Space Time Coding

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Abstract

This paper descrides about space time coding. A short summary of space time coding which is one of the different transmit diversity techniques is given and a matlab code for performance characterstics of space time coded systems is also provided.

1 INTRODUCTION

Wireless systems should have high quality, better reliability, larger coverage, more power and bandwidth efficiency, provide high bit rate data services and at the same time remote units are supposed to be small lightweight pocket communicators. Therefore, transferring the complexity of the remote units to the base station will have a merit towards getting simple lightweight receiving devices.

We broadly define diversity as the method of conveying information through multiple independent Instantiations of random fades. There are several forms of diversity; frequency diversity, Time interleaving and spatial diversity. However, time interleaving results in large delays when the channel is slowly varying or low Doppler spread. Equivalently, frequency diversity are ineffective when the coherence bandwidth of the channel is larger than the spreading bandwidth or, equivalently, where there is relatively small delay spread in the channel. In most scattering environments, antenna diversity (spatial diversity) is a practical, effective and, hence, a widely applied technique for reducing the effect of multipath fading.

Information theory has been used to show that multiple antennas have the potential to dramatically increase achievable bit rates. The earliest form of spatial diversity was receiver diversity. i.e. to use multiple antennas at the receiver and perform selection combining or switched combining or maximum ratio combining to achieve better quality of received signal. But, this approach makes the remote units, which should have been cheap, simple and small lightweight and power efficient, to be expensive in cost, larger in size and higher power requirement. However, transferring this technique to the base station will be economical and will maintain the needed small size of the remote unit due to equipment reduction in the remote unit. For this reason, transmit diversity become superior over receive diversity.

The earliest form of spatial transmit diversity is the delay diversity scheme proposed in Uddenfeldt and Raith, 1992; Wittneben, 1993 By viewing multiple-antenna diversity as independent information streams, more sophisticated transmission (coding) schemes can be designed to get closer to theoretical performance limits. Using this approach, I will focus on space-time coding (STC) schemes defined by Tarokh et al., 1998a and Alamouti, 1998a which introduce temporal and spatial correlation into the signals transmitted from different antennas without increasing the total transmitted power or the transmission bandwidth. There is in fact a diversity gain that results from multiple paths between the base station and user terminal, and a coding gain that results from how symbols are correlated across transmit antennas.

2 TRANSMIT DIVERSITY SCHEME

Transmit diversity is more challenging to provision and realize than receive diversity because it involves the design of multiple correlated signals from a single information signal without utilizing CSI (typically not available accurately at the transmitter). Furthermore, transmit diversity must be coupled with effective receiver signal processing techniques that can extract the desired information signal from the distorted and noisy received signal. Transmit diversity is more practical than receive diversity for enhancing the downlink (which is the bottleneck in broadband asymmetric applications such as Internet browsing and downloading) to preserve the small size and low power consumption features of the user terminal. There are two main classes of multiple-antennas transmitter techniques : closed-loop and open-loop. The former uses a feedback channel to send CSI acquired at the receiver back to the transmitter to be used in signal design while the latter does not require CSI.

For transmit diversity systems with feedback, modulated signals are transmitted from multiple transmit antennas with different weighting factors. The weighting factors for the transmit antennas are chosen adaptively so that the received signal power or channel capacity is maximized. For transmit diversity schemes without feedback, messages to be transmitted are usually processed at the transmitter and then sent from multiple transmit antennas. Signal processing at the transmitter is designed appropriately to enable the receiver exploiting the embedded diversity from the received signals. A typical example is a delay diversity scheme. In this scheme, copies of the same symbol are transmitted through multiple antennas in different times.

Assuming availability of ideal (i.e. error free and instantaneous) CSI at the transmitter, closed-loop techniques have an SNR advantage of 10 log10(Mt) dB over open-loop techniques due to the "array gain" factor Alamouti, 1998a. However, several practical factors degrade the performance of closed-loop techniques including channel estimation errors at the receiver, errors in feedback link (due to noise, interference, and quantization effects), and feedback delay which causes a mismatch between available and actual CSI. All of these factors combined with the extra bandwidth and system complexity resources needed for the feedback link make open-loop techniques more attractive as a robust means for improving downlink performance for high-mobility applications while closed-loop techniques (such as beamforming) become attractive under low-mobility conditions.

In open loop transmit diversity, improvement in error performance of the multiple antennas transmission can be gained by a joint design of error control coding, modulation and transmit diversity. Coding techniques designed for multiple antenna transmission are called space-time coding. In particular, coding is performed by adding properly designed redundancy in both spatial and temporal domains, which introduces correlation into the transmitted signals. Due to joint design, space-time codes can achieve transmit diversity as well as a coding gain without sacrificing bandwidth. My focus in this paper will be exclusively on space time coding .

3 SPACE TIME CODED SYSTEMS

Space-time coding is a powerful transmit diversity technique that relies on coding across space (transmit antennas) and time to extract diversity. Space-time coding has received considerable attention in academic and industrial circles due to its many advantages. First, it improves the downlink performance without the need for multiple receive antennas at the terminals. Second, it can be elegantly combined with channel coding, realizing a coding gain in addition to the spatial diversity gain. Third, it does not require CSI (channel state information) at the transmitter, i.e. operates in open-loop mode. Finally, they have been shown to be robust against non-ideal operating conditions such as antenna correlation, channel estimation errors, and Doppler effects.

There are various approaches in coding structures, including space-time block codes (STBC), space-time trellis codes (STTC), space-time turbo trellis codes and layered space-time (LST) codes. A central issue in all these schemes is the exploitation of multipath effects in order to achieve high spectral efficiencies and performance gains.

let us consider a baseband space-time coded communication system with n_T transmit antennas and n_R receive antennas, as shown in Fig.1. The transmitted data are encoded by a space-time encoder. At each time instant t , a block of m binary information symbols, denoted by:

$$\mathbf{c}_t = (c_t^1, c_t^2, ..., c_t^m) \tag{1}$$

is fed into the space-time encoder. The space-time encoder maps the block of m binary input data into n_T modulation symbols from a signal set of $M = 2^m$ points. The coded data are applied to a serial-to-parallel (S/P) converter producing a sequence of n_T parallel symbols, arranged into an n_T 1 column vector:

$$\mathbf{x}_t = (x_t^1, x_t^2, ..., x_t^{n_T})^T \tag{2}$$

where T means the transpose of a matrix. The nT parallel outputs are simultaneously transmitted by n_T different antennas, whereby symbol x_t^i, $1 \leq i \leq n_T$, is transmitted by antenna i and all transmitted symbols have the same duration of Tsec. The vector of coded modulation symbols from different antennas, as shown in eq. 2, is called a space-time symbol. The spectral efficiency of the system is:

$$\eta = \frac{r_b}{B} = mbits/sec/Hz \tag{3}$$

where rb is the data rate and B is the channel bandwidth. The spectral efficiency in eq. 3 is equal to the spectral efficiency of a reference uncoded system with one transmit antenna.

The multiple antennas at both the transmitter and the receiver create a MIMO channel. For wireless mobile communications, each link from a transmit antenna to a receive antenna can be modeled by flat fading, if we assume that the channel is memoryless. The MIMO channel with n_T transmit and n_R receive antennas can be represented by an (n_Rxn_T) channel matrix H. At time t , the channel matrix is given by:

$$\mathbf{H}_t = \begin{pmatrix} h_{1,1}^t & h_{1,2}^t & \cdots & h_{1,n_T}^t \\ h_{2,1}^t & h_{2,2}^t & \cdots & h_{2,n_T}^t \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_R,1}^t & h_{n_R,2}^t & \cdots & h_{n_R,n_T}^t \end{pmatrix} \tag{4}$$

where the ji-th element, denoted by $h^t_{j,i}$, is the fading attenuation coefficient for the path from transmit antenna i to receive antenna j.

At the receiver, the signal at each of the n_R receive antennas is a noisy superposition of the n_T transmitted signals degraded by channel fading. At time t , the received signal at antenna j, j = 1, 2,......, n_R, denoted by r^j_t , is given by:

$$r^j_t = \sum_{i=1}^{n_T} h^t_{j,i} x^i_t + n^j_t \tag{5}$$

where n^j_t is the noise component of receive antenna j at time t.

Let us represent the received signals from n_R receive antennas at time t by an n_R x 1 column vector.

$$\mathbf{r}_t = (r^1_t, r^2_t, ..., r^{n_R}_t)^T \tag{6}$$

The noise at the receiver can be described by an n_R 1 column vector, denoted by $\mathbf{n}_t$

$$\mathbf{n}_t = (n^1_t, n^2_t, ..., n^{n_R}_t)^T \tag{7}$$

where each component refers to a sample of the noise at a receive antenna. Thus, the received signal vector can be represented as:

$$\mathbf{r}_t = \mathbf{H}_t \mathbf{x}_t + \mathbf{n}_t \tag{8}$$

3.1 Space-Time Block Codes

We first introduce the Alamouti code [1], which is a simple twobranch transmit diversity scheme. The key feature of the scheme is that it achieves a full diversity gain with a simple maximum-likelihood decoding algorithm. In this section, we also present space-time block codes with a large number of transmit antennas based on orthogonal designs.

3.1.1 Alamouti Space-Time Code

The Alamouti scheme is historically the first space-time block code to provide full transmit diversity for systems with two transmit antennas [1]. It is worthwhile to mention that delay diversity schemes can also achieve a full diversity, but they introduce interference between symbols and complex detectors are required at the receiver. In this section, we present Alamoutis transmit diversity technique, including encoding and decoding algorithms and its performance.

Figure 1 shows the block diagram of the Alamouti space-time encoder. Let us assume that an M-ary modulation scheme is used. In the Alamouti space-time encoder, each group of m information bits is first modulated, where m = $\log 2M$. Then, the encoder takes a block of two modulated symbols x1 and x2 in each encoding operation and maps them to the transmit antennas according to a code matrix given by

$$\mathbf{X} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \tag{9}$$

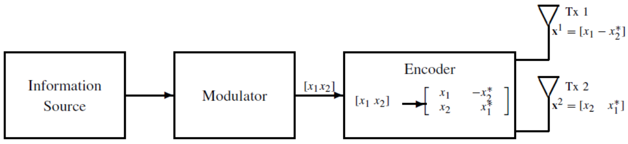

Figure 1: A block diagram of the Alamouti space-time encoder

The encoder outputs are transmitted in two consecutive transmission periods from two transmit antennas. During the first transmission period, two signals x_1 and x_2 are transmitted simultaneously from antenna one and antenna two, respectively. In the second transmission period, signal -x_2^* is transmitted from transmit antenna one and signal x_1^* from transmit antenna two, where x_1^* is the complex conjugate of x_1.

It is clear that the encoding is done in both the space and time domains. Let us denote the transmit sequence from antennas one and two by x_1 and x_2, respectively.

$$\mathbf{x}^1 = [x_1, -x_2^*]\mathbf{x}^2 = [x_2, x_1^*] \tag{10}$$

The key feature of the Alamouti scheme is that the transmit sequences from the two transmit antennas are orthogonal, since the inner product of the sequences $\mathbf{x}^1$ and $\mathbf{x}^2$ is zero.

At the receive antenna, the received signals over two consecutive symbol periods, denoted by r_1 and r_2 for time t and t + T (T is symbol duration) , respectively, can be expressed as:

$$r_1 = h_1x_1 + h_2x_2 + n_1r_2 = -h_1x_2^* + h_2x_1^* + n_2 \tag{11}$$

where h_1 and h_2 are the fading channel coefficeints from the 1^{st} and 2^{nd} transmit antennas to the receiver antenna at time t and n_1 and n_2 are independent complex variables with zero mean and power spectral density N_0/2 per dimension, representing additive white Gaussian noise samples at time t and t + T , respectively. If the channel fading coefficients, h_1 and h_2, can be perfectly recovered at the receiver, the decoder will use them as the channel state information (CSI). Assuming that all the signals in the modulation constellation are equiprobable, a maximum likelihood decoder chooses a pair of signals ($\hat{x}_1$, $\hat{x}_2$) from the signal modulation constellation to minimize the distance metric

$$d^2(r_1, h_1\hat{x}_1 + h_2\hat{x}_2) + d^2(r_2, -h_1\hat{x}_2^* + h_2\hat{x}_1^*) = |r_1 - h_1\hat{x}_1 - h_2\hat{x}_2|^2 + |r_2 + h_1\hat{x}_2^* - h_2\hat{x}_1^*|^2 \tag{12}$$

over all possible values of $\hat{x}_1$ and $\hat{x}_2$.

The Alamouti scheme can be applied for a system with two transmit and n_R receive antennas. The encoding and transmission for this configuration is identical to the case of a single receive antenna. Let us denote by r_1^j and r_2^j the received signals at the j^{th} receive antenna at time t and t +T, respectively.

$$r_1^j = h_{j,1}x_1 + h_{j,2}x_2 + n_1^jr_2^j = -h_{j,1}x_2^* + h_{j,2}x_1^* + n_2^j \tag{13}$$

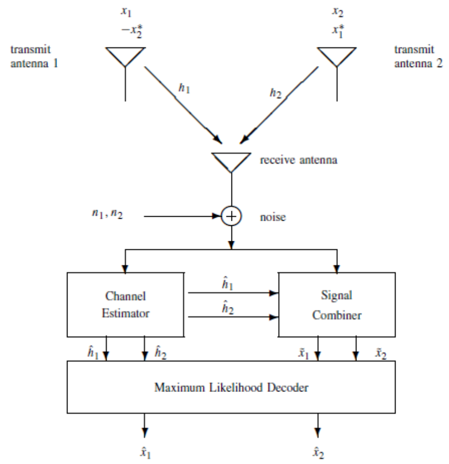

Figure 2: Receiver for the Alamouti scheme

where $h_{j,i}$,i=1,2 and j=1,2,....,n_R, is the fading coefficient for the path from transmit antenna i to receive antenna j, and n_1^j and n_2^j are the noise signals for receive antenna j at time t and t + T , respectively.

The receiver constructs two decision statistics based on the linear combination of the received signals.

3.1.2 Space-Time Block Codes (STBC)

Figure 3 shows an encoder structure for space-time block codes. The Alamouti scheme achieves the full diversity with a very simple maximum-likelihood decoding algorithm. The key feature of the scheme is orthogonality between the sequences generated by the two transmit antennas. This scheme was generalized to an arbitrary number of transmit antennas by applying the theory of orthogonal designs. The generalized schemes are referred to as space-time block codes (STBCs). The space-time block codes can achieve the full transmit diversity specified by the number of the transmit antennas n_T , while allowing a very simple maximum-likelihood decoding algorithm, based only on linear processing of the received signals.

a space-time block code is defined by an n_T x p transmission matrix **X**. Here n_T represents the number of transmit antennas and p represents the number of time periods for transmission of one block of coded symbols.

Let us assume that the signal constellation consists of 2^m points. At each encoding operation, a block of km information bits are mapped into the signal constellation to select k modulated signals x_1, x_2, . . . , x_k, where each group of m bits selects a constellation signal. The k modulated signals are encoded by a space-time block encoder to generate n_T parallel signal sequences of length p according to the transmission matrix **X**. These sequences are transmitted through n_T transmit antennas simultaneously in p time periods.

In the space-time block code, the number of symbols the encoder takes as its input in each encoding operation is k. The number of transmission periods required to transmit the space-time coded symbols through the multiple transmit antennas is p. In other words, there are p space-time symbols transmitted from each antennas for each block of k input symbols. The rate of a space-time block code is defined as the ratio between the number of symbols the encoder takes as its input and the number of space-time coded symbols transmitted from each antenna. The entries of the transmission matrix **X** are linear combinations of the k modulated symbols

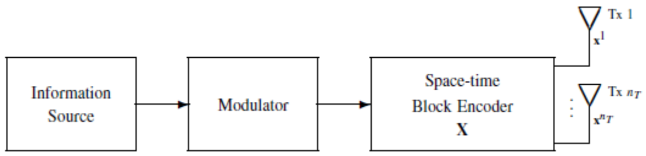

Figure 3: Encoder for STBC

$x_1, x_2, \ldots, x_k$ and their conjugates x_1^*, x_2^*, ..., x_k^*. In order to achieve the full transmit diversity of n_T , the transmission matrix **X** is constructed based on orthogonal designs such that:

$$\mathbf{X}\mathbf{X}^H = c(|x_1|^2 + |x_2|^2 + ... + |x_k|^2)\mathbf{I_{n_T}} \tag{14}$$

where c is a constant, $\mathbf{X}^H$ is the Hermitian of **X** and $\mathbf{I}_{n_T}$ is an n_T x n_T identity matrix. The i_{th} row of **X** represents the symbols transmitted from the i_{th} transmit antenna consecutively in p transmission periods, while the j_{th} column of **X** represents the symbols transmitted simultaneously through n_T transmit antennas at time j. $x_{i,j}$, i=1,2,......,n_T, j=1,2,......, p, represents the signal transmitted from the antenna i at time j. The rows of the transmission matrix $\mathbf{X}_{nT}$ are orthogonal to each other. This means that in each block, the signal sequences from any two transmit antennas are orthogonal.

It has been shown that the rate of a space-time block code with full transmit diversity is less than or equal to one, $R \leq 1$. The code with a full rate R = 1 requires no bandwidth expansion, while the code with rate R <1 requires a bandwidth expansion of 1/R. For space-time block codes with n_T transmit antennas, the transmission matrix is denoted by $\mathbf{X}_{nT}$. The code is called the space-time block code with size n_T.

3.1.3 Performance of the Alamouti Scheme

The performance of the Alamouti transmit diversity scheme on slow Rayleigh fading channels is evaluated by simulation. In the simulation, it is assumed that fading from each transmit antenna to each receive antenna is mutually independent and that the receiver has the perfect knowledge of the channel coefficients. Figure 4 shows the bit error rate (BER) performance of the Alamouti scheme with coherent BPSK modulation against the signalto- noise ratio (SNR) per receive antenna. The BER performance of two receive diversity schemes with single transmit antenna and maximal ratio combining (MRC) is also shown in the figure for comparison. Furthermore, we assume that the total transmit power from two antennas for the Alamouti scheme is the same as the transmit power from the single transmit antenna for the MRC receiver diversity scheme.

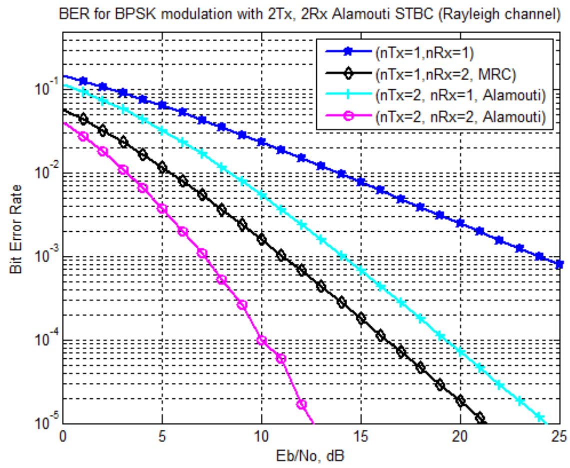

Figure 4: The BER performance of the BPSK Alamouti scheme with one and two receive antennas on slow Rayleigh fading channels

The simulation results show that the Alamouti scheme with two transmit antennas and a single receive antenna achieves the same diversity order as a two-branch MRC receive diversity scheme. In general, the Alamouti scheme with two transmit and n_R receive antennas has the same diversity gain as an MRC receive diversity scheme with one transmit and $2n_R$ receive antennas. The sample matlab code is given in APPENDIX.

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4 APPENDIX

19pt % Script for computing the BER for BPSK modulation in a
% Rayleigh fading channel with Alamouti Space Time Block Coding
% Two transmit antenna, Two Receive antenna

clear
N = 10 $\wedge$ 6; % number of bits or symbols
Eb_N0_dB = [0:25]; % multiple Eb/N0 values
nRx = 2;
for ii = 1:length(Eb_N0_dB)

% Transmitter
ip = rand(1,N)>0.5; % generating 0,1 with equal probability
s = 2*ip-1; % BPSK modulation 0 $\longrightarrow$ -1; 1 $\longrightarrow$ 0

% Alamouti STBC
sCode = 1/sqrt(2)*kron(reshape(s,2,N/2),ones(1,2)) ;

% channel
h = 1/sqrt(2)*[randn(nRx,N) + j*randn(nRx,N)]; % Rayleigh channel
n = 1/sqrt(2)*[randn(nRx,N) + j*randn(nRx,N)]; % white gaussian noise, 0dB variance

y = zeros(nRx,N);
yMod = zeros(nRx*2,N);
hMod = zeros(nRx*2,N);
for kk = 1:nRx

hMod = kron(reshape(h(kk,:),2,N/2),ones(1,2)); % repeating the same channel for two symbols
hMod = kron(reshape(h(kk,:),2,N/2),ones(1,2));
temp = hMod;
hMod(1,[2:2:end]) = conj(temp(2,[2:2:end]));
hMod(2,[2:2:end]) = -conj(temp(1,[2:2:end]));

% Channel and noise Noise addition
y(kk,:) = sum(hMod.*sCode,1) + 10 $\wedge$ $(-Eb\_N0\_dB(ii)/20) * n(kk,:)$;

% Receiver
yMod([2*kk-1:2*kk],:) = kron(reshape(y(kk,:),2,N/2),ones(1,2));

% forming the equalization matrix
hEq([2*kk-1:2*kk],:) = hMod;
hEq(2*kk-1,[1:2:end]) = conj(hEq(2*kk-1,[1:2:end]));
hEq(2*kk, [2:2:end]) = conj(hEq(2*kk, [2:2:end]));

```
end

% equalization
hEqPower = sum(hEq.*conj(hEq),1);
yHat = sum(hEq.*yMod,1)./hEqPower; % [h1*y1 + h2y2*, h2*y1 -h1y2*, ... ]
yHat(2:2:end) = conj(yHat(2:2:end));

% receiver - hard decision decoding
ipHat = real(yHat)>0;

% counting the errors
nErr(ii) = size(find([ip- ipHat]),2);

end

simBer = nErr/N; % simulated ber
EbN0Lin = 10.^(Eb_N0_dB/10);
theoryBer_nRx1 = 0.5.*(1-1*(1+1./EbN0Lin).^(-0.5));

p = 1/2 - 1/2*(1+1./EbN0Lin).^(-1/2);
theoryBerMRC_nRx2 = p.^2.*(1+2*(1-p));

pAlamouti = 1/2 - 1/2*(1+2./EbN0Lin).^(-1/2);
theoryBerAlamouti_nTx2_nRx1 = pAlamouti.^2.*(1+2*(1-pAlamouti));

close all
figure
semilogy(Eb_N0_dB,theoryBer_nRx1,'bp-','LineWidth',2);
hold on
semilogy(Eb_N0_dB,theoryBerMRC_nRx2,'kd-','LineWidth',2);
semilogy(Eb_N0_dB,theoryBerAlamouti_nTx2_nRx1,'c+-','LineWidth',2);
semilogy(Eb_N0_dB,simBer,'mo-','LineWidth',2);
axis([0 25 10^-5 0.5])
grid on
legend('(nTx=1,nRx=1)', '(nTx=1,nRx=2, MRC)', '(nTx=2, nRx=1, Alamouti)', '(nTx=2, nRx=2, Alamouti)');
xlabel('Eb/No, dB');
ylabel('Bit Error Rate');
title('BER for BPSK modulation with 2Tx, 2Rx Alamouti STBC (Rayleigh channel)');
```