PRML Assignment I

Dataset

Click here to visit the dataset.

The dataset is constructed using the ***create_dataset*** function, who write the samples generated by the function ***generate_gaussian_distributed_samples*** to a text file. You can load the dataset by calling the function ***load_dataset*** and split it into training set and testing set by calling the function ***split_dataset***.

Description of models

A super class called ***LinearModel*** is defined in ***source.py*** for the two models and it contains attributes and method for plotting the classification result.

The linear discriminative model is defined by Python class ***LinearDiscriminativeModel*** in **source.py**. On the linear classification of n dimensional C classes, an instance of the model has C n $\times$ 1 weight vectors, i.e. an n $\times$ C weight matrix. The weight matrix is specified on calling the method ***train*** to train the model. The method train uses **gradient descent strategy** and you can specify the training options. Once the model is trained, you can classify samples by calling the ***classify*** method, who uses softmax function to generate the probability that samples belong to each class.

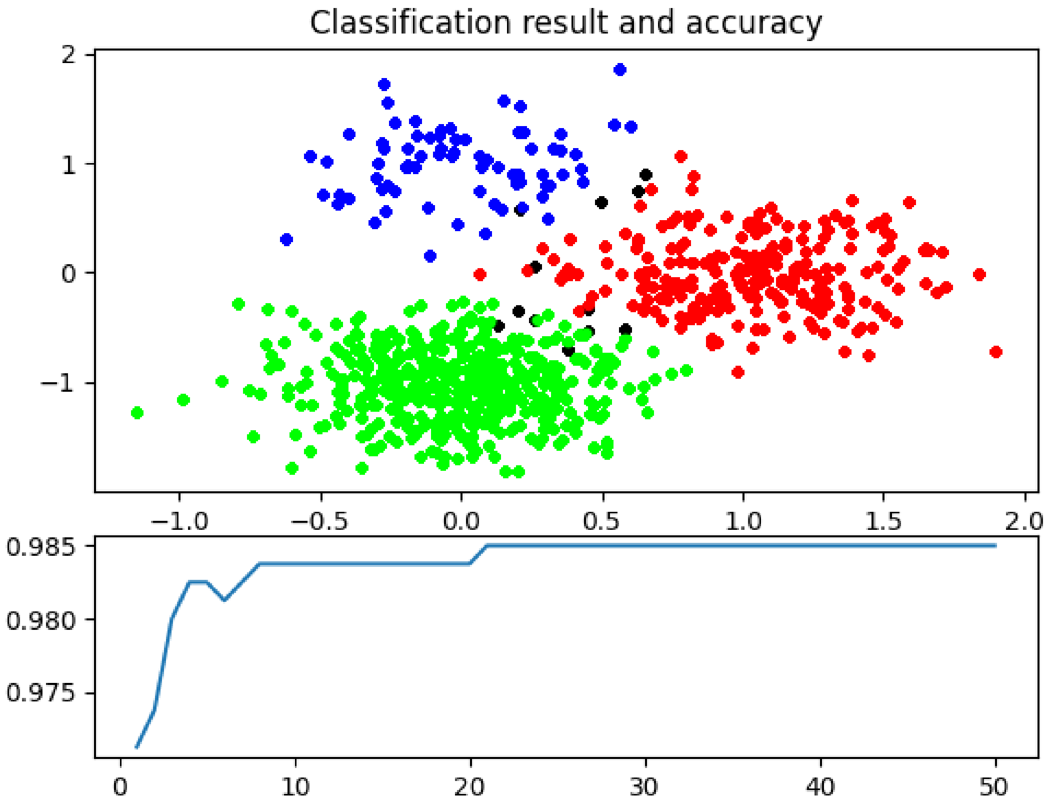
The linear generative model is defined by Python class ***LinearGenerativeModel*** in **source.py**. On the linear classification of n dimensional C classes, an instance of the model has C n $\times$ 1 weight vectors, C biases and one covariance matrix, i.e. an n $\times$ C weight matrix, a C $\times$ 1 bias vector and an n $\times$ n covariance matrix. The weight matrix, bias vector and covariance matrix are specified on calling the method ***train*** to train the model. The method train calculates mathematical expectations and covariance matrices of each class using maximum likelihood estimation. Once the model is trained, you can classify samples by calling the ***classify*** method, who uses ***softmax*** function to generate the probability that samples belong to each class.

The **major differences** between these two models are the **training method** and **classification method**. On the **training method**, the **discriminative model** uses gradient descent strategy, i.e. an **optimization method**, to approach an ideal weight matrix, while the **generative model** tries to generalize the statistical properties of given samples using **statistical method**. As a result, the **generative model** doesn't need to train the model by iteration and can generate new samples from its estimation of samples' mathematical expectations and covariances while the **discriminative model** have to train the model by means of iteration or other approaches and can not generate new samples. On the **classification method**, the **discriminative model** assumes the probability that samples belong to each class has the form of $softmax(w^T x)$, which comes from the **linear property** of the classifier. However, the **generative model** adopts $softmax(w^T x + b)$ as the form of probability that samples belong to each class because it assumes that **samples are compiled to Gaussian distributions**. Thus the **generative model** is extremely relied on its assumption of samples' statistical distribution and **is sensitive to outliers**

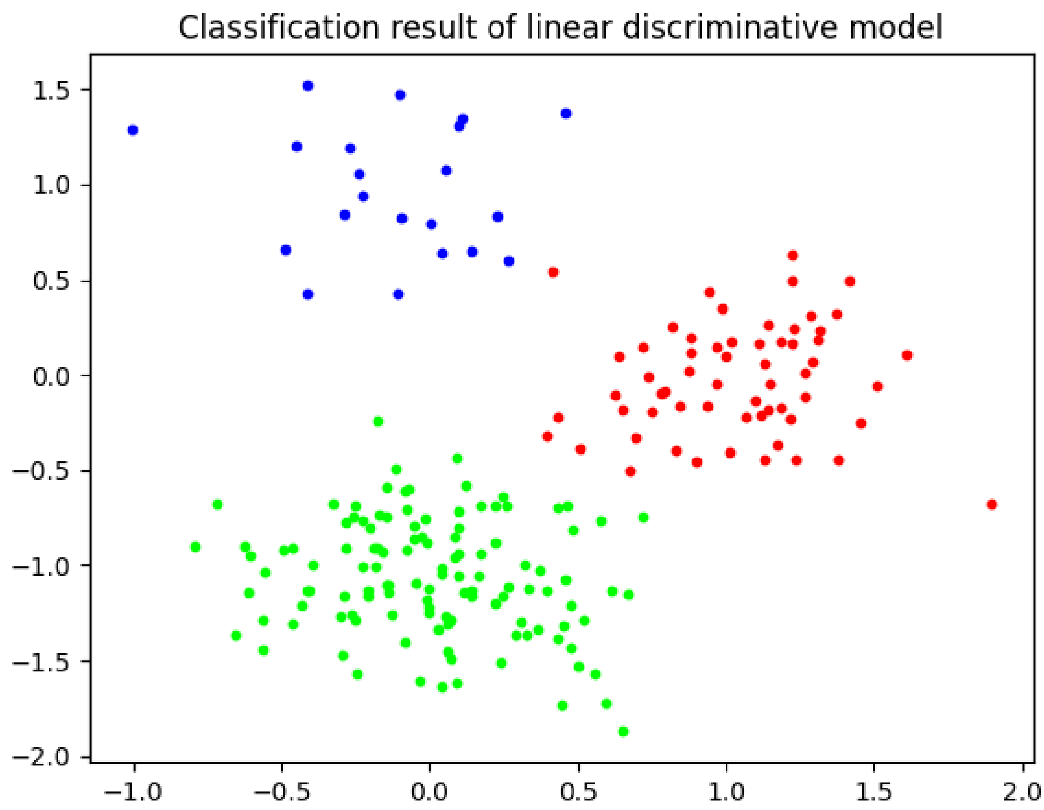
or samples compiled to multi-center Gaussian distribution while the **discriminative model** can distinguish samples compiled to any distribution as long as they are linearly separable. The performance of two models is similar.

Performance of models

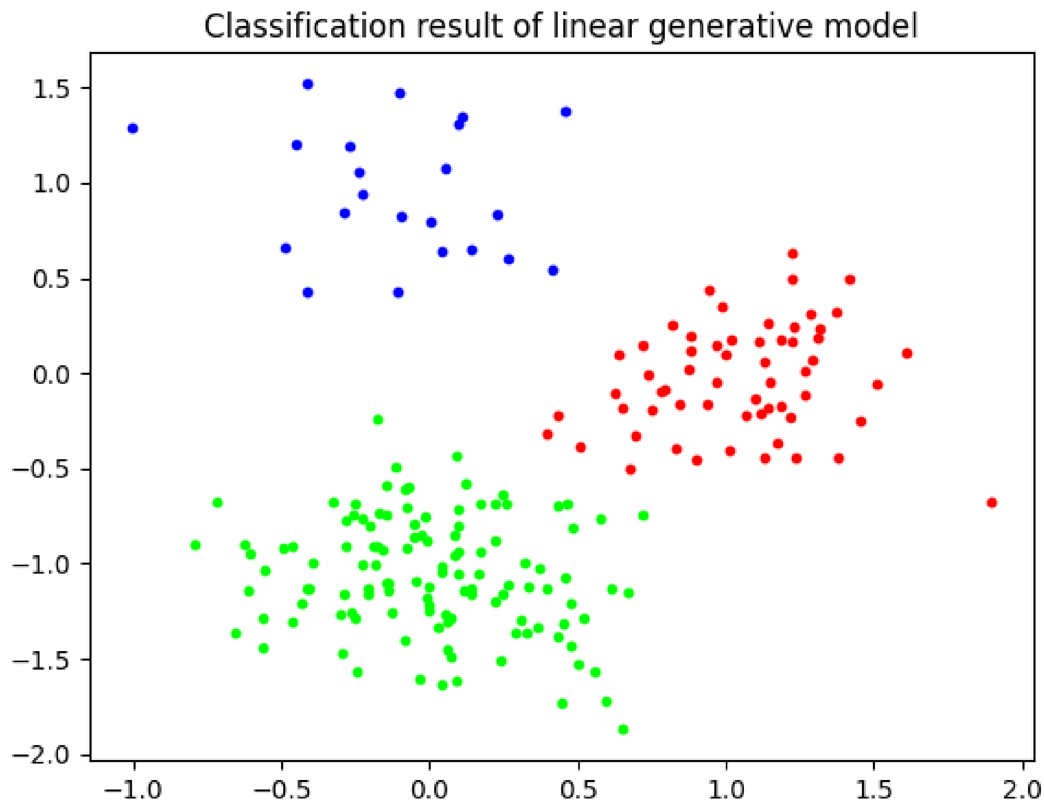
Use samples to train the **generative model**, the training process is shown in this picture. The misclassified samples are shown in black. It's evident that the model reach a high accuracy according to the lower part of the picture.



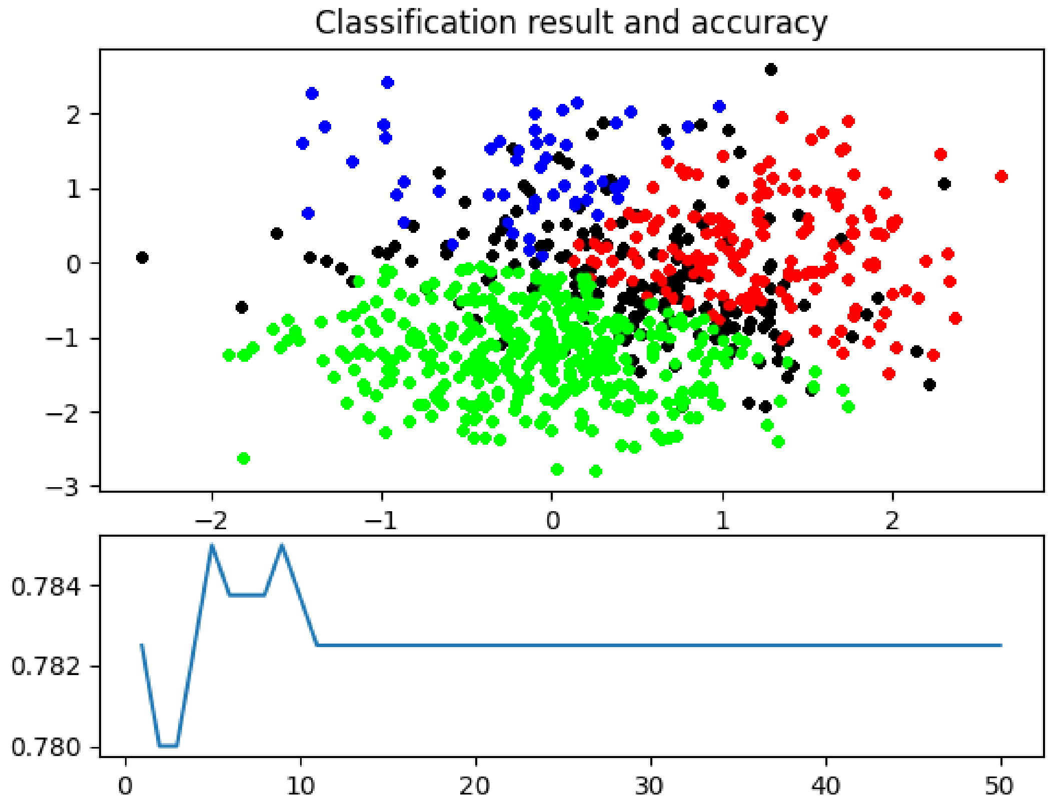
Then test the model on the testing dataset, the model can identified 99% of samples correctly. Note that the misclassified samples are **not** shown in the following picture since we won't tell the model the correct labels when calling the ***classify*** method.

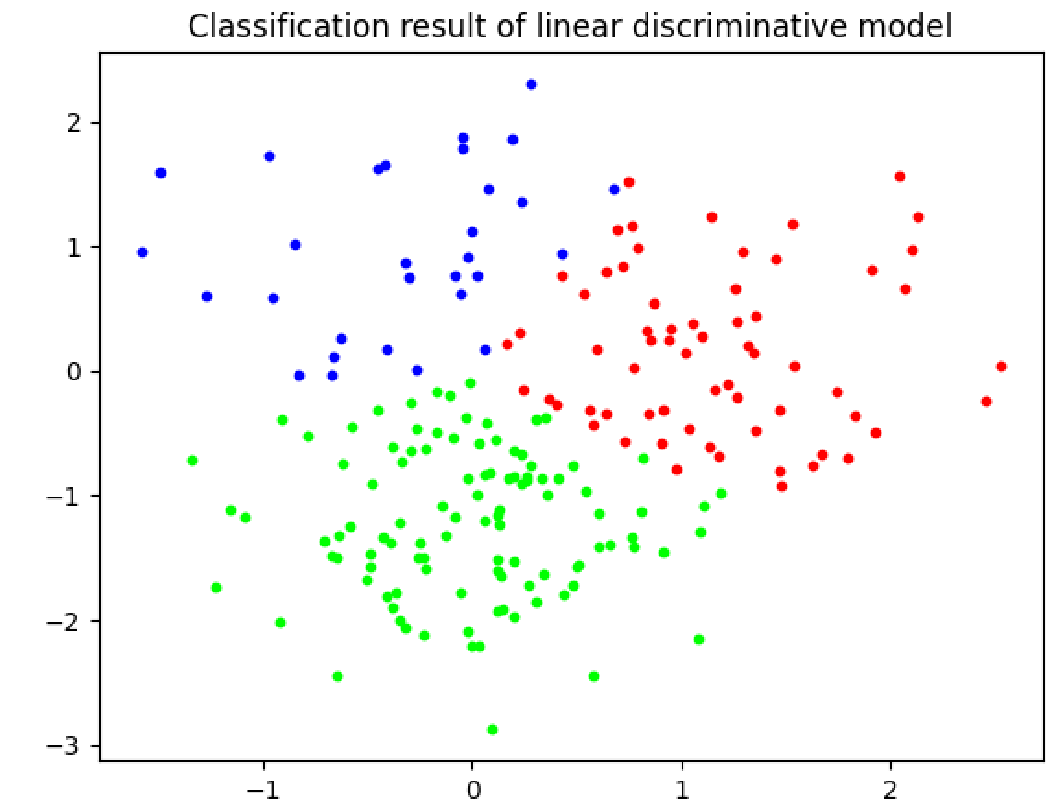


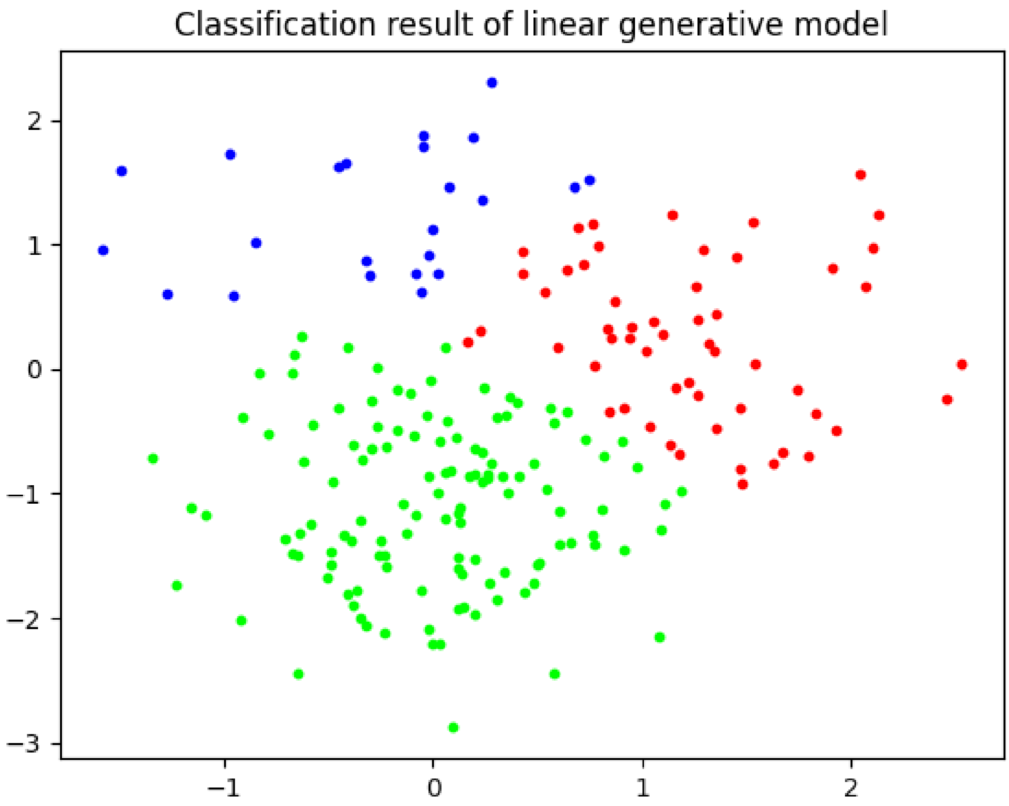
Apply the **generative model** to the same training set and testing set, the model can also identified 98.5% percent of samples correctly.



Then **increase the covariance** of Gaussian distributions and repeat above. Both of **generative model** and **discriminative model** deteriorated. The proportion of black points in the classification result increases rapidly when samples of different classes start to mix together. These two models can identify 81% and 80.5% of samples respectively.


Classification result and accuracy
2
1
0
−1
−2
−3
−2
−1
0
1
2
0.784
0.782
0.780
0
10
20
30
40
50


Classification result of linear discriminative model
2
1
0
−1
−2
−3
−1
0
1
2



Demonstration

Run this demonstration in **Python Console**. This demo is also at the beginning of **source.py**.

```
Python 3.6.9 (default, MM dd yyyy, hh:mm:ss)
Type "help", "copyright", "credits" or "license" for more information.
>>> from source import *
>>> create_dataset()
>>> samples, labels = load_dataset()
>>> set_of_samples, set_of_labels = split_dataset(samples, labels, [0.7])
>>> training_samples, testing_samples = set_of_samples
>>> training_labels, testing_labels = set_of_labels
>>> model1 = LinearDiscriminativeModel()
>>> model1.train(training_samples, training_labels, max_epochs=20, plot_training_process=False)
Epoch   Accuracy
1      0.9714285714285714
2      0.9785714285714285
3      0.9814285714285714
4      0.9828571428571429
5      0.9828571428571429
6      0.9828571428571429
7      0.9828571428571429
8      0.9814285714285714
9      0.9814285714285714
10      0.9842857142857143
11      0.9842857142857143
12      0.9842857142857143
13      0.9842857142857143
14      0.9842857142857143
15      0.9842857142857143
16      0.9842857142857143
```

```
17    0.9842857142857143
18    0.9842857142857143
19    0.9842857142857143
20    0.9842857142857143
>>> labels1 = model1.classify(testing_samples)
>>> acc = sum(testing_labels == labels1) / testing_labels.size
>>> acc[0, 0]
0.98
>>> model1.w
matrix([[ 2.89275597, -1.44460552, -1.44815044],
        [ 0.77766718, -4.04375914,  3.26609195]])
>>> model2 = LinearGenerativeModel()
>>> model2.train(training_samples, training_labels)
>>> labels2 = model2.classify(testing_samples)
>>> acc = sum(testing_labels == labels2) / testing_labels.size
>>> acc[0, 0]
0.98
>>> model2.w
matrix([[  9.57567184,   0.06218358,   0.07589503],
        [ -0.11014837, -10.67945524,  11.09769756]])
>>> model2.b
matrix([[-5.85848281],
        [-5.9195002 ],
        [-8.16539317]])
```