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About the service model of two classes of multicast traffic

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Purpose of study

Constructing the model for the performance measures analysis of the access point servicing the multicast traffic generated by the users located in its coverage:

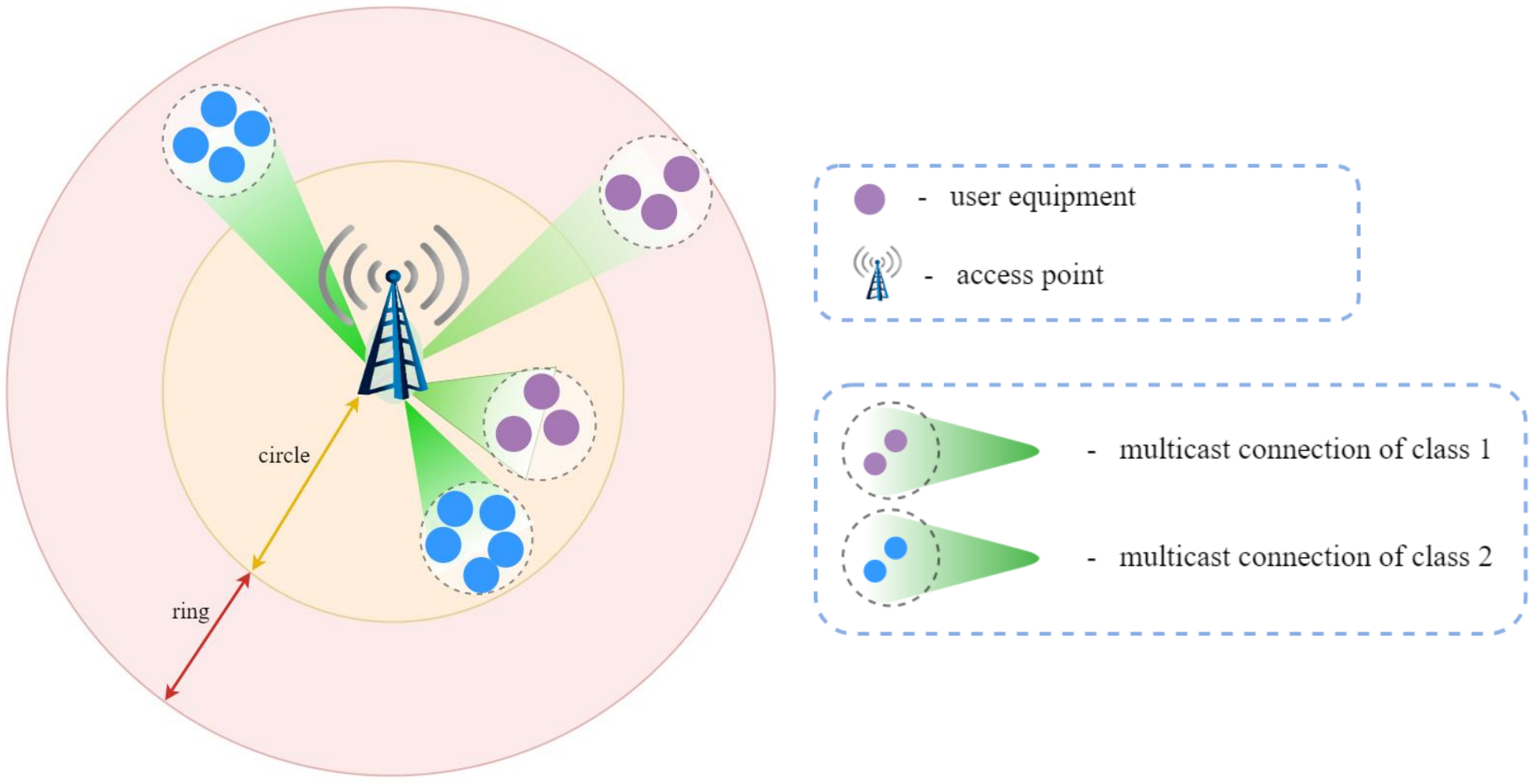
- two classes of requests corresponding to two services
- two coverage areas corresponding to signal propagation

Tasks

- Constructing the model in the form of Markov process
- Obtaining analytical expressions for stationary probabilities in the product form
- Calculating the main characteristics of the model:
  - blocking probabilities
  - average number of occupied resources
- Conducting a numerical analysis for illustration

System model

- Access point (AP) coverage divided into circle and ring areas
- Random distribution of stationary users generating multicast service requests
- AP dynamically allocates resources to serve multicast requests in both circle and ring areas
- Multicast requests throughout the entire BS coverage area are serviced on the same resource



Performance measures:

- access blocking probabilities
- average number of occupied resources

Mathematical model (1/2)

Simplifying assumptions

1. Modeling the service process using a queuing system with limited resources, considering two classes of multicast traffic in two coverage areas.

2. The system's functioning is represented by a random process, enabling analysis of stationary probabilities and performance metrics.

Parameters of the model

- R – number of resource units available for servicing multicast requests
- m – index of multicast traffic class: $m = 1$ for service 1, $m = 2$ for service 2
- l – index of area: l = 1 for circle, $l = 2$ for ring
- λ_{ml} – arrival intensity for requests of class $m, m = 1,2$, in area $l, l = 1,2$
- μ_{ml} – parameter of the exponential distribution of servicing duration for requests of class $m, m = 1,2$, in area $l, l = 1,2$
- b_{ml} - resource requirements for requests of class $m, m = 1,2$, in area $l, l = 1,2$

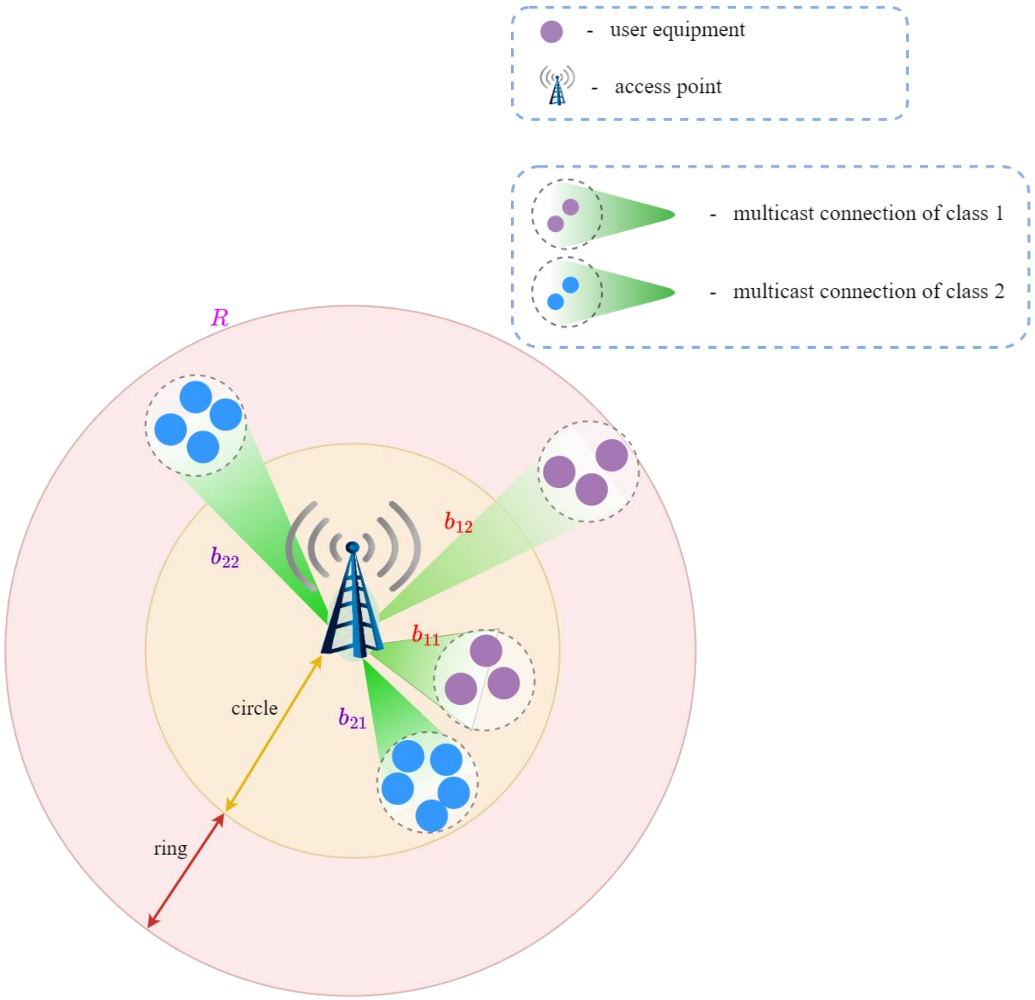
Mathematical model (2/2)

Random process

$$X(t) = \{(I_{11}(t), I_{12}(t), I_{21}(t), I_{22}(t)), t \geq 0\}, \qquad (1)$$

where:

- $I_{ml}(t) \in \{0,1\}$ –
indicator of the multicast connection of class m in area l
- $m \in \{1,2\}$ – class of multicast traffic
- $l \in \{1,2\}$ – area

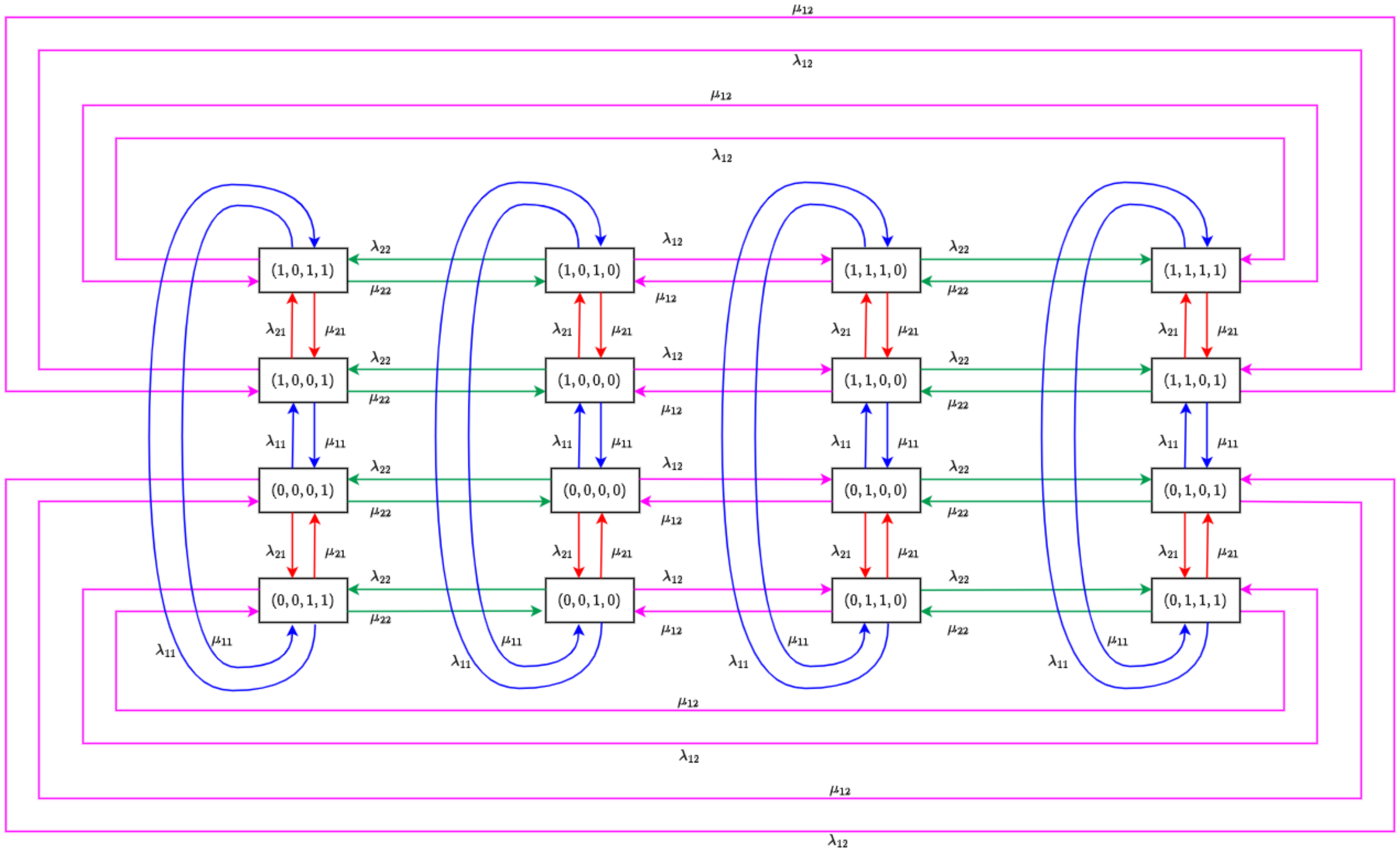


State Space

$$\mathcal{X} = \{\mathbf{x} = (i_{11}, i_{12}, i_{21}, i_{22}): \quad i_{ml} \in \{0,1\}, m, l = 1, 2,$$
$$i_{11}b_{11} + i_{12}(b_{12} - b_{11}) + i_{21}b_{21} + i_{22}(b_{22} - b_{21}) \leq R\} \qquad (2)$$

State Transition Graph with Transition Intensities

- λ_{ml} – arrival intensities for multicast requests of class m in area l
- μ_{ml} – parameters of the exponential distribution of service durations for multicast requests of class m in area l
- – transitions in the circle for class 1 , occur when a request of class 1 either arrives or departs the circle, without affecting the state of class 1 in the ring
- – transitions in the ring for class 1, triggered by the arrival or departure of class 1 requests in the ring, which do not impact the state of class 1 requests in the circle
- – transitions in the circle for class 2, occur when a request of class 2 either arrives or departs the circle, without affecting the state of class 2 in the ring
- – transitions in the ring for class 2, triggered by the arrival or departure of class 2 requests in the ring, which do not impact the state of class 2 requests in the circle



System of Equilibrium Equations

$p(\mathbf{x}) = p(i_{11}, i_{12}, i_{21}, i_{22}) = \lim_{t\to\infty} P\{\mathcal{X}(t) = (i_{11}, i_{12}, i_{21}, i_{22})\}$ – stationary probability of state $\mathbf{x} = (i_{11}, i_{12}, i_{21}, i_{22}) \in \mathcal{X}$

$$\begin{cases}
(\lambda_{11}+\lambda_{12}+\lambda_{21}+\lambda_{22})p(0,0,0,0) = \mu_{11}p(1,0,0,0)+\mu_{12}p(0,1,0,0)+\mu_{21}p(0,0,1,0)+\mu_{22}p(0,0,0,1),\\
(\lambda_{11}+\lambda_{12}+\lambda_{21}+\mu_{22})p(0,0,0,1) = \mu_{11}p(1,0,0,1)+\mu_{12}p(0,1,0,1)+\mu_{21}p(0,0,1,1)+\lambda_{22}p(0,0,0,0),\\
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(\lambda_{11}+\mu_{12}+\lambda_{21}+\mu_{22})p(0,1,0,1) = \mu_{11}p(1,1,0,1)+\lambda_{12}p(0,0,0,1)+\mu_{12}p(0,1,1,1)+\lambda_{22}p(0,1,0,0),\\
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(\mu_{11}+\lambda_{12}+\lambda_{21}+\lambda_{22})p(1,0,0,0) = \lambda_{11}p(0,0,0,0)+\mu_{12}p(1,1,0,0)+\mu_{21}p(1,0,1,0)+\mu_{22}p(1,0,0,1),\\
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(\mu_{11}+\mu_{12}+\mu_{21}+\lambda_{22})p(1,1,1,0) = \lambda_{11}p(0,1,1,0)+\lambda_{12}p(1,0,1,0)+\lambda_{21}p(1,1,0,0)+\mu_{22}p(1,1,1,1),\\
(\mu_{11}+\mu_{12}+\mu_{21}+\mu_{22})p(1,1,1,1) = \lambda_{11}p(0,1,1,1)+\lambda_{12}p(1,0,1,1)+\lambda_{21}p(1,1,0,1)+\lambda_{22}p(1,1,1,0)
\end{cases} \qquad (3)$$

Stationary Probability Distribution

The stationary probability of state $\mathbf{x}$, $\mathbf{x} = (i_{11}, i_{12}, i_{21}, i_{22}) \in \mathcal{X}$, is defined as follows:

$$p(\mathbf{x}) = \frac{\prod_{l=1}^{2} \prod_{m=1}^{2} \rho_{ml}^{i_{ml}}}{\sum_{(j_{11}, j_{12}, j_{21}, j_{22}) \in \mathcal{X}} \prod_{l=1}^{2} \prod_{m=1}^{2} \rho_{ml}^{j_{ml}}}, \quad \mathbf{x} = (i_{11}, i_{12}, i_{21}, i_{22}) \in \mathcal{X} \tag{4}$$

Blocking probabilities

Number of occupied resources $$U(\mathbf{x}) = i_{11}b_{11} + i_{12}(b_{12} - b_{11}) + i_{21}b_{21} + i_{22}(b_{22} - b_{21}) \quad (5)$$

Blocking spaces:

- In circle $$\mathcal{B}_{m1} = \{\mathbf{x} \in \mathcal{X}: i_{m2} = 0: U(\mathbf{x}) + b_{m1} > R\} \quad (6)$$
- In ring $$\mathcal{B}_{m2} = \{\mathbf{x} \in \mathcal{X}: U(\mathbf{x}) + (b_{m2} - i_{m1}b_{m1}) > R\} \quad (7)$$

Blocking probabilities:

- In circle $$\mathcal{B}_{m1} = \sum_{\mathbf{x} \in \mathcal{B}_{m1}} p(\mathbf{x}) \quad (8)$$
- In ring $$\mathcal{B}_{m2} = \sum_{\mathbf{x} \in \mathcal{B}_{m2}} p(\mathbf{x}) \quad (9)$$

Average number of occupied resources:

$$R = \sum_{\mathbf{x} \in \mathcal{X}} U(\mathbf{x}) p(\mathbf{x}) \quad (10)$$

Numerical analysis: input parameters

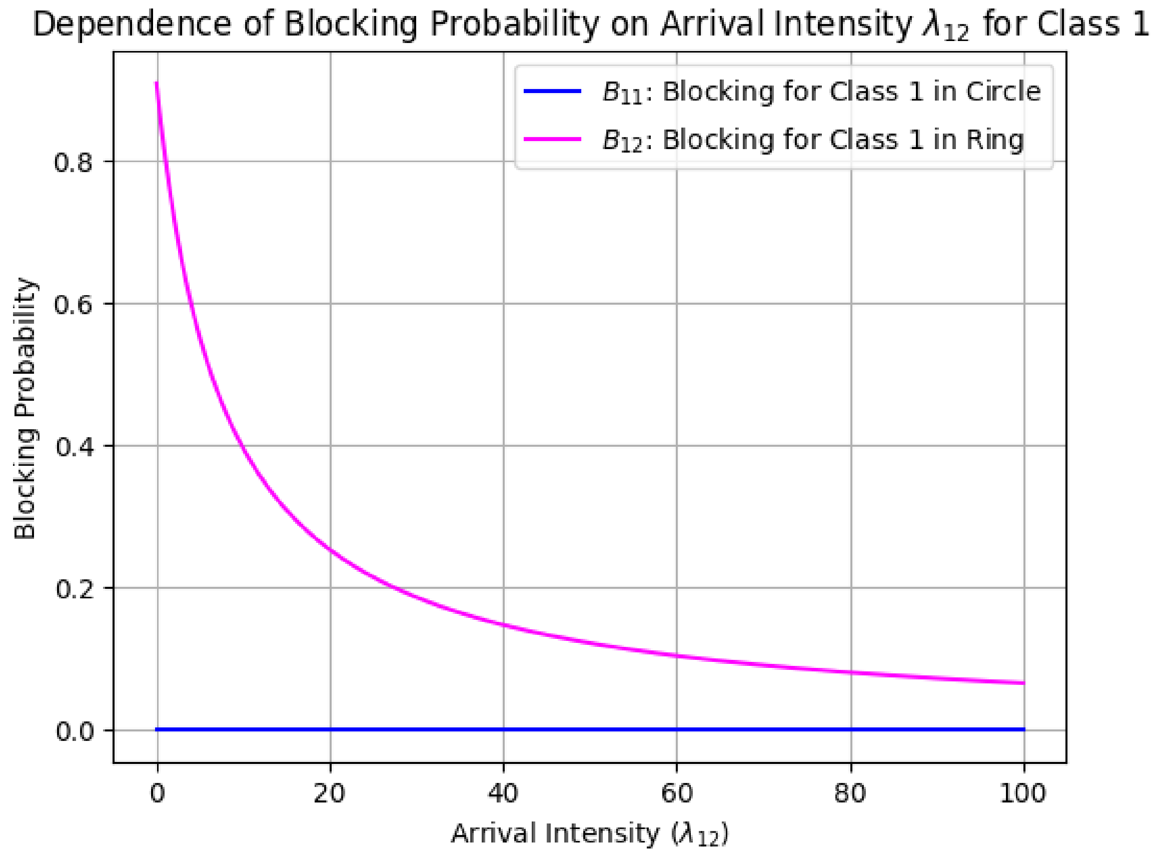
$R = 5$ – number of resources available for multicast requests service

$b_{11} = 3$ – resource requirement for class 1 ($m = 1$) in ring area ($l = 1$)
$b_{12} = 5$ – resource requirement for class 1 ($m = 1$) in circle area ($l = 2$)
$b_{21} = 2$ – resource requirement for class 2 ($m = 2$) in ring area ($l = 1$)
$b_{22} = 6$ – resource requirement for class 2 ($m = 2$) in circle area ($l = 2$)

$\mu_{11} = 0.4$ – intensity of service for class 1 ($m = 1$) in ring area ($l = 1$)
$\mu_{12} = 0.7$ – intensity of service for class 1 ($m = 1$) in circle area ($l = 2$)
$\mu_{21} = 0.5$ – intensity of service for class 2 ($m = 2$) in ring area ($l = 1$)
$\mu_{22} = 0.3$ – intensity of service for class 2 ($m = 2$) in circle area ($l = 2$)

Numerical analysis:
class 1 blocking probability

The dependence of the class 1 blocking probability on the intensity of request arrival λ_{12}



$\lambda_{11} = 4$

$\lambda_{21} = 5$

$\lambda_{22} = 6$

$R = 5$

$\mu_{11} = 0.4$ $\quad b_{11} = 3$

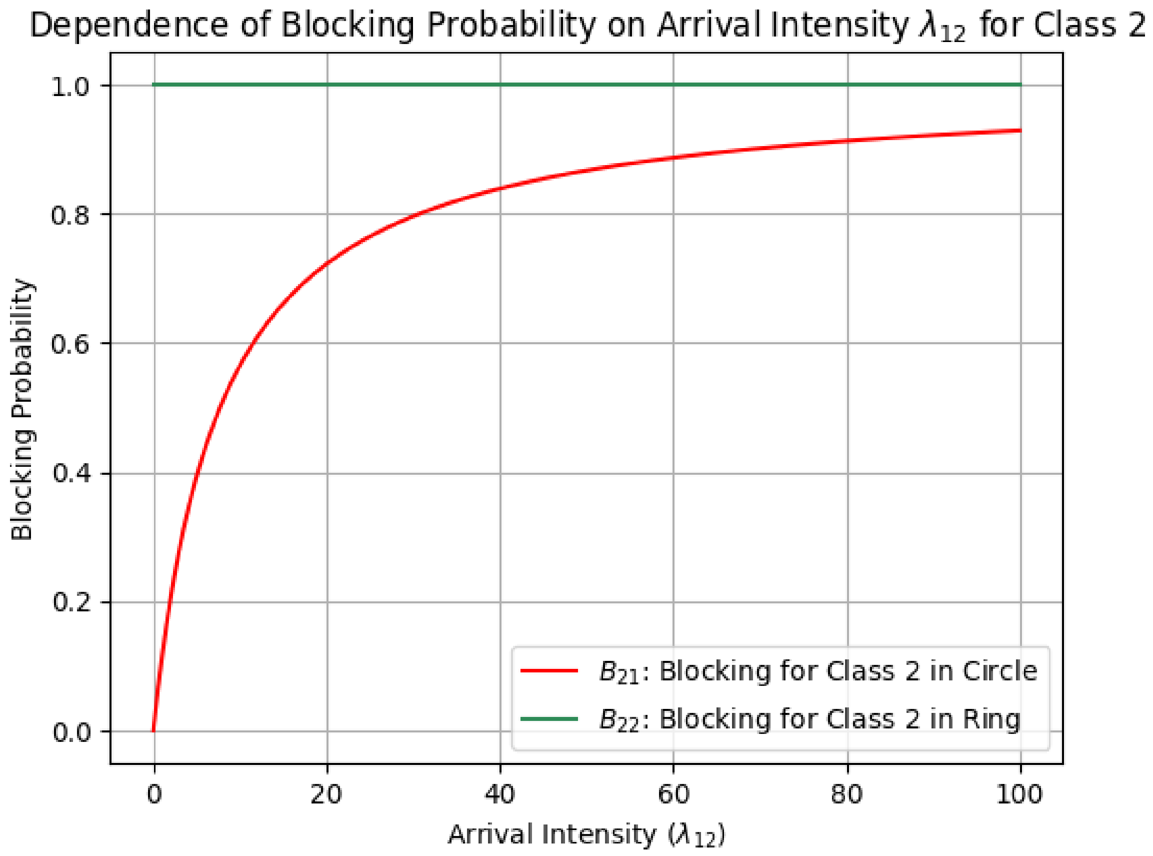
$\mu_{12} = 0.7$ $\quad b_{12} = 5$

$\mu_{21} = 0.5$ $\quad b_{21} = 2$

$\mu_{22} = 0.3$ $\quad b_{22} = 6$

Numerical analysis:
class 2 blocking probability

The dependence of the class 2 blocking probability on the intensity of request arrival λ_{12}



$\lambda_{11} = 4$

$\lambda_{21} = 5$

$\lambda_{22} = 6$

$R = 5$

$\mu_{11} = 0.4$ $\quad b_{11} = 3$

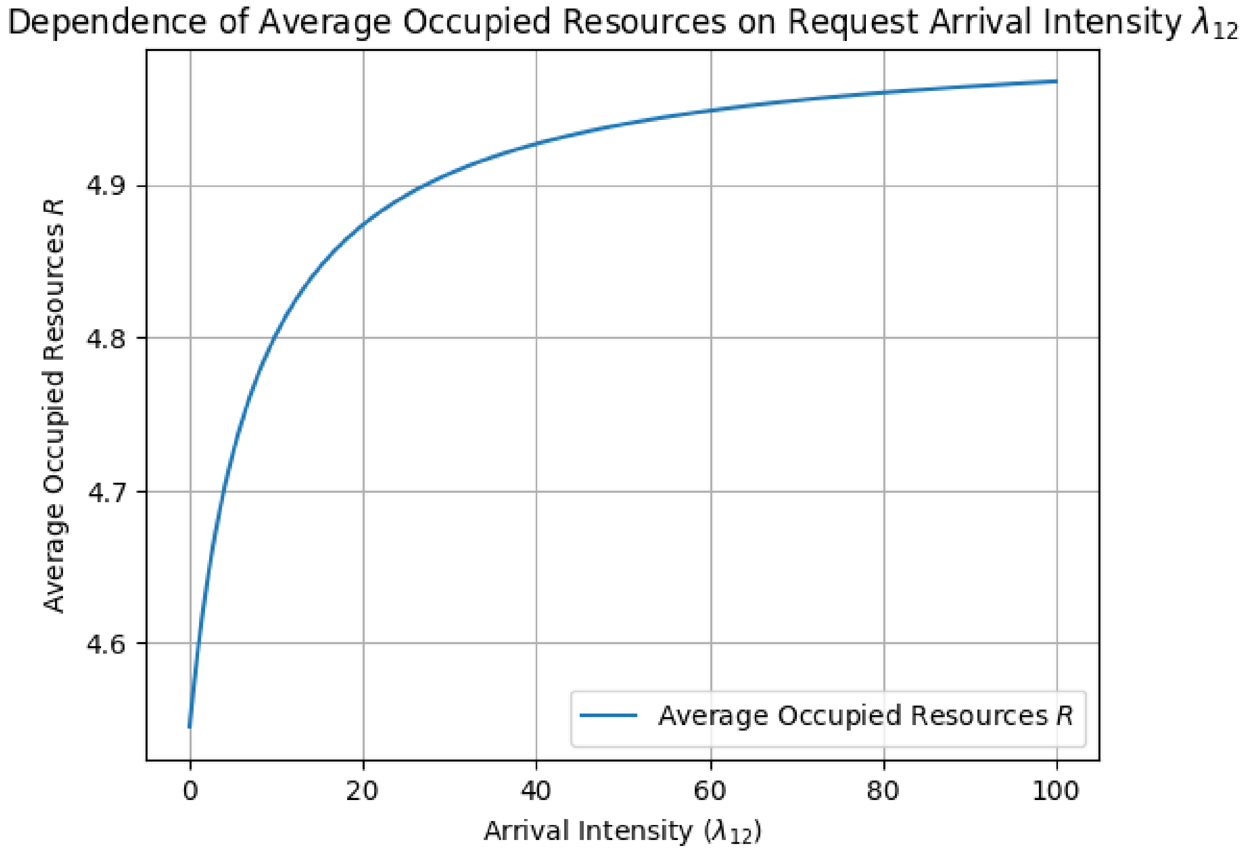
$\mu_{12} = 0.7$ $\quad b_{12} = 5$

$\mu_{21} = 0.5$ $\quad b_{21} = 2$

$\mu_{22} = 0.3$ $\quad b_{22} = 6$

Numerical analysis: average number of occupied resources

The dependence of average number of occupied resources on the intensity of request arrival λ_{12}



$\lambda_{11} = 4$

$\lambda_{21} = 5$

$\lambda_{22} = 6$

$R = 5$

$\mu_{11} = 0.4$ $\quad b_{11} = 3$

$\mu_{12} = 0.7$ $\quad b_{12} = 5$

$\mu_{21} = 0.5$ $\quad b_{21} = 2$

$\mu_{22} = 0.3$ $\quad b_{22} = 6$

Conclusion

- A mathematical model servicing two classes of multicast traffic across different areas of a single access point is developed, enabling the analysis of stationary probabilities and blocking scenarios, with its novelty lying in handling multiple traffic classes.

Tasks for further research

- Expanding the model to encompass M classes of multicast traffic across L access points, integrating maintenance blockers, and incorporating unicast traffic analysis