

CONCEPTS OF  
PROGRAMMING LANGUAGES

# Chapter 15

## Functional Programming Languages



ROBERT W. SEBESTA

12/E

ISBN 0-321-49362-1

# Chapter 15 Topics

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- Introduction
- Mathematical Functions
- Fundamentals of Functional Programming Languages
- The First Functional Programming Language: LISP
- Introduction to Scheme
- Common LISP
- ML
- Haskell
- F#
- Support for Functional Programming in Primarily Imperative Languages
- Comparison of Functional and Imperative Languages

# 서론

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- 명령형 언어는 *von Neumann* 구조를 기반으로 설계됨
  - 소프트웨어 개발에 적합한 언어 관정보다는 효율성이 주 관심
- 함수형 언어는 수학 함수에 기반하여 설계

# 수학 함수

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- 수학함수는 정의역 집합의 원소를 치역 집합으로의 사상(mapping)이다.
- 함수정의는 함수명, 매개변수리스트, 사상표현식으로 구성  
(예) 함수  $\text{cube}(x) \equiv x * x * x$ , 단  $x$ 는 실수
- 특성
  - 사상표현식의 제어는 재귀나 조건식
  - 부작용(side effect) 없음
  - 함수의 상태라는 개념이 없다.

# 람다 표현식(Lambda expressions)

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- 람다 표현식은 이름이 없는 함수를 기술하는 방법

(예)  $\lambda(x) \ x * x * x$

- 람다 표현식의 적용

-표현식 이후에 매개변수 값을 위치시켜 적용

(예)  $(\lambda(x) \ x * x * x) (2)$

➔ 평가결과는 8

# 합성 함수(function composition)

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- 두 개의 함수 매개변수를 취하여 두 번째 실 매개변수 함수의 결과에 첫 번째 실 매개변수 함수를 적용한 결과를 값으로 갖는 함수를 산출

**Form:**  $h \equiv f \circ g$

**which means**  $h(x) \equiv f(g(x))$

**(예) For**  $f(x) \equiv x + 2$  **and**  $g(x) \equiv 3 * x$ ,

$h \equiv f \circ g$  **yields**  $(3 * x) + 2$

# 모두-적용(Apply-to-all) 함수

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- 매개변수로서 단일 함수를 취하는 범함수 형태;
- 인자 리스트에 적용했다면, 모두-적용 함수는 인자 리스트에 있는 각 값에 범함수 매개변수를 적용하여 결과를 리스트나 수열로 수집

Form:  $\alpha$

For  $h(x) \equiv x * x$

$\alpha(h, (2, 3, 4))$  yields (4, 9, 16)

# 함수형 프로그래밍 언어의 원리

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- **FPL**의 설계 목적은 가능한 최대한로 수학 함수를 모방하자는 것임
- **FPL**에서 계산의 기본 처리과정은 명령형 언어에서 사용한 방법과 근본적으로 다르다.
  - 명령형 언어에서는, 연산이 수행되고 결과를 변수에 저장하여 나중에 사용하도록 함
  - 변수의 관리는 지속적인 관심의 대상이며 명령형 언어의 복잡성의 원인이 된다.
- **FPL**에서는, 수학 함수처럼 변수가 필수적이 아님
- 참조 투명성(*Referential Transparency*) – **FPL**에서, 함수의 평가는 동일한 매개변수가 주어졌을 때 항상 동일한 결과를 내준다.



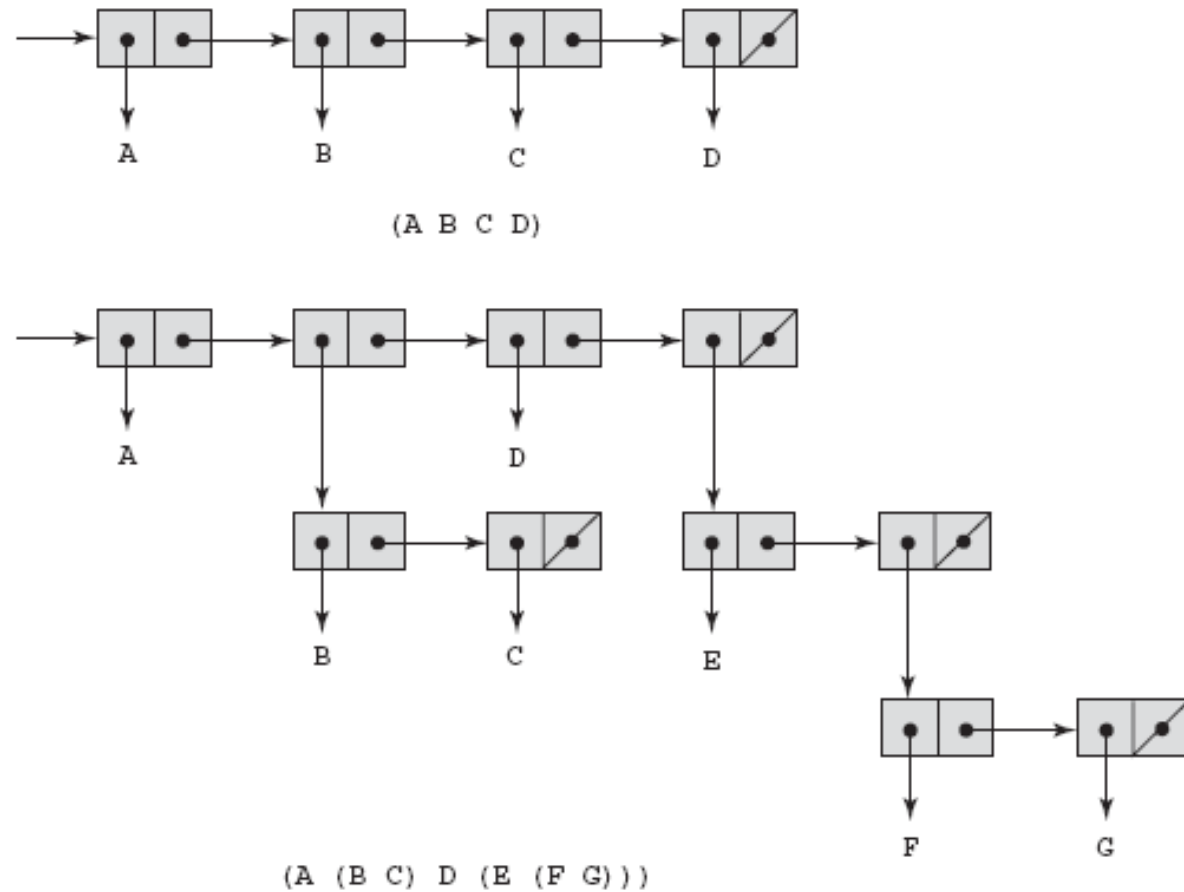
# LISP 데이터 타입과 구조

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- *데이터 객체 타입*: 원자, 리스트
- *리스트의 형식*: (서브리스트 and/or 원자)  
e.g., (A B (C D) E)
- 원래 LISP는 타입이 없는 언어였음
- 리스트는 내부적으로 단일 연결 리스트로 저장
- 리스트 원소들은 쌍으로 구성
  - 첫 부분: 원소의 데이터; 원자나 중첩 리스트에 대한 포인터
  - 두 번째 부분: 원자에 대한 포인터; 다른 원자에 대한 포인터 또는 널리스트

**Figure 15.1**

Internal representation  
of two LISP lists



# Lisp Interpretation

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- Lambda notation is used to specify functions and function definitions. Function applications and data have the same form.

e.g., If the list (A B C) is interpreted as data it is a simple list of three atoms, A, B, and C

If it is interpreted as a function application, it means that the function named A is applied to the two parameters, B and C

# Scheme의 기원

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- 1970년대 중반, LISP의 파생어; cleaner, more modern, and simpler version than the contemporary dialects of LISP
- 정적 영역 규칙 사용
- 함수는 일등급 개체로 취급
  - 표현식의 값, 리스트의 원소
  - 변수로 대입, 매개변수로 전달, 함수로부터 반환

# Scheme 인터프리터

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- In interactive mode, the Scheme interpreter is an infinite read-evaluate-print loop (REPL)
  - This form of interpreter is also used by Python and Ruby
- Expressions are interpreted by the function `EVAL`
- Literals evaluate to themselves

# Primitive Function Evaluation

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- Parameters are evaluated, in no particular order
- The values of the parameters are substituted into the function body
- The function body is evaluated
- The value of the last expression in the body is the value of the function

# Primitive Functions & LAMBDA Expressions

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- Primitive Arithmetic Functions : **+**, **-**, **\***, **/**, **ABS**, **SQRT**, **REMAINDER**, **MIN**, **MAX**  
e.g., `(+ 5 2)` yields 7
- Lambda Expressions
  - Form is based on  $\lambda$  notation  
e.g., `(LAMBDA (x) (* x x))`  
x is called a bound variable
- Lambda expressions can be applied to parameters  
e.g., `((LAMBDA (x) (* x x)) 7)`
- LAMBDA expressions can have any number of parameters  
`(LAMBDA (a b x) (+ (* a x x) (* b x)))`

# 특수형 함수: DEFINE

---

- DEFINE – Two forms:

1. To bind a symbol to an expression

e.g., (DEFINE pi 3.141593)

Example use: (DEFINE two\_pi (\* 2 pi))

These symbols are not variables – they are like the names bound by Java's `final` declarations

2. To bind names to lambda expressions (LAMBDA is implicit)

e.g., (DEFINE (square x) (\* x x))

Example use: (square 5)

- The evaluation process for `DEFINE` is different! The first parameter is never evaluated. The second parameter is evaluated and bound to the first parameter.



# Output Functions

---

- `(DISPLAY expression)`
- `(NEWLINE)`

# Output Functions

---

- Usually not needed, because the interpreter always displays the result of a function evaluated at the top level (not nested)
- Scheme has `PRINTF`, which is similar to the `printf` function of C
- Note: explicit input and output are not part of the pure functional programming model, because input operations change the state of the program and output operations are side effects

# 수치 술어 함수

---

- 술어 함수는 불리안 값을 반환하는 함수
- #T (or #t) is true and #F (or #f) is false (sometimes () is used for false)
- =, <>, >, <, >=, <=
- EVEN?, ODD?, ZERO?, NEGATIVE?
- The NOT function inverts the logic of a Boolean expression

# 제어 흐름

---

- 이중선택 구조 - IF

(IF predicate then\_exp else\_exp)

**(Ex)** (IF (<> count 0)  
          (/ sum count)  
          )

- 다중선택 구조 - COND function (8장에서 다룸)

**(Ex)** DEFINE (leap? year)  
          (COND  
            ((ZERO? (MODULO year 400)) #T)  
            ((ZERO? (MODULO year 100)) #F)  
            (ELSE (ZERO? (MODULO year 4))))  
          ))

# 리스트 함수

---

- `QUOTE` – takes one parameter; returns the parameter without evaluation
  - `QUOTE` is required because the Scheme interpreter, named `EVAL`, always evaluates parameters to function applications before applying the function. `QUOTE` is used to avoid parameter evaluation when it is not appropriate
  - `QUOTE` can be abbreviated with the apostrophe prefix operator
    - ' (A B) is equivalent to (`QUOTE` (A B))
- Recall that `CAR`, `CDR`, and `CONS` were covered in Chapter 6

# 리스트 함수(continued)

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- **Examples:**

(CAR ' ( (A B) C D) ) **returns** (A B)

(CAR 'A) **is an error**

(CDR ' ( (A B) C D) ) **returns** (C D)

(CDR 'A) **is an error**

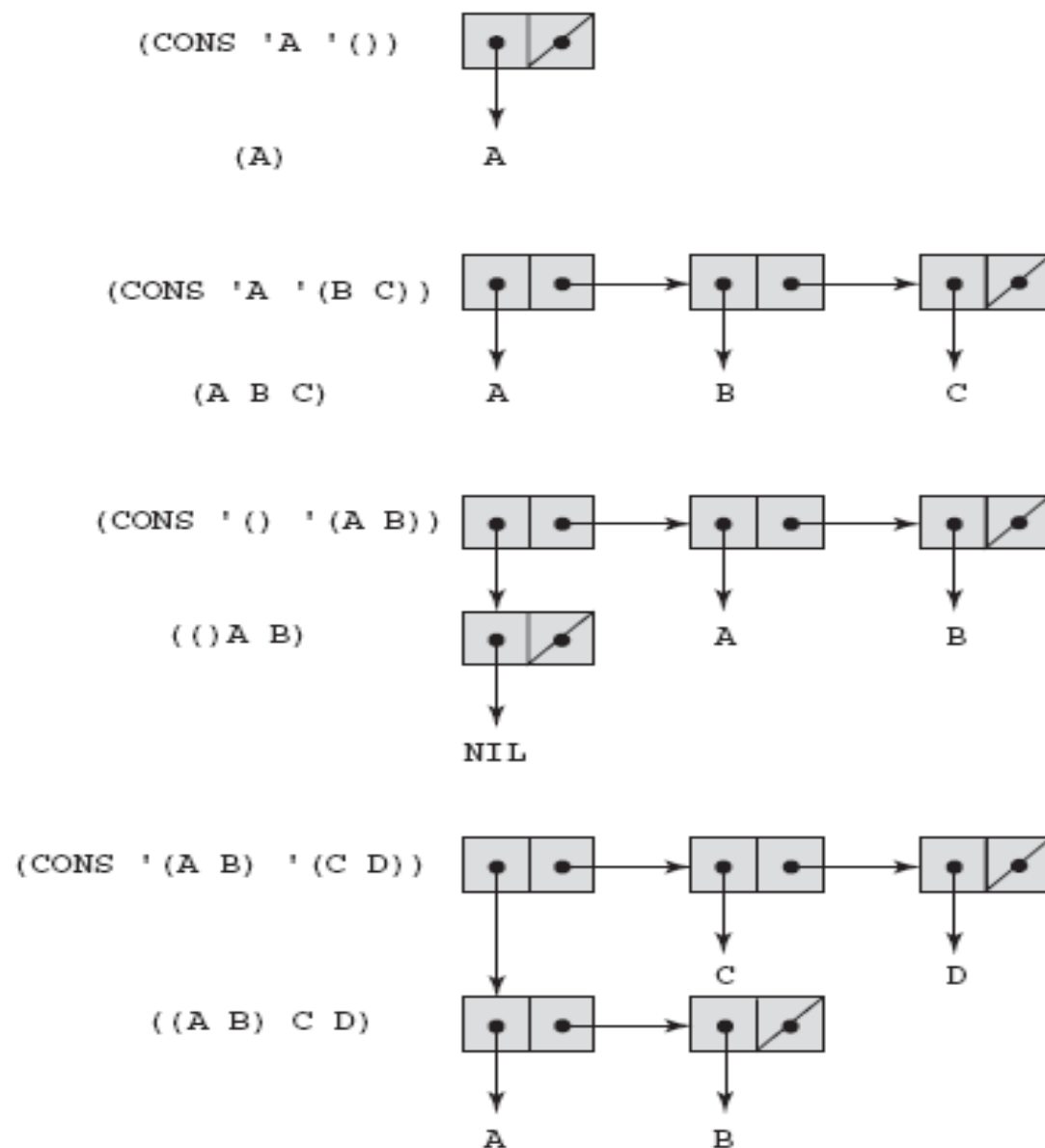
(CDR ' (A) ) **returns** ( )

(CONS ' ( ) ' (A B) ) **returns** ( ( ) A B)

(CONS ' (A B) ' (C D) ) **returns** ( (A B) C D)

**Figure 15.2**

The result of several  
CONS operations



## 리스트 함수 (continued)

---

- `LIST` is a function for building a list from any number of parameters

`(LIST 'apple 'orange 'grape)` returns

`(apple orange grape)`



# 술어 함수: EQ?

---

- EQ? takes two expressions as parameters (usually two atoms); it returns #T if both parameters have the same pointer value; otherwise #F

(EQ? 'A 'A) yields #T

(EQ? 'A 'B) yields #F

(EQ? 'A '(A B)) yields #F

(EQ? '(A B) '(A B)) yields #T or #F

(EQ? 3.4 (+ 3 0.4)) yields #T or #F

## 술어 함수: EQV?

---

- `EQV?` is like `EQ?`, except that it works for both symbolic and numeric atoms; it is a value comparison, not a pointer comparison

`(EQV? 3 3) yields #T`

`(EQV? 'A 3) yields #F`

`(EQV 3.4 (+ 3 0.4)) yields #T`

`(EQV? 3.0 3) yields #F` (floats and integers are different)

## 술어 함수: LIST? and NULL?

---

- LIST? takes one parameter; it returns #T if the parameter is a list; otherwise #F  
(LIST? ' ( ) ) yields #T
- NULL? takes one parameter; it returns #T if the parameter is the empty list; otherwise #F  
(NULL? ' ( ( ) ) ) yields #F

# Example Scheme Function: `member`

---

- `member` takes an atom and a simple list; returns `#T` if the atom is in the list; `#F` otherwise

```
DEFINE (member atm a_list)
  (COND
    ((NULL? a_list) #F)
    ((EQ? atm (CAR a_list)) #T)
    ((ELSE (member atm (CDR a_list))))
  ))
```

# Example Scheme Function: `equalsimp`

---

- `equalsimp` takes two simple lists as parameters; returns `#T` if the two simple lists are equal; `#F` otherwise

```
(DEFINE (equalsimp list1 list2)
  (COND
    ((NULL? list1) (NULL? list2))
    ((NULL? list2) #F)
    ((EQ? (CAR list1) (CAR list2))
      (equalsimp (CDR list1) (CDR list2)))
    (ELSE #F)
  ))
```

# Example Scheme Function: `append`

---

- `append` takes two lists as parameters; returns the first parameter list with the elements of the second parameter list appended at the end

```
(DEFINE (append list1 list2)
  (COND
    ((NULL? list1) list2)
    (ELSE (CONS (CAR list1)
                 (append (CDR list1) list2))))
))
```

# Example Scheme Function: LET

---

- Recall that `LET` was discussed in Chapter 5
- `LET` is actually shorthand for a `LAMBDA` expression applied to a parameter

```
(LET ((alpha 7)) (* 5 alpha))
```

is the same as:

```
((LAMBDA (alpha) (* 5 alpha)) 7)
```

# LET Example

---

```
(DEFINE (quadratic_roots a b c)
  (LET (
    (root_part_over_2a
      (/ (SQRT (- (* b b) (* 4 a c))) (* 2 a)))
    (minus_b_over_2a (/ (- 0 b) (* 2 a)))
    (DISPLAY (+ minus_b_over_2a root_part_over_2a))
    (NEWLINE)
    (DISPLAY (- minus_b_over_2a root_part_over_2a))
  ))
```



# Tail Recursion in Scheme

---

- Definition: A function is *tail recursive* if its recursive call is the last operation in the function
- Why the tail recursion is important?
  - tail recursions can be easily converted into loops
- A tail recursive function can be automatically converted by a compiler to use iteration, making it faster
- what is important is not the location of the call but the time of the call
- Scheme language definition requires that Scheme language systems convert all tail recursive functions to use iteration

# Tail Recursion in Scheme – continued

---

- Example of rewriting a function to make it tail recursive, using helper a function

Original:

```
(DEFINE (factorial n)
  (IF (<= n 0)
      1
      (* n (factorial (- n 1)))
  ))
```

Tail recursive:

```
(DEFINE (facthelper n factpartial)
  (IF (<= n 0)
      factpartial
      facthelper((- n 1) (* n factpartial)))
  ))

(DEFINE (factorial n)
  (facthelper n 1))
```

# Functional Sum

---

## Imperative sum

```
int sum(int i, int j)
{ int k, temp;
  temp = 0;
  for(k = i; k <= j; k++)
    temp += k;
  return temp;
}
```

## Functional sum

```
int sum(int i, int j)
{ if (i > j)
  return 0;
else
  return i + sum(i+1, j);
}
```

Is this call tail recursive?

Though the recursive call is located at the end of the function body, this is not a tail recursion.

# Accumulating Parameters

---

- Conversion to Tail-Recursion
  - Some recursive functions may be converted into tail-recursive ones using “accumulating parameters.”
  - In this cases, helper functions may be needed.

```
int sum1(int i, int j, int sumSoFar)
{ if (i > j)
    return sumSoFar;
  else
    return sum1(i+1, j, sumSoFar+i);
};

int sum(int i, int j)
{ return sum1(i, j, 0);
}
```

accumulating  
parameter

helper function  
(helping procedure)

**This is a tail call !**

# 범함수형 - 합성함수(Composition)

---

- Composition

- If  $h$  is the composition of  $f$  and  $g$ ,  $h(x) = f(g(x))$

```
(DEFINE (g x) (* 3 x))
```

```
(DEFINE (f x) (+ 2 x))
```

```
(DEFINE h x) (+ 2 (* 3 x)) (The composition)
```

- In Scheme, the functional composition function `compose` can be written:

```
(DEFINE (compose f g) (LAMBDA (x) (f (g x))))
```

```
((compose CAR CDR) '((a b) c d)) yields c → CADR
```

```
((compose CDR CAR) '((a b) c d)) yields (b) → CDAR
```

```
(DEFINE (third a_list)
```

```
((compose CAR (compose CDR CDR)) a_list))
```

**is equivalent to** CADDR

# 모두-적용(Apply-to-All) 범함수 형태

---

- Apply to All – one form in Scheme is `map`
  - Applies the given function to all elements of the given list;

```
(DEFINE (map fun a_list)
  (COND
    ((NULL? a_list) '())
    (ELSE (CONS (fun (CAR a_list))
                  (map fun (CDR a_list))))
  ))
```

```
(map (LAMBDA (num) (* num num num)) '(3 4 2 6)) yields
(27 64 8 216)
```

# COMMON LISP

---

- A combination of many of the features of the popular dialects of LISP around in the early 1980s
- A large and complex language--the opposite of Scheme
- Features include:
  - records
  - arrays
  - complex numbers
  - character strings
  - powerful I/O capabilities
  - packages with access control
  - iterative control statements

# ML

---

- A static-scoped functional language with syntax that is closer to Pascal than to Lisp
- Uses type declarations, but also does *type inferencing* to determine the types of undeclared variables
- It is strongly typed (whereas Scheme is essentially typeless) and has no type coercions
- Does not have imperative-style variables
- Its identifiers are untyped names for values
- Includes exception handling and a module facility for implementing abstract data types
- Includes lists and list operations



# Haskell

---

- Similar to ML (syntax, static scoped, strongly typed, type inferencing)
- Different from ML (and most other functional languages) in that it is *purely* functional (e.g., no variables, no assignment statements, and no side effects of any kind)
- Most Important Features
  - Uses *lazy evaluation* (evaluate no subexpression until the value is needed)
  - Has *list comprehensions*, which allow it to deal with infinite lists

# F#

---

- Based on Ocaml, which is a descendant of ML and Haskell
- Fundamentally a functional language, but with imperative features and supports OOP
- Has a full-featured IDE, an extensive library of utilities, and interoperates with other .NET languages
- Includes tuples, lists, discriminated unions, records, and both mutable and immutable arrays
- Supports generic sequences, whose values can be created with generators and through iteration

# F# (continued)

---

- Why F# is Interesting:
  - It builds on previous functional languages
  - It supports virtually all programming methodologies in widespread use today
  - It is the first functional language that is designed for interoperability with other widely used languages
  - At its release, it had an elaborate and well-developed IDE and library of utility software

# Support for Functional Programming in Primarily Imperative Languages

---

- Support for functional programming is increasingly creeping into imperative languages
  - Anonymous functions (lambda expressions)
    - JavaScript: leave the name out of a function definition
    - C#: `i => (i % 2) == 0` (returns true or false depending on whether the parameter is even or odd)
    - Python: `lambda a, b : 2 * a - b`

# Support for Functional Programming in Primarily Imperative Languages (continued)

---

- Python supports the higher-order functions filter and map (often use lambda expressions as their first parameters)

```
map(lambda x : x ** 3, [2, 4, 6, 8])
```

Returns [8, 64, 216, 512]

- Python supports partial function applications

```
from operator import add
```

```
add5 = partial (add, 5)
```

(the first line imports add as a function)

Use: add5(15)

# Support for Functional Programming in Primarily Imperative Languages (continued)

---

- Ruby Blocks

- Are effectively subprograms that are sent to methods, which makes the method a higher-order subprogram
- A block can be converted to a subprogram object with `lambda`

```
times = lambda {|a, b| a * b}
```

**Use:** `x = times.(3, 4)` (sets `x` to 12)

- Times can be curried with

```
times5 = times.curry.(5)
```

**Use:** `x5 = times5.(3)` (sets `x5` to 15)

# Comparing Functional and Imperative Languages

---

- Imperative Languages:
  - Efficient execution
  - Complex semantics
  - Complex syntax
  - Concurrency is programmer designed
- Functional Languages:
  - Simple semantics
  - Simple syntax
  - Less efficient execution
  - Programs can automatically be made concurrent

# Summary

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- Functional programming languages use function application, conditional expressions, recursion, and functional forms to control program execution
- Lisp began as a purely functional language and later included imperative features
- Scheme is a relatively simple dialect of Lisp that uses static scoping exclusively
- Common Lisp is a large Lisp-based language
- ML is a static-scoped and strongly typed functional language that uses type inference
- Haskell is a lazy functional language supporting infinite lists and set comprehension.
- F# is a .NET functional language that also supports imperative and object-oriented programming
- Some primarily imperative languages now incorporate some support for functional programming
- Purely functional languages have advantages over imperative alternatives, but still are not very widely used