ch_15_assignment

May 9, 2023

Ch_15_assignment

[ ]:
```
from IPython.core.interactiveshell import InteractiveShell
InteractiveShell.ast_node_interactivity = 'all'
```

1 Equation Solving

1.1 Importing Modules

[ ]:
```
from scipy import linalg as la
from scipy import optimize
import sympy
sympy.init_printing()
import numpy as np
import matplotlib.pyplot as plt
```

1.2 Inner Product

[ ]:
```
# Innervect.py

import numpy as np

def innerVec(a,b):
    rowA = a.shape[0]
    rowB = b.shape[0]

    if (rowA != rowB):
        print('Wrong input data')
        return None

    c = 0.0
    for i in range(rowA):
        c += a[i]*b[i]
    return c

if __name__ == '__main__':
```

```
    a = np.array([1.1, 1.2, 1.3])
    b = np.array([2.1, 2.2, 2.3])

    c = innerVec(a, b)
    print("Inner product of vectors=", c)
```

```
Inner product of vectors= 7.940000000000001
```

[ ]:
```
# InnerVect2.py

import numpy as np

a = np.array([1.1, 1.2, 1.3])
b = np.array([2.1, 2.2, 2.3])

c = np.dot(a, b)
print("inner product = ", c)
```

```
inner product =  7.94
```

1.3 Square Systems

[ ]:
```
A = sympy.Matrix([[2, 3],[5, 4]])
B = sympy.Matrix([4, 3])

A.rank()

A.condition_number()

sympy.N(_)

A.norm()
```

[ ]: 2

[ ]:
$$\frac{\sqrt{2\sqrt{170}+27}}{\sqrt{27-2\sqrt{170}}}$$

[ ]: 7.58240137440151

[ ]: $3\sqrt{6}$

[ ]:
```
A = np.array([[2, 3], [5, 4]])
b = np.array([4, 3])

np.linalg.matrix_rank(A)

np.linalg.cond(A)
```

```
np.linalg.norm(A)
```

[ ]: 2

[ ]: 7.58240137440151

[ ]: 7.34846922834953

1.3.1 Gaussian Elimination

[ ]:
```
# GaussElimin.py
'''
x = GaussElimin(A, b)
    Equiation Ax=b
'''

import numpy as np

def GaussElimin(A, b):
    n = len(b)

    # Elimination
    for k in range(0, n-1):
        for i in range(k+1, n):
            if A[i, k ] != 0.0:
                lam = A[i, k] / A[k, k]
                A[i, k+1:n] -= lam * A[k, k+1:n]
                b[i] -= lam *b[k]

    # Substition
    for k in range(n-1, -1, -1):
        b[k] = (b[k] - np.dot(A[k, k+1:n], b[k+1:n])) / A[k, k]

    return b
```

[ ]: '\nx = GaussElimin(A, b)\n   Equiation Ax=b\n'

[ ]:
```
# GaussElimin_examply.py
import numpy as np
from GaussElimin import *

if __name__ == '__main__':
    A = np.array([[6.0, -4.0, 1.0],
                  [-4.0, 6.0, -4.0],
                  [1.0, -4.0, 6.0]])
    b = np.array([-14.0, 36.0, 6.0])

    Aorg = A.copy()
```

```
    borg = b.copy()

    x = GaussElimin(A, b)

    print('\nresult: x=', x)
    print('\ncheck: Ax-b =', np.dot(Aorg, x) - borg)
```

```
result: x= [10. 22. 14.]

check: Ax-b = [ 1.77635684e-15  7.10542736e-15 -3.55271368e-15]
```

1.3.2 LU decomposion

[ ]:
```
A = sympy.Matrix([[2, 3],[5, 4]])
b = sympy.Matrix([4, 3])
L, U, _ = A.LUdecomposition()

L

U
L*U

x = A.solve(b); x
```

[ ]: $\begin{bmatrix} 1 & 0 \\ \frac{5}{2} & 1 \end{bmatrix}$

[ ]: $\begin{bmatrix} 2 & 3 \\ 0 & -\frac{7}{2} \end{bmatrix}$

[ ]: $\begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}$

[ ]: $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

[ ]:
```
A = np.array([[2, 3], [5, 4]])
b = np.array([4, 3])
P, L, U = la.lu(A)

L
U
P.dot(L.dot(U))

la.solve(A, b)
```

[ ]:
```
array([[1. , 0. ],
       [0.4, 1. ]])
```

```
[ ]: array([[5. , 4. ],
            [0. , 1.4]])
```

```
[ ]: array([[2., 3.],
            [5., 4.]])
```

```
[ ]: array([-1.,  2.])
```

1.4 Rectangular Systems

```
[ ]: x_vars = sympy.symbols("x_1, x_2, x_3")
     A = sympy.Matrix([[1, 2, 3], [4, 5, 7]])
     x = sympy.Matrix(x_vars)
     b = sympy.Matrix([7, 8])
     sympy.solve(A*x - b, x_vars)
```

[ ]: $\left\{x_1 : \frac{x_3}{3} - \frac{19}{3},\ x_2 : \frac{20}{3} - \frac{5x_3}{3}\right\}$

1.4.1 Least-square solution

```
[ ]: # la_lstsq_examply.py

     import numpy as np
     from scipy import linalg as linalg

     x = np.array([0, 1, 2, 3])
     y = np.array([-1, 0.2, 0.9, 2.1])

     A = np.vstack([x, np.ones(len(x))])
     B = A.T
     print(np.ones(len(x)))
     print('\n', A)
     print('\n', B)

     sol, r, rank, sv = la.lstsq(B, y)
     print(sol, r, rank, sv)
```

```
[1. 1. 1. 1.]

 [[0. 1. 2. 3.]
 [1. 1. 1. 1.]]

 [[0. 1.]
 [1. 1.]
 [2. 1.]
 [3. 1.]]
[ 1.   -0.95] 0.05 2 [4.10003045 1.09075677]
```

[ ]:
```python
import matplotlib.pyplot as plt
# define true model parameters
x = np.linspace(-1, 1, 100)
a, b, c = 1, 2, 3
y_exact = a + b * x + c * x**2

# simulate noisy data
m = 100
X = 1 - 2 * np.random.rand(m)
Y = a + b * X + c * X**2 + np.random.randn(m)

# fit the data to the model using linear leqst square
A = np.vstack([X**0, X**1, X**2])
sol, r, rank, sv = la.lstsq(A.T, Y)

y_fit = sol[0] + sol[1] * sol[2] * x**2
fig, ax = plt.subplots(figsize=(12, 4))

ax.plot(X, Y, 'go', alpha = 0.5, label = 'Simulated data')
ax.plot(x, y_exact, 'k', lw=2, label = 'True value $y = 1 + 2x + 3x^2$')
ax.plot(x, y_fit, 'b', lw=2, label ='Least square fit')
ax.set_xlabel(r"$x$", fontsize=18)
ax.set_ylabel(r"$y$", fontsize=18)
ax.legend(loc=2)
```

[ ]:
```
[<matplotlib.lines.Line2D at 0x1663b9820>]
```

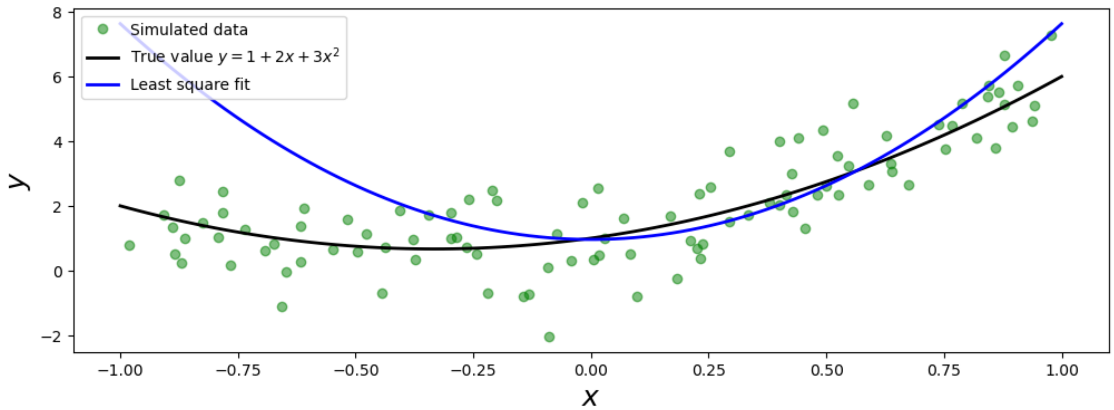
[ ]:
```
[<matplotlib.lines.Line2D at 0x1663b9a60>]
```

[ ]:
```
[<matplotlib.lines.Line2D at 0x1663b9c70>]
```

[ ]:
```
Text(0.5, 0, '$x$')
```

[ ]:
```
Text(0, 0.5, '$y$')
```

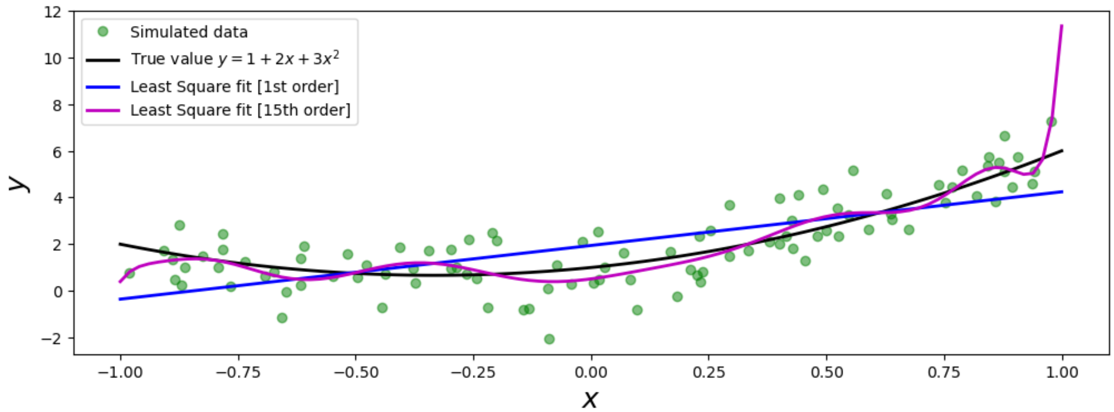
[ ]:
```
<matplotlib.legend.Legend at 0x1663b9c10>
```




```
[ ]: # fit the data to the model using linear least square:
     # ist order polynomial
     A = np.vstack([X**n for n in range(2)])
     sol, r, rank, sv = la.lstsq(A.T, Y)
     y_fit1 = sum([s * x**n for n, s in enumerate(sol)])

     # 15th order polynomial
     A = np.vstack([X**n for n in range(16)])
     sol, r, rank, sv = la.lstsq(A.T, Y)
     y_fit15 = sum([s * x**n for n, s in enumerate(sol)])

     fig, ax = plt.subplots(figsize=(12, 4))
     ax.plot(X, Y, 'go', alpha = 0.5, label='Simulated data')
     ax.plot(x, y_exact, 'k', lw=2, label='True value $y = 1 + 2x + 3x^2$')
     ax.plot(x, y_fit1, 'b', lw=2, label='Least Square fit [1st order]')
     ax.plot(x, y_fit15, 'm', lw=2, label='Least Square fit [15th order]')
     ax.set_xlabel(r"$x$", fontsize=18)
     ax.set_ylabel(r"$y$", fontsize=18)
     ax.legend(loc=2)
```

```
[ ]: [<matplotlib.lines.Line2D at 0x166629850>]

[ ]: [<matplotlib.lines.Line2D at 0x166627910>]

[ ]: [<matplotlib.lines.Line2D at 0x166629a90>]

[ ]: [<matplotlib.lines.Line2D at 0x166629ee0>]

[ ]: Text(0.5, 0, '$x$')

[ ]: Text(0, 0.5, '$y$')
```

[ ]:
```
<matplotlib.legend.Legend at 0x1666238b0>
```



1.5 Eigenvalue Problems

[ ]:
```python
eps, delta = sympy.symbols("epsilon, Delta")
H = sympy.Matrix([[eps, delta], [delta, -eps]])
H

H.eigenvals()

H.eigenvects()
```

[ ]: $$\begin{bmatrix}\epsilon & \Delta\\ \Delta & -\epsilon\end{bmatrix}$$

[ ]: $$\left\{-\sqrt{\Delta^{2}+\epsilon^{2}} : 1,\ \sqrt{\Delta^{2}+\epsilon^{2}} : 1\right\}$$

[ ]: $$\left[\left(-\sqrt{\Delta^{2}+\epsilon^{2}},\ 1,\ \left[\begin{bmatrix}\frac{\epsilon}{\Delta}-\frac{\sqrt{\Delta^{2}+\epsilon^{2}}}{\Delta}\\ 1\end{bmatrix}\right]\right),\ \left(\sqrt{\Delta^{2}+\epsilon^{2}},\ 1,\ \left[\begin{bmatrix}\frac{\epsilon}{\Delta}+\frac{\sqrt{\Delta^{2}+\epsilon^{2}}}{\Delta}\\ 1\end{bmatrix}\right]\right)\right]$$

[ ]:
```python
A = np.array([[1, 3, 5], [3, 5, 3], [5, 3, 9]])
evals, evecs = la.eig(A)
evals

evecs

la.eigvalsh(A)
```

[ ]:
```
array([13.35310908+0.j, -1.75902942+0.j,  3.40592034+0.j])
```

[ ]:
```
array([[ 0.42663918,  0.90353276, -0.04009445],
       [ 0.43751227, -0.24498225, -0.8651975 ],
       [ 0.79155671, -0.35158534,  0.49982569]])
```

[ ]: array([-1.75902942, 3.40592034, 13.35310908])

Reference * Title: Physics Programming Lecture Note (INU) * Author: Jeongwoo Kim, Ph.D. * Availability: https://sites.google.com/view/jeongwookim

---