

## Australian National University

#### Workshop on Decomposition Methods

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# Kitagawa and Vaupel & Canudas-Romo Decomposition

**CDR** 

## $CDR_{JPN} - CDR_{KOR} =$

$$\sum_{x} \Delta m_x \bar{c_x} + \sum_{x} \bar{m}_x \Delta c_x$$

#### Direct component

$$\Delta m_x =$$

#### Direct component

$$\overline{c}_x =$$

#### Indirect component

$$\Delta c_x =$$

#### Indirect component

$$\overline{m}_x =$$

#### CDR decomposition

Table 1: CDR decomposition (Kitagawa)

CDR in Japan	11.08
CDR in Korea	5.94
Difference	5.14
Direct component	
Indirect component	
Total estimated difference	

Note: Results are multiplied by 1000

#### Population growth rates

$$\bar{r}(t) = \int_0^\omega r(x,t)c(x,t).$$

Sex-gap in growth rate

$$\Delta \bar{r}(t) =$$

#### Sex-gap in growth rate

Table 2: Growth Difference between Male and Female, Japan

	Japan
Growth of Male Population 2010-2020	-0.22
Growth of Female Population 2010-2020	-0.18
Observed difference	-0.04
Direct Component	
Indirect Component	
Estimated Difference	

Note: Results are multiplied by 1000





## DEMOGRAPHIC RESEARCH

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Decomposing demographic change into direct vs. compositional components

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#### Abstract

We present and prove a formula for decomposing change in a population average into two components. One component captures the effect of direct change in the characteristic of interest, and the other captures the effect of compositional change. The decomposition is applied to time derivatives of averages over age and over subpopulations.



















#### Vaupel & Canudas-Romo Decomposition

$$\bar{v}(t) = \frac{\int_0^\omega v(x,t) w(x,t) dx}{\int_0^\omega w(x,t) dx}$$

#### **Derivatives**

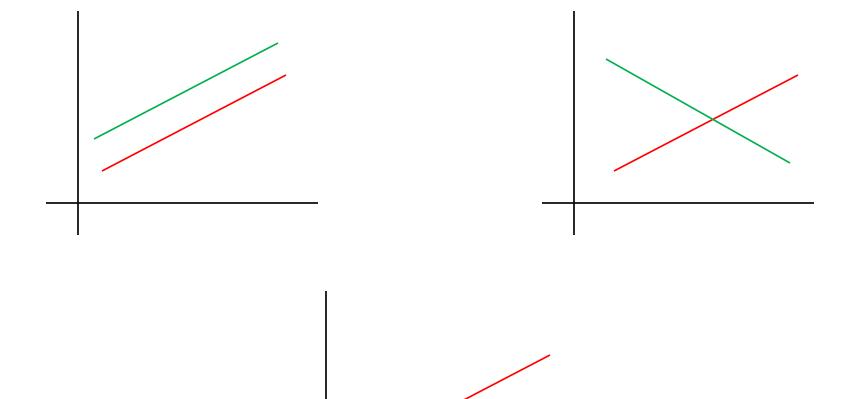
$$\dot{v}(x,t) = \frac{\partial}{\partial t}v(x,t)$$

#### Relative derivatives

$$\label{eq:weights} \acute{w}(x,t) = \frac{\frac{\partial}{\partial t} w(x,t)}{w(x,t)}$$

#### VCR decomposition

## **CO-variance**



#### Relative derivatives estimation

$$\acute{v}(x,t) = \frac{\frac{\partial}{\partial t} v(x,t)}{v(x,t)}$$

#### Relative derivatives estimation

$$\dot{v}(x,t) = \frac{\partial}{\partial t} \ln[v(x,t)]$$

#### Relative derivatives estimation

$$\dot{v}(x,t) \approx \ln \left[ \frac{v(x,t+h)}{v(x,t)} \right] / h$$

#### Mid point estimation

$$v\left(x,t+\frac{h}{2}\right) \approx$$

$$[v(x,t)v(x,t+h)]^{\frac{1}{2}}$$

#### Derivatives estimation

$$\dot{v}(x,t) = \acute{v}(x,t)v\left(x,t + \frac{h}{2}\right)$$

#### Crude Death Rate

#### **CDR**

Table 3: CDR decomposition (VCR)

	Korea
CDR in 2010	4.54
CDR in 2020	5.42
CDR annualized difference	0.09
Direct component	
Compositional component	
Total estimated difference	

Note: Results are multiplied by 1000

#### GFR decomposition

#### GFR decomposition

Table 4: GFR decomposition (VCR)

	Korea
GFR in 2010	28.24
GFR in 2020	18.06
GFR annualized difference	-1.01
Direct component	
Compositional component	
Total estimated difference	

Note: Results are multiplied by 1000

Mean Age of the Population (MAP)

$$\bar{a} = \frac{\int_0^\omega a P(a,t) da}{\int_0^\omega P(a,t) da}$$

#### Only covariance

$$\dot{\bar{a}} = Cov(a,r)$$

#### MAP decomposition

Table 5: MAP decomposition (VCR)

	Korea
MAP for Korea	43.14
MAP for Japan	48.72
Difference in MAP	
Estimated Difference	

Note: Results are multiplied by 1000

#### Assignment 2

Select one of the measures in the examples of this section and apply to a different population from HMD or HFD.

Submit ONE page: one Figure (or Table) and a brief paragraph describing the results that you find.