Отчёт по лабораторной работе №8

Оптимизация

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Содержание

Список иллюстраций

1 Цель работы

Основная цель работа — освоить пакеты Julia для решения задач оптимизации.

2 Выполнение лабораторной работы

2.1 Линейное программирование

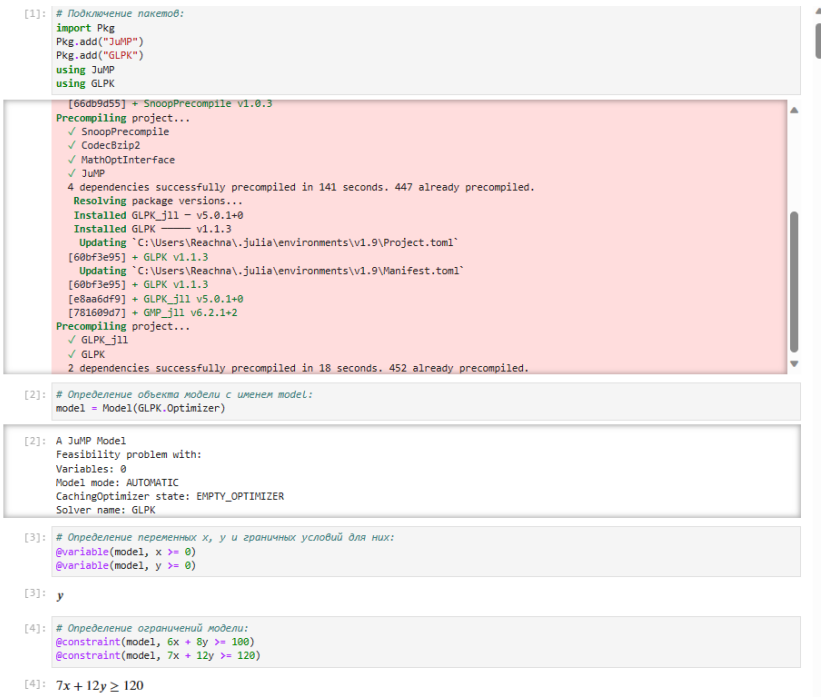


Рис. 2.1: Примеры линейного программирования

```
[5]: # Определение целевой функции:
     @objective(model, Min, 12x + 20y)

[5]: 12x + 20y

[6]: # Вызов функции оптимизации:
     optimize!(model)

[7]: # Определение причины завершения работы оптимизатора:
     termination_status(model)

[7]: OPTIMAL::TerminationStatusCode = 1

[8]: # Демонстрация первичных результирующих значений переменных x и y:
     @show value(x);
     @show value(y);
     # Демонстрация результата оптимизации:
     @show objective_value(model);

     value(x) = 14.999999999999993
     value(y) = 1.2500000000000047
     objective_value(model) = 205.0
```

Рис. 2.2: Примеры линейного программирования

2.2 Векторизованные ограничения и целевая функция оптимизации

```
[9]: # Определение объекта модели с именем vector_model:
     vector_model = Model(GLPK.Optimizer)

[9]: A JuMP Model
     Feasibility problem with:
     Variables: 0
     Model mode: AUTOMATIC
     CachingOptimizer state: EMPTY_OPTIMIZER
     Solver name: GLPK

[10]: # Определение начальных данных:
      A= [ 1 1 9 5;
           3 5 0 8;
           2 0 6 13]
      b = [7; 3; 5]
      c = [1; 3; 5; 2]

[10]: 4-element Vector{Int64}:
       1
       3
       5
       2

[11]: # Определение вектора переменных:
      @variable(vector_model, x[1:4] >= 0)

[11]: 4-element Vector{VariableRef}:
       x[1]
       x[2]
       x[3]
       x[4]

[12]: # Определение ограничений модели:
      @constraint(vector_model, A * x .== b)

[12]: 3-element Vector{ConstraintRef{Model, MathOptInterface.ConstraintIndex{MathOptInterface.ScalarAffineFunction{Float64}, MathOptInterface.EqualTo{Float64}}, ScalarShape}}:
       x[1] + x[2] + 9 x[3] + 5 x[4] == 7
       3 x[1] + 5 x[2] + 8 x[4] == 3
       2 x[1] + 6 x[3] + 13 x[4] == 5

[13]: # Определение целевой функции:
      @objective(vector_model, Min, c' * x)

[13]: x1 + 3x2 + 5x3 + 2x4

[14]: # Вызов функции оптимизации:
      optimize!(vector_model)

[15]: # Определение причины завершения работы оптимизатора:
      termination_status(vector_model)

[15]: OPTIMAL::TerminationStatusCode = 1

[16]: # Демонстрация результата оптимизации:
      @show objective_value(vector_model);

      objective_value(vector_model) = 4.923076923076925
```

Рис. 2.3: Векторизованные ограничения и целевая функция оптимизации

2.3 Оптимизация рациона питания

```
[17]: # Контейнер для хранения данных об ограничениях на количество потребляемых калорий, белков, жиров и соли:
category_data = JuMP.Containers.DenseAxisArray(
    [1800 2200;
    91 Inf;
    0 65;
    0 1779],
    ["calories", "protein", "fat", "sodium"],
    ["min", "max"])
```

```
[17]: 2-dimensional DenseAxisArray{Float64,2,...} with index sets:
    Dimension 1, ["calories", "protein", "fat", "sodium"]
    Dimension 2, ["min", "max"]
And data, a 4×2 Matrix{Float64}:
 1800.0  2200.0
   91.0    Inf
    0.0    65.0
    0.0  1779.0
```

```
[18]: # массив данных с наименованиями продуктов:
foods = ["hamburger", "chicken", "hot dog", "fries", "macaroni","pizza","salad", "milk", "ice cream"]
```

```
[18]: 9-element Vector{String}:
 "hamburger"
 "chicken"
 "hot dog"
 "fries"
 "macaroni"
 "pizza"
 "salad"
 "milk"
 "ice cream"
```

```
[19]: # Массив стоимости продуктов:
cost = JuMP.Containers.DenseAxisArray(
[2.49, 2.89, 1.50, 1.89, 2.09, 1.99, 2.49, 0.89, 1.59], foods)
```

```
[19]: 1-dimensional DenseAxisArray{Float64,1,...} with index sets:
    Dimension 1, ["hamburger", "chicken", "hot dog", "fries", "macaroni", "pizza", "salad", "milk", "ice cream"]
And data, a 9-element Vector{Float64}:
 2.49
 2.89
 1.5
 1.89
 2.09
 1.99
 2.49
 0.89
 1.59
```

Рис. 2.4: Примеры оптимизации рациона питания

```
[20]: # Массив данных о содержании калорий, белков, жиров и соли в продуктах питания:
food_data = JuMP.Containers.DenseAxisArray(
    [410 24 26 730;
    420 32 10 1190;
    560 20 32 1800;
    380 4 19 270;
    320 12 10 930;
    320 15 12 820;
    320 31 12 1230;
    100 8 2.5 125;
    330 8 10 180],
    foods,
    ["calories", "protein", "fat", "sodium"])
```

```
[20]: 2-dimensional DenseAxisArray{Float64,2,...} with index sets:
    Dimension 1, ["hamburger", "chicken", "hot dog", "fries", "macaroni", "pizza", "salad", "milk", "ice cream"]
    Dimension 2, ["calories", "protein", "fat", "sodium"]
And data, a 9×4 Matrix{Float64}:
 410.0  24.0  26.0   730.0
 420.0  32.0  10.0  1190.0
 560.0  20.0  32.0  1800.0
 380.0   4.0  19.0   270.0
 320.0  12.0  10.0   930.0
 320.0  15.0  12.0   820.0
 320.0  31.0  12.0  1230.0
 100.0   8.0   2.5   125.0
 330.0   8.0  10.0   180.0
```

```
[21]: # Определение объекта модели с именем model:
model = Model(GLPK.Optimizer)
```

```
[21]: A JuMP Model
Feasibility problem with:
Variables: 0
Model mode: AUTOMATIC
CachingOptimizer state: EMPTY_OPTIMIZER
Solver name: GLPK
```

```
[22]: # Определим массив:
categories = ["calories", "protein", "fat", "sodium"]
```

```
[22]: 4-element Vector{String}:
 "calories"
 "protein"
 "fat"
 "sodium"
```

Рис. 2.5: Примеры оптимизации рациона питания

```
[23]: # Определение переменных:
@variables(model, begin
    category_data[c, "min"] <= nutrition[c = categories] <= category_data[c, "max"]
    # Сколько покупать продуктов:
    buy[foods] >= 0
end)
```

```
[23]: (1-dimensional DenseAxisArray{VariableRef,1,...} with index sets:
    Dimension 1, ["calories", "protein", "fat", "sodium"]
And data, a 4-element Vector{VariableRef}:
 nutrition[calories]
 nutrition[protein]
 nutrition[fat]
 nutrition[sodium], 1-dimensional DenseAxisArray{VariableRef,1,...} with index sets:
    Dimension 1, ["hamburger", "chicken", "hot dog", "fries", "macaroni", "pizza", "salad", "milk", "ice cream"]
And data, a 9-element Vector{VariableRef}:
 buy[hamburger]
 buy[chicken]
 buy[hot dog]
 buy[fries]
 buy[macaroni]
 buy[pizza]
 buy[salad]
 buy[milk]
 buy[ice cream])
```

```
[24]: # Определение целевой функции:
@objective(model, Min, sum(cost[f] * buy[f] for f in foods))
```

[24]: $2.49buy_{hamburger} + 2.89buy_{chicken} + 1.5buy_{hotdog} + 1.89buy_{fries} + 2.09buy_{macaroni} + 1.99buy_{pizza} + 2.49buy_{salad} + 0.89buy_{milk} + 1.59buy_{icecream}$

```
[25]: # Определение ограничений модели:
@constraint(model, [c in categories], sum(food_data[f, c] * buy[f] for f in foods) == nutrition[c])
```

```
[25]: 1-dimensional DenseAxisArray{ConstraintRef{Model, MathOptInterface.ConstraintIndex{MathOptInterface.ScalarAffineFunction{F
loat64}, MathOptInterface.EqualTo{Float64}}, ScalarShape},1,...} with index sets:
    Dimension 1, ["calories", "protein", "fat", "sodium"]
And data, a 4-element Vector{ConstraintRef{Model, MathOptInterface.ConstraintIndex{MathOptInterface.ScalarAffineFunction{F
loat64}, MathOptInterface.EqualTo{Float64}}, ScalarShape}}:
 -nutrition[calories] + 410 buy[hamburger] + 420 buy[chicken] + 560 buy[hot dog] + 380 buy[fries] + 320 buy[macaroni] + 32
0 buy[pizza] + 320 buy[salad] + 100 buy[milk] + 330 buy[ice cream] == 0
 -nutrition[protein] + 24 buy[hamburger] + 32 buy[chicken] + 20 buy[hot dog] + 4 buy[fries] + 12 buy[macaroni] + 15 buy[pi
zza] + 31 buy[salad] + 8 buy[milk] + 8 buy[ice cream] == 0
 -nutrition[fat] + 26 buy[hamburger] + 10 buy[chicken] + 32 buy[hot dog] + 19 buy[fries] + 10 buy[macaroni] + 12 buy[pizz
a] + 12 buy[salad] + 2.5 buy[milk] + 10 buy[ice cream] == 0
 -nutrition[sodium] + 730 buy[hamburger] + 1190 buy[chicken] + 1800 buy[hot dog] + 270 buy[fries] + 930 buy[macaroni] + 82
0 buy[pizza] + 1230 buy[salad] + 125 buy[milk] + 180 buy[ice cream] == 0
```

Рис. 2.6: Примеры оптимизации рациона питания

```
[26]: # Вызов функции оптимизации:
      JuMP.optimize!(model)
      term_status = JuMP.termination_status(model)

[26]: OPTIMAL::TerminationStatusCode = 1

[27]: hcat(buy.data,JuMP.value.(buy.data))

[27]: 9×2 Matrix{AffExpr}:
       buy[hamburger]  0.6045138888888888
       buy[chicken]    0
       buy[hot dog]    0
       buy[fries]      0
       buy[macaroni]   0
       buy[pizza]      0
       buy[salad]      0
       buy[milk]       6.9701388888888935
       buy[ice cream]  2.591319444444441
```

Рис. 2.7: Примеры оптимизации рациона питания

2.4 Путешествие по миру

```
[28]: # Подключение пакетов:
      import Pkg
      Pkg.add("DelimitedFiles")
      Pkg.add("CSV")
      using DelimitedFiles
      using CSV

        Resolving package versions...
         Updating `C:\Users\Reachna\.julia\environments\v1.9\Project.toml`
       [8bb1440f] + DelimitedFiles v1.9.1
        No Changes to `C:\Users\Reachna\.julia\environments\v1.9\Manifest.toml`
        Resolving package versions...
        No Changes to `C:\Users\Reachna\.julia\environments\v1.9\Project.toml`
        No Changes to `C:\Users\Reachna\.julia\environments\v1.9\Manifest.toml`

[29]: # Считывание данных:
      passportdata = readdlm("passport-index-matrix.csv", ',')

[29]: 200×200 Matrix{Any}:
       "Passport"                "Albania"        …  "Afghanistan"
       "Afghanistan"             "visa required"     -1
       "Albania"                -1                    "visa required"
       "Algeria"                 "visa required"      "visa required"
       "Andorra"                90                    "visa required"
       "Angola"                  "visa required"   …  "visa required"
       "Antigua and Barbuda"    90                    "visa required"
       "Argentina"              90                    "visa required"
       "Armenia"                90                    "visa required"
       "Australia"              90                    "visa required"
       "Austria"                90                …   "visa required"
       "Azerbaijan"             90                    "visa required"
       "Bahamas"                90                    "visa required"
       ⋮                                           ⋱
       "United Arab Emirates"   90                    "visa required"
       "United Kingdom"         90                    "visa required"
       "United States"          90                …   "visa required"
       "Uruguay"                90                    "visa required"
       "Uzbekistan"              "visa required"      "visa required"
       "Vanuatu"                 "visa required"      "visa required"
       "Vatican"                90                    "visa required"
       "Venezuela"              90                …   "visa required"
       "Vietnam"                 "visa required"      "visa required"
       "Yemen"                   "visa required"      "visa required"
       "Zambia"                  "visa required"      "visa required"
       "Zimbabwe"                "visa required"      "visa required"

[30]: # Задаём переменные:
      cntr = passportdata[2:end,1]
      vf = (x -> typeof(x)==Int64 || x == "VF" || x == "VOA" ? 1 : 0).(passportdata[2:end, 2:end]);
```

Рис. 2.8: Путешествие по миру

[31]:
```
# Определение объекта модели с именем model:
model = Model(GLPK.Optimizer)
```

[31]:
```
A JuMP Model
Feasibility problem with:
Variables: 0
Model mode: AUTOMATIC
CachingOptimizer state: EMPTY_OPTIMIZER
Solver name: GLPK
```

[32]:
```
# Переменные, ограничения и целевая функция:
@variable(model, pass[1:length(cntr)], Bin)
@constraint(model, [j=1:length(cntr)], sum( vf[i,j]*pass[i] for i in 1:length(cntr)) >= 1)
@objective(model, Min, sum(pass))
```

[32]:
$$pass_1 + pass_2 + pass_3 + pass_4 + pass_5 + pass_6 + pass_7 + pass_8 + pass_9 + pass_{10} + pass_{11} + pass_{12} + pass_{13} + pass_{14} + pass_{15} + pass_{16} + pass_{17} + pass_{18} + pass_{19} + pass_{20} + pass_{21} + pass_{22} + pass_{23} + pass_{24} + pass_{25} + pass_{26} + pass_{27} + pass_{28} + pass_{29} + pass_{30} + [[\ldots \text{139 terms omitted} \ldots]] + pass_{170} + pass_{171} + pass_{172} + pass_{173} + pass_{174} + pass_{175} + pass_{176} + pass_{177} + pass_{178} + pass_{179} + pass_{180} + pass_{181} + pass_{182} + pass_{183} + pass_{184} + pass_{185} + pass_{186} + pass_{187} + pass_{188} + pass_{189} + pass_{190} + pass_{191} + pass_{192} + pass_{193} + pass_{194} + pass_{195} + pass_{196} + pass_{197} + pass_{198} + pass_{199}$$

[33]:
```
# Вызов функции оптимизации:
JuMP.optimize!(model)
termination_status(model)
```

[33]:
```
OPTIMAL::TerminationStatusCode = 1
```

[34]:
```
# Просмотр результата:
print(JuMP.objective_value(model)," passports: ",join(cntr[findall(JuMP.value.(pass) .== 1)],", "))
```

```
63.0 passports: Afghanistan, Andorra, Argentina, Australia, Azerbaijan, Bahrain, Brunei, Cambodia, Cameroon, Canada, Chil
e, Colombia, Comoros, DR Congo, Djibouti, Equatorial Guinea, Eritrea, Fiji, Gabon, Georgia, Guinea, Guinea-Bissau, Hong Ko
ng, Hungary, Indonesia, Iraq, Ireland, Israel, Jamaica, Japan, Kuwait, Laos, Liberia, Libya, Macao, Madagascar, Malaysia,
Maldives, Marshall Islands, Mauritania, Mauritius, Mongolia, Mozambique, Nauru, Nepal, New Zealand, North Korea, Palestin
e, Papua New Guinea, Qatar, Saudi Arabia, Solomon Islands, Somalia, South Sudan, Sri Lanka, Syria, Taiwan, Timor-Leste, To
go, Turkmenistan, United States, Uruguay, Vietnam
```

Рис. 2.9: Путешествие по миру

2.5 Портфельные инвестиции

[31]:
```
# Определение объекта модели с именем model:
model = Model(GLPK.Optimizer)
```

[31]:
```
A JuMP Model
Feasibility problem with:
Variables: 0
Model mode: AUTOMATIC
CachingOptimizer state: EMPTY_OPTIMIZER
Solver name: GLPK
```

[32]:
```
# Переменные, ограничения и целевая функция:
@variable(model, pass[1:length(cntr)], Bin)
@constraint(model, [j=1:length(cntr)], sum( vf[i,j]*pass[i] for i in 1:length(cntr)) >= 1)
@objective(model, Min, sum(pass))
```

[32]:
$$pass_1 + pass_2 + pass_3 + pass_4 + pass_5 + pass_6 + pass_7 + pass_8 + pass_9 + pass_{10} + pass_{11} + pass_{12} + pass_{13} + pass_{14} + pass_{15} + pass_{16} + pass_{17} + pass_{18} + pass_{19} + pass_{20} + pass_{21} + pass_{22} + pass_{23} + pass_{24} + pass_{25} + pass_{26} + pass_{27} + pass_{28} + pass_{29} + pass_{30} + [[\ldots \text{139 terms omitted} \ldots]] + pass_{170} + pass_{171} + pass_{172} + pass_{173} + pass_{174} + pass_{175} + pass_{176} + pass_{177} + pass_{178} + pass_{179} + pass_{180} + pass_{181} + pass_{182} + pass_{183} + pass_{184} + pass_{185} + pass_{186} + pass_{187} + pass_{188} + pass_{189} + pass_{190} + pass_{191} + pass_{192} + pass_{193} + pass_{194} + pass_{195} + pass_{196} + pass_{197} + pass_{198} + pass_{199}$$

[33]:
```
# Вызов функции оптимизации:
JuMP.optimize!(model)
termination_status(model)
```

[33]:
```
OPTIMAL::TerminationStatusCode = 1
```

[34]:
```
# Просмотр результата:
print(JuMP.objective_value(model)," passports: ",join(cntr[findall(JuMP.value.(pass) .== 1)],", "))
```

```
63.0 passports: Afghanistan, Andorra, Argentina, Australia, Azerbaijan, Bahrain, Brunei, Cambodia, Cameroon, Canada, Chil
e, Colombia, Comoros, DR Congo, Djibouti, Equatorial Guinea, Eritrea, Fiji, Gabon, Georgia, Guinea, Guinea-Bissau, Hong Ko
ng, Hungary, Indonesia, Iraq, Ireland, Israel, Jamaica, Japan, Kuwait, Laos, Liberia, Libya, Macao, Madagascar, Malaysia,
Maldives, Marshall Islands, Mauritania, Mauritius, Mongolia, Mozambique, Nauru, Nepal, New Zealand, North Korea, Palestin
e, Papua New Guinea, Qatar, Saudi Arabia, Solomon Islands, Somalia, South Sudan, Sri Lanka, Syria, Taiwan, Timor-Leste, To
go, Turkmenistan, United States, Uruguay, Vietnam
```


Рис. 2.10: Портфельные инвестиции

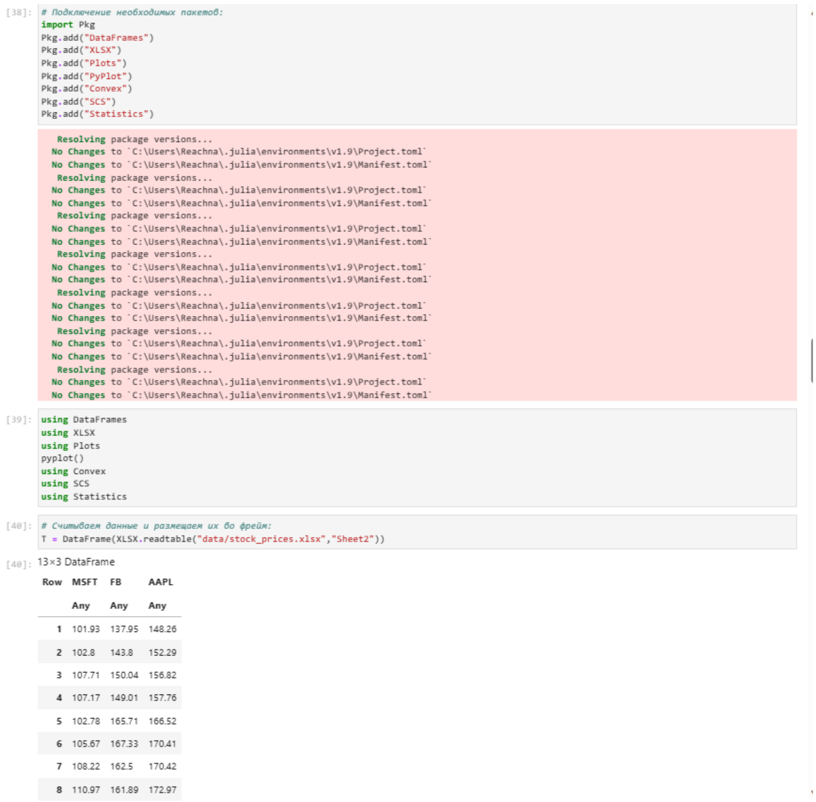
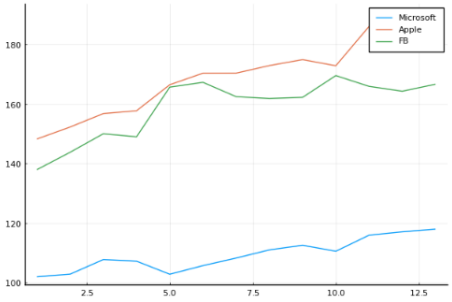


Рис. 2.11: Портфельные инвестиции

9	112.53	162.28	174.97
10	110.51	169.6	172.91
11	115.91	165.98	186.12
12	117.05	164.34	191.05
13	117.94	166.69	189.95

[41]:
```
# Построение графика:
plot(T[!,:MSFT],label="Microsoft")
plot!(T[!,:AAPL],label="Apple")
plot!(T[!,:FB],label="FB")
```

[41]:



[42]:
```
# Данные о ценах на акции размещаем в матрице:
prices_matrix = Matrix(T)
```

[42]:
```
13×3 Matrix{Any}:
 101.93  137.95  148.26
 102.8   143.8   152.29
 107.71  150.04  156.82
 107.17  149.01  157.76
 102.78  165.71  166.52
 105.67  167.33  170.41
 108.22  162.5   170.42
 110.97  161.89  172.97
 112.53  162.28  174.97
 110.51  169.6   172.91
 115.91  165.98  186.12
 117.05  164.34  191.05
 117.94  166.69  189.95
```

Рис. 2.12: Портфельные инвестиции

[43]:
```
# Вычисление матрицы доходности за период времени:
M1 = prices_matrix[1:end-1,:]
M2 = prices_matrix[2:end,:]
# Матрица доходности:
R = (M2.-M1)./M1
```

[43]:
```
12×3 Matrix{Float64}:
   0.00853527   0.0424067    0.027182
   0.0477626    0.0433936    0.0297459
  -0.00501346  -0.00686484   0.00599413
  -0.040963     0.112073     0.0555274
   0.0281183    0.00977611   0.0233606
   0.0241317   -0.0288651    5.8682e-5
   0.0254112   -0.00375385   0.014963
   0.0140579    0.00240904   0.0115627
  -0.0179508    0.0451072   -0.0117734
   0.0488644   -0.0213443    0.0763981
   0.00983522  -0.00988071   0.0264883
   0.00760359   0.0142996   -0.00575766
```

[44]:
```
# Матрица рисков:
risk_matrix = cov(R)
# Проверка положительной определённости матрицы рисков:
isposdef(risk_matrix)
```

[44]:
```
true
```

[45]:
```
# Доход от каждой из компаний:
r = mean(R,dims=1)[:]
```

[45]:
```
3-element Vector{Float64}:
 0.012532748705136572
 0.016563036855293173
 0.02114580465503291
```

[46]:
```
# Вектор инвестиций:
x = Variable(length(r))
```

[46]:
```
Variable
size: (3, 1)
sign: real
vexity: affine
id: 121…312
```

Рис. 2.13: Портфельные инвестиции

```
[47]: # Объект модели:
problem = minimize(Convex.quadform(x,risk_matrix),[sum(x)==1;r'*x>=0.02;x.>=0])
```

```
[47]: minimize
└─ * (convex; positive)
   ├─ 1
   └─ qol_elem (convex; positive)
      ├─ norm2 (convex; positive)
      │  └─ …
      └─ [1.0;;]
subject to
├─ == constraint (affine)
│  ├─ sum (affine; real)
│  │  └─ 3-element real variable (id: 121…312)
│  └─ 1
├─ >= constraint (affine)
│  ├─ * (affine; real)
│  │  ├─ [0.0125327 0.016563 0.0211458]
│  │  └─ 3-element real variable (id: 121…312)
│  └─ 0.02
├─ >= constraint (affine)
│  ├─ index (affine; real)
│  │  └─ 3-element real variable (id: 121…312)
│  └─ 0
├─ >= constraint (affine)
│  ├─ index (affine; real)
│  │  └─ 3-element real variable (id: 121…312)
│  └─ 0
└─ >= constraint (affine)
   ├─ index (affine; real)
   │  └─ 3-element real variable (id: 121…312)
   └─ 0

status: `solve!` not called yet
```

Рис. 2.14: Портфельные инвестиции

```
[48]: # Находим решение:
solve!(problem, SCS.Optimizer)
```

```
------------------------------------------------------------------
               SCS v3.2.4 - Splitting Conic Solver
        (c) Brendan O'Donoghue, Stanford University, 2012
------------------------------------------------------------------
problem:  variables n: 6, constraints m: 14
cones:    z: primal zero / dual free vars: 2
          l: linear vars: 5
          q: soc vars: 7, qsize: 2
settings: eps_abs: 1.0e-004, eps_rel: 1.0e-004, eps_infeas: 1.0e-007
          alpha: 1.50, scale: 1.00e-001, adaptive_scale: 1
          max_iters: 100000, normalize: 1, rho_x: 1.00e-006
          acceleration_lookback: 10, acceleration_interval: 10
lin-sys:  sparse-direct-amd-qdldl
          nnz(A): 24, nnz(P): 0
------------------------------------------------------------------
 iter | pri res | dua res |   gap   |   obj   |  scale  | time (s)
------------------------------------------------------------------
     0|1.71e+001 1.00e+000 1.62e+001 -8.03e+000 1.00e-001 2.46e-003
    75|8.16e-005 1.46e-004 5.60e-005 5.56e-004 1.00e-001 2.51e-003
------------------------------------------------------------------
status:  solved
timings: total: 2.51e-003s = setup: 2.42e-003s + solve: 9.60e-005s
         lin-sys: 2.39e-005s, cones: 1.58e-005s, accel: 4.80e-006s
------------------------------------------------------------------
objective = 0.000556
------------------------------------------------------------------
```

```
[49]: sum(x.value)
```

```
[49]: 0.9999994443731297
```

```
[50]: r'*x.value
```

```
[50]: 1×1 adjoint(::Vector{Float64}) with eltype Float64:
 0.02001195936160116
```

```
[51]: x.value .* 1000
```

```
[51]: 3×1 Matrix{Float64}:
  69.22834751660402
 117.301582202275
 813.4695146542507
```

Рис. 2.15: Портфельные инвестиции

2.6 Восстановление изображения

```
[52]: # Подключение необходимых пакетов:
import Pkg
Pkg.add("ImageMagick")
Pkg.add("Convex")
Pkg.add("SCS")
using Images
using Convex
using SCS
```

```
  Resolving package versions...
 No Changes to `C:\Users\Reachna\.julia\environments\v1.9\Project.toml`
 No Changes to `C:\Users\Reachna\.julia\environments\v1.9\Manifest.toml`
  Resolving package versions...
 No Changes to `C:\Users\Reachna\.julia\environments\v1.9\Project.toml`
 No Changes to `C:\Users\Reachna\.julia\environments\v1.9\Manifest.toml`
  Resolving package versions...
 No Changes to `C:\Users\Reachna\.julia\environments\v1.9\Project.toml`
 No Changes to `C:\Users\Reachna\.julia\environments\v1.9\Manifest.toml`
```

```
[53]: # Считывание исходного изображения:
Kref = load("data/khiam-small.jpg")
```

[53]:

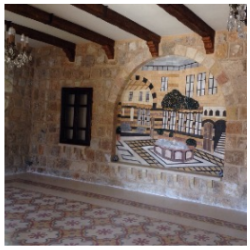


Рис. 2.16: Примеры восстановления изображений

```
[54]: K = copy(Kref)
p = prod(size(K))
missingids = rand(1:p,400)
K[missingids] .= RGBX{N0f8}(0.0,0.0,0.0)
K
Gray.(K)
```

[54]:

```
[55]: # Матрица цветов:
Y = Float64.(Gray.(K));
```

```
[56]: correctids = findall(Y[:].!=0)
X = Convex.Variable(size(Y))
problem = minimize(nuclearnorm(X))
problem.constraints += X[correctids]==Y[correctids]
```

```
[56]: 1-element Vector{Constraint}:
 == constraint (affine)
├─ index (affine; real)
│  └─ 283×283 real variable (id: 159…846)
└─ 79690-element Vector{Float64}
```

Рис. 2.17: Примеры восстановления изображений

```
[57]: # Находим решение:
solve!(problem, SCS.Optimizer())

------------------------------------------------------------------
              SCS v3.2.4 - Splitting Conic Solver
       (c) Brendan O'Donoghue, Stanford University, 2012
------------------------------------------------------------------
problem:  variables n: 240268, constraints m: 400047
cones:    z: primal zero / dual free vars: 239586
          s: psd vars: 160461, ssize: 1
settings: eps_abs: 1.0e-004, eps_rel: 1.0e-004, eps_infeas: 1.0e-007
          alpha: 1.50, scale: 1.00e-001, adaptive_scale: 1
          max_iters: 100000, normalize: 1, rho_x: 1.00e-006
          acceleration_lookback: 10, acceleration_interval: 10
lin-sys:  sparse-direct-amd-qdldl
          nnz(A): 400330, nnz(P): 0
------------------------------------------------------------------
 iter | pri res | dua res |   gap   |   obj   |  scale  | time (s)
------------------------------------------------------------------
     0|1.50e+001 9.96e-001 8.34e+003 1.76e+002 1.00e-001 8.40e-001
   250|6.08e-004 2.05e-005 1.10e-005 4.46e+002 3.36e-001 8.52e+001
   275|3.73e-004 1.95e-005 6.92e-006 4.46e+002 3.36e-001 9.42e+001
------------------------------------------------------------------
status:  solved
timings: total: 9.42e+001s = setup: 4.42e-001s + solve: 9.38e+001s
         lin-sys: 4.46e+000s, cones: 8.68e+001s, accel: 4.08e-001s
------------------------------------------------------------------
objective = 445.611251
------------------------------------------------------------------

[58]: @show norm(float.(Gray.(Kref))-X.value)
@show norm(-X.value)
colorview(Gray, X.value)

norm(float.(Gray.(Kref)) - X.value) = 1.1511800929439586
norm(-(X.value)) = 124.34122745625774
[58]:
```

Рис. 2.18: Примеры восстановления изображений

2.7 Задания для самостоятельного выполнения

2.7.1 Линейное программирование

Решите задачу линейного программирования: $x_1 + 2x_2 + 5x_3 \to max$, при заданных ограничениях: $-x_1 + x_2 + 3x_3 \leqslant -5, x_1 + 3x_2 - 7x_3 \leqslant 10, 0 \leqslant x_1 \leqslant 10, x_2 \geqslant 0, x_3 \geqslant 0$

Для начала определяю объект модели с именем model. Затем задаю переменные и граничные условия для них с помощью $@variable$. Далее применяем ограничения модели с помощью $@consatraint$.$@objective$ определяем функцию для нахождения максимума, передаем также модель со всей информацией, переданной до этого. Затем вызываем функцию оптимизации и проверяем причину остановки оптимизатора. Выдан результат OPTIMAL, что означает завершение

нахождения оптимального значения для заданной функции:

Рис. 2.19: Линейное программирование

2.7.2 Линейное программирование. Использование массивов

Решите предыдущее задание, используя массивы вместо скалярных переменных.

Рис. 2.20: Линейное программирование. Использование массивов

Результат совпадает с предыдущим пунктом – делаю вывод, что оптимизация проведена верно.

2.7.3 Выпуклое программирование

Решите задачу оптимизации: $\|A\vec{x} - \vec{b}\|_2^2 \to min$ при заданных ограничениях: $\vec{x} \succeq 0$:

[74]:
```
using Convex
using SCS
```

[75]:
```
m = 5
n = 4

A = rand(m, n)
b = rand(m)

display(A)
println()
display(b)
```

```
5×4 Matrix{Float64}:
 0.849155  0.936487  0.74695   0.513831
 0.401326  0.875853  0.874495  0.631306
 0.800074  0.526601  0.666555  0.145633
 0.483516  0.305902  0.636353  0.582247
 0.336612  0.354855  0.968644  0.92455

5-element Vector{Float64}:
 0.45183142669440834
 0.6667329596934422
 0.7275304988925214
 0.8926481804606815
 0.3173554782366863
```

[76]:
```
x = Variable(n)
display(x)
```

```
Variable
size: (4, 1)
sign: real
vexity: affine
id: 173…816
```

[77]:
```
model = minimize(Convex.sumsquares(A*x - b), [x >= 0])
```

[77]:
```
minimize
└─ qol_elem (convex; positive)
   ├─ norm2 (convex; positive)
   │  └─ + (affine; real)
   │     ├─ …
   │     └─ …
   └─ [1.0;;]
subject to
└─ >= constraint (affine)
   ├─ 4-element real variable (id: 173…816)
   └─ 0

status: `solve!` not called yet
```

Рис. 2.21: Выпуклое программирование

[78]:
```
solve!(model, SCS.Optimizer)
```

```
------------------------------------------------------------------
               SCS v3.2.4 - Splitting Conic Solver
        (c) Brendan O'Donoghue, Stanford University, 2012
------------------------------------------------------------------
problem:  variables n: 7, constraints m: 15
cones:    z: primal zero / dual free vars: 1
          l: linear vars: 5
          q: soc vars: 9, qsize: 2
settings: eps_abs: 1.0e-004, eps_rel: 1.0e-004, eps_infeas: 1.0e-007
          alpha: 1.50, scale: 1.00e-001, adaptive_scale: 1
          max_iters: 100000, normalize: 1, rho_x: 1.00e-006
          acceleration_lookback: 10, acceleration_interval: 10
lin-sys:  sparse-direct-amd-qdldl
          nnz(A): 30, nnz(P): 0
------------------------------------------------------------------
 iter | pri res | dua res |   gap   |   obj   |  scale  | time (s)
------------------------------------------------------------------
     0|1.71e+001 1.00e+000 1.62e+001 -8.04e+000 1.00e-001 7.85e-005
   125|4.22e-007 3.39e-007 1.28e-006 3.04e-001 7.57e-001 1.63e-004
------------------------------------------------------------------
status:  solved
timings: total: 1.64e-004s = setup: 6.40e-005s + solve: 9.98e-005s
         lin-sys: 3.49e-005s, cones: 1.40e-005s, accel: 7.20e-006s
------------------------------------------------------------------
objective = 0.304356
------------------------------------------------------------------
```

[79]:
```
model.status
```

[79]:
```
OPTIMAL::TerminationStatusCode = 1
```

[80]:
```
model.optval
```

[80]:
```
0.30435506756440117
```

Рис. 2.22: Выпуклое программирование

2.7.4 Оптимальная рассадка по залам

[81]:
```
using JuMP
using GLPK

# Заданные параметры
num_sections = 5
num_rooms = 5
min_capacity = 180
max_capacity = 250
target_capacity = 220
num_participants = 1000

# Генерация случайных приоритетов для слушателей
using Random
Random.seed!(42)
priorities = rand(1:3, num_participants, num_sections)

# Создание модели оптимизации
model = Model(optimizer_with_attributes(GLPK.Optimizer, "msg_lev" => GLPK.GLP_MSG_ALL))

# Переменные решения: x[i, j] = 1, если слушатель i посещает секцию j
@variable(model, x[1:num_participants, 1:num_sections], Bin)

# Условия для вместимости залов
room_capacities = [200, 210, 220, 230, 240]  # Пример вместимости залов
for j in 1:num_rooms
    @constraint(model, sum(x[i, j] for i in 1:num_participants) <= room_capacities[j])
end

# Условие, чтобы у каждого слушателя была одна и только одна секция с максимальным приоритетом
for i in 1:num_participants
    @constraint(model, sum(x[i, :]) == 1)
end

# Условия для учета приоритетов
for j in 1:num_sections
    for k in 1:3
        @constraint(model, sum(x[i, j] for i in findall(priorities[:, j] .== k)) == 0)
    end
end
```

Рис. 2.23: Оптимальная рассадка по залам

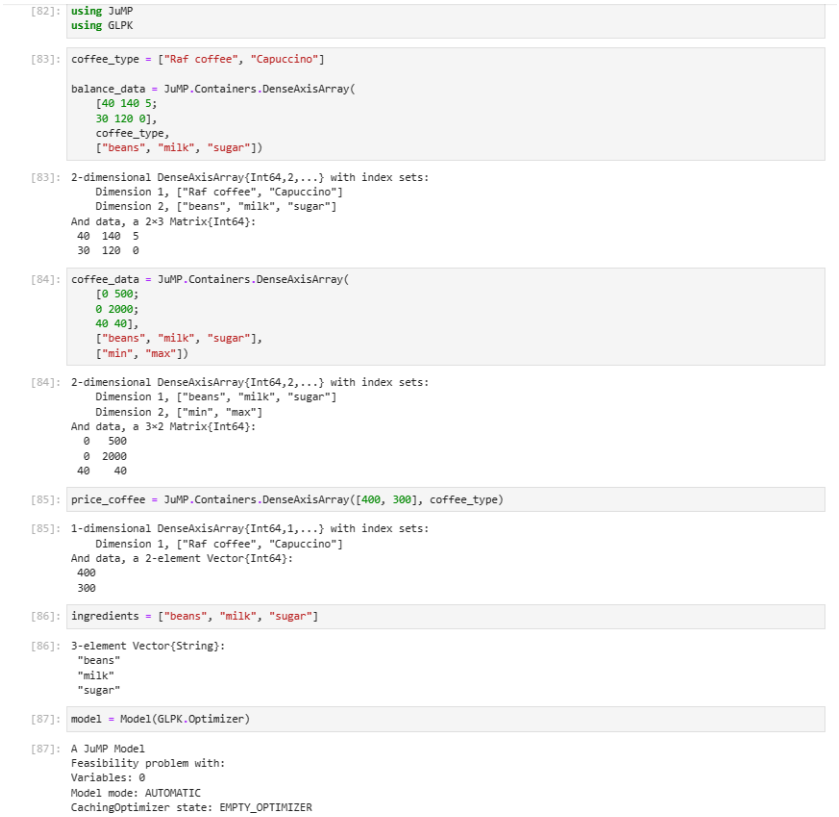
```
# Условия для учета приоритетов
for j in 1:num_sections
    for k in 1:3
        @constraint(model, sum(x[i, j] for i in findall(priorities[:, j] .== k)) == 0)
    end
end

# Условие для третьей секции, где нужно ровно 220 человек
@constraint(model, sum(x[i, 3] for i in 1:num_participants) == target_capacity)

# Функция цели: максимизация общего числа посетителей
@objective(model, Max, sum(x))

# Решение задачи
optimize!(model)

# Вывод результатов
println("Status: ", termination_status(model))

if termination_status(model) == MOI.OPTIMAL
    println("Objective value: ", objective_value(model))

    allocation = argmax(value.(x), dims=2)
    for i in 1:num_participants
        println("Слушатель $i посещает секцию $(allocation[i])")
    end
else
    println("Решение не найдено")
end
```

```
GLPK Simplex Optimizer 5.0
1021 rows, 5000 columns, 16000 non-zeros
      0: obj = -0.000000000e+000 inf =  1.220e+003 (1001)
     15: obj = -0.000000000e+000 inf =  1.220e+003 (1001)
LP HAS NO PRIMAL FEASIBLE SOLUTION
GLPK Integer Optimizer 5.0
1021 rows, 5000 columns, 16000 non-zeros
5000 integer variables, all of which are binary
Preprocessing...
PROBLEM HAS NO PRIMAL FEASIBLE SOLUTION
Status: INFEASIBLE
Решение не найдено
```

Рис. 2.24: Оптимальная рассадка по залам

2.7.5 План приготовления кофе

Кофейня готовит два вида кофе «Раф кофе» за 400 рублей и «Капучино» за 300. Чтобы сварить 1 чашку «Раф кофе» необходимо: 40 гр. зёрен, 140 гр. молока и 5 гр. ванильного сахара. Для того чтобы получить одну чашку «Капучино» необходимо потратить: 30 гр. зёрен, 120 гр. молока. На складе есть: 500 гр. зёрен, 2000 гр. молока и 40 гр. ванильного сахара.



```
[82]: using JuMP
      using GLPK

[83]: coffee_type = ["Raf coffee", "Capuccino"]

      balance_data = JuMP.Containers.DenseAxisArray(
          [40 140 5;
          30 120 0],
          coffee_type,
          ["beans", "milk", "sugar"])

[83]: 2-dimensional DenseAxisArray{Int64,2,...} with index sets:
          Dimension 1, ["Raf coffee", "Capuccino"]
          Dimension 2, ["beans", "milk", "sugar"]
      And data, a 2×3 Matrix{Int64}:
       40  140  5
       30  120  0

[84]: coffee_data = JuMP.Containers.DenseAxisArray(
          [0 500;
          0 2000;
          40 40],
          ["beans", "milk", "sugar"],
          ["min", "max"])

[84]: 2-dimensional DenseAxisArray{Int64,2,...} with index sets:
          Dimension 1, ["beans", "milk", "sugar"]
          Dimension 2, ["min", "max"]
      And data, a 3×2 Matrix{Int64}:
        0   500
        0  2000
       40    40

[85]: price_coffee = JuMP.Containers.DenseAxisArray([400, 300], coffee_type)

[85]: 1-dimensional DenseAxisArray{Int64,1,...} with index sets:
          Dimension 1, ["Raf coffee", "Capuccino"]
      And data, a 2-element Vector{Int64}:
       400
       300

[86]: ingredients = ["beans", "milk", "sugar"]

[86]: 3-element Vector{String}:
       "beans"
       "milk"
       "sugar"

[87]: model = Model(GLPK.Optimizer)

[87]: A JuMP Model
      Feasibility problem with:
      Variables: 0
      Model mode: AUTOMATIC
      CachingOptimizer state: EMPTY_OPTIMIZER
```

Рис. 2.25: План приготовления кофе

```
CachingOptimizer state: EMPTY_OPTIMIZER
Solver name: GLPK
```

```
[88]: # Определение переменных:
@variables(model,
    begin coffee_data[i, "min"] <= nutrition[i = ingredients] <= coffee_data[i, "max"]
        # Сколько использовать продуктов:
        use[coffee_type] >= 0
    end)
```

```
[88]: (1-dimensional DenseAxisArray{VariableRef,1,...} with index sets:
    Dimension 1, ["beans", "milk", "sugar"]
And data, a 3-element Vector{VariableRef}:
 nutrition[beans]
 nutrition[milk]
 nutrition[sugar], 1-dimensional DenseAxisArray{VariableRef,1,...} with index sets:
    Dimension 1, ["Raf coffee", "Capuccino"]
And data, a 2-element Vector{VariableRef}:
 use[Raf coffee]
 use[Capuccino])
```

```
[89]: # Определение целевой функции:
@objective(model, Max, sum(price_coffee[c] * use[c] for c in coffee_type))
```

[89]: $400use_{Raf\,coffee} + 300use_{Capuccino}$

```
[90]: # Определение ограничений модели:
@constraint(model, [i in ingredients],
    sum(balance_data[c, i] * use[c] for c in coffee_type) == nutrition[i])
```

```
[90]: 1-dimensional DenseAxisArray{ConstraintRef{Model, MathOptInterface.ConstraintIndex{MathOptInterface.ScalarAffineFunction{F
loat64}, MathOptInterface.EqualTo{Float64}}, ScalarShape},1,...} with index sets:
    Dimension 1, ["beans", "milk", "sugar"]
And data, a 3-element Vector{ConstraintRef{Model, MathOptInterface.ConstraintIndex{MathOptInterface.ScalarAffineFunction{F
loat64}, MathOptInterface.EqualTo{Float64}}, ScalarShape}}:
 -nutrition[beans] + 40 use[Raf coffee] + 30 use[Capuccino] == 0
 -nutrition[milk] + 140 use[Raf coffee] + 120 use[Capuccino] == 0
 -nutrition[sugar] + 5 use[Raf coffee] == 0
```

```
[91]: # Вызов функции оптимизации:
JuMP.optimize!(model)
term_status = JuMP.termination_status(model)
```

```
[91]: OPTIMAL::TerminationStatusCode = 1
```

```
[92]: hcat(use.data,JuMP.value.(use.data))
```

```
[92]: 2×2 Matrix{AffExpr}:
 use[Raf coffee]  8
 use[Capuccino]   6
```

Рис. 2.26: План приготовления кофе

3 Листинги программы

```
# Подключение пакетов:
import Pkg
Pkg.add("JuMP")
Pkg.add("GLPK")
using JuMP
using GLPK
# Определение объекта модели с именем model:
model = Model(GLPK.Optimizer)
A JuMP Model
Feasibility problem with:
Variables: 0
Model mode: AUTOMATIC
CachingOptimizer state: EMPTY_OPTIMIZER
Solver name: GLPK
# Определение переменных x, y и граничных условий для них:
@variable(model, x >= 0)
@variable(model, y >= 0)
# Определение ограничений модели:
@constraint(model, 6x + 8y >= 100)
@constraint(model, 7x + 12y >= 120)
# Определение целевой функции:
@objective(model, Min, 12x + 20y)
```

```
# Вызов функции оптимизации:
optimize!(model)
# Определение причины завершения работы оптимизатора:
termination_status(model)
OPTIMAL::TerminationStatusCode = 1
# Демонстрация первичных результирующих значений переменных x и y:
@show value(x);
@show value(y);
# Демонстрация результата оптимизации:
@show objective_value(model);
# Демонстрация первичных результирующих значений переменных x и y:
@show value(x);
@show value(y);
# Демонстрация результата оптимизации:
@show objective_value(model);
value(x) = 14.999999999999993
value(y) = 1.2500000000000047
objective_value(model) = 205.0
## Векторизованные ограничения и целевая функция оптимизации
# Подключение пакетов:
import Pkg
Pkg.add("JuMP")
Pkg.add("GLPK")
using JuMP
using GLPK
   Resolving package versions...
  No Changes to `C:\Users\Reachna\.julia\environments\v1.9\Project.toml`
  No Changes to `C:\Users\Reachna\.julia\environments\v1.9\Manifest.toml`
   Resolving package versions...
```

```
  No Changes to `C:\Users\Reachna\.julia\environments\v1.9\Project.toml`
  No Changes to `C:\Users\Reachna\.julia\environments\v1.9\Manifest.toml`
# Определение объекта модели с именем vector_model:
vector_model = Model(GLPK.Optimizer)
A JuMP Model
Feasibility problem with:
Variables: 0
Model mode: AUTOMATIC
CachingOptimizer state: EMPTY_OPTIMIZER
Solver name: GLPK
# Определение начальных данных:
A= [ 1 1 9 5;
     3 5 0 8;
     2 0 6 13]
b = [7; 3; 5]
c = [1; 3; 5; 2]
4-element Vector{Int64}:
 1
 3
 5
 2
# Определение вектора переменных:
@variable(vector_model, x[1:4] >= 0)
4-element Vector{VariableRef}:
 x[1]
 x[2]
 x[3]
 x[4]
# Определение ограничений модели:
```

```
@constraint(vector_model, A * x .== b)
# Определение целевой функции:
@objective(vector_model, Min, c' * x)
# Вызов функции оптимизации:
optimize!(vector_model)
# Определение причины завершения работы оптимизатора:
termination_status(vector_model)
OPTIMAL::TerminationStatusCode = 1
# Демонстрация результата оптимизации:
@show objective_value(vector_model);
objective_value(vector_model) = 4.9230769230769225
```

Оптимизация рациона питания

```
# Подключение пакетов:
import Pkg
Pkg.add("JuMP")
Pkg.add("GLPK")
using JuMP
using GLPK
# Контейнер для хранения данных об ограничениях на количество
# потребляемых калорий, белков, жиров и соли:
category_data = JuMP.Containers.DenseAxisArray(
    [1800 2200;
    91 Inf;
    0 65;
    0 1779],
    ["calories", "protein", "fat", "sodium"],
    ["min", "max"])
2-dimensional DenseAxisArray{Float64,2,...} with index sets:
    Dimension 1, ["calories", "protein", "fat", "sodium"]
```

```
    Dimension 2, ["min", "max"]
And data, a 4×2 Matrix{Float64}:
 1800.0  2200.0
   91.0    Inf
    0.0    65.0
    0.0  1779.0
# массив данных с наименованиями продуктов:
foods = ["hamburger", "chicken", "hot dog", "fries", "macaroni",
"pizza","salad", "milk", "ice cream"]
9-element Vector{String}:
 "hamburger"
 "chicken"
 "hot dog"
 "fries"
 "macaroni"
 "pizza"
 "salad"
 "milk"
 "ice cream"
# Массив стоимости продуктов:
cost = JuMP.Containers.DenseAxisArray(
    [2.49, 2.89, 1.50, 1.89, 2.09, 1.99, 2.49, 0.89, 1.59],foods)
1-dimensional DenseAxisArray{Float64,1,...} with index sets:
    Dimension 1, ["hamburger", "chicken", "hot dog", "fries",
    "macaroni", "pizza", "salad", "milk", "ice cream"]
And data, a 9-element Vector{Float64}:
 2.49
 2.89
 1.5
```

```
 1.89
 2.09
 1.99
 2.49
 0.89
 1.59
# Массив данных о содержании калорий, белков, жиров и соли в продуктах питания:
food_data = JuMP.Containers.DenseAxisArray(
    [410 24 26 730;
    420 32 10 1190;
    560 20 32 1800;
    380 4 19 270;
    320 12 10 930;
    320 15 12 820;
    320 31 12 1230;
    100 8 2.5 125;
    330 8 10 180],
    foods,
    ["calories", "protein", "fat", "sodium"])
2-dimensional DenseAxisArray{Float64,2,...} with index sets:
    Dimension 1, ["hamburger", "chicken", "hot dog", "fries",
    "macaroni", "pizza", "salad", "milk", "ice cream"]
    Dimension 2, ["calories", "protein", "fat", "sodium"]
And data, a 9×4 Matrix{Float64}:
 410.0  24.0  26.0   730.0
 420.0  32.0  10.0  1190.0
 560.0  20.0  32.0  1800.0
 380.0   4.0  19.0   270.0
 320.0  12.0  10.0   930.0
```

```
 320.0  15.0  12.0   820.0
 320.0  31.0  12.0  1230.0
 100.0   8.0   2.5   125.0
 330.0   8.0  10.0   180.0
# Определение объекта модели с именем model:
model = Model(GLPK.Optimizer)
A JuMP Model
Feasibility problem with:
Variables: 0
Model mode: AUTOMATIC
CachingOptimizer state: EMPTY_OPTIMIZER
Solver name: GLPK
# Определим массив:
categories = ["calories", "protein", "fat", "sodium"]
4-element Vector{String}:
 "calories"
 "protein"
 "fat"
 "sodium"
# Определение переменных:
@variables(model, begin
    category_data[c, "min"] <= nutrition[c = categories] <= category_data[c, "max
    # Сколько покупать продуктов:
    buy[foods] >= 0
end)
(1-dimensional DenseAxisArray{VariableRef,1,...} with index sets:
    Dimension 1, ["calories", "protein", "fat", "sodium"]
And data, a 4-element Vector{VariableRef}:
 nutrition[calories]
```

```
 nutrition[protein]
 nutrition[fat]
And data, a 9-element Vector{VariableRef}:
 buy[hamburger]
 buy[chicken]
 buy[hot dog]
 buy[fries]
 buy[macaroni]
 buy[pizza]
 buy[salad]
 buy[milk]
 buy[ice cream])
# Определение целевой функции:
@objective(model, Min, sum(cost[f] * buy[f] for f in foods))
# Определение ограничений модели:
@constraint(model, [c in categories], sum(food_data[f, c] * buy[f] for f in foods
# Вызов функции оптимизации:
JuMP.optimize!(model)
term_status = JuMP.termination_status(model)
OPTIMAL::TerminationStatusCode = 1
hcat(buy.data,JuMP.value.(buy.data))
9×2 Matrix{AffExpr}:
 buy[hamburger]  0.6045138888888888
 buy[chicken]    0
 buy[hot dog]    0
 buy[fries]      0
 buy[macaroni]   0
 buy[pizza]      0
 buy[salad]      0
```

```
 buy[milk]       6.9701388888888935
 buy[ice cream]  2.591319444444441
9×2 Matrix{AffExpr}:
 buy[hamburger]  0.6045138888888888
 buy[chicken]    0
 buy[hot dog]    0
 buy[fries]      0
 buy[macaroni]   0
 buy[pizza]      0
 buy[salad]      0
 buy[milk]       6.9701388888888935
 buy[ice cream]  2.591319444444441
```

```
# Подключение пакетов:
import Pkg
Pkg.add("DelimitedFiles")
Pkg.add("CSV")
using DelimitedFiles
using CSV
# Считывание данных:
passportdata = readdlm("passport-index-matrix.csv",',')
200×200 Matrix{Any}:
 "Passport"                "Albania"         .    "Afghanistan"
 "Afghanistan"             "visa required"     -1
 "Albania"                -1                       "visa required"
 "Algeria"                 "visa required"         "visa required"
 "Andorra"                90                       "visa required"
 "Angola"                  "visa required"   .     "visa required"
 "Antigua and Barbuda"    90                       "visa required"
 "Argentina"              90                       "visa required"
```

```
 "Armenia"                90                    "visa required"
 "Australia"              90                    "visa required"
 "Austria"                90                .   "visa required"
 "Azerbaijan"             90                    "visa required"
 "Bahamas"                90                    "visa required"
 .                                          .
 "United Arab Emirates"   90                    "visa required"
 "United Kingdom"         90                    "visa required"
 "United States"          90                .   "visa required"
 "Uruguay"                90                    "visa required"
 "Uzbekistan"               "visa required"     "visa required"
 "Vanuatu"                  "visa required"     "visa required"
 "Vatican"                90                    "visa required"
 "Venezuela"              90                .   "visa required"
 "Vietnam"                  "visa required"     "visa required"
 "Yemen"                    "visa required"     "visa required"
 "Zambia"                   "visa required"     "visa required"
 "Zimbabwe"                 "visa required"     "visa required"
# Задаём переменные:
cntr = passportdata[2:end,1]
vf = (x -> typeof(x)==Int64 || x == "VF" || x == "VOA" ? 1 : 0).(passportdata[2:e
# Определение объекта модели с именем model:
model = Model(GLPK.Optimizer)
A JuMP Model
Feasibility problem with:
Variables: 0
Model mode: AUTOMATIC
CachingOptimizer state: EMPTY_OPTIMIZER
Solver name: GLPK
```

```
# Переменные, ограничения и целевая функция:
@variable(model, pass[1:length(cntr)], Bin)
@constraint(model, [j=1:length(cntr)], sum( vf[i,j]*pass[i] for i in 1:length(cnt
@objective(model, Min, sum(pass))
# Вызов функции оптимизации:
JuMP.optimize!(model)
termination_status(model)
OPTIMAL::TerminationStatusCode = 1
# Просмотр результата:
print(JuMP.objective_value(model)," passports: ",join(cntr[findall(JuMP.value.(pa
# Подключение необходимых пакетов:
import Pkg
Pkg.add("DataFrames")
Pkg.add("XLSX")
Pkg.add("Plots")
Pkg.add("PyPlot")
Pkg.add("Convex")
Pkg.add("SCS")
Pkg.add("Statistics")
using DataFrames
using XLSX
using Plots
pyplot()
using Convex
using SCS
using Statistics
# Считываем данные и размещаем их во фрейм:
T = DataFrame(XLSX.readtable("data/stock_prices.xlsx","Sheet2"))
# Построение графика:
```

```
plot(T[!,:MSFT],label="Microsoft")
plot!(T[!,:AAPL],label="Apple")
plot!(T[!,:FB],label="FB")
# Данные о ценах на акции размещаем в матрице:
prices_matrix = Matrix(T)
# Данные о ценах на акции размещаем в матрице:
prices_matrix = Matrix(T)
13×3 Matrix{Any}:
 101.93  137.95  148.26
 102.8   143.8   152.29
 107.71  150.04  156.82
 107.17  149.01  157.76
 102.78  165.71  166.52
 105.67  167.33  170.41
 108.22  162.5   170.42
 110.97  161.89  172.97
 112.53  162.28  174.97
 110.51  169.6   172.91
 115.91  165.98  186.12
 117.05  164.34  191.05
 117.94  166.69  189.95
# Вычисление матрицы доходности за период времени:
M1 = prices_matrix[1:end-1,:]
M2 = prices_matrix[2:end,:]
# Матрица доходности:
R = (M2.-M1)./M1
12×3 Matrix{Float64}:
  0.00853527   0.0424067    0.027182
  0.0477626    0.0433936    0.0297459
```

```
 -0.00501346  -0.00686484   0.00599413
 -0.040963     0.112073     0.0555274
  0.0281183    0.00977611   0.0233606
  0.0241317   -0.0288651    5.8682e-5
  0.0254112   -0.00375385   0.014963
  0.0140579    0.00240904   0.0115627
 -0.0179508    0.0451072   -0.0117734
  0.0488644   -0.0213443    0.0763981
  0.00983522  -0.00988071   0.0264883
  0.00760359   0.0142996   -0.00575766
# Матрица рисков:
risk_matrix = cov(R)
# Проверка положительной определённости матрицы рисков:
isposdef(risk_matrix)
true
# Доход от каждой из компаний:
r = mean(R,dims=1)[:]
3-element Vector{Float64}:
 0.012532748705136572
 0.016563036855293173
 0.02114580465503291
# Вектор инвестиций:
x = Variable(length(r))
Variable
size: (3, 1)
sign: real
vexity: affine
id: 121…312
# Объект модели:
```

```
problem = minimize(Convex.quadform(x,risk_matrix),[sum(x)==1;r'*x>=0.02;x.>=0])
minimize
└─ * (convex; positive)
   ├─ 1
   └─ qol_elem (convex; positive)
      ├─ norm2 (convex; positive)
      │  └─ …
      └─ [1.0;;]
subject to
├─ == constraint (affine)
│  ├─ sum (affine; real)
│  │  └─ 3-element real variable (id: 121…312)
│  └─ 1
├─ >= constraint (affine)
│  ├─ * (affine; real)
│  │  ├─ [0.0125327 0.016563 0.0211458]
│  │  └─ 3-element real variable (id: 121…312)
│  └─ 0.02
├─ >= constraint (affine)
│  ├─ index (affine; real)
│  │  └─ 3-element real variable (id: 121…312)
│  └─ 0
├─ >= constraint (affine)
│  ├─ index (affine; real)
│  │  └─ 3-element real variable (id: 121…312)
│  └─ 0
└─ >= constraint (affine)
   ├─ index (affine; real)
   │  └─ 3-element real variable (id: 121…312)
```

```
  └─ 0
status: `solve!` not called yet
# Находим решение:
solve!(problem, SCS.Optimizer)
------------------------------------------------------------------
           SCS v3.2.4 - Splitting Conic Solver
    (c) Brendan O'Donoghue, Stanford University, 2012
------------------------------------------------------------------
problem:  variables n: 6, constraints m: 14
cones:    z: primal zero / dual free vars: 2
      l: linear vars: 5
      q: soc vars: 7, qsize: 2
settings: eps_abs: 1.0e-004, eps_rel: 1.0e-004, eps_infeas: 1.0e-007
      alpha: 1.50, scale: 1.00e-001, adaptive_scale: 1
      max_iters: 100000, normalize: 1, rho_x: 1.00e-006
      acceleration_lookback: 10, acceleration_interval: 10
lin-sys:  sparse-direct-amd-qdldl
      nnz(A): 24, nnz(P): 0
------------------------------------------------------------------
 iter | pri res | dua res |   gap   |   obj   |  scale  | time (s)
------------------------------------------------------------------
     0|1.71e+001 1.00e+000 1.62e+001 -8.03e+000 1.00e-001 2.46e-003
    75|8.16e-005 1.46e-004 5.60e-005 5.56e-004 1.00e-001 2.51e-003
------------------------------------------------------------------
status:  solved
timings: total: 2.51e-003s = setup: 2.42e-003s + solve: 9.60e-005s
     lin-sys: 2.39e-005s, cones: 1.58e-005s, accel: 4.80e-006s
------------------------------------------------------------------
objective = 0.000556
```

```
------------------------------------------------------------------
sum(x.value)
0.9999994443731297
r'*x.value
1×1 adjoint(::Vector{Float64}) with eltype Float64:
 0.02001195936160116
x.value .* 1000
3×1 Matrix{Float64}:
  69.22834751660402
 117.301582202275
 813.4695146542507
# Подключение необходимых пакетов:
import Pkg
Pkg.add("ImageMagick")
Pkg.add("Convex")
Pkg.add("SCS")
using Images
using Convex
using SCS
# Считывание исходного изображения:
Kref = load("data/khiam-small.jpg")
K = copy(Kref)
p = prod(size(K))
missingids = rand(1:p,400)
K[missingids] .= RGBX{N0f8}(0.0,0.0,0.0)
K
Gray.(K)
# Матрица цветов:
Y = Float64.(Gray.(K));
```

```
correctids = findall(Y[:].!=0)
X = Convex.Variable(size(Y))
problem = minimize(nuclearnorm(X))
problem.constraints += X[correctids]==Y[correctids]
# Находим решение:
solve!(problem, SCS.Optimizer())
@show norm(float.(Gray.(Kref))-X.value)
@show norm(-X.value)
colorview(Gray, X.value)
## Линейное программирование
# Определение объекта модели с именем model:
model = Model(GLPK.Optimizer)
A JuMP Model
Feasibility problem with:
Variables: 0
Model mode: AUTOMATIC
CachingOptimizer state: EMPTY_OPTIMIZER
Solver name: GLPK
@variable(model, 0 <= x1 <= 10)
@variable(model, x2 >= 0)
@variable(model, x3 >= 0)
@constraint(model, -x1 + x2 + 3x3 <= -5)
@constraint(model, x1 + 3x2 - 7x3 <= 10)
@objective(model, Max, x1 + 2x2 + 5x3)
optimize!(model)
termination_status(model)
OPTIMAL::TerminationStatusCode = 1
@show value(x1);
@show value(x2);
```

```
@show value(x3);

@show objective_value(model);
value(x1) = 10.0
value(x2) = 2.1875
value(x3) = 0.9375
objective_value(model) = 19.0625
Линейное программирование. Использование массивов
vector_model_2 = Model(GLPK.Optimizer)
A JuMP Model
Feasibility problem with:
Variables: 0
Model mode: AUTOMATIC
CachingOptimizer state: EMPTY_OPTIMIZER
Solver name: GLPK
A = [-1 1 3;
    1 3 -7]
b = [-5; 10]
c = [1; 2; 5]
3-element Vector{Int64}:
 1
 2
 5
@variable(vector_model_2, x[1:3] >= 0)
set_upper_bound(x[1], 10)
@constraint(vector_model_2, A * x .== b)
@objective(vector_model_2, Max, c' * x)
optimize!(vector_model_2)
termination_status(vector_model_2)
```

```
OPTIMAL::TerminationStatusCode = 1
@show value(x1);
@show value(x2);
@show value(x3);
@show objective_value(vector_model_2);
value(x1) = 10.0
value(x2) = 2.1875
value(x3) = 0.9375
objective_value(vector_model_2) = 19.0625
```

Выпуклое программирование

```
using Convex
using SCS
m = 5
n = 4
A = rand(m, n)
b = rand(m)
display(A)
println()
display(b)
x = Variable(n)
display(x)
```

```
Variable
size: (4, 1)
sign: real
vexity: affine
id: 173…816
```

```
model = minimize(Convex.sumsquares(A*x - b), [x >= 0])
```

```
minimize
└─ qol_elem (convex; positive)
```

```
   ├─ norm2 (convex; positive)
   │  └─ + (affine; real)
   │     ├─ …
   │     └─ …
   └─ [1.0;;]
subject to
└─ >= constraint (affine)
   ├─ 4-element real variable (id: 173…816)
   └─ 0

status: `solve!` not called yet
solve!(model, SCS.Optimizer)
------------------------------------------------------------------
           SCS v3.2.4 - Splitting Conic Solver
    (c) Brendan O'Donoghue, Stanford University, 2012
------------------------------------------------------------------
problem:  variables n: 7, constraints m: 15
cones:    z: primal zero / dual free vars: 1
      l: linear vars: 5
      q: soc vars: 9, qsize: 2
settings: eps_abs: 1.0e-004, eps_rel: 1.0e-004, eps_infeas: 1.0e-007
      alpha: 1.50, scale: 1.00e-001, adaptive_scale: 1
      max_iters: 100000, normalize: 1, rho_x: 1.00e-006
      acceleration_lookback: 10, acceleration_interval: 10
lin-sys:  sparse-direct-amd-qdldl
      nnz(A): 30, nnz(P): 0
------------------------------------------------------------------
 iter | pri res | dua res |   gap   |   obj   |  scale  | time (s)
------------------------------------------------------------------
```

```
     0|1.71e+001 1.00e+000 1.62e+001 -8.04e+000 1.00e-001 7.85e-005
   125|4.22e-007 3.39e-007 1.28e-006 3.04e-001 7.57e-001 1.63e-004
------------------------------------------------------------------
status:  solved
timings: total: 1.64e-004s = setup: 6.40e-005s + solve: 9.98e-005s
     lin-sys: 3.49e-005s, cones: 1.40e-005s, accel: 7.20e-006s
------------------------------------------------------------------
objective = 0.304356
------------------------------------------------------------------
model.status
OPTIMAL::TerminationStatusCode = 1
model.optval
0.30435506756440117
Оптимальная рассадка по залам
using JuMP
using GLPK
# Заданные параметры
num_sections = 5
num_rooms = 5
min_capacity = 180
max_capacity = 250
target_capacity = 220
num_participants = 1000
# Генерация случайных приоритетов для слушателей
using Random
Random.seed!(42)
priorities = rand(1:3, num_participants, num_sections)
# Создание модели оптимизации
model = Model(optimizer_with_attributes(GLPK.Optimizer,
```

```
"msg_lev" => GLPK.GLP_MSG_ALL))

# Переменные решения: x[i, j] = 1, если слушатель i посещает секцию j
@variable(model, x[1:num_participants, 1:num_sections], Bin)

# Условия для вместимости залов
room_capacities = [200, 210, 220, 230, 240]  # Пример вместимости залов
for j in 1:num_rooms
    @constraint(model, sum(x[i, j] for i in 1:num_participants) <= room_capacitie
end

# Условие, чтобы у каждого слушателя была одна и только одна
for i in 1:num_participants
    @constraint(model, sum(x[i, :]) == 1)
end

# Условия для учета приоритетов
for j in 1:num_sections
    for k in 1:3
        @constraint(model, sum(x[i, j] for i in findall(priorities[:, j] .== k))
    end
end

# Условие для третьей секции, где нужно ровно 220 человек
@constraint(model, sum(x[i, 3] for i in 1:num_participants) == target_capacity)

# Функция цели: максимизация общего числа посетителей
@objective(model, Max, sum(x))
```

```
# Решение задачи
optimize!(model)

# Вывод результатов
println("Status: ", termination_status(model))

if termination_status(model) == MOI.OPTIMAL
    println("Objective value: ", objective_value(model))

    allocation = argmax(value.(x), dims=2)
    for i in 1:num_participants
        println("Слушатель $i посещает секцию $(allocation[i])")
    end
else
    println("Решение не найдено")
end
GLPK Simplex Optimizer 5.0
1021 rows, 5000 columns, 16000 non-zeros
      0: obj = -0.000000000e+000 inf =  1.220e+003 (1001)
     15: obj = -0.000000000e+000 inf =  1.220e+003 (1001)
LP HAS NO PRIMAL FEASIBLE SOLUTION
GLPK Integer Optimizer 5.0
1021 rows, 5000 columns, 16000 non-zeros
5000 integer variables, all of which are binary
Preprocessing...
PROBLEM HAS NO PRIMAL FEASIBLE SOLUTION
Status: INFEASIBLE
Решение не найдено
План приготовления кофе
```

```
using JuMP
using GLPK
coffee_type = ["Raf coffee", "Capuccino"]
balance_data = JuMP.Containers.DenseAxisArray(
    [40 140 5;
    30 120 0],
    coffee_type,
    ["beans", "milk", "sugar"])
2-dimensional DenseAxisArray{Int64,2,...} with index sets:
    Dimension 1, ["Raf coffee", "Capuccino"]
    Dimension 2, ["beans", "milk", "sugar"]
And data, a 2×3 Matrix{Int64}:
 40  140  5
 30  120  0
coffee_data = JuMP.Containers.DenseAxisArray(
    [0 500;
    0 2000;
    40 40],
    ["beans", "milk", "sugar"],
    ["min", "max"])
2-dimensional DenseAxisArray{Int64,2,...} with index sets:
    Dimension 1, ["beans", "milk", "sugar"]
    Dimension 2, ["min", "max"]
And data, a 3×2 Matrix{Int64}:
  0   500
  0  2000
 40    40
price_coffee = JuMP.Containers.DenseAxisArray([400, 300], coffee_type)
1-dimensional DenseAxisArray{Int64,1,...} with index sets:
```

```
    Dimension 1, ["Raf coffee", "Capuccino"]
And data, a 2-element Vector{Int64}:
 400
 300
ingredients = ["beans", "milk", "sugar"]
3-element Vector{String}:
 "beans"
 "milk"
 "sugar"
model = Model(GLPK.Optimizer)
A JuMP Model
Feasibility problem with:
Variables: 0
Model mode: AUTOMATIC
CachingOptimizer state: EMPTY_OPTIMIZER
Solver name: GLPK
# Определение переменных:
@variables(model,
    begin coffee_data[i, "min"] <= nutrition[i = ingredients]
    <= coffee_data[i, "max"]
        # Сколько использовать продуктов:
        use[coffee_type] >= 0
    end)
(1-dimensional DenseAxisArray{VariableRef,1,...} with index sets:
    Dimension 1, ["beans", "milk", "sugar"]
And data, a 3-element Vector{VariableRef}:
 nutrition[beans]
 nutrition[milk]
 nutrition[sugar], 1-dimensional DenseAxisArray{VariableRef,1,...} with index set
```

```
    Dimension 1, ["Raf coffee", "Capuccino"]
And data, a 2-element Vector{VariableRef}:
 use[Raf coffee]
 use[Capuccino])
# Определение целевой функции:
@objective(model, Max, sum(price_coffee[c] * use[c] for c in coffee_type))
# Определение ограничений модели:
@constraint(model, [i in ingredients],
    sum(balance_data[c, i] * use[c] for c in coffee_type) == nutrition[i])
# Вызов функции оптимизации:
JuMP.optimize!(model)
term_status = JuMP.termination_status(model)
OPTIMAL::TerminationStatusCode = 1
hcat(use.data,JuMP.value.(use.data))
2×2 Matrix{AffExpr}:
 use[Raf coffee]  8
 use[Capuccino]   6
```

4 Вывод

Освоила пакеты Julia для решения задач оптимизации.