Link-level Performance Evaluation of SISO/SIMO/MISO

Yang Zhao

I. Introduction

Wireless systems have experienced significant improvement and popularity in the past decades. However, more emerging techniques as artificial intelligence, blockchain, and the Internet of things are rechallenging the network regarding delay, capacity, and reliability. As a feasible solution, multiple-antenna schemes as spatial-temporal beamforming and massive-MIMO techniques have been widely used in contemporary wireless communication systems. By utilising reasonable splitting and combination strategies, they are demonstrated to achieve large data rate, low outage probability, and high spectrum and power efficiency. This article investigates the bit error rate (BER) performance of fundamental single-input single-output (SISO), single-input multi-output (SIMO), and multi-input single-output (MISO) systems based on uncoded and Alamouti quadrature phase shift keying (QPSK) modulation. First, the performance of SISO with Rayleigh fading channels is simulated. Second, we simulate maximal ratio combining (MRC) combination technique on SIMO fading channel. Third, the maximal ratio transmission (MRT) and Alamouti space-time block coding (STBC) scheme are analysed regarding BER and feasibility. Finally, the link level performance of all cases is compared. We also examines the array and diversity gains in SIMO and MISO. It assume that in MRT the perfect channel state information at the transmitter (CSIT) can be obtained, and in other cases the channel state information at the receiver (CSIR) is available.

II. Theory and Methods

A. Key concepts

1) Symbol error rate (SER), bit error rate (BER), and Union Bound: SER can be utilised to approximate BER. If at least one bit within a symbol experience error, a symbol error occurs. For QPSK modulation, the probability that error occurs on both bits leads to the SER being smaller than twice the BER. It can be negligible in most cases, and the error count is approximately doubled.

Boole's inequality suggests that the sum of pairwise error probability $P_e\left(s_j, s_k\right)$ of all possible error cases is greater than the actual SER:

$$P\left(e|s_k\right) \leq \sum_{j=1, j\neq k}^{M} P_e\left(s_j, s_k\right) \tag{1}$$

where $P\left(e|s_k\right)$ is the SER when s_k is transmitted and M is the number of symbols ($M=4$ for QPSK). The right side of the inequation is defined as the union bound and can be used to estimate the actual SER.

In this report, SNR ρ refers to signal-to-noise ratio per bit (i.e. $\frac{E_b}{N_0}$), which is different from the notation in [1]. As a result, all formulas involving SER are replaced by BER instead.

2) Diversity in Multiple Antennas Systems: Fading brings about uncertainty, and the principle of diversity is to produce multiple copies of the original signal via different channels. Signal diversity comes from channel diversity, which can be created in time, frequency, and space domains. With suitable splitting and combining schemes, diversity can be utilised to stabilise the link and reduce BER, and the benefit for SIMO and MISO can be expressed in array gain and diversity gain.

Array gain g_a indicates the growth in average output signal-to-noise ratio (SNR) over the single-branch average SNR. It can be observed from the BER plot as the SNR shift [1]:

$$g_a = \frac{\bar{\rho}_{out}}{\rho} \tag{2}$$

where ρ is single-branch SNR and $\bar{\rho}_{out}$ is the average output SNR.

Diversity gain g_d refers to the increase in the BER slope concerning SNR. It can be obtained by calculating the negative asymptotic slope of the BER curve [1]:

$$g_d = -\frac{\log_2\left(P_e\right)}{\log_2\left(\rho\right)} \tag{3}$$

where P_e is the BER.

B. Single-input Single-output (SISO)

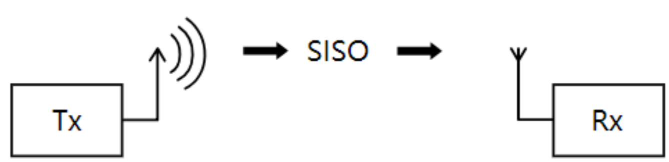

Fig. 1. Illustration of SISO [2]

Figure 1 illustrates a simplified SISO system. In SISO there are one transmit antenna and one receive antenna, hence only one channel and no diversity. The system estimates CSIR and utilises zero-forcing to suppress the influence of fading.

Zero-forcing is a conventional approach in signal recovery. It cancels the impact of fading on the transmitted symbols by weighting $w=\frac{1}{h}$ the received signal with inverse channel response h but the noise n is affected simultaneously, and the signal for detection is $z = wy = c + \frac{n}{h}$.

For QPSK transmission through a SISO Rayleigh fading channel, the BER is bounded by:

$$P_e = \int_0^\infty Q\left(\sqrt{2\rho s}\right) p_s\left(s\right) ds = \frac{1}{2}\left(1 - \sqrt{\frac{\rho}{1+\rho}}\right) \approx \frac{1}{4\rho} \tag{4}$$

with $Q\left(.\right)$ being the q-function, s being the amplitude of channel response, and p_s being the probability density distribution (PDF). Notice that for a SISO Rayleigh fading channel, when SNR is high, BER approximately decreases linearly when the reciprocal of SNR decrease.

C. Single-input Multi-output (SIMO)

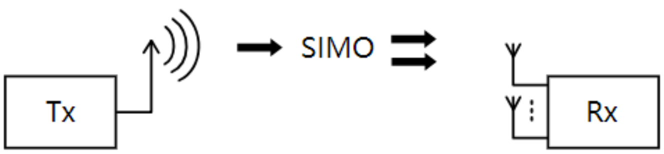

Fig. 2. Illustration of SIMO [2]

We investigate SIMO with two receive antennas as denoted by Figure 2. The diversity combining methods can be categorised into selection combining and gain combing, with the former choosing the branch with the highest SNR among received copies and the latter combining them linearly with specific gain vector. We focus on the MRC strategy that maximises the average output SNR for additive white Gaussian noise (AWGN), although equal gain combining is low-complexity and minimum mean square error (MMSE) combining is optimal with interference.

MRC chooses the weight as $g = h^*$ which is optimum for the channel with AWGN only. By linearly combining all branches with weight g, the merged signal used for detection can be expressed as $z = \mathbf{gy} = \sqrt{E_s}\mathbf{gh}c + \mathbf{gn}$.

It can be concluded that the effect of fading on phase shift is cancelled out, and the transmitted symbol is scaled by a constant. Moreover, the influence of noise is reduced since both noise and channel response are CSCG distributed.

The accurate BER expression is given in [3]:

$$P_e = p^N \sum_{k=0}^{N-1} \begin{pmatrix} N-1+k \\ k \end{pmatrix} (1-p)^k \tag{5}$$

where $p = \frac{1}{2} - \frac{1}{2}\left(1+\frac{1}{\rho}\right)^{-\frac{1}{2}}$ is the PDF and N is the number of receive antennas. For 2-antenna case, it reduces to $P_e = p^2[1+2(1-p)]$ which can be further approximated at high SNR [1]:

$$P_e = (4\rho)^{-N} \begin{pmatrix} 2N-1 \\ N \end{pmatrix} \tag{6}$$

Equation 6 indicates that the array gain g_a and the diversity gain g_d are both N. Note for a SIMO Rayleigh fading channel, N degrees of freedom at the receiver provides N array gain and N diversity gain. Also, BER is approximately proportional to ρ^{-N}, which means quadratic improvement over SNR for the two-antenna receiver.

D. Multi-input Single-output (MISO)

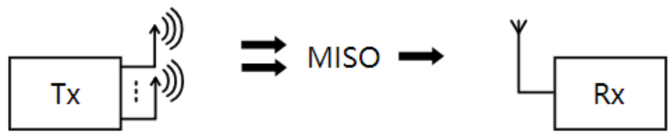

Fig. 3. Illustration of MISO [2]

Consider a two-transmitter MISO system. The diversity is utilised at the transmitter by precoding the data stream on N transmit antennas. Theoretically, the combining strategies of SIMO can be transformed to the splitting approaches of MISO by symmetry. However, obtaining perfect CSIT has been a challenging topic especially for the antennas with relatively high speed. Therefore, the splitting techniques with or without CSIT are divided into two categories.

1) MRT (Perfect CSIT): MRT is the symmetry of MRC. Instead of merging the copies by weight at the receiver, MRT splits the transmitted signal c into multiple streams, then applies different weight w on each branch before transmission. The received signal is directly used for detection. Similar to MRC, the impact of fading on symbols are removed, and the noise tends to be reduced by averaging. The received signal is $y = \sqrt{E_s}\mathbf{hc} + n = \sqrt{E_s}\mathbf{hw}c + n$ where the gain vector that maximise the receive SNR is $\mathbf{w} = \frac{\mathbf{h}^H}{\|\mathbf{h}\|}$. Due to the symmetry, the BER, array gain and diversity gain are expected to be the same with MRC.

2) Alamouti Scheme (No CSIT): As a popular STBC that transmit data in both time and space domains [4], the Alamouti scheme does not require CSIT but rely on the channel being constant over several consecutive symbol periods. For our 2-transmitter MISO model, the assumption is that the two channels h_1, h_2 remain unchanged over two successive symbol slots, and the Alamouti code is [5]:

$$\mathbf{c} = \begin{pmatrix} c_1 & -c_2^* \\ c_2 & c_1^* \end{pmatrix} \tag{7}$$

where c_1, c_2 are adjacent symbols, the column corresponds to time slots and the row indicates antenna index. Therefore, the received patterns in two consecutive periods can be expressed as:

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2^* \end{pmatrix} = \sqrt{E_s}\begin{pmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{pmatrix}\begin{pmatrix} \frac{c_1}{\sqrt{2}} \\ \frac{c_2}{\sqrt{2}} \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2^* \end{pmatrix} \tag{8}$$

Define $\begin{pmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{pmatrix}$ as $\mathbf{H}_{eff}$ and apply the matched filter $\mathbf{H}_{eff}^H$ on the received signal every two symbols, the transmitted symbols can be decoupled effectively. Based on the

TABLE I
PARAMETERS IN SIMULATION

Number of bits	$D = 10^4$
Number of channels	$M = 10^4$
SNR per bit	$\rho = 0 - 20$ dB
Noise power	$\sigma = 1$

assumption, $\mathbf{H}_{eff}^H \mathbf{H}_{eff}$ reduces to a 2-by-2 diagonal matrix with diagonal elements being $|h_1|^2 + |h_2|^2$. Therefore, the symbol pair to detect becomes:

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \frac{\sqrt{E_s}}{\left(|h_1|^2 + |h_2|^2\right)} \mathbf{I}_2 \begin{pmatrix} \frac{c_1}{\sqrt{2}} \\ \frac{c_2}{\sqrt{2}} \end{pmatrix} + \mathbf{H}_{eff}^H \begin{pmatrix} n_1 \\ n_2^* \end{pmatrix} \tag{9}$$

The result indicates that although the actual value of h_1, h_2 are unknown, the Alamouti coding forces the influence factor to be constant and averages the noise as well. Thus, the system can maintain successful communication without the knowledge of CSIT. Nevertheless, the equation also suggests that the symbol power is reduced to half, and therefore the average output SNR is $\bar{\rho}_{out} = \rho\varepsilon \left\{ \frac{\left(\|\mathbf{h}\|^2\right)^2}{2\|\mathbf{h}\|^2} \right\} = \rho$, which denotes the array gain g_a is one and not influenced by the number of transmit antennas. The BER according to [3] is $P_e = p^2[1 + 2(1-p)]$ with $p = \frac{1}{2} - \frac{1}{2}\left(1 + \frac{2}{\rho}\right)^{-\frac{1}{2}}$ and can be approximated as $P_e \leq \bar{N}_e \left(\frac{\rho d_{\min}^2}{8}\right)^{-2}$, where $\bar{N}_e$ is the number of closest constellations and $d_{\min}$ is the minimum distance. The diversity gain g_d is N.

Note for a MISO Rayleigh fading channel without CSIT, N degrees of freedom at the transmitter provides no array gain and N diversity gain. Also, BER is approximately proportional to ρ^{-N}. Compared with MRT, Alamouti does not require CSIT and has 3 dB loss for our 2-by-1 system.

It can be concluded that diversity in multiple antennas systems is beneficial to the BER performance regarding array gain and diversity gain. The former one is associated with the knowledge of channel state information (CSI), and the latter is determined by the number of channels.

III. RESULT AND ANALYSIS

A. Parameters

The simulation is based on the parameters in Table I.

B. SISO

Figure 4 shows the BER performance of uncoded QPSK transmission over a SISO Rayleigh fading channel. The numerical BER of BPSK and QPSK decrease linearly with the increase of SNR per bit in log-log scale, which is in line with theory. The result denotes that even at a relatively high SNR that equals 20 dB, the BER reaches 2.035×10^{-3} which is still high. In other words, the SISO system provides no diversity with only one transmit and receive antenna thus large BER.

There are no array gain and diversity gain defined in the SISO system since it acts as the reference. Also, the simulated

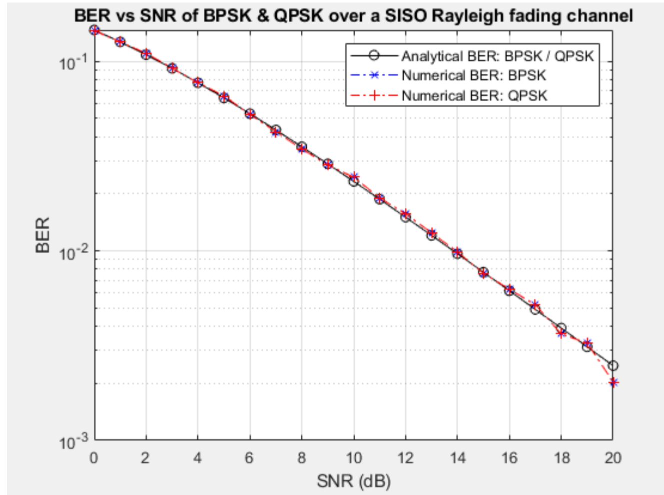

Fig. 4. BER vs SNR of BPSK and QPSK over a SISO Rayleigh fading channel

result is slightly different from the theoretical values. The main reason is the fixed estimation error by union bound mentioned in the theory section, although the amount is small. Also, the randomness brought by Monte Carlo method (MCM) influences the result especially at high SNR cases where one or two error difference can lead to a significant deviation from the expectation. The error can be mitigated by increasing the number of bits to transmit and generating more channel response for ergodicity.

C. SIMO

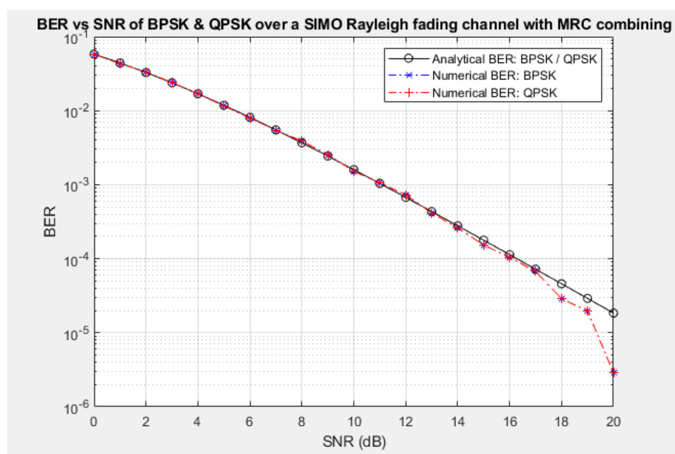

Fig. 5. BER vs SNR of BPSK and QPSK over a SIMO Rayleigh fading channel with MRC combining

BER-SNR plots of a two-antenna SIMO Rayleigh fading channel with MRC combining are illustrated by Figure 5. Again BER of BPSK and QPSK denoted by blue and red curves approximately decrease linearly when the reciprocal of SNR decreases. Notice that instead of using the approximation in [1], a more accurate analytical expression is found in [3] as equation 5 which is used in the simulation. The difference between theoretical and practical values comes from the estimation error of union bound and the randomness of MCM. Increasing the size of the sample can reduce the error. The diversity in multiple antennas systems is proved to provide quadratic decrease for a two-antenna receiver when the reciprocal of SNR decrease.

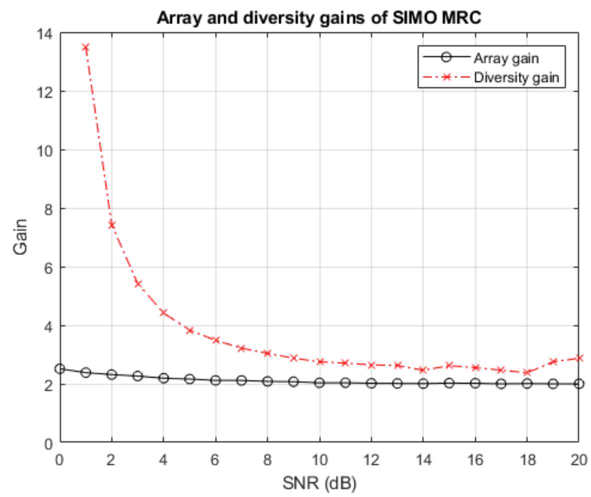

Fig. 6. Array and diversity gains of SIMO MRC

Figure 6 presents the array and diversity gains of our 1-by-2 system. It can be concluded that as SNR approaches infinity, both gains tend to reach 2 which equals the number of receive antennas as anticipated.

D. MISO

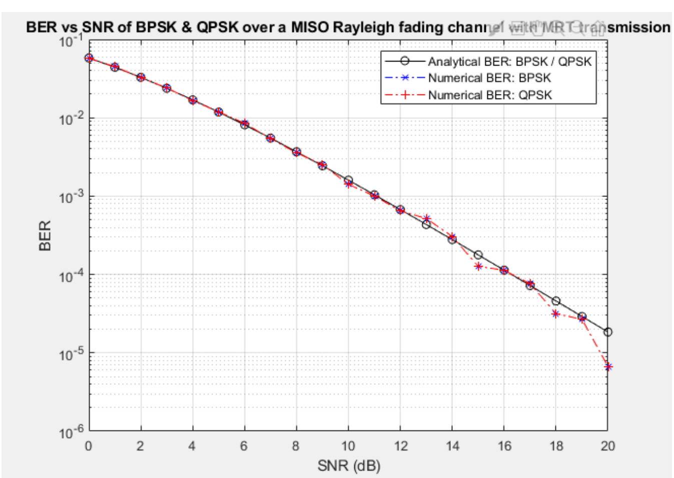

Fig. 7. BER vs SNR of BPSK and QPSK over a MISO Rayleigh fading channel with MRT transmission

1) MRT: With the knowledge of perfect CSIT, MRT transmission is a feasible approach at the transmitter. Figure 7 compares theoretical and practical the error rate of a 2-by-1 MISO system with MRT strategy. It indicates that the simulation result corresponds to the theory that the BER is inversely proportional to the square of SNR, which is the same as the 1-by-2 SIMO MRC case. The sources of error are the same (union bound and MCM) which can be reduced by enlarging the data set. Similar to the MRC case, two degrees of freedom in transmitter provides significant enhancement in BER that is inversely proportional to the SNR square.

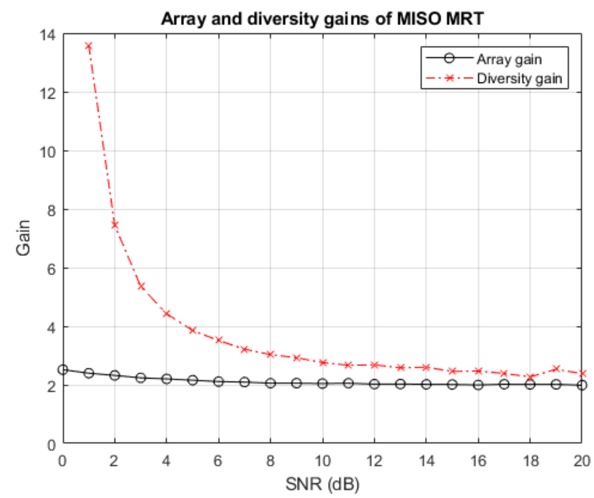

Fig. 8. Array and diversity gains of MISO MRT

The array gain and diversity gain are shown in Figure 8, which become close to the number of transmit antennas as SNR increases. The result corresponds to the expectation.

MRT utilises transmitter beamforming to achieve signal diversity at the transmitter and maintains the same beamforming and diversity gains with MRC. Nevertheless, the requirement on perfect CSIT can be unavailable in some situation as for rapid movements. The Alamouti scheme can overcome this problem.

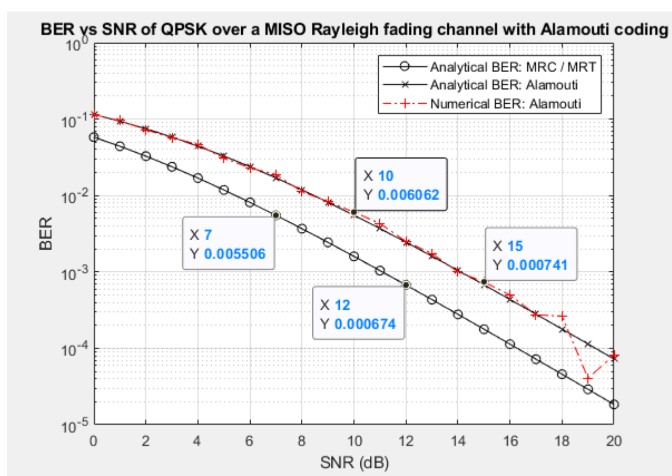

Fig. 9. BER vs SNR of QPSK over a MISO Rayleigh fading channel with Alamouti coding

2) Alamouti Scheme: Figure 9 illustrates the analytical and numerical BER curves of Alamouti coding in contrast with that of MRC and MRT. The substantial deviation from the analytical value when SNR equals 18 and 19 dB is due to the randomness in MCM where the sample is not large enough. For a fixed BER, the SNR is 3 dB worse than MRC and MRT, which suggests that the array gain is halved. In other words, there is no array gain for Alamouti scheme since the array gain of the two-antenna MRC and MRT is 2.

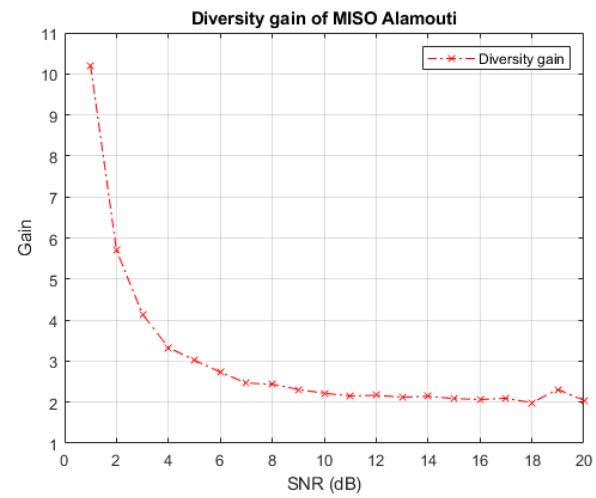

Fig. 10. Diversity gain of MISO Alamouti

Figure 10 illustrates the diversity gain. It approaches 2 as SNR increases which confirms the conclusion that Alamouti coding can produce a diversity gain that equals the number of transmit antennas.

Alamouti coding utilises the system diversity to reduce error rate without the knowledge of CSIT, although the performance is not as good as MRT. However, the overall complexity increases significantly as the number of antennas grows. Therefore, which approach to use should be determined based on the specific requirements of the system.

E. Comparison

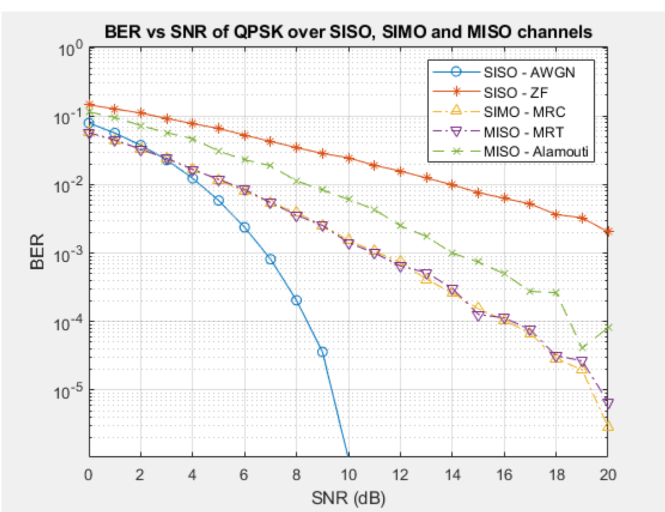

Fig. 11. BER vs SNR of QPSK over SISO, SIMO and MISO channels

Figure 11 compares the error rate performance of all models used in this task. The BER of the fading channel is significantly larger than that of the AWGN channel, and using multiple antennas can provide channel and signal diversity and thus smaller BER. Also, SIMO and MISO systems introduce array gain that describes the improvement in average SNR over single-branch SNR and the diversity gain which examines the enhancement in BER over single-branch SNR. The former is related to the CSI knowledge while the latter is associated with the channel counts. It can be interpreted that the multiple antennas approach reduces the system uncertainty (noise and channel) by utilising the diversity, and both gains can quantise the improvement.

IV. Conclusion

In this article, we explored and compared the error rate performance of SISO, SIMO and MISO system based on uncoded and Alamouti QPSK modulation with Rayleigh fading channel. It has been demonstrated that the diversity by multiple-antenna approach can provide additional reliability to the wireless communication system with proper transmit splitting and receive combining schemes. Also, the array gain and diversity gain are related to CSI knowledge and channel diversity respectively. For SIMO system, MRC combining is optimal if the noise is AWGN without interference. At the receiver, MRT transmission can achieve the same performance as MRC with CSIT. If CSIT is unavailable but the channel is constant over several symbol periods (2 for the two-antenna case), Alamouti scheme can achieve the same diversity gain but no array gain. Hence, the SNR of Alamouti is 3 dB worse than MRC and MRT for a 2-by-1 MISO system. The main limitations are that (1) only two-antenna case is considered for SIMO and MISO at the receiver and transmitter respectively (2) MIMO system is not examined. The random error by MCM can be further reduced by increasing the number of bits and generating more channels for ergodicity. With the code provided in the appendix, the system can be extended to more antennas cases.

V. Appendix: MATLAB code

The source code can be retrieved from https://github.com/SnowzTail/.

References

[1] B. Clerckx, "Wireless communications," January 2019.
[2] Wikimedia, "Mimo simo miso siso explanation without confusion," March 2018.
[3] J. Proakis, *Digital Communications*, ser. Electrical engineering series. McGraw-Hill, 2001. [Online]. Available: https://books.google.co.uk/books?id=sbr8QwAACAAJ
[4] A. Paulraj, R. Nabar, and D. Gore, *Introduction to space-time wireless communications*. Cambridge university press, 2003.
[5] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE Journal on selected areas in communications*, vol. 16, no. 8, pp. 1451–1458, 1998.