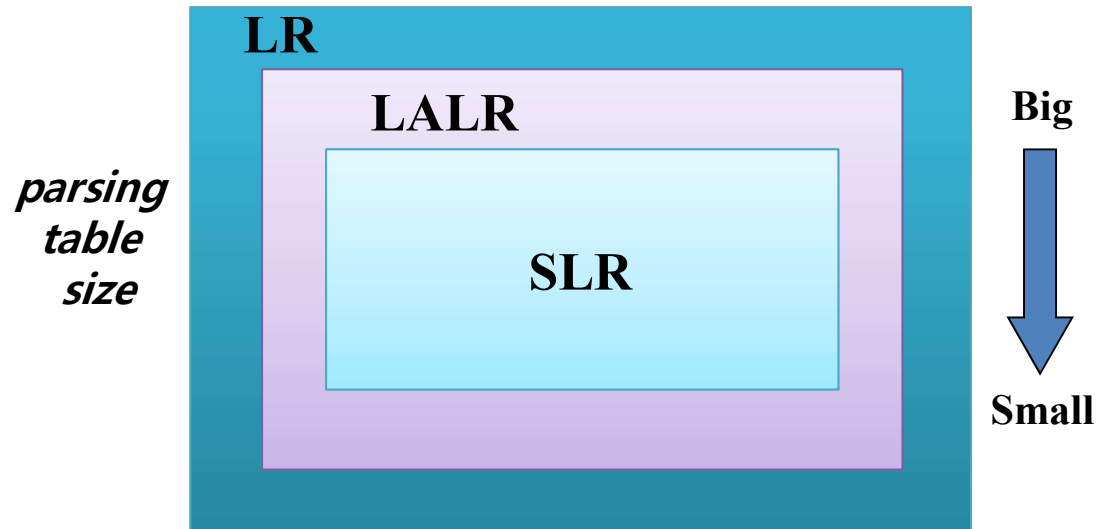
상태	구문 분석기 행동				GOTO 함수		
	=	*	id	\$	S	R	L
0		s4	s5		1	3	2
1				acc			
2	r5, s6			r5			
3				r2			
4		s4	s5			7	8
5	r4			r4			
6		s4	s5			9	8
7	r3			r3			
8	r5			r5			
9				r1			

0. $S' \rightarrow S$
1. $S \rightarrow L = R$
2. $S \rightarrow R$
3. $L \rightarrow * R$
4. $L \rightarrow id$
5. $R \rightarrow L$

FOLLOW(S) = {\$}
FOLLOW(R) = {\$, =}
FOLLOW(L) = {\$, =}

LR Parsing

- SLR(*Simple* LR)
- LALR (*Lookahead* LR)
 - *more general* (used in Yacc)
- CLR (*Canonical* LR) 또는 general LR



SLR(1) *versus* LR(1)

■ SLR(1)의 경우

- DFA를 구성하고 나서 lookahead를 적용
 - 즉, DFA를 구성할 때 lookahead를 무시

■ LR(1)의 경우

- DFA를 구성할 때부터 lookahead를 포함시킴
 - LR(0) item + *lookahead* = LR(1) item

■ 엉뚱한 기호 말고 진짜 나타날 기호는 무엇?

- 식당에서 full-course 요리를 시켰을 때
 - 전채(appetizer) 를 먹고 나면 다음 번엔 ?
 - 후식 ?

SLR(1) Parsing 단점

SLR(1) treats all occurrences of a RHS on stack as identical.
Only a few of these reductions may lead to a successful parse.

Example:

$$\begin{array}{lcl} S & \longrightarrow & AaAb \quad A \longrightarrow \epsilon \\ S & \longrightarrow & BbBa \quad B \longrightarrow \epsilon \end{array}$$

$$I_0 = \{[S' \rightarrow \bullet S], [S \rightarrow \bullet AaAb], [S \rightarrow \bullet BbBa], [A \rightarrow \bullet], [B \rightarrow \bullet]\}$$

Since $FOLLOW(A) = FOLLOW(B)$, we have reduce/reduce conflict in state 0.

LR(1) Item Sets

Construct **LR(1) items** of the form $[A \rightarrow \alpha \bullet \beta, a]$, which means:

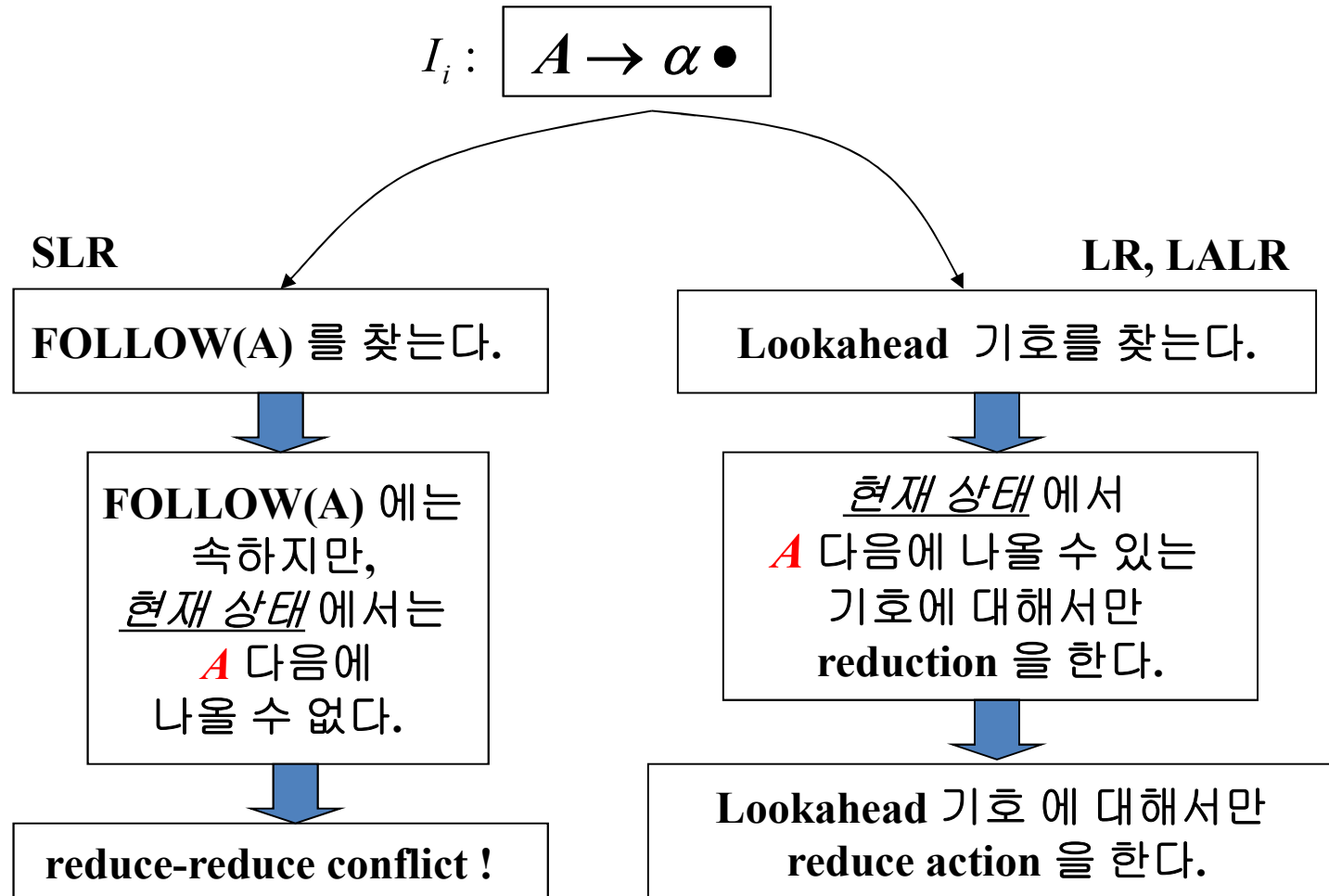
The item $A \rightarrow \alpha \bullet \beta$, can be chosen
when the next token on input stream is a .

$S \rightarrow AaAb$	$A \rightarrow \epsilon$
$S \rightarrow BbBa$	$B \rightarrow \epsilon$

An example LR(1) item set:

$$I_0 = \{[S' \rightarrow \bullet S, \$], [S \rightarrow \bullet AaAb, \$], [S \rightarrow \bullet BbBa, \$], \\ [A \rightarrow \bullet, a], [B \rightarrow \bullet, b]\}.$$

어떤 input symbol 에 대해 reduce 할 것인가?



Finite Automata of LR(1) Items(1/2)

■ LR(1) item

- $[A \rightarrow \alpha \bullet \beta, a]$ 로 표기
 - $A \rightarrow \alpha \bullet \beta$ 는 LR(0) item, $A \rightarrow \alpha\beta \in P$
 - a 는 lookahead (token), $a \in \{V_T \cup \$\}$

■ Definition of LR(1) transitions

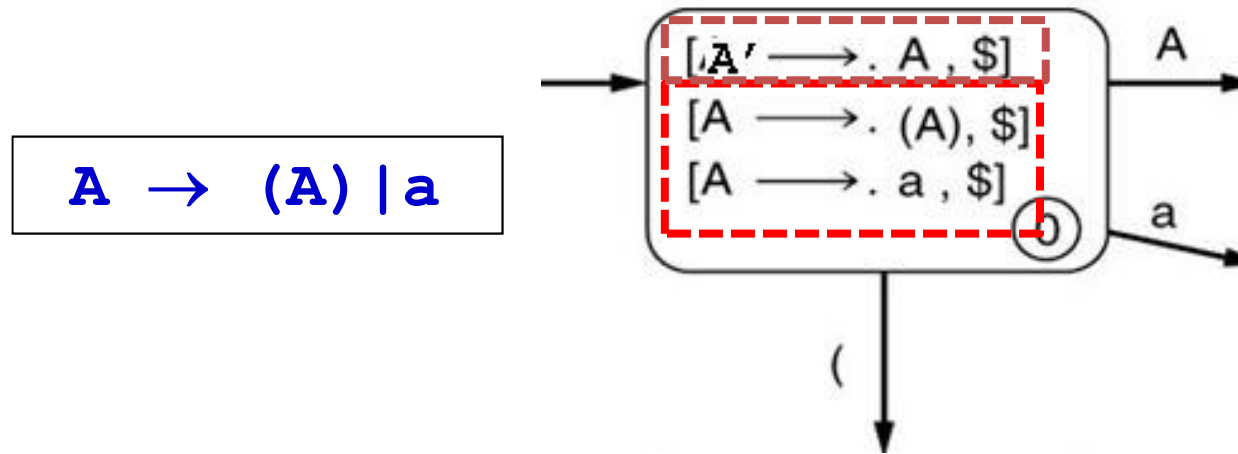
Start State: $[S' \rightarrow \bullet S, \$]$

■ Definition of LR(1) transitions

$$[A \rightarrow \alpha \bullet B \gamma \textcolor{red}{a}] \xrightarrow[\text{(closure item을 구할 때)}]{\text{\textcolor{blue}{\mathcal{E}}-transition}} [B \rightarrow \bullet \beta, \textcolor{red}{b}]$$

for every token $\textcolor{red}{b}$ in $\text{First}(\underline{\gamma a})$

예 14 (1/2)

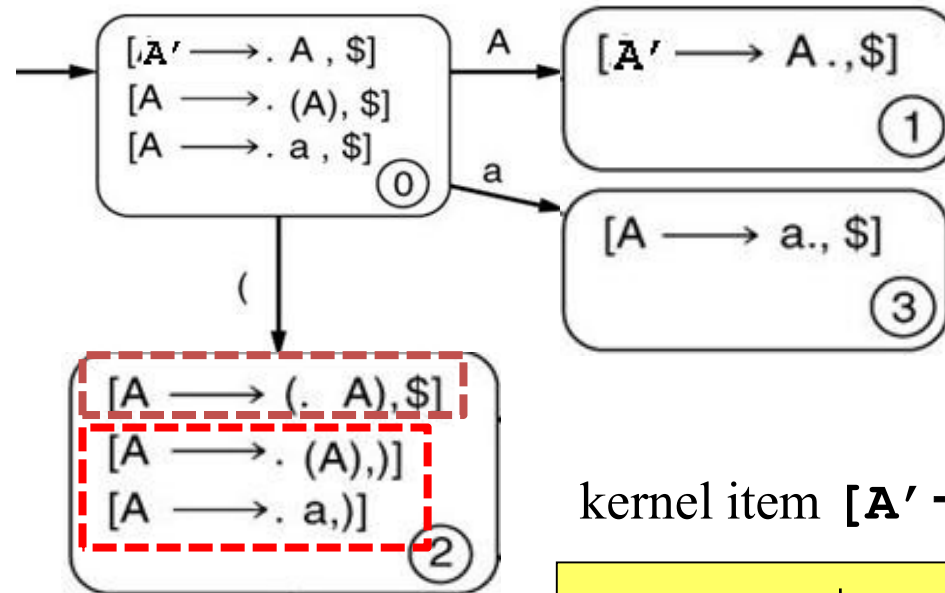


초기 상태(0)에서
kernel item $[A' \rightarrow \cdot A, \$]$ 의 closure item 구하기

$$\begin{array}{ccc}
 [A' \rightarrow \bullet A, \$] & \xrightarrow{\varepsilon} & [A \rightarrow \bullet (A), \$] \\
 B=A, \gamma=\varepsilon, a=\$ & & \text{First}(\gamma a) = \text{First}(\varepsilon \$) = \{\$ \}
 \end{array}$$

$$[A \rightarrow \alpha \bullet B \gamma, a] \xrightarrow{\varepsilon} [B \rightarrow \bullet \beta, \text{First}(\gamma a)]$$

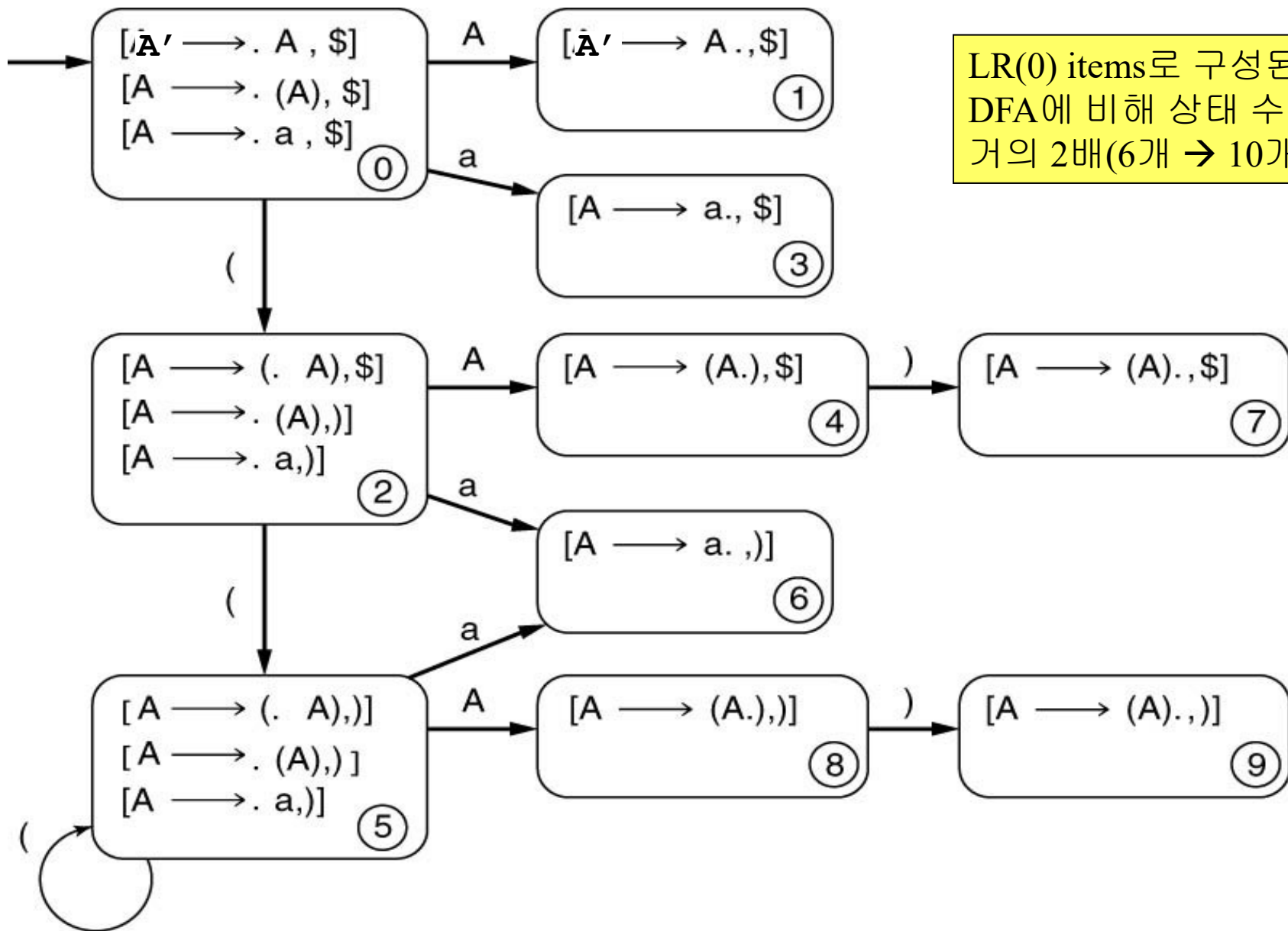
예 14 (2/2)



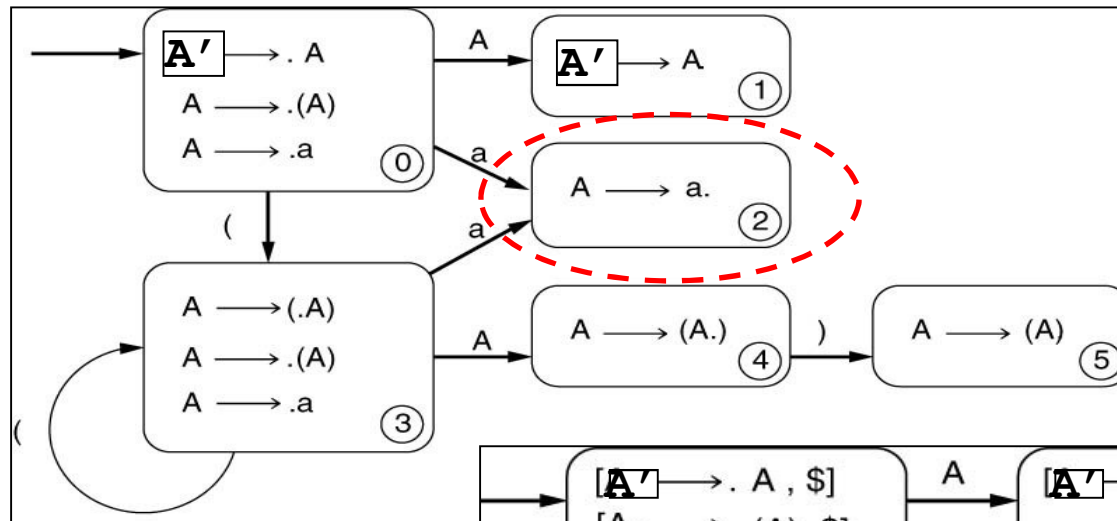
상태 2에서
kernel item $[A' \rightarrow (\cdot A), \$]$ 의 closure item 구하기

$$\begin{array}{lcl}
 [A \rightarrow (\bullet A), \$] & \xrightarrow{\varepsilon} & [A \rightarrow \bullet (A),)] \\
 B = A, \gamma =) , a = \$ & & \text{First}(\gamma a) = \text{First}()\$ = \{) \}
 \end{array}$$

$$[A \rightarrow \alpha \bullet B \gamma, a] \xrightarrow{\varepsilon} [B \rightarrow \bullet \beta, \text{First}(\gamma a)]$$

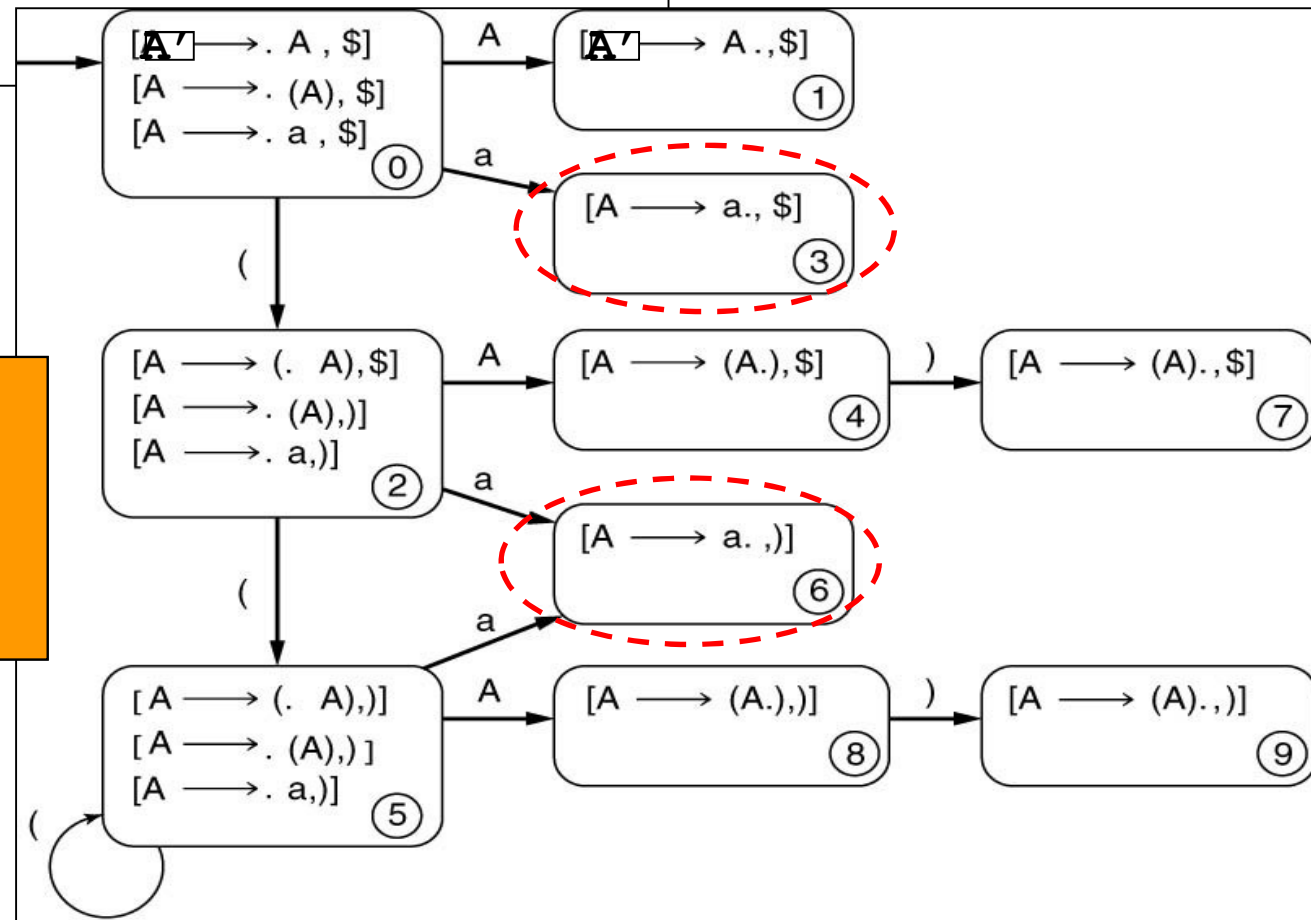


LR(0) items로 구성된
DFA에 비해 상태 수가
거의 2배(6개 → 10개).



DFA of sets of LR(0) items

DFA of sets of LR(1) items



예 15: CLR(1) parsing (1/4) – Canonical Collection of LR(1) items

■ 문법

1. $S \rightarrow CC$ 2. $C \rightarrow cC$ 3. $C \rightarrow d$

■ 확장 문법

0. $S' \rightarrow S$ 1. $S \rightarrow CC$ 2. $C \rightarrow cC$ 3. $C \rightarrow d$

■ Canonical Collection of LR(1) items

$I_0 = \text{CLOSURE}([S' \rightarrow \bullet S, \$])$

$= \{[S' \rightarrow \bullet S, \$], [S \rightarrow \bullet CC, \$], [C \rightarrow \bullet cC, c/d], [C \rightarrow \bullet d, c/d]\}$

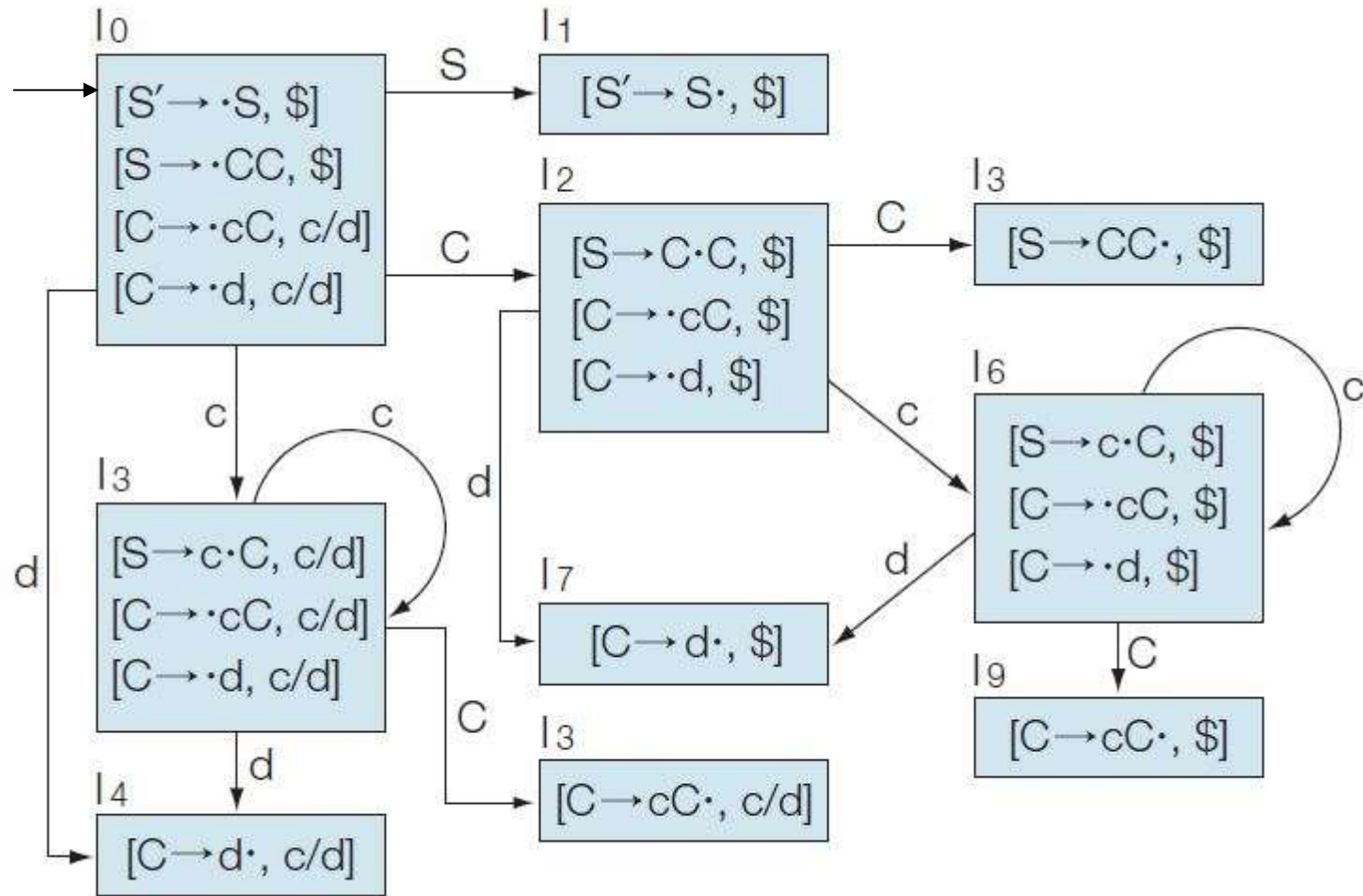
$\text{CLOSURE}(I_0, S) = I_1 = \{[S' \rightarrow S \bullet, \$]\}$

$\text{CLOSURE}(I_0, C) = I_2 = \{[S \rightarrow C \bullet C, \$], [C \rightarrow \bullet cC, \$], [C \rightarrow \bullet d, \$]\}$

$\text{CLOSURE}(I_0, c) = I_3 = \{[C \rightarrow c \bullet C, c/d], [C \rightarrow \bullet cC, c/d], [C \rightarrow \bullet d, c/d]\}$

$\text{CLOSURE}(I_0, d) = I_4 = \{[C \rightarrow d \bullet, c/d]\}$

예 15: CLR(1) parsing (2/4) - DFA of LR(1) items



예 15: CLR(1) parsing (3/4) – parsing table

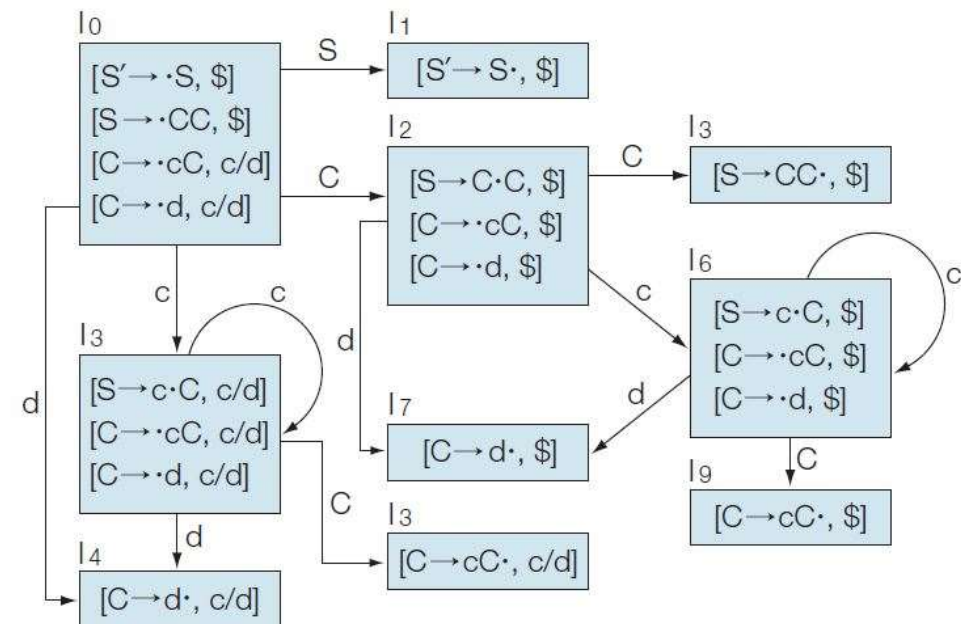
상태	구문 분석기 행동			GOTO 함수	
	c	d	\$	S	C
0	s3	s4		1	2
1			acc		
2	s6	s7			5
3	s3	s4			8
4	r3	r3			
5			r1		
6	s6	s7			9
7			r3		
8	r2	r2			
9			r2		

0. $S' \rightarrow S$

1. $S \rightarrow CC$

2. $C \rightarrow cC$

3. $C \rightarrow d$



예 15: CLR(1) parsing (4/4) – syntax analysis

단계	스택	입력 기호	구문 분석 내용
0	0	ccdd\$	이동 3
1	0c3	cdd\$	이동 3
2	0c3c3	dd\$	이동 4
3	0c3c3d4	d\$	감축 3
4	0c3c3C	d\$	GOTO 8
5	0c3c3C8	d\$	감축 2
6	0c3C	d\$	GOTO 8
7	0c3C8	d\$	감축 2
8	0C	d\$	GOTO 2
9	0C2	d\$	이동 7
10	0C2d7	\$	감축 3
11	0C2C	\$	GOTO 5
12	0C2C5	\$	감축 1
13	0S	\$	GOTO 1
14	0S1	\$	수락

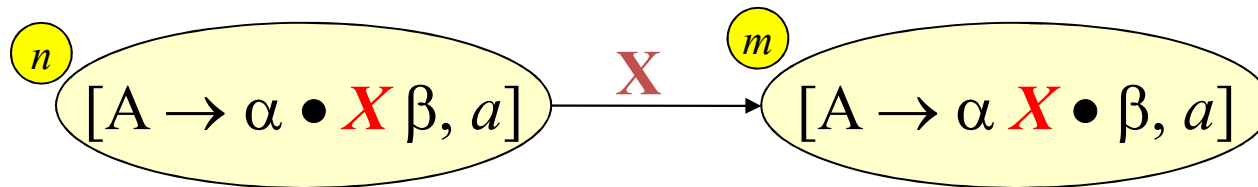
상태	구문 분석기 행동			GOTO 함수	
	c	d	\$	S	C
0	s3	s4		1	2
1			acc		
2	s6	s7			5
3	s3	s4			8
4	r3	r3			
5			r1		
6	s6	s7			9
7			r3		
8	r2	r2			
9			r2		

General LR(1) Parsing Algorithm (1/3)

■ 현재 상태 n 이

1. $[A \rightarrow \alpha \bullet X \beta, a]$ 형태의 item을 갖고 있는 경우

- X 가 *terminal*이고, 입력에서 읽어 올 *next token* 도 X 이면,
 - **shift action** : *input lookahead* 를 stack에 push
- 다음 상태 m 은 $[A \rightarrow \alpha X \bullet \beta, a]$ 의 item을 포함

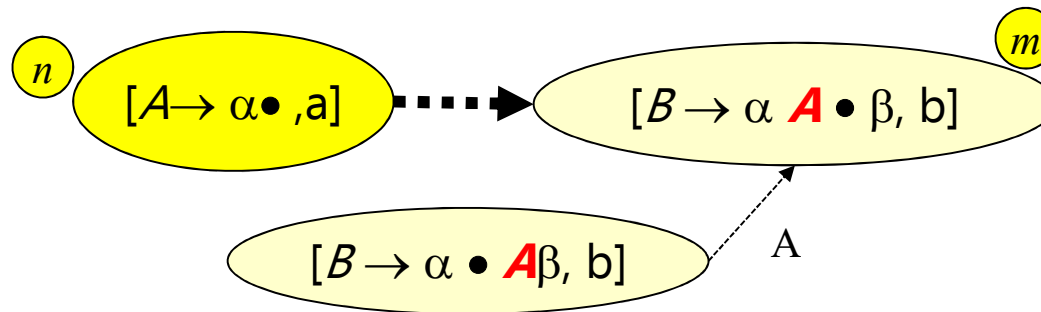


General LR(1) Parsing Algorithm (2/3)

■ 현재 상태 n 이

2. complete item $[A \rightarrow \alpha \bullet, a]$ 을 가지며, 입력에서 읽어 온 token이 a 이면,

- **reduce action:** reduction by $A \rightarrow \alpha \bullet$
 - Stack에서 기호(α) 와 상태 번호를 함께 제거(*pop*)
 - nonterminal A 를 stack에 push
- $B \rightarrow \alpha A \bullet \beta$ ($B \rightarrow \alpha \bullet A\beta$ 에서 reduction직후의 item) 를 포함하는
- 상태 번호 m 을 push



General LR(1) Parsing Algorithm (3/3)

- 현재 상태 n 이
- 2. complete item [$S' \rightarrow S \bullet, \$$] 을 가지며,
입력에서 읽어 온 기호가 $\$$ 이면 accept!

LR(1) Grammar

- A grammar is LR(1) if and only if the following two conditions are satisfied:

for any state s ,

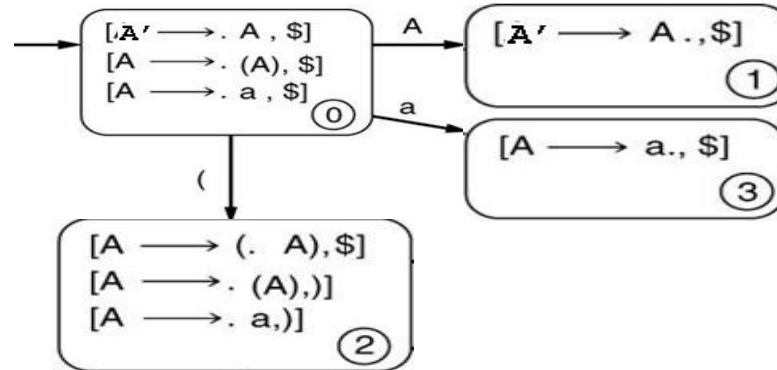
1. no shift-reduce conflict

- For any item $[A \rightarrow \alpha \bullet \textcolor{red}{X}\beta, a]$ in s with X a terminal,
 - there is no complete item $[B \rightarrow \gamma \bullet, \textcolor{red}{X}]$ in s .

2. no reduce-reduce conflict

- There are no two items in s of the form
 - $[A \rightarrow \alpha \bullet, \textcolor{red}{a}]$ and $[B \rightarrow \beta \bullet, \textcolor{red}{a}]$.

예 16: General LR(1) Parsing Table

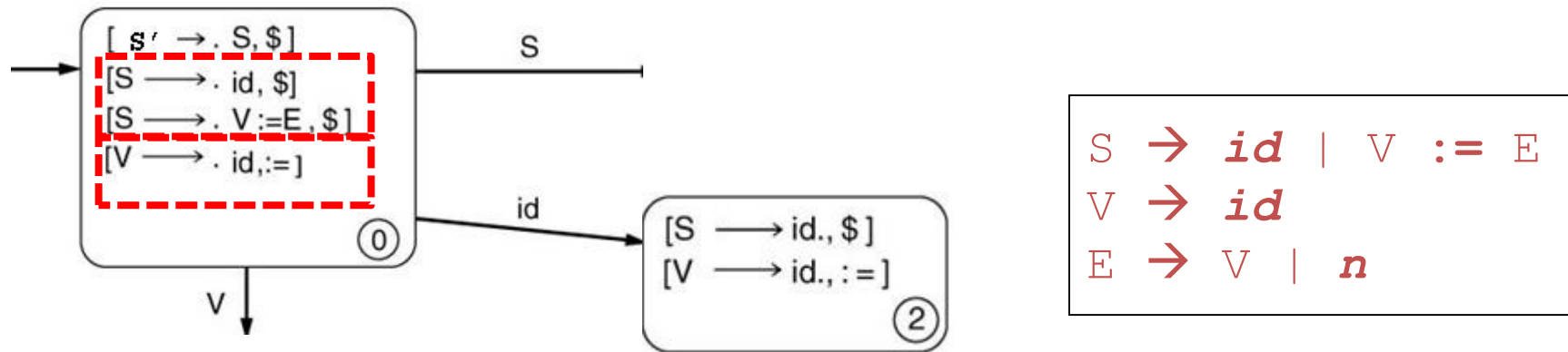


State	Input				Goto
	(a)	\$	A
0	s2	s3			1
1				accept	
2	s5	s6			4
3				r2	

(1) $A \rightarrow (A)$

(2) $A \rightarrow a$

예 17 (1/2)



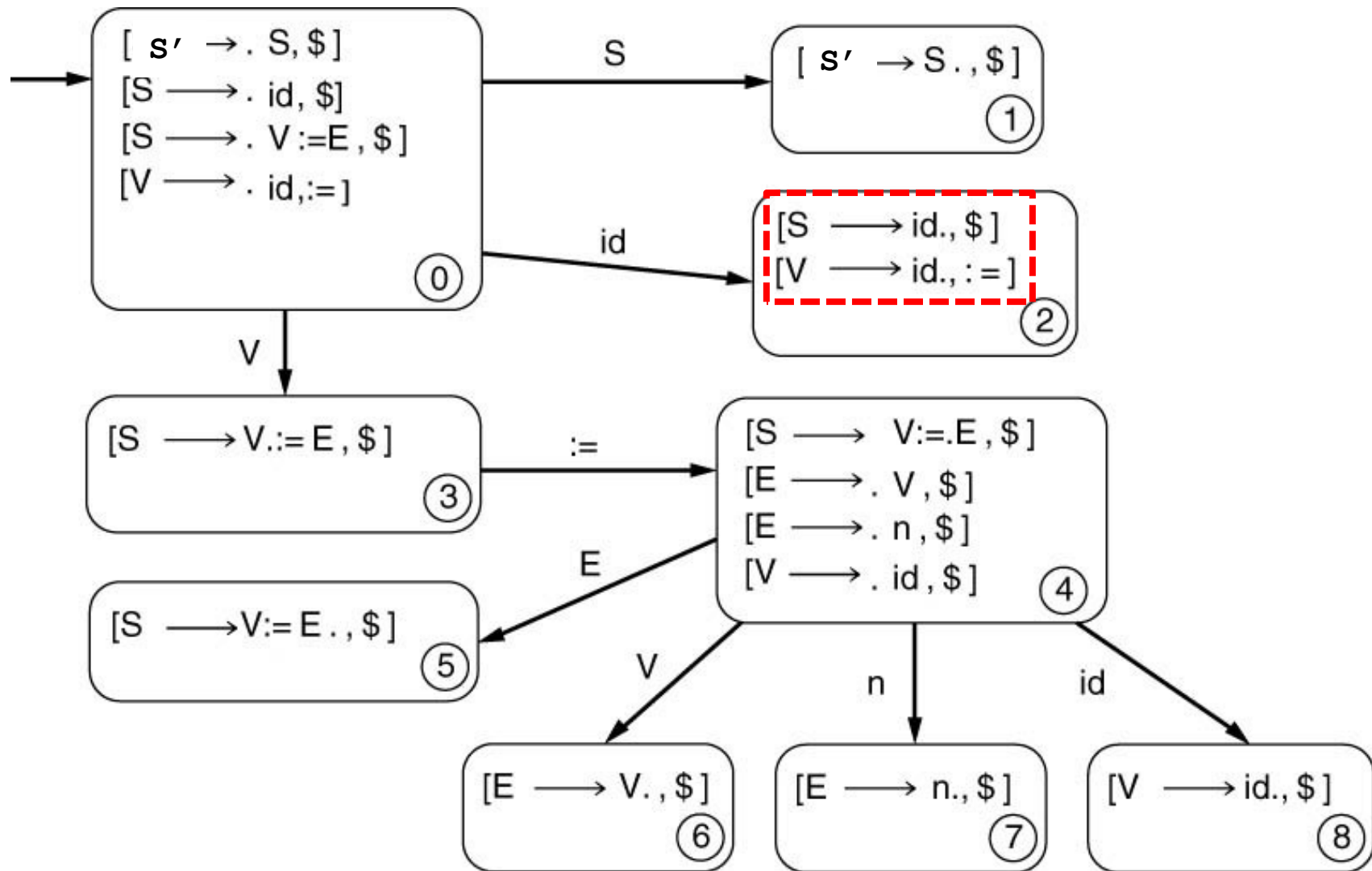
초기 상태(0)에서
kernel item $[S' \rightarrow \bullet S, \$]$ 의 Closure item 구하기

$$[S \rightarrow \bullet V := E, \$] \xrightarrow{\varepsilon} [V \rightarrow \bullet id, :=]$$

$\alpha = \varepsilon, B = V, \gamma = :=$ $\text{First}(\gamma a) = \text{First}(:= \$) = \{:=\}$

$$[A \rightarrow \alpha \bullet B \gamma, a] \xrightarrow{\varepsilon} [B \rightarrow \bullet \beta, \text{First}(\gamma a)]$$

예 17 (2/2)



예 18: CLR(1) parsing (1/4)

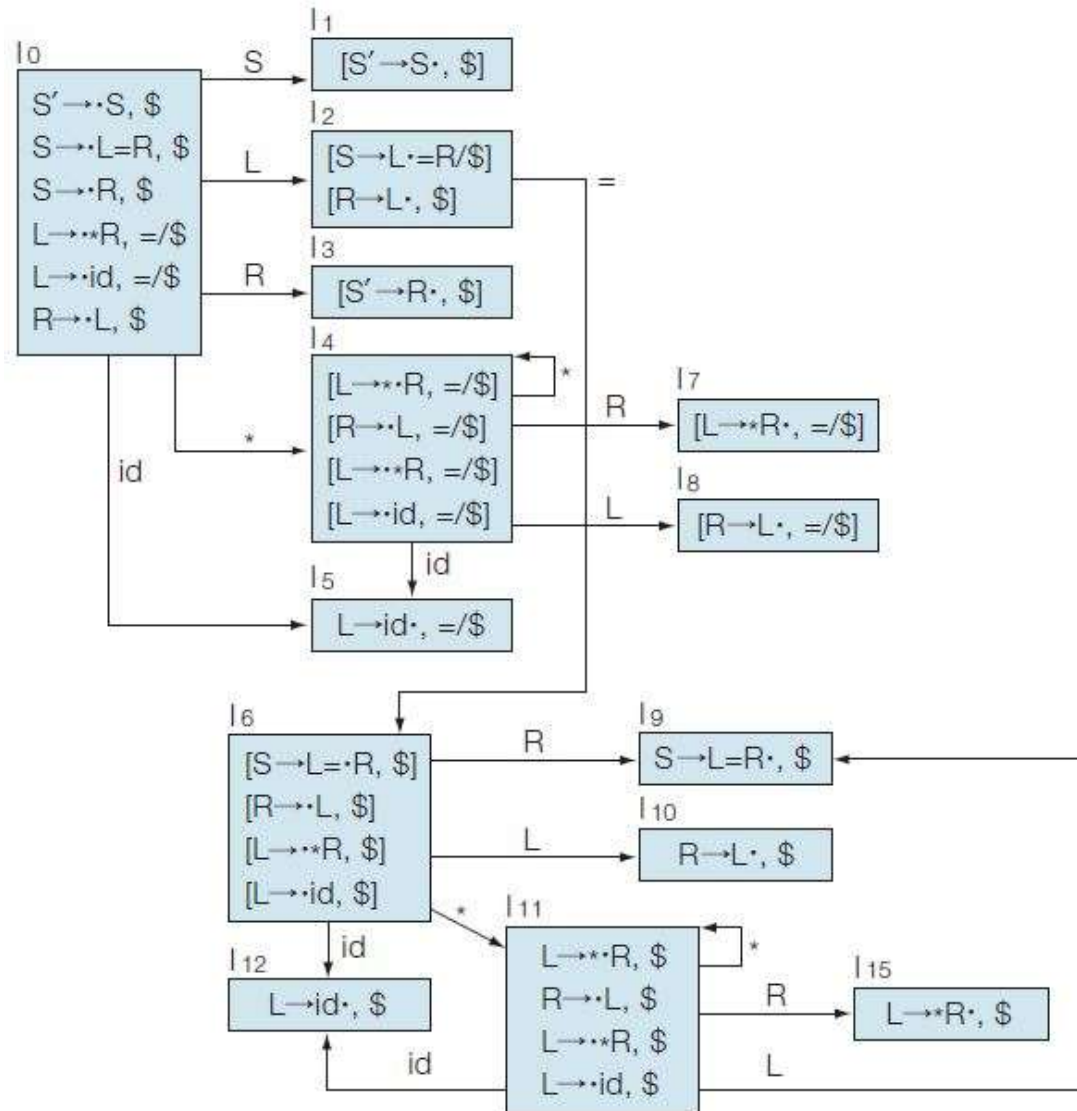
■ 문법

1. $S \rightarrow L = R$ 2. $S \rightarrow R$ 3. $L \rightarrow * R$ 4. $L \rightarrow \text{id}$ 5. $R \rightarrow L$

■ Augmented grammar

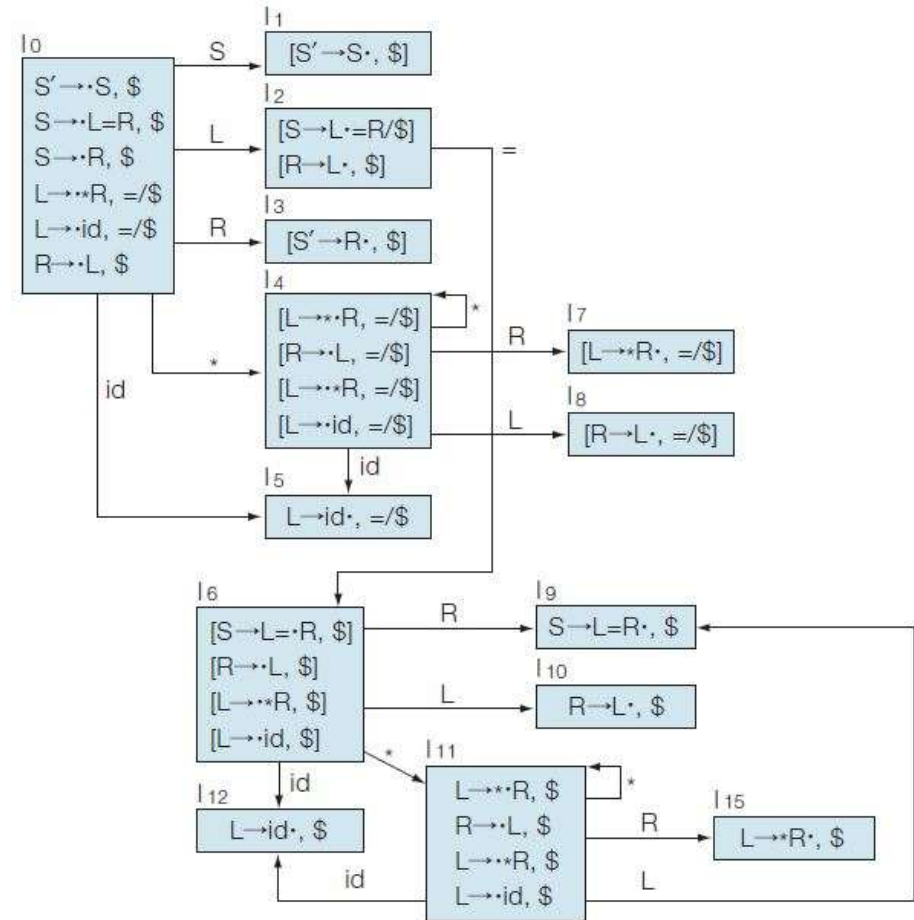
0. $S' \rightarrow S$ 1. $S \rightarrow L = R$ 2. $S \rightarrow R$ 3. $L \rightarrow * R$ 4. $L \rightarrow \text{id}$ 5. $R \rightarrow L$

예 18: CLR(1) parsing (2/4) – DFA of LR(1) items



예 18: CLR(1) parsing (3/4) – parsing table

상태	구문 분석기 행동				GOTO 함수		
	=	*	id	\$	S	R	L
0		s4	s5		1	3	2
1				acc			
2	s6			r5			
3				r2			
4		s4	s5			7	8
5	r4						
6		s11	s12			9	10
7	r3			r3			
8	r5			r3			
9				r1			
10				r5			
11		s11	s12			13	10
12				r4			
13				r3			

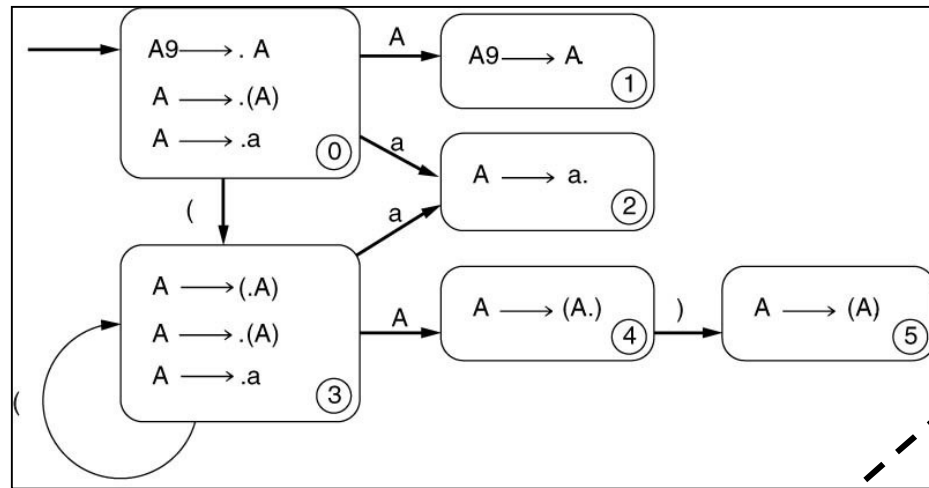


예 18: CLR(1) parsing (4/4) – syntax analysis

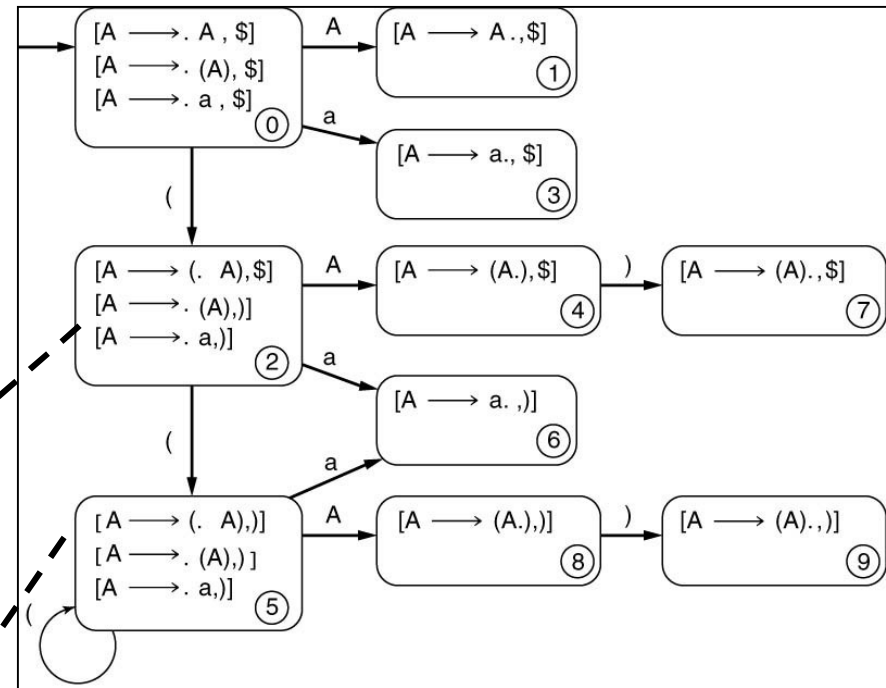
단계	스택	입력 기호	구문 분석 내용
0	0	* id = id\$	이동 4
1	0 * 4	id = id\$	이동 5
2	0 * 4id5	= id\$	감축 4
3	0 * 4L	= id\$	GOTO 8
4	0 * 4L8	= id\$	감축 5
5	0 * 4R	= id\$	GOTO 7
6	0 * 4R7	= id\$	감축 3
7	0L	= id\$	GOTO 2
8	0L2	= id\$	이동 6
9	0L2 = 6	id\$	이동 12
10	0L2 = 6id12	\$	감축 4
11	0L2 = 6L	\$	GOTO 10
12	0L2 = 6L10	\$	감축 5
13	0L2 = 6R	\$	GOTO 9
14	0L2 = 6R9	\$	감축 1
15	0S	\$	GOTO 1
16	0S1	\$	수락

상태	구문 분석기 행동				GOTO 함수		
	=	*	id	\$	S	R	L
0		s4	s5		1	3	2
1				acc			
2	s6			r5			
3				r2			
4		s4	s5			7	8
5	r4						
6		s11	s12			9	10
7	r3			r3			
8	r5			r3			
9				r1			
10				r5			
11		s11	s12			13	10
12				r4			
13				r3			

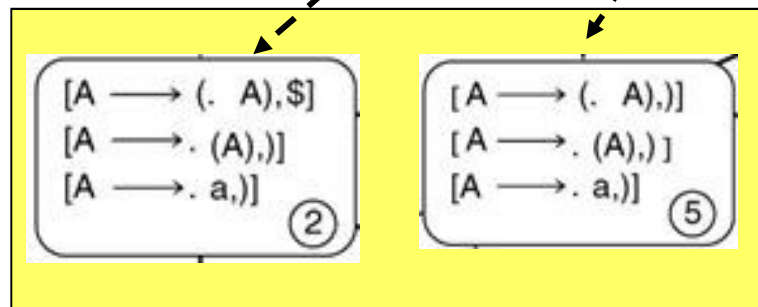
What's the problem?



DFA of LR(0) items



DFA of LR(1) items



What's the difference?

LALR(1) Parsing

■ LR(1) has so many states!

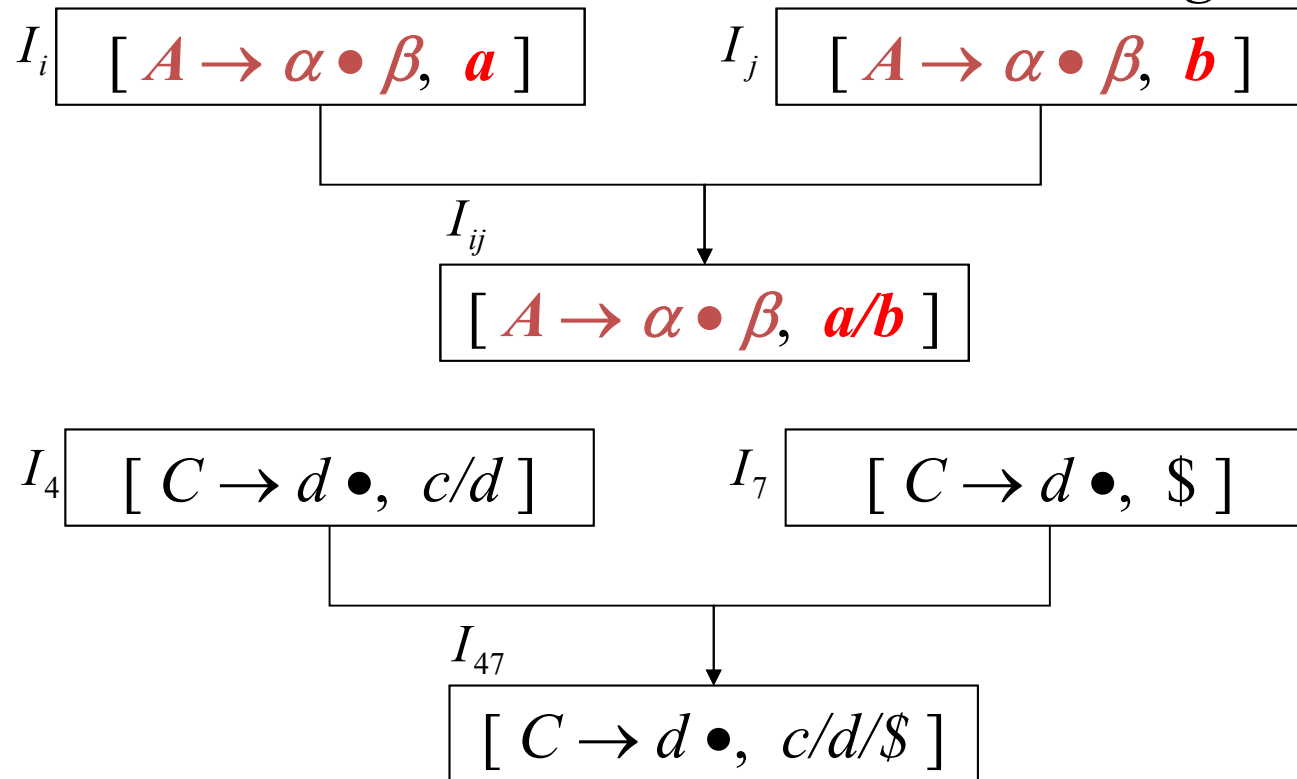
- LR(0) item은 같지만, *lookahead* 만 다른 상태들이 많음
 - 이 상태들을 모두 합치면 LR(0) item으로 이루어진 DFA로 축약됨.
- LR(0) item으로 이루어진 DFA와 LR(1) item 으로 이루어진 DFA의 차이점은
 - lookahead 포함 여부

■ 상태 수를 줄이기 위해

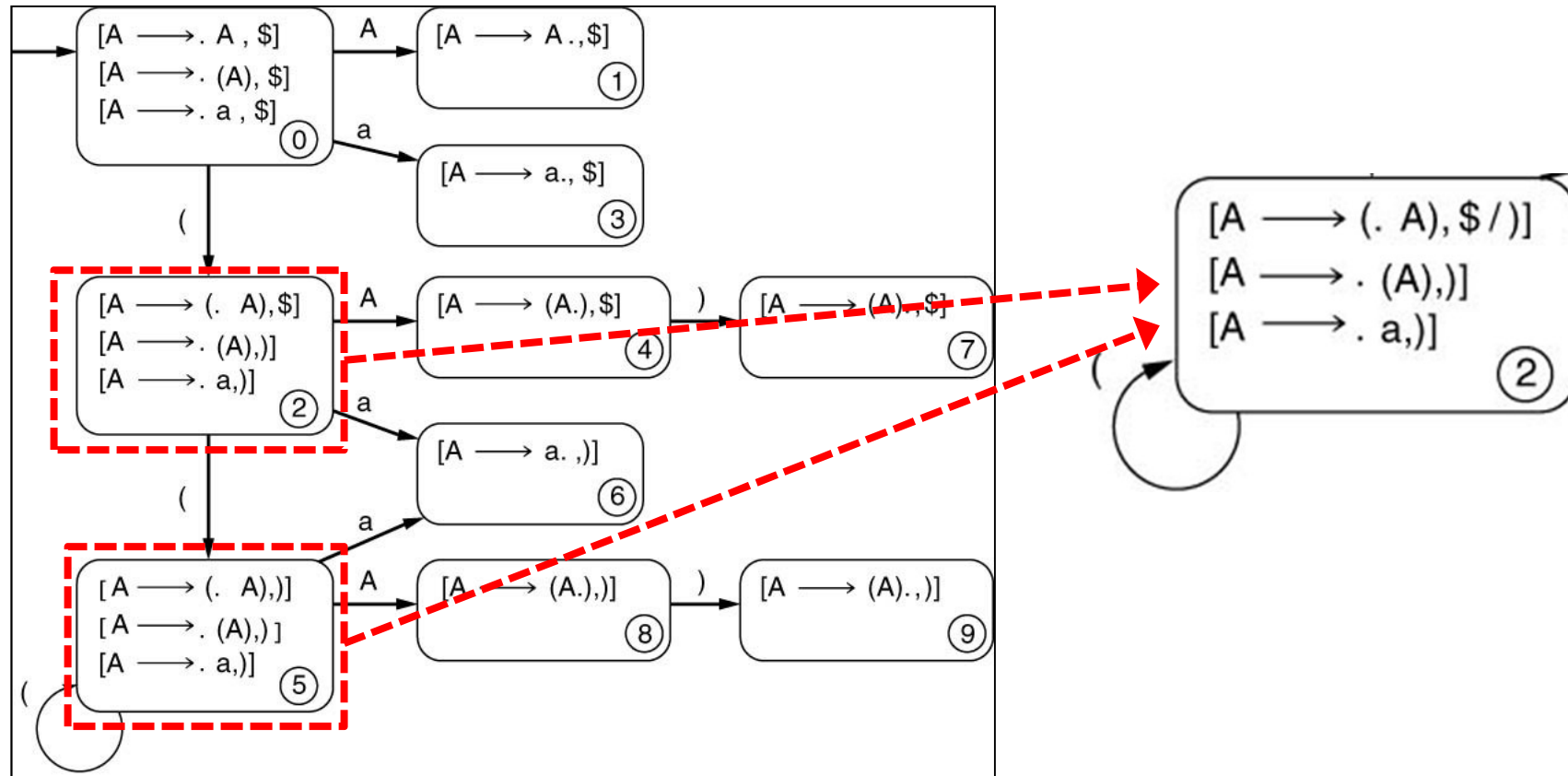
- core item이 같은 상태들을 merge 시킴
 - lookahead 가 달라 따로 취급했던 상태들을 하나의 상태로 합침
- DFA of LALR(1) items

LALR(1) Parsing

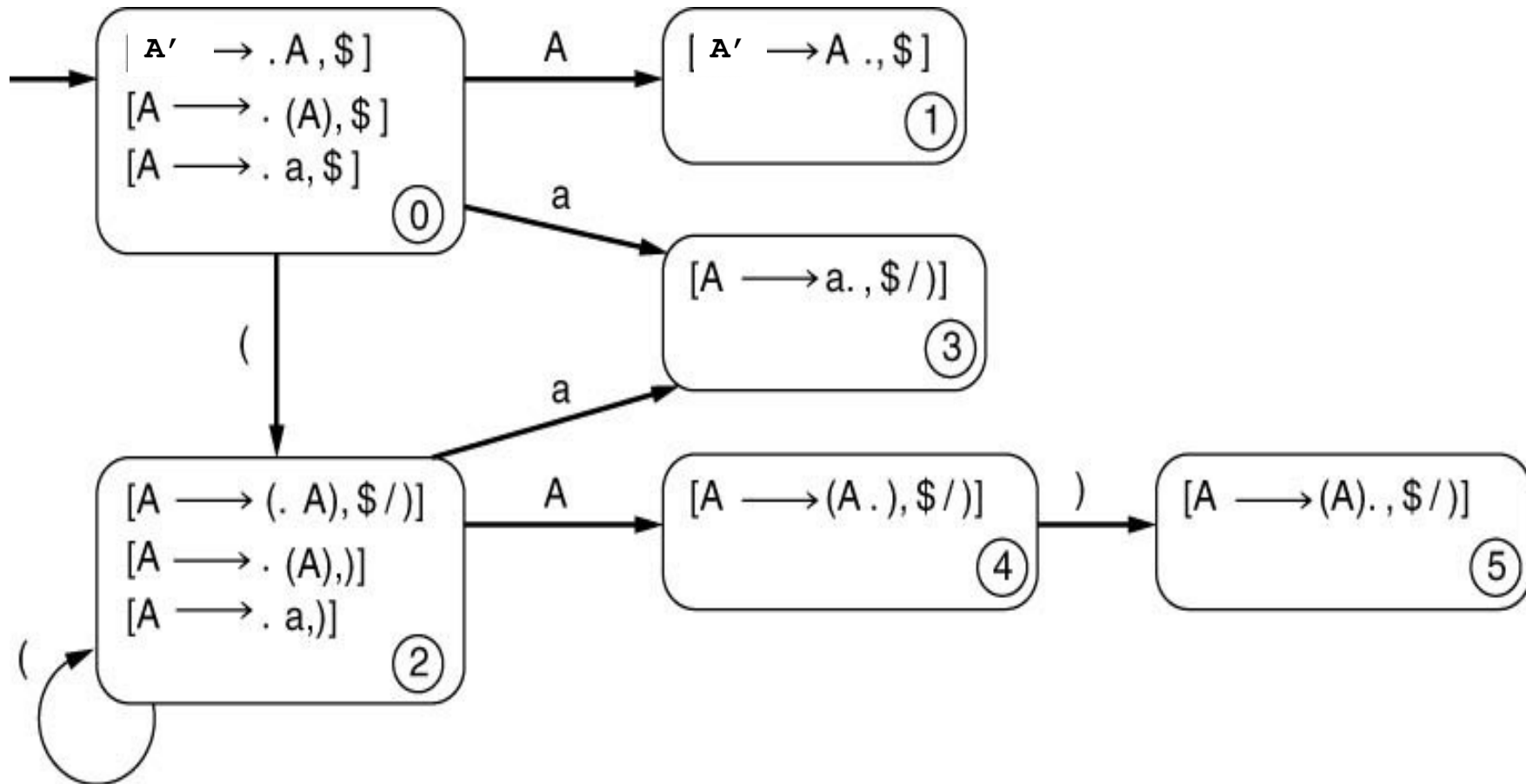
같은 *core* 를 갖고 있는 서로 다른 상태를 *merge* 시킴.



Identifying states 2 and 4 gives the state of LALR(1) items



예 19: DFA of LALR(1) items



예 20: CLR table을 LALR table로 바꾸기 (1/2)

표 6-5 CLR 파싱표

상태	구문 분석기 행동			GOTO 함수	
	c	d	\$	S	C
0	s3	s4		1	2
1			acc		
2	s6	s7			5
3	s3	s4			8
4	r3	r3			
5			r1		
6	s6	s7			9
7			r3		
8	r2	r2			
9			r2		

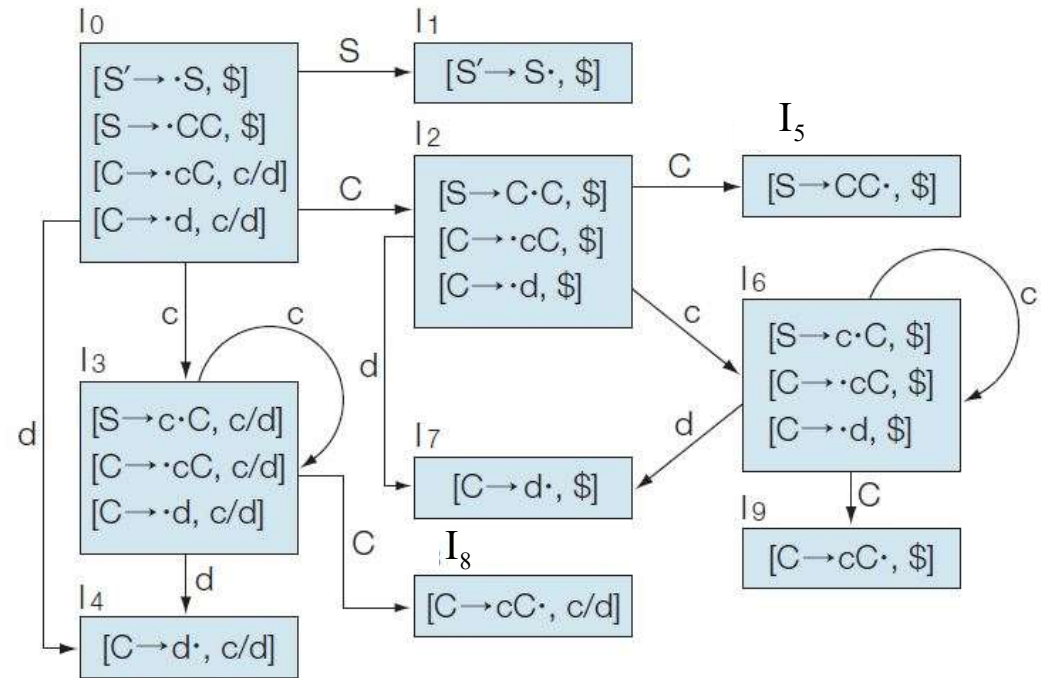


그림 6-7 CLR 정규 항목 집합과 GOTO 그래프

$$I_3 \cup I_6 \Rightarrow I_{36} = \{[C \rightarrow c \cdot C, c/d/\$], [C \rightarrow \cdot cC, c/d/\$], [C \rightarrow \cdot d, c/d/\$]\}$$

$$I_4 \cup I_7 \Rightarrow I_{47} = \{[C \rightarrow d \cdot, c/d/\$]\}$$

$$I_8 \cup I_9 \Rightarrow I_{89} = \{[C \rightarrow cC \cdot, c/d/\$]\}$$

예 20: CLR table을 LALR table로 바꾸기 (2/2)

표 6-5 CLR 파싱표

상태	구문 분석기 행동			GOTO 함수	
	c	d	\$	S	C
0	s3	s4		1	2
1			acc		
2	s6	s7			5
3	s3	s4			8
4	r3	r3			
5			r1		
6	s6	s7			9
7			r3		
8	r2	r2			
9			r2		

표 6-8 LALR 파싱표

상태	구문 분석기 행동			GOTO 함수	
	c	d	\$	S	C
0	s36	s47		1	2
1			acc		
2	s36	s47			5
36	s36	s47			89
47	r3	r3	r3		
5			r1		
89	r2	r2	r2		

LR(1) and LALR(1) Parsing

LR(1) parsing: Parse tables built using LR(1) item sets.

LALR(1) parsing: Look Ahead LR(1)

Merge LR(1) item sets; then build parsing table.

Typically, LALR(1) parsing tables are much smaller than LR(1) parsing table.

$SLR(1) \subset LALR(1) \subset LR(1)$.

$LL(1) \not\subset SLR(1)$, but $LL(1) \subset LR(1)$.

Construct the DFA of LR(0) items for the following grammar

$$P \rightarrow b D ; S e$$
$$D \rightarrow d ; D \mid d$$
$$S \rightarrow s ; S \mid s$$