ch_17_assignment

May 22, 2023

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[ ]:
```
from IPython.core.interactiveshell import InteractiveShell
InteractiveShell.ast_node_interactivity = 'all'
```

1 Ordinary Differential Equation

1.1 Population Growth Programming

[ ]:
```
import numpy as np
```

[ ]:
```
N_0 = int(input('Give initial populatioin size N_0: '))
r = float(input('Give net growth rate r: '))
dt = float(input('Give time step size: '))
N_t = int(input('Give number of steps: '))

print("N_0 : %.2f, r : %.2f, dt : %.2f, N_t: %.2f" %(N_0, r, dt, N_t))
from numpy import linspace, zeros
t = linspace(0, (N_t+1) * dt, N_t+2)
N = zeros(N_t+2)

N[0] = N_0
for n in range(N_t+1):
    N[n+1] = N[n] + r*dt*N[n]

import matplotlib.pyplot as plt
numerical_sol = 'bo' if N_t < 70 else 'b-'
plt.plot(t, N, numerical_sol, t, N_0*np.exp(r*t), 'r-')
plt.legend(['Numerical', 'exact'], loc = 'upper left')
plt.xlabel('t'); plt.ylabel('N(t)')
filestem = 'growth1_%dsteps' %N_t
plt.savefig('%s.png' % filestem); plt.savefig('%s.pdf' % filestem)
```

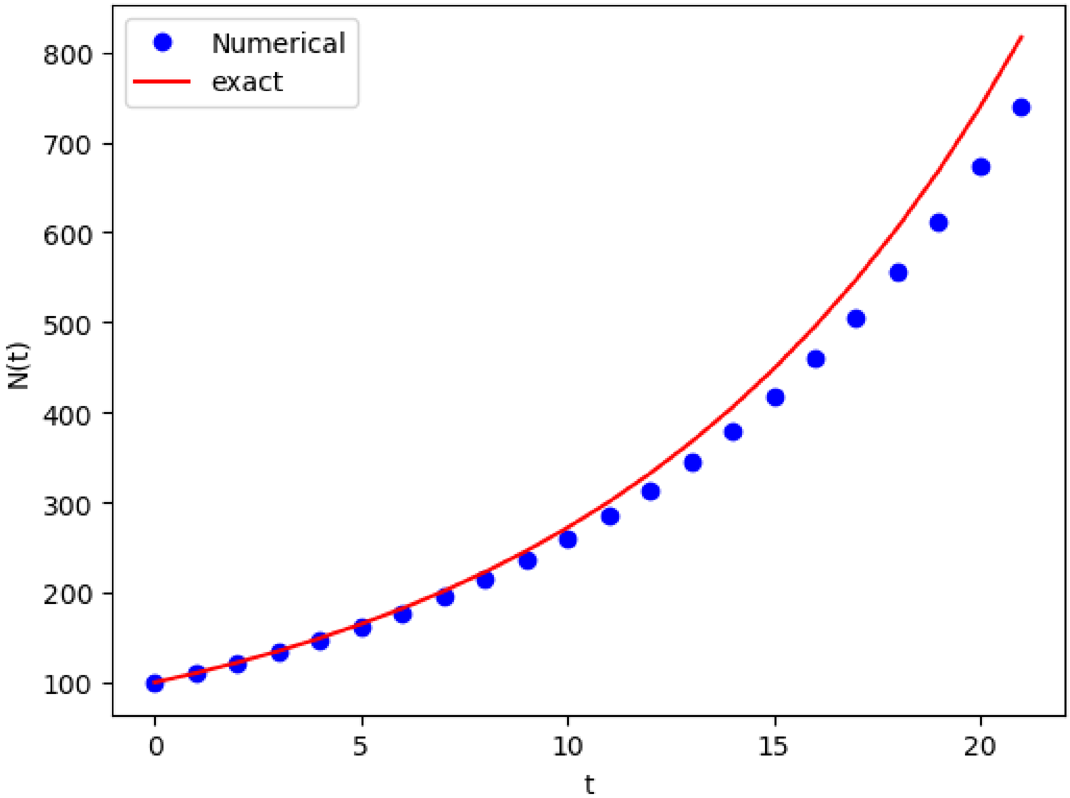
```
N_0 : 100.00, r : 0.10, dt : 1.00, N_t: 20.00
```

```
[ ]: [<matplotlib.lines.Line2D at 0x157539040>,
      <matplotlib.lines.Line2D at 0x157539100>]
```

```
[ ]: <matplotlib.legend.Legend at 0x1524bf970>
```

```
[ ]: Text(0.5, 0, 't')
```

```
[ ]: Text(0, 0.5, 'N(t)')
```

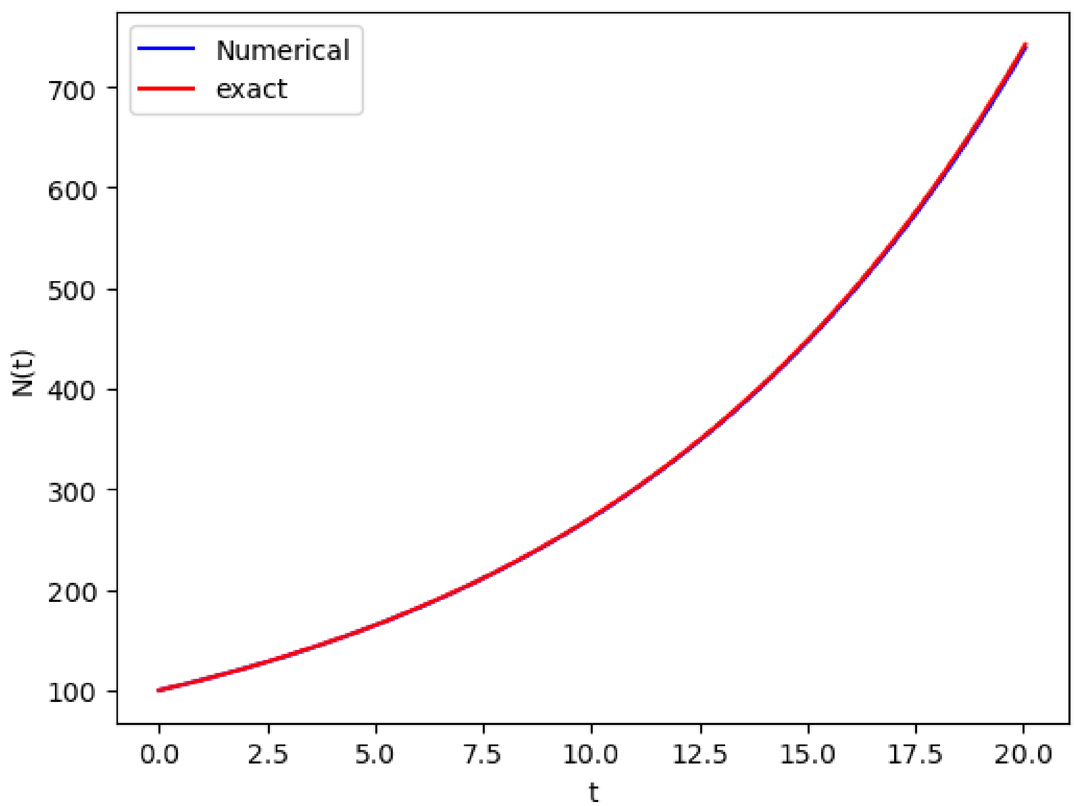



```
[ ]: N_0 = int(input('Give initial populatioin size N_0: '))
     r = float(input('Give net growth rate r: '))
     dt = float(input('Give time step size: '))
     N_t = int(input('Give number of steps: '))

     print("N_0 : %.2f, r : %.2f, dt : %.2f, N_t: %.2f" %(N_0, r, dt, N_t))
     from numpy import linspace, zeros
     t = linspace(0, (N_t+1) * dt, N_t+2)
     N = zeros(N_t+2)

     N[0] = N_0
     for n in range(N_t+1):
```

```
    N[n+1] = N[n] + r*dt*N[n]

import matplotlib.pyplot as plt
numerical_sol = 'b-' if N_t < 70 else 'b-'
plt.plot(t, N, numerical_sol, t, N_0*np.exp(r*t), 'r-')
plt.legend(['Numerical', 'exact'], loc = 'upper left')
plt.xlabel('t'); plt.ylabel('N(t)')
filestem = 'growth2_%dsteps' %N_t
plt.savefig('%s.png' % filestem); plt.savefig('%s.pdf' % filestem)
```

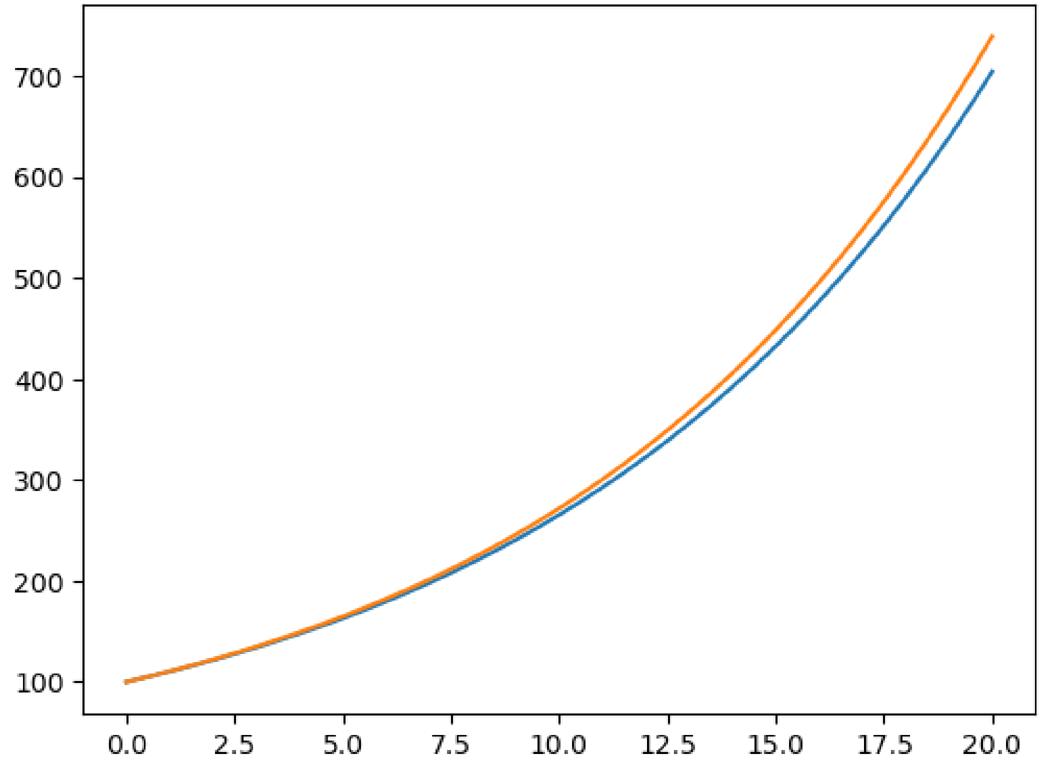
```
N_0 : 100.00, r : 0.10, dt : 0.05, N_t: 400.00
```

```
[ ]: [<matplotlib.lines.Line2D at 0x157594400>,
      <matplotlib.lines.Line2D at 0x1575944f0>]

[ ]: <matplotlib.legend.Legend at 0x1524f1c70>

[ ]: Text(0.5, 0, 't')

[ ]: Text(0, 0.5, 'N(t)')
```



1.2 General ODE case

[ ]:
```
# ode_FE.py
from numpy import linspace, zeros, exp
import matplotlib.pyplot as plt

def ode_FE(f, U_0, dt, T):
    N_t = int(round(float(T)/dt))
    u = zeros(N_t+1)
    t = linspace(0, N_t *dt, len(u))
    u[0] = U_0
    for n in range(N_t):
        u[n+1] = u[n] + dt*f(u[n], t[n])
    return u, t

def demo_polulation_growth():
    """Test case : u=r*u, u(0) = 100 """

    def f(u, t):
        return 0.1*u

    u, t = ode_FE(f=f, U_0 = 100, dt = 0.5, T=20)
    plt.plot(t, u, t, 100*exp(0.1*t))
    plt.show()

if __name__ == '__main__':
    demo_polulation_growth()
```



1.2.1 More realistic model

[ ]:
```
from ode_FE import ode_FE
import matplotlib.pyplot as plt

for dt, T in zip((0.5, 20), (60, 100)):
    u, t = ode_FE(f=lambda u, t: 0.1*(1-u/500.) *u,
                  U_0 = 100, dt=dt, T=T)
    plt.figure() # Make separate figures for each pass in the loop
    plt.plot(t, u, 'b-')
    plt.xlabel('t'); plt.ylabel('N(t)')
    plt.savefig('tmp_%g.png' %dt); plt.savefig('tmp_%g.pdf' %dt)
```

[ ]:
```
<Figure size 640x480 with 0 Axes>
```

[ ]:
```
[<matplotlib.lines.Line2D at 0x1576c2970>]
```

[ ]:
```
Text(0.5, 0, 't')
```

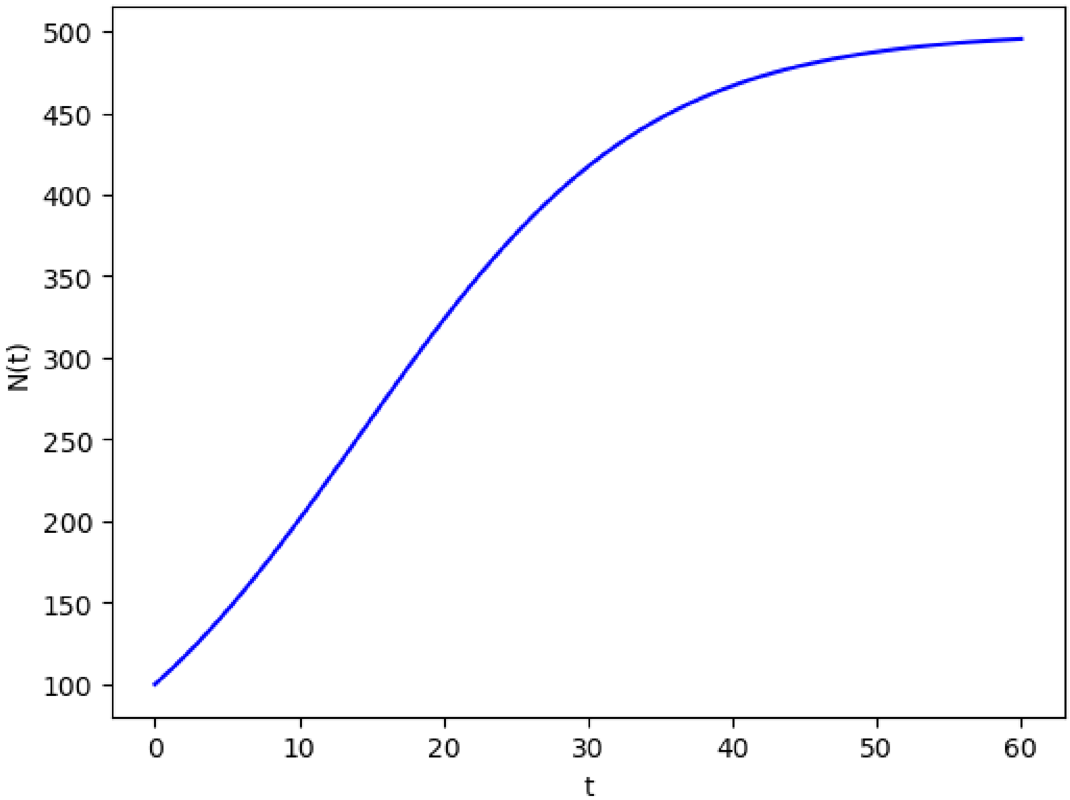
[ ]:
```
Text(0, 0.5, 'N(t)')
```

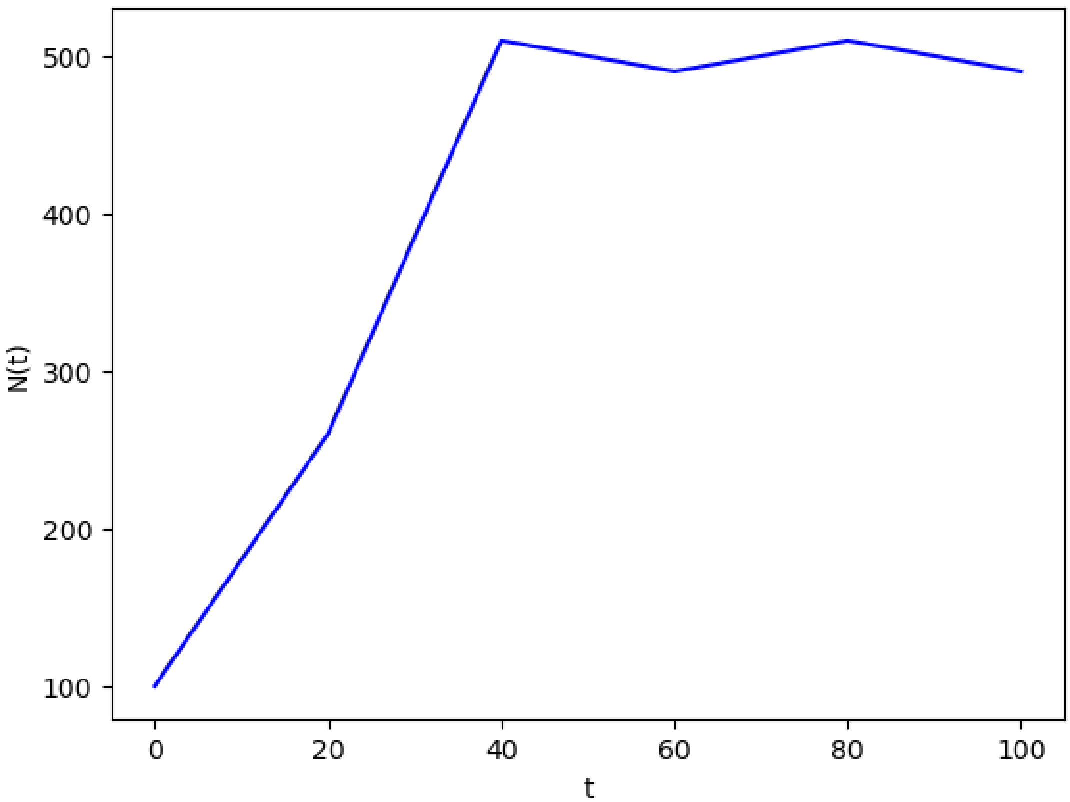
[ ]:
```
<Figure size 640x480 with 0 Axes>
```

```
[ ]: [<matplotlib.lines.Line2D at 0x157770eb0>]
```

```
[ ]: Text(0.5, 0, 't')
```

```
[ ]: Text(0, 0.5, 'N(t)')
```





1.3 Importing Modules

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy import integrate
import sympy

sympy.init_printing()
```

1.4 Symbolic Solution to ODEs

1.4.1 Newton's cooling law

[ ]:
```
t, k, T0, Ta = sympy.symbols("t, k, T_0, T_a")
T = sympy.Function("T")

ode = T(t).diff(t) + k*(T(t) - Ta)
sympy.Eq(ode, 0) # Eq(ode) -> Eq(ode, 0)

ode_sol = sympy.dsolve(ode)
ode_sol
```

[ ]:
$$k\left(-T_a + T(t)\right) + \frac{d}{dt}T(t) = 0$$

[ ]:
$$T(t) = C_1 e^{-kt} + T_a$$

[ ]:
```
ode_sol = sympy.dsolve(ode)
ode_sol

ode_sol.lhs
ode_sol.rhs

ics = {T(0): T0}
ics

C_eq = ode_sol.subs(t, 0).subs(ics)
C_eq
```

[ ]:
$$T(t) = C_1 e^{-kt} + T_a$$

[ ]:
$$T(t)$$

[ ]:
$$C_1 e^{-kt} + T_a$$

[ ]:
$$\{T(0) : T_0\}$$

[ ]:
$$T_0 = C_1 + T_a$$

[ ]:
```
C_sol = sympy.solve(C_eq)
C_sol

ode_sol.subs(C_sol[0])
```

[ ]:
$$[\{C_1 : T_0 - T_a\}]$$

[ ]:
$$T(t) = T_a + (T_0 - T_a)\, e^{-kt}$$

1.5 Numerical Integraion of ODEs

1.5.1 Example.1

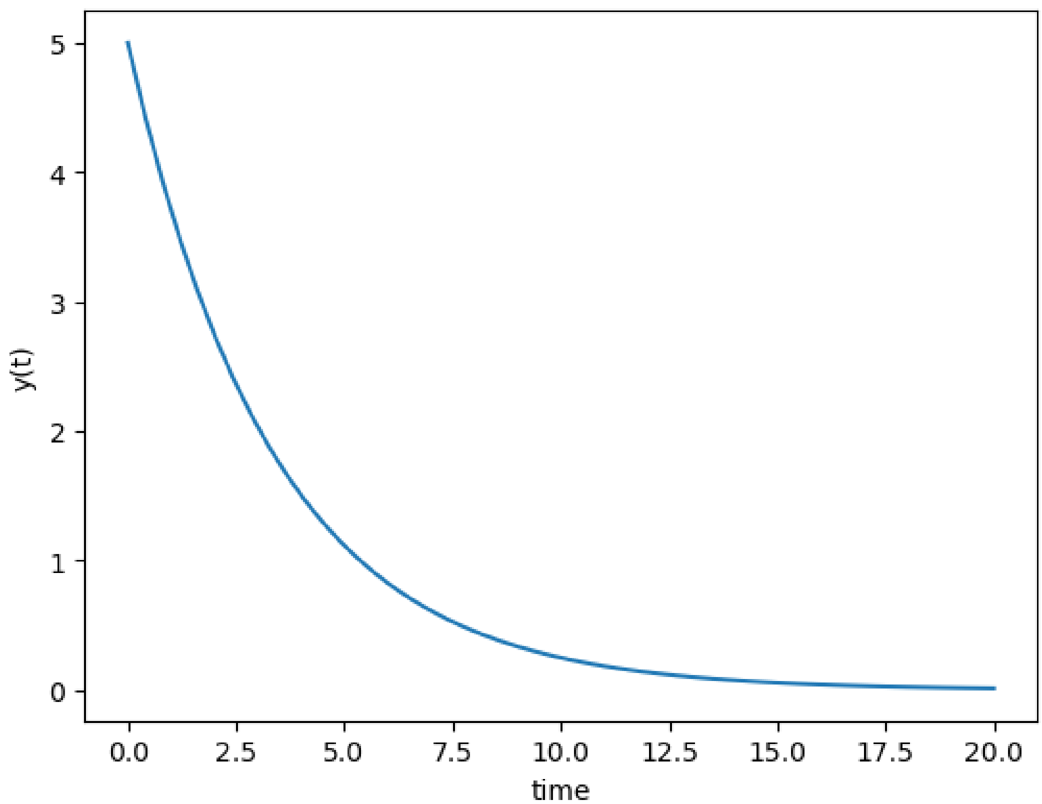
[ ]:
```
import numpy as np
from scipy.integrate import odeint
import matplotlib.pyplot as plt

# function that returns dy/dt
def model(y, t):
    k = 0.3
    dydt = -k * y
    return dydt

# initial condition
```

```
y0 = 5

# time points
t = np.linspace(0, 20)
y = odeint(model, y0, t)

# plot results
plt.plot(t, y)
plt.xlabel('time'); plt.ylabel('y(t)')
plt.show()
```

```
[ ]: [<matplotlib.lines.Line2D at 0x1577f02b0>]
```

```
[ ]: Text(0.5, 0, 'time')
```

```
[ ]: Text(0, 0.5, 'y(t)')
```



1.5.2 Example.2

```
[ ]: # https://colab.research.google.com/github/jrjohansson/
     ↪numerical-python-book-code/blob/master/ch09-code-listing.
     ↪ipynb#scrollTo=sqzc7DWOrWhd
     def  plot_direction_field(x, y_x, f_xy, x_lim=(-5, 5), y_lim=(-5, 5), ax=None):

         f_np = sympy.lambdify((x, y_x), f_xy, 'numpy')

         x_vec = np.linspace(x_lim[0], x_lim[1], 20)
         y_vec = np.linspace(y_lim[0], y_lim[1], 20)

         if ax is None:
             _, ax = plt.subplots(figsize=(4, 4))

         dx = x_vec[1] - x_vec[0]
         dy = y_vec[1] - y_vec[0]

         for m, xx in enumerate(x_vec):
             for n, yy in enumerate(y_vec):
                 Dy = f_np(xx, yy) * dx
                 Dx = 0.8 * dx**2 / np.sqrt(dx**2 + Dy**2)
                 Dy = 0.8 * Dy*dy / np.sqrt(dx**2 + Dy**2)
                 ax.plot([xx - Dx/2, xx + Dx/2],
                         [yy - Dy/2, yy + Dy/2], 'b', lw=0.5)
         ax.axis('tight')

         ax.set_title(r"$%s$" %
                      (sympy.latex(sympy.Eq(y(x).diff(x), f_xy))),
                      fontsize=18)

         return ax
```

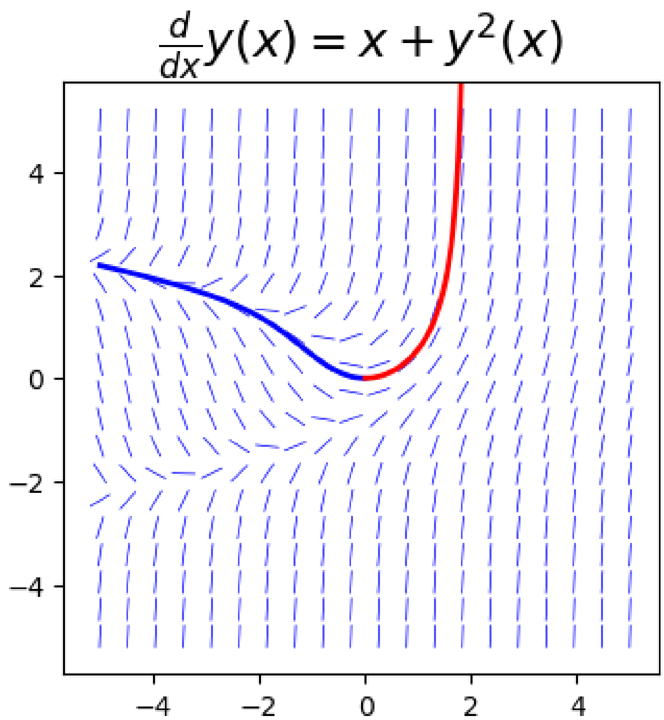
```
[ ]: x = sympy.symbols("x")
     y = sympy.Function("y")
     f = y(x) ** 2 + x

     f_np = sympy.lambdify((y(x), x), f)

     # positive directions
     y0 = 0
     xp = np.linspace(0, 1.9, 100)
     yp = integrate.odeint(f_np, y0, xp)

     # negative directions
     xm = np.linspace(0, -5, 100)
     ym = integrate.odeint(f_np, y0, xm)
```

```
fig, ax = plt.subplots(1, 1, figsize = (4, 4))
plot_direction_field(x, y(x), f, ax= ax)
ax.plot(xm, ym, 'b', lw=2)
ax.plot(xp, yp, 'r', lw=2)
```

[ ]: <Axes: title={'center': '$\\frac{d}{d x} y{\\left(x \\right)} = x + y^{2}{\\left(x \\right)}$'}>

[ ]: [<matplotlib.lines.Line2D at 0x157a5ed00>]

[ ]: [<matplotlib.lines.Line2D at 0x157a5eeb0>]

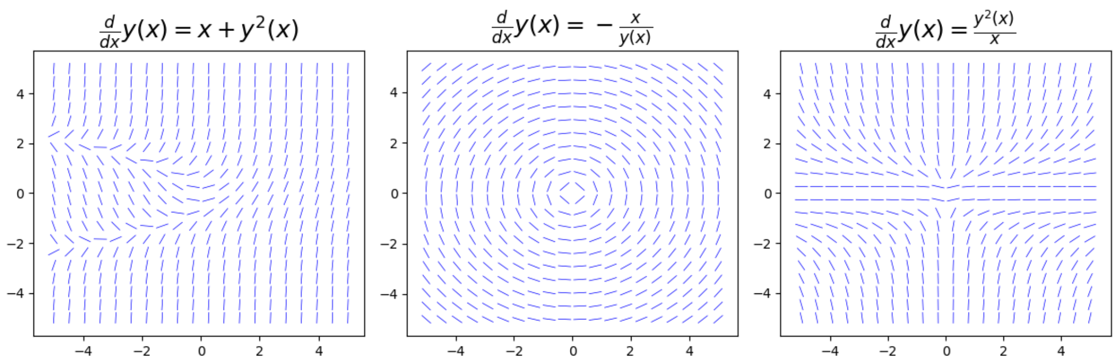


[ ]:
```
fig, axes = plt.subplots(1, 3, figsize=(12, 4))

plot_direction_field(x, y(x), y(x)**2 + x, ax=axes[0])
plot_direction_field(x, y(x), -x / y(x), ax=axes[1])
plot_direction_field(x, y(x), y(x)**2 / x, ax=axes[2])

fig.tight_layout()
# fig.savefig('ch9-direction-field.pdf')
```

```
[ ]: <Axes: title={'center': '$\\frac{d}{d x} y{\\left(x \\right)} = x +
     y^{2}{\\left(x \\right)}$'}>
```

```
[ ]: <Axes: title={'center': '$\\frac{d}{d x} y{\\left(x \\right)} = -
     \\frac{x}{y{\\left(x \\right)}}$'}>
```

```
[ ]: <Axes: title={'center': '$\\frac{d}{d x} y{\\left(x \\right)} =
     \\frac{y^{2}{\\left(x \\right)}}{x}$'}>
```



Reference * Title: Physics Programming Lecture Note (INU) * Author: Jeongwoo Kim, Ph.D. * Availability: https://sites.google.com/view/jeongwookim

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