MAKERERE UNIVERSITY

SCHOOL OF ENGINEERING
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING
B.Sc (CE), SECOND YEAR, SECOND SEMESTER 2015/16 FINAL EXAMINATIONS
CMP2201: DISCRETE MATHEMATICS AND RANDOM PROCESSES

Date: Tuesday 10th May 2016 Time: 9:00 AM – 12:00 PM

Instructions:

1. This paper consists of **FIVE (5)** questions. Attempt only **FOUR (4)** questions.
2. All questions carry 25 marks

Question 1 (25 Marks)

a) Distinguish between continuous and discrete variables with examples. **(2 Marks)**

b) If $A = \{2, 4, 6, 8\}$ write down P(A) by listing its elements. What is the value of $|P(A)|$? **(5 Marks)**

c) Simplify the following set notations and state clearly the set laws you are using at each stage.

i) $B \cup (\emptyset \cap A)$ **(4 Marks)**

ii) $(A \cap B) \cup (A \cap \bar{B})$ **(4 Marks)**

d) A chess board's 8 rows are labelled using numbers starting at 1 and its 8 columns using letters starting at a. Each square of the board is described by the ordered pair (column letter, row number).

i) A knight is positioned at (d, 3). Write down its possible positions after a single move of the knight. Give an illustration. Hint: A knight moves either two squares horizontally then one square vertically, or one square horizontally then two squares vertically—i e. in an "L" pattern. Refer to figure 1. The crosses show some of the possible positions of the knight after a single move. **(8 Marks)**

ii) If $R = \{1, 2, \ldots, 8\}$, $C = \{a, b, \ldots, h\}$ and $P = \{coordinates\ of\ all\ squares\ on\ the\ chess\ board\}$, use set notation to express P in terms of R and C. **(2 Marks)**

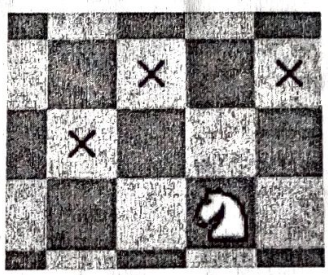


Figure 1

Question 2 (25 Marks)

a) Find a subset Y of the set of real numbers X such that the function $g: X \rightarrow Y$ defined by the following is a one to one correspondence. Then compute g^{-1}.

i) $g(x) = {}^{-1}/_{x}$ **(3 Marks)**

ii) $g(x) = 8(2^{x+1})$ **(3 Marks)**

b) If $f(x) = 2x + 3$ *and* $g(x) = 1 - 4x^2$ Determine fg and gf. Is fg = gf? **(6 Marks)**

c) Determine the properties of the following relations: **(8 Marks)**

i) $S = \{0, 1, 2, 3\}, (x, y) \in R\ if\ \max\{x, y\} = 3$

ii) S is the set of positive integers and xRy if and only if $y = n^2x$ for some integer n.

d) An investor is going to invest $ 20,000 in 5 stocks chosen from a list of 12 prepared by her broker. How many different investments are possible if: **(5 Marks)**

i) $ 4000 is to be invested in each stock

ii) $ 8000 is to be invested in one stock, $ 6000 in another, $ 3000 in the third, $ 2000 in the fourth and $ 1000 in the fifth.

Question 3 (25 Marks)

a) Prove by mathematical induction: For all positive integers n. **(8 Marks)**

$1^2 + 3^2 + \ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

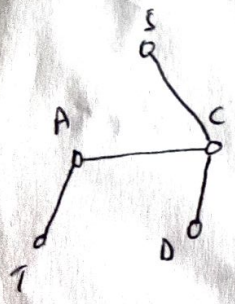
b) Prove or disprove the following result: **(5 Marks)**

If a and b are odd integers then $(a^2 + b^2)$ is even.

c) There is a group of 6 students: Alice, Bob, Carol, Dean, Santos and Tom where Alice and Carol are always fighting, likewise for Dean and Carol and for Santos, Tom and Alice. Draw the graph to represent this situation. **(5 Marks)**

d) Let $S = \{3, 5, 8, 10, 15, 24\}$ and R be the relation defined on S by xRy whenever x divides y. Draw the directed multigraph of this relation. **(4 Marks)**

e) Five colleges of Makerere University want to develop a computer telecommunications network to facilitate communication for research. They want to use as few lines as possible but still allow communication between any two colleges. Indicate how this can be done for the map in Figure 2. Redraw the map in your answer booklet. **(3 Marks)**



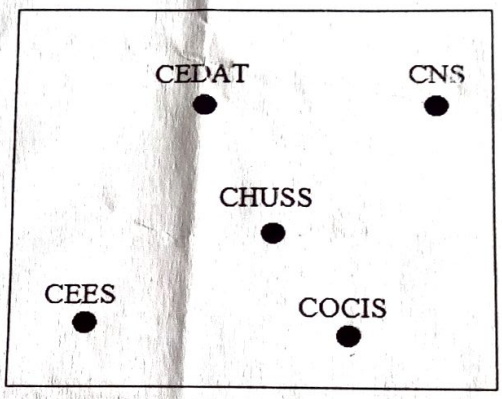


Figure 2

Question 4 (25 Marks)

a) Construct a truth table to determine whether the following compound propositions are logically equivalent. **(8 Marks)**

$(\sim p \wedge q) \vee (p \wedge \sim q)$ and $(\sim p \wedge \sim q) \vee (p \wedge q)$

b) Use the laws of logic or truth tables to simplify the following proposition as far as possible.

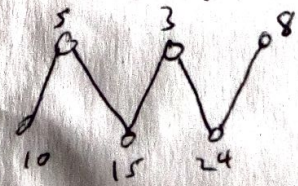
$(a \wedge b) \vee (a \wedge \sim b) \vee (\sim a \wedge b)$ **(5 Marks)**

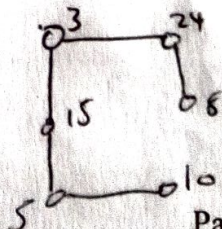
c) Determine whether the following proposition is a tautology.

$q \leftrightarrow (\sim p \vee \sim q)$ **(5 Marks)**

d) For the following the universe of discourse is people. One place predicates are defined as follows:

cheats is "… cheats at cards"
plaits is "… has plaited hair"
scout is "… is a boy scout"





(x, y)

$(1, n^2), (2, 2n^2), (3, 3n^2)$

Write the following propositions as sensible English sentences:

i) $\exists x, scout(x) \wedge cheats(x)$ **(4 Marks)**

ii) $\forall x, scout(x) \rightarrow \sim (plaits(x) \vee cheats(x))$

e) The process for cleaning up waste in a nuclear reactor core room eliminates 85 % of the waste present in the area each week. If there is 1.5 kg of waste in the room at the beginning of the monitoring period and 2.3 kg of additional waste are generated each week, determine a recurrence relation and initial conditions describing the amount w_n of waste in the core room at the end of week n of the monitoring period. **(3 Marks)**

Question 5 (25 Marks)

a) Write brief notes about the following as applied to data in statistics. **(7 Marks)**

i) Measures of central tendency

ii) Measures of dispersion

iii) estimation

b) A large stock of microprocessors is known to have 15 % defectives. If 6 microprocessors are drawn at random determine

i. the probability that none is defective **(3 Marks)**

ii. at least two are defective **(5 Marks)**

iii. mean and standard deviation of the distribution of defects **(2 Marks)**

c) A packet contains 100 washers, 24 of which are brass, 36 copper and the remainder steel. One washer is taken at random, retained and a second washer similarly drawn. Determine the probability that:

i) Both washers are steel **(2 Marks)**

ii) The first is brass and the second copper **(2 Marks)**

iii) One is brass and one is steel **(4 Marks)**

END

$B \cup (\phi \cap A)$

$(B \cup \phi) \cap (B \cup A)$

$B \cap (B \cup A)$

$(B \cap A)$

CMP2201: Discrete Mathematics and random processes

CAT#1 **DURATION: $1\frac{1}{2}$ HOURS**

DATE: April 8, 2016

Instructions: *This paper consists of objective type questions. Write down the correct options for each question on the answer sheet provided. Write clearly. Marks will be deducted for poorly presented work.*

1) One of the following statements is **TRUE.**
A) A discrete variable is one for which there is a fixed set of values
B) A discrete variable is one that takes on one fixed value
C) A continuous variable is a variable that can be measured
D) none

2) Which of the following are discrete variables?
A) The number of components in a machine
B) The size of workforce in a factory
C) The number of students in the discrete math class
D) all

3) Augusta Ada Byron is recognized for her contribution in:
A) Development of algorithms for computing devices
B) Defining relations between sets
C) Establishment of causal graphs
D) none

4) Give an example of sets for which $A \cup C = B \cup C$ *but* $A \neq B$.
A) $A = \{1,2,3\}, B = \{2,3\}, C = \{3\}$
B) $A = \{1\}, B = \{2\}, C = \{3\}$
C) $A = \{1\}, B = \{2\}, C = \{1,2\}$
D) None

5) A and B are subsets of a universal set U. If $A \subseteq B$ then
A) $(B \subseteq A)$
B) $(\bar{B} \subseteq A)$
C) $(\bar{B} \subseteq \bar{A})$
D) none

For questions 6 – 9 determine the properties of the relations.

6) S is the set of all teenagers and xRy means that x has a grandmother in common with y.
A) Transitive
B) Symmetric
C) Not reflexive
D) Transitive, symmetric and reflexive

7) S = {0, 1, 2, 3}. $(x, y) \in R$ if max {x, y} =3.
A) reflexive
B) symmetric
C) transitive
D) all

8) S is the set of nonzero real numbers and xRy means that $xy > 0$.
A) All
B) symmetric
C) Transitive
D) reflexive

CMP2201: Discrete Mathematics and random processes

CAT#1 **DURATION: $1\frac{1}{2}$ HOURS**

DATE: April 8, 2016

Instructions: *This paper consists of objective type questions. Write down the correct options for each question on the answer sheet provided. Write clearly. Marks will be deducted for poorly presented work.*

1) One of the following statements is **TRUE**.
A) A discrete variable is one for which there is a fixed set of values
B) A discrete variable is one that takes on one fixed value
C) A continuous variable is a variable that can be measured
D) none

2) Which of the following are discrete variables?
A) The number of components in a machine
B) The size of workforce in a factory
C) The number of students in the discrete math class
D) all

3) Augusta Ada Byron is recognized for her contribution in:
A) Development of algorithms for computing devices
B) Defining relations between sets
C) Establishment of causal graphs
D) none

4) Give an example of sets for which $A \cup C = B \cup C$ *but* $A \neq B$.
A) $A = \{1,2,3\}, B = \{2,3\}, C = \{3\}$
B) $A = \{1\}, B = \{2\}, C = \{3\}$
C) $A = \{1\}, B = \{2\}, C = \{1,2\}$
D) None

5) A and B are subsets of a universal set U. If $A \subseteq B$ then
A) $(B \subseteq A)$
B) $(\bar{B} \subseteq A)$
C) $(\bar{B} \subseteq \bar{A})$
D) none

For questions 6 – 9 determine the properties of the relations.

6) S is the set of all teenagers and xRy means that x has a grandmother in common with y.
A) Transitive
B) Symmetric
C) Not reflexive
D) Transitive, symmetric and reflexive

7) S ={0, 1, 2, 3}. $(x, y) \in R$ if max {x, y} =3.
A) reflexive
B) symmetric
C) transitive
D) all

8) S is the set of nonzero real numbers and xRy means that $xy > 0$.
A) All
B) symmetric
C) Transitive
D) reflexive

9) S is the set of positive integers and xRy if and only if $y = n^2x$ for some integer n.

A) Irreflexive
B) not transitive
C) symmetric
D) antisymmetric

For questions 10 – 11 determine the necessary conditions for the functions.

10) For $x \in X, g(x) = 10log_2x$

A) Function if and only if X is a set of positive real numbers
B) Function if X is a set of real numbers
C) Function with a pervasive domain
D) none

11) X is the set of _____ currently in use at Makerere university campus and for $x \in X$, g(x) is the operating system that x is running

A) Landline phones i.e. with wires
B) Computers
C) Identity cards
D) none

For questions 12 and 13 let Z denote the set of integers. Which properties does the following function have: $g: Z \rightarrow Z$

12) $g(x) = 10 - 2x$

A) One to one correspondence
B) Onto
C) Not one to one
D) none

13) $g(x) = \begin{cases} \frac{1}{2}(x+1) \text{ if } x \text{ is odd} \\ \frac{1}{2}x \text{ if } x \text{ is even} \end{cases}$

A) One to one correspondence
B) Onto
C) One to one
D) none

14) Let X denote the set of real numbers. Obtain the inverse of the function $f: X \rightarrow X$. $f(x) = 3\,(2^{x+1})$.

A) $-1 + log_2\left(\frac{x}{3}\right)$
B) $[3\,(2^{x+1})]^{-1}$
C) $\sqrt[x+1]{6}$
D) Does not exist

15) $f(x) = 2^x$ and $g(x) = 5x + 7$. Evaluate fg(-10).

A) 7
B) -8×10^{12}
C) -493
D) 0.1×10^{-12}

16) The statement

$1 + 4 + 9 + \cdots + n^2 =$ is equal to:

A) $\frac{(n^2+n)(2n+1)}{6}$ for every positive integer n
B) $\frac{2n(n+1)}{8}$ for every integer n
C) $\frac{n(n+1)^2}{3}$ for every positive integer n
D) none

17) Which of the following sentences is a statement?

(A) Silicon valley is the home of electronics
B) What's the answer?
C) Have a nice day
D) none

18) The statement $\sim(p \rightarrow q)$ is logically equivalent to:

A) $q \vee (p \wedge q)$
(B) $\sim q \wedge (p \vee q)$
C) $(p \leftrightarrow q)$
D) none

19) Which of the following makes the following statement true?

If x is even and x is a perfect square; then:

(A) x is divisible by 4
B) x is divisible by 10
C) 2x is a perfect square
D) none

20) Which method of proof assumes that the conclusion is false and therefore the hypothesis is also false?

(A) Law of contradiction
B) Law of contrapositive
C) Proof by cases
D) None

The End

Makerere University

College of Engineering, Design, Art and Technology

Department of Electrical and Computer Engineering

CMP2203: DIGITAL LOGIC

Date: 13th June, 2017 **Time:** 09:00 A.M -12:00 P.M

INSTRUCTIONS

- This Examination contains Six (6) questions.
- Attempt any four (4) Questions for full Marks
- The first four questions shall be marked if more than four questions are attempted.
- All Questions carry equal marks of 25 %.

Begin each Question on a fresh page

Question One [25 Marks]

a) Give five reasons for minimization of Boolean expressions during digital circuit design. **(5 Marks)**

b) Simplify the following logic function using Boolean algebra and compare your results using K-Map. Draw the resulting logic circuit using NAND gates only. **(8 Marks)**

$$Y = \overline{\overline{A}\overline{B}\overline{C}} + \overline{A}C + \overline{A}B$$

c) Consider the following Boolean expression as stated below.

$$Y = f(A,B,C) = \overline{(A+B)}(\overline{B}+C)$$

(i) Without simplifying the expression, draw the logic circuit realization of the above expression and give the corresponding complete truth table. **(5 Marks)**

(ii) Convert the Boolean expression above to its Demorgan equivalent. Hence draw the logic circuit for the realised Demorgan equivalent Boolean equation. **(6 Marks)**

Question Two [25 Marks]

a) Explain the difference between a timer and a counter. Give a practical example where each of these finds application. **(5 Marks)**

b) Consider a counter circuit that contains six FFs wired as shown in the arrangement in fig Q2-b below.

(i) Give the state transition diagram of the counter. **(5 Marks)**

(ii) Determine the frequency at the output of FF Q2 when the input clock frequency is 1 GHz . **(3 Marks)**

(iii) Determine the MOD number and counting range for this counter. Also determine the counters state after 130 pulses assuming an initial state of 000000. **(7 Marks)**

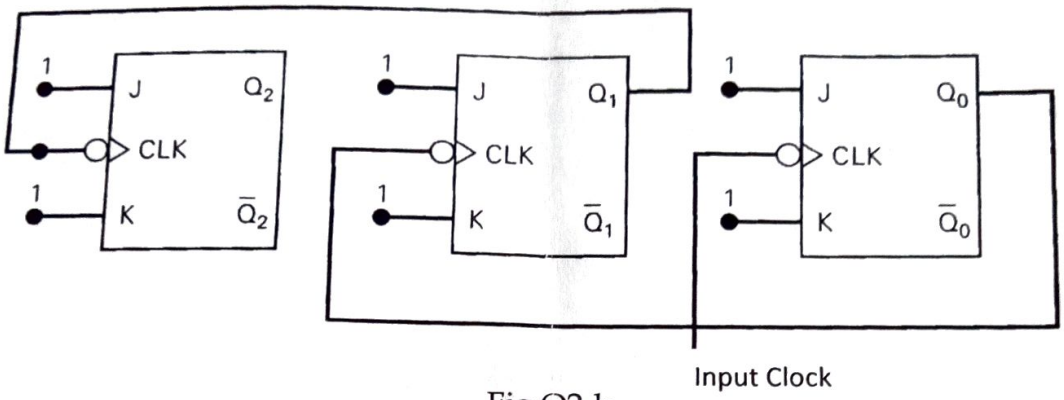


Fig Q2-b

c) Consider five seats, numbered 0 to 4, arranged in a circle and described by Boolean variables i_o *to* i_4 .Boolean variable i_o is true if seat 0 is occupied and i_o is false if the seat is not occupied, likewise for i_1, i_2, i_3 *and* i_4 . Write a Boolean expression that's true if at least two people are sitting next to each other and at least one seat is not occupied. Draw the logic circuit to implement this expression. **(3 Marks)**

Question Three [25 Marks]

a) State the function of the following components of the digital arithmetic circuit:

(i) Accumulator register. **(2 Marks)**

(ii) B register. **(2 Marks)**

(iii) Control unit. **(2 Marks)**

b) Draw a complete logic circuit of the 4-bit parallel adder including the associated registers. By using LOAD, CLEAR and TRANSFER signal lines, describe the sequence of operations by which the logic circuit will add binary numbers 1001_{LSB} and 0101_{LSB}. **(9 Marks)**

c) Write **brief** notes about the following shift registers with focus on logic circuits, operation, advantages and draw backs:

(i) Ring counters **(5 Marks)**

(ii) Johnson Counters **(5 Marks)**

Question Four [25 Marks]

a) Indicate how a Nor gate can be used to implement each of the following logic gates:
   (i) Inverter **(2 Marks)**
   (ii) AND gate **(2 Marks)**
   (iii) OR gate **(2 Marks)**

b) (i) What is the difference between a latch and a flip-flop? Hence draw a block diagram of a D latch and a D flip-flop and give their corresponding truth tables. **(5 Marks)**

(ii) On the graph Q5-b below, inputs CLK and D are inputs to both a D latch and a D flip-flop. CLK goes into the EN or C input of the D latch. Ignoring setup and hold time requirements and assuming both outputs are initially 0 at the start of the graph, write the output of the D latch as Q_{DL} and the output of the D flip-flop as Q_{DFF} on the same graph. Do the two outputs differ, and if so, why? **(4 Marks)**

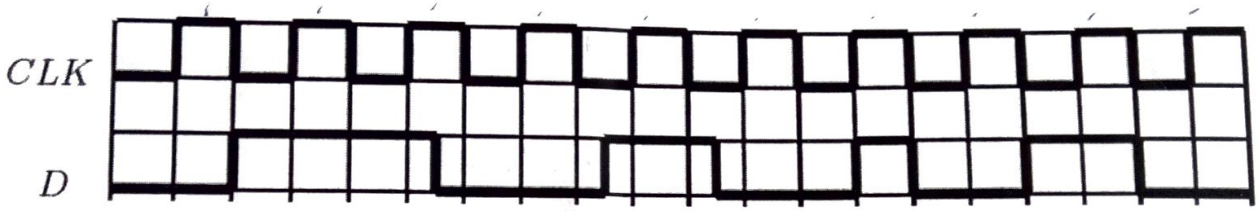


Fig Q5-b

c) (i) Draw a logic circuit used for serial transfer of a DATA word $X_2X_1X_0 = 101$ from one register X to another register Y leaving register X with a DATA word $X_2X_1X_0 = 000$ at the end of the transfer operation. Consider X_0 to be the LSB of the data word and use D Flip-flops. **(6 Marks)**

(ii) Modify the logic circuit in part c(i) above so that the original DATA word stored in register X is present in both registers at the end of the transfer operation. **(4 Marks)**

Question Five [25 Marks]

a) With reference to sequential circuit design, explain the following concepts and give their significance:
   (i) State diagram. **(2 Marks)**
   (ii) State reduction. **(2 Marks)**
   (iii) Transition equation. **(2 Marks)**

b) Consider a sequential circuit implemented using T-Flip-flops as shown in Fig Q5-b below. For this circuit, derive the corresponding;

(i) Excitation equations. **(4 Marks)**

(ii) Next state equation and state table. **(6 Marks)**

(iii) State diagram. **(3 Marks)**

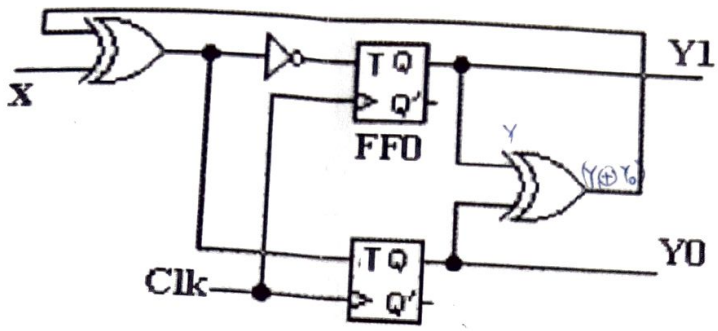


Fig Q5-b

c) Using the 74ALS163 counter shown in fig Q5-c below and logic gates, design a counter that counts in the sequence 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 3, 4,... Connect all unused inputs. The counter may cycle through several unwanted states before settling into the final count sequence. Q_D is the most significant bit (MSB) of the counter output.

(6 Marks)

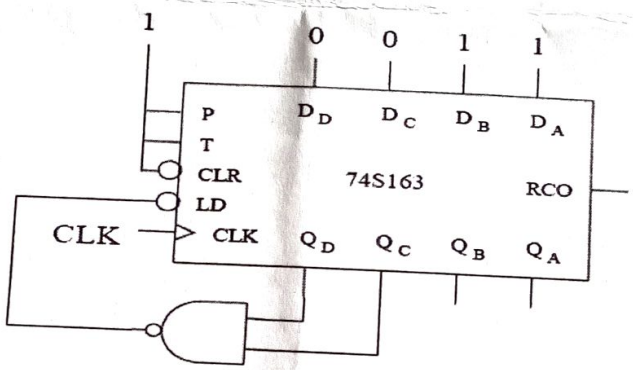


Fig Q5-c

Question Six [25 Marks]

a) Give one application of multiplexers and encoders in digital circuitry. Draw a block diagram of a 4-to-2 encoder labeling all inputs and outputs. How is this encoder different from a 4-to-1 multiplexer? **(6 Marks)**

b) Show how the 8-input multiplexers can be implemented using the components indicated below. In each case, the three select input bits should be labeled s_2, s_1, s_0 with s_0 being the least significant bit. The data inputs should be labeled 0 to 7:

(i) Two 4-input multiplexers and a 2-input multiplexer **(4 Marks)**

(ii) Four 2-input multiplexers and a 4-input multiplexer **(4 Marks)**

(iii) A decoder and logic gates. **(4 Marks)**

Mukasa has half adders and full adders available to use as components in his tool bag. If Mukasa wishes to add two 4-bit numbers;

(i) Draw a **block** diagram for his 4-bit adder using half adder and full adders. Show and label all inputs and outputs. **(5 Marks)**

(ii) Assume that a half adder has a maximum propagation delay of Δ, and a full adder has a maximum propagation delay of 2Δ. What is the maximum propagation delay for his 4-bit adder, from LSB to MSB output? **(3 Marks)**

END