MODULE II

THE SET OF REAL NUMBERS

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A real number is any number that has a decimal representation

SYMBOL	NAME	DESCRIPTION	EXAMPLE
$\mathbb{N}$	Natural	Counting numbers	1, 2, 3, 4 …
	Whole	Natural numbers and O	0,1,2,3
	Prime number	Natural number greater than 1 and divisible by 1 and itself	2,3,5,7…
	Composite number	Natural number greater than 1 that are not prime	1,4,6,8,9,10
$\mathbb{Z}$	Integers	Whole numbers and negatives	-2,-1,0,1,2
$\mathbb{Q}$	Rational number	Numbers that can be represented In the form $\frac{a}{b}$, *b≠0; a* and *b* are integers	..-2,-1,0,1,2, 3, 4 …
	Irrational number	Numbers that cannot be represented in the form $\frac{a}{b}$, *b≠0; a* and *b* are integers	$\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \pi$,..
$\mathbb{R}$	Real numbers	All the rational and irrational numbers.	...-2,-1, 0, 1, 2, 3, 4, $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \pi$,...

OPERATIONAL WITH REAL NUMBERS

Real numbers can be added, subtracted, multiplied and divided

BASIC PROPERTIES OF THE SET OF REAL NUMBERS

Let R be the set of real numbers, and let a, b, and c be arbitrary elements of R.

ADDITION PROPERTIES

1) Closure : a + b is a unique element in R.
2) Association : $(a+b)-c = a+(b+c)$
3) Commutative : $a+b=b+a$
4) Identity : 0 is the additive identity; that is $0+a=a+0=a$ for all a in R, and 0 is the only element in R with this property.
5) Inverse : For each a in R, (-a) is its unique inverse, that is $a+(-a)=(-a)+a=0$, and $(-a)$ is the only element in R relative to a with this property.

Note that (–a) is not necessarily a negative number, it is positive if a is negative and negative if a is positive.

MULTIPLICATION PROPERTIES

1) Closure : ab is a unique element in R.
2) Associative : $(ab)c = a(bc)$
3) Commutative : $ab = ba$
4) Identity : 1 is the multiplicative identity; that is for any a in R, $1 \times a = a \times 1 = a$, and 1 is the only element in R with this property.
5) Inverse : for each a in R, $a \neq 0, \frac{1}{a}$ is its unique multiplicative inverse, that is $a \times \frac{1}{a} = \frac{1}{a} \times a = 1, and\ \frac{1}{a}$ is the only element in R relative to a with this property
Combined property
6) Distributive:
$a(b+c) = ab + ac$ *and* $(a+b)c = ac + bc$

SUBTRACTION AND DIVISION

Subtraction and division can be defined in terms of addition and multiplication respectively.

For all real numbers a and b

1) Subtraction: $a - b = a + (-b)$

2) Division:

$$b \div a = \sqrt[b]{a} = \frac{a}{b} = a\left(\frac{1}{b}\right)$$

$$where\, b \neq 0$$

Note: Division by 0 is never allowed.

If $a \neq 0, \frac{0}{a} = 0\ but\ \frac{a}{b}$ is undefined; $\frac{a}{0} = \infty(\text{inf}\ inity)$

PROPERTIES OF NEGATIVES

For all real numbers a and b

1) $-(-a) = a$
2) $(-a)b = -(ab) = a(-b) = -ab$
3) $(-a)(-b) = ab$
4) $(-1)a = -a$

5) $\frac{-a}{b} = -\frac{a}{b} = \frac{a}{-b} = \frac{a}{b}, b \neq 0$

6) $\frac{-a}{-b} = -\frac{-a}{b} = \frac{-a}{-b} = \frac{a}{b}; b \neq 0$

ZERO PROPERTIES

For all real numbers a and b

1) $a \times 0 = 0$

$ab = 0 \ \mathit{iff} \ \ a = 0 \, or \, b = 0 \, or \, a = b = 0 (both)$