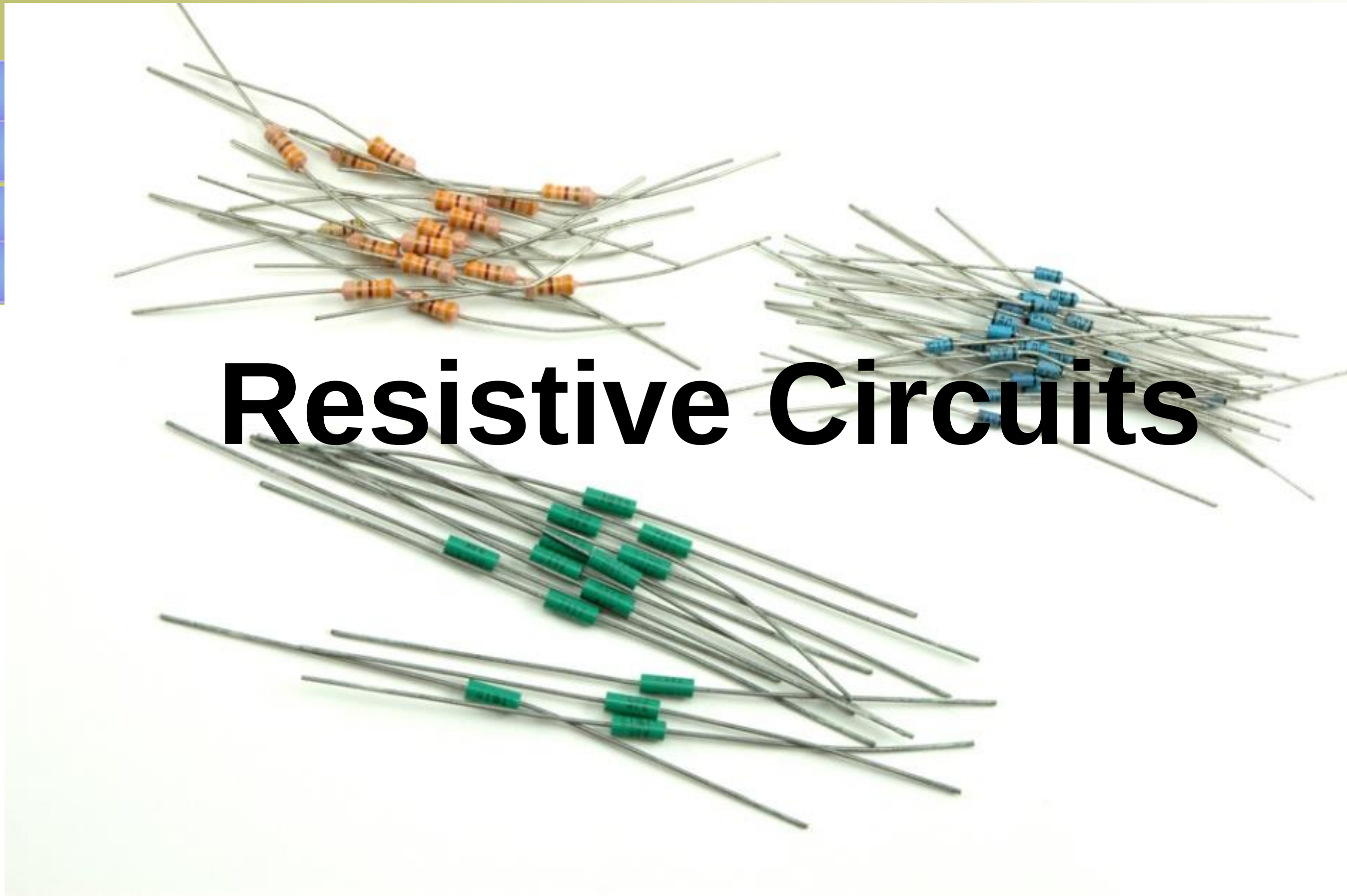
Chapter 2



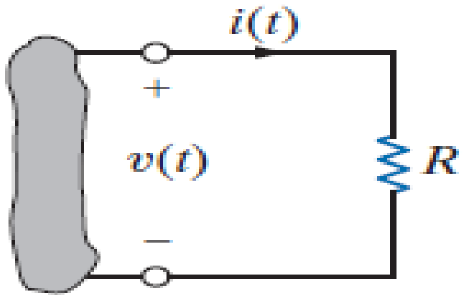
Resistive Circuits

Learning goals

- By the end of this chapter, the students should be able to:
- Use Ohm's law, KVL and KCL to calculate the voltages and currents in electric circuits.
- Determine the equivalent resistance of a resistor network where the resistors are in series and parallel.
- Calculate the voltages and currents in a simple electric circuit using voltage and current division.
- Transform the basic wye resistor network to a delta resistor network, and visa versa.
- Analyze electric circuits to determine the voltages and currents in electric circuits that contain dependent sources

Ohm's Law

- *Ohm's law states that the voltage across a resistance is directly proportional to the current flowing through it*. The resistance, measured in ohms, Ω, is the constant of proportionality between the voltage and current.
- A circuit element whose electrical characteristic is primarily resistive is called a resistor and is represented by the symbol shown in Fig. below.



Symbol for the resistor

Ohm's Law

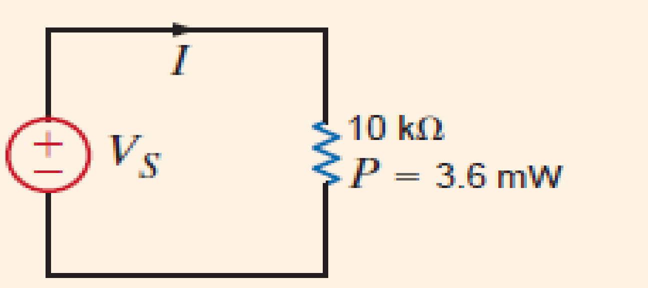
- Mathematically, $v(t) = Ri(t)$ $\quad where\ R \geq 0$
- Since a resistor is a passive element, the proper current–voltage relationship is illustrated in Fig. above.
- The power supplied to the terminals is absorbed by the resistor.
- Note that the charge moves from the higher to the lower potential as it passes through the resistor and the energy absorbed is dissipated by the resistor in the form of heat.

Ohm's Law

- The rate of energy dissipation is the instantaneous power, and therefore
- $P(t) = v(t)i(t)$
- $P(t) = Ri^2(t) = \frac{v^2(t)}{R}$
- This equation illustrates that the power is a nonlinear function of either current or voltage and that it is always a positive quantity.

Ohm's Law

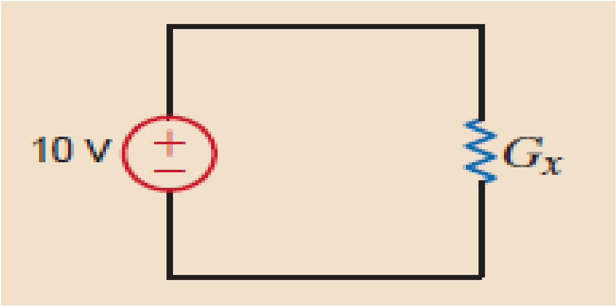
- **Examples**
- The power absorbed by the 10-kΩ resistor in Fig. below is 3.6 mW. Determine the voltage and the current in the circuit.



(V=6 V, I=0.6 mA)

Ohm's Law

- **Examples**
- The power absorbed by G_x in Fig. below is 50 mW. Find G_x.



(500 µS)

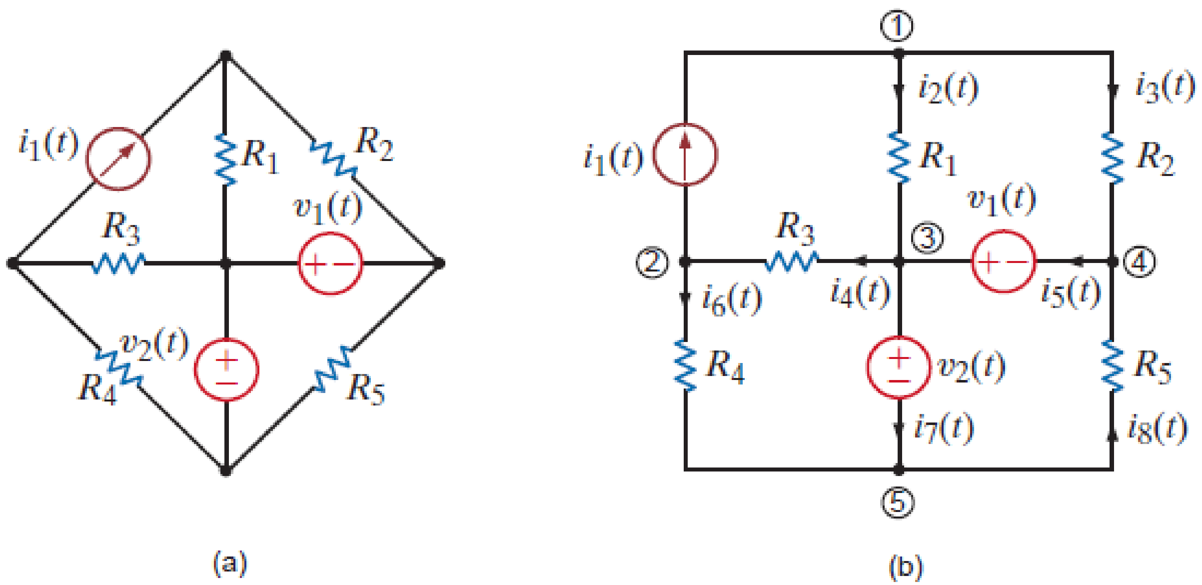
Kirchhoff’S Laws

- The first law is *Kirchhoff ’s current law* (KCL), which states that *the algebraic sum of the currents entering any node is zero*
- Alternatively
- *The sum of the currents entering a node is equal to the sum of the currents leaving the node*.

Kirchhoff'S Laws

- The KCL equations for nodes 1 through 5 are:
- $-i_1(t) + i_2(t) + i_3(t) = 0$
- $i_1(t) - i_4(t) + i_6(t) = 0$
- $-i_2(t) + i_4(t) - i_5(t) + i_7(t) = 0$
- $-i_3(t) + i_5(t) - i_8(t) = 0$
- $-i_6(t) - i_7(t) + i_8(t) = 0$

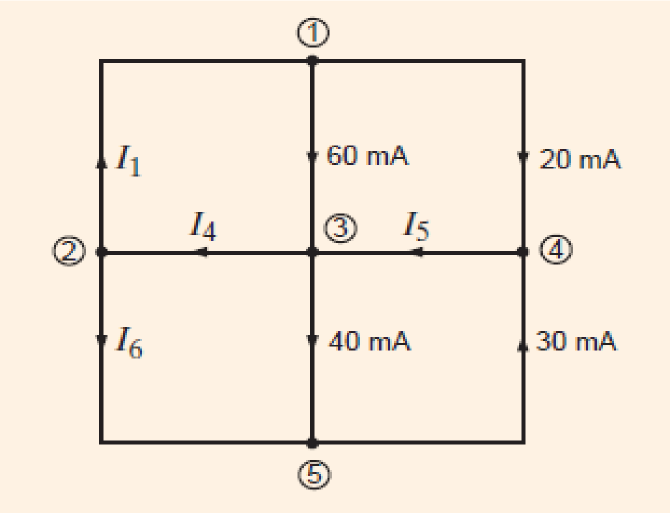
Kirchhoff'S Laws



Circuit to illustrate KCL

Kirchhoff'S Laws

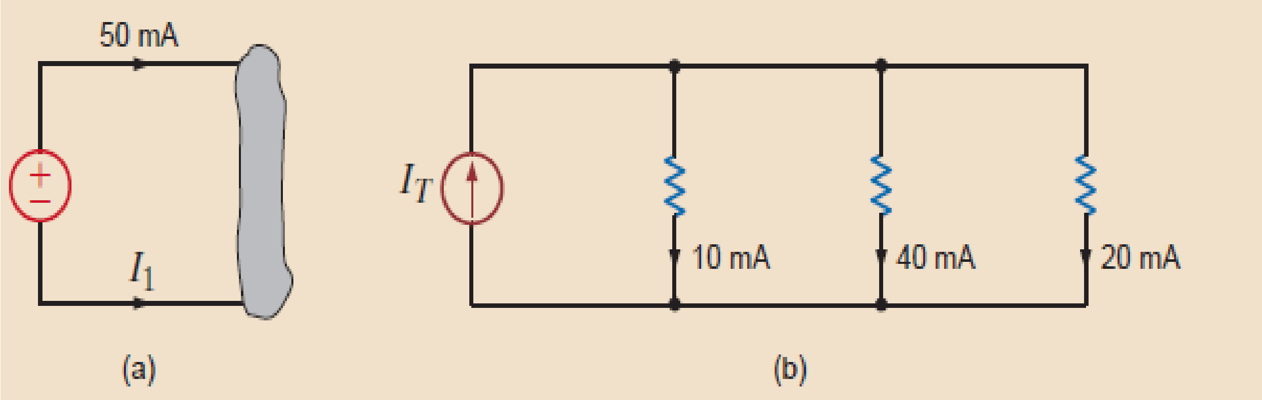
- The network in Fig. below is represented by the topological diagram shown in Fig. below. Determine the unknown currents in the network.



(80 mA, 70 mA, 50 mA, -10 mA)

Kirchhoff'S Laws

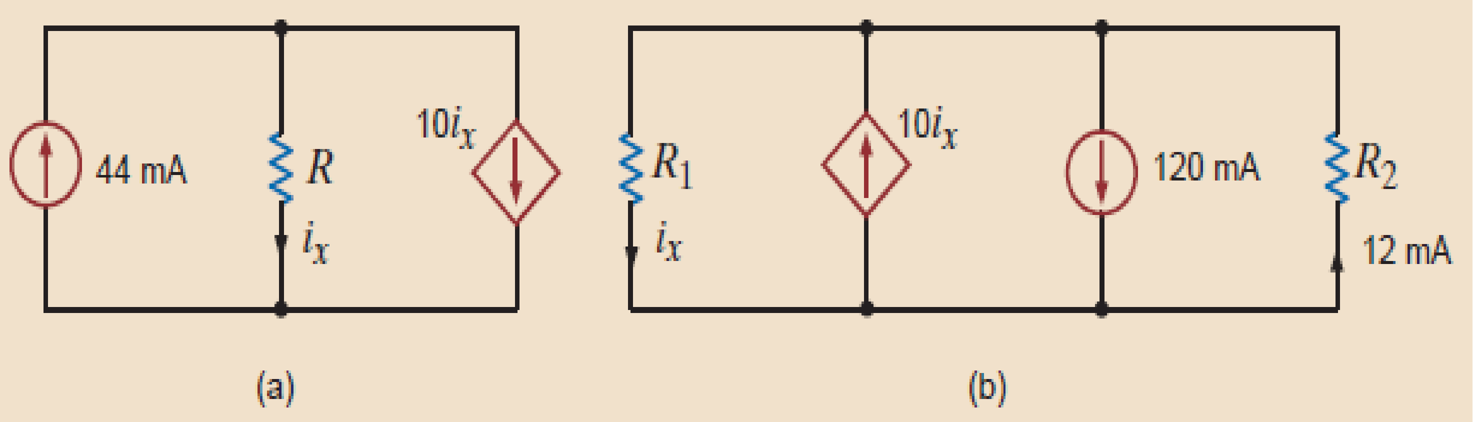
- Kirchhoff's second law, called *Kirchhoff's voltage law* (KVL), states that the *algebraic sum of the voltages around any loop is zero*.
- **Examples**
- Given the networks below, find (a) I_1 in (a) and (b) I_T in (b)



(-50 mA, 70 mA)

Kirchhoff'S Laws

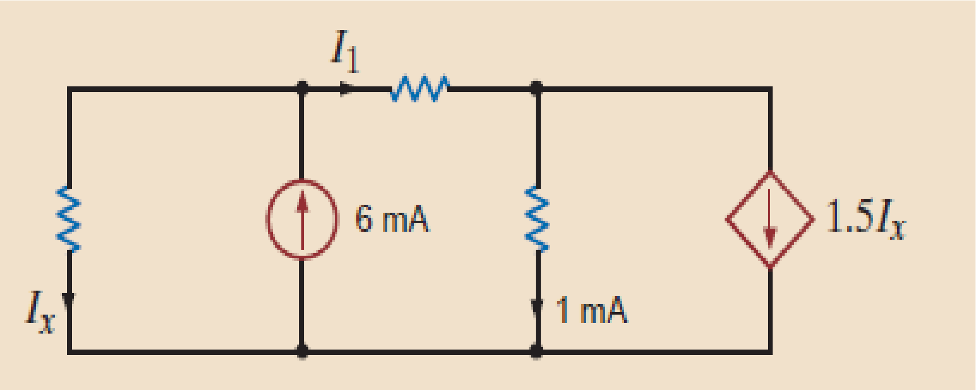
- Find I_x in figure below



(4 mA, 12 mA)

Kirchhoff'S Laws

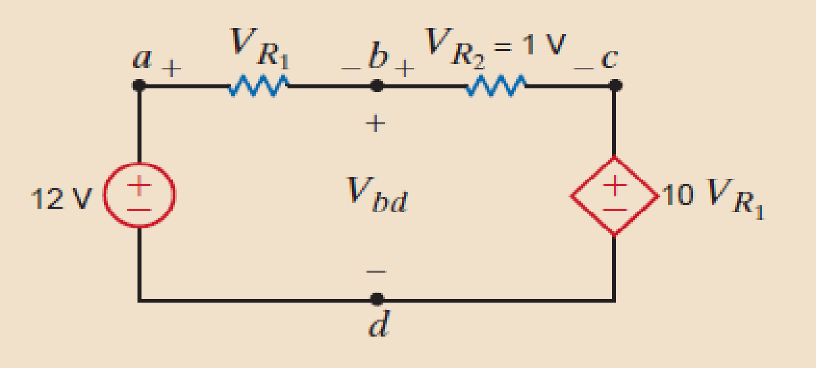
- Find I_x and I_1 in figure below



(2 mA, 4 mA)

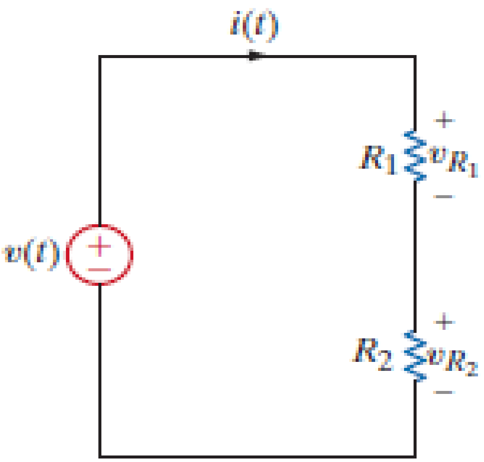
Kirchhoff'S Laws

- Find V_{bd} in the circuit below



(11V)

Voltage Division



Single loop Circuit

$$-v(t) + v_{R1} + v_{R2} = 0$$

$$Or\ v(t) = v_{R1} + v_{R2}$$

$$v_{R1} = R_1 i(t)\ \ and\ v_{R2}$$
$$= R_2 i(t)\ \ (Ohm's\ Law)$$

Therefore $v(t) = R_1 i(t)\ + R_2 i(t)$

Solving for $i(t)$ yields

$$i(t) = \frac{v(t)}{R_1 + R_2}$$

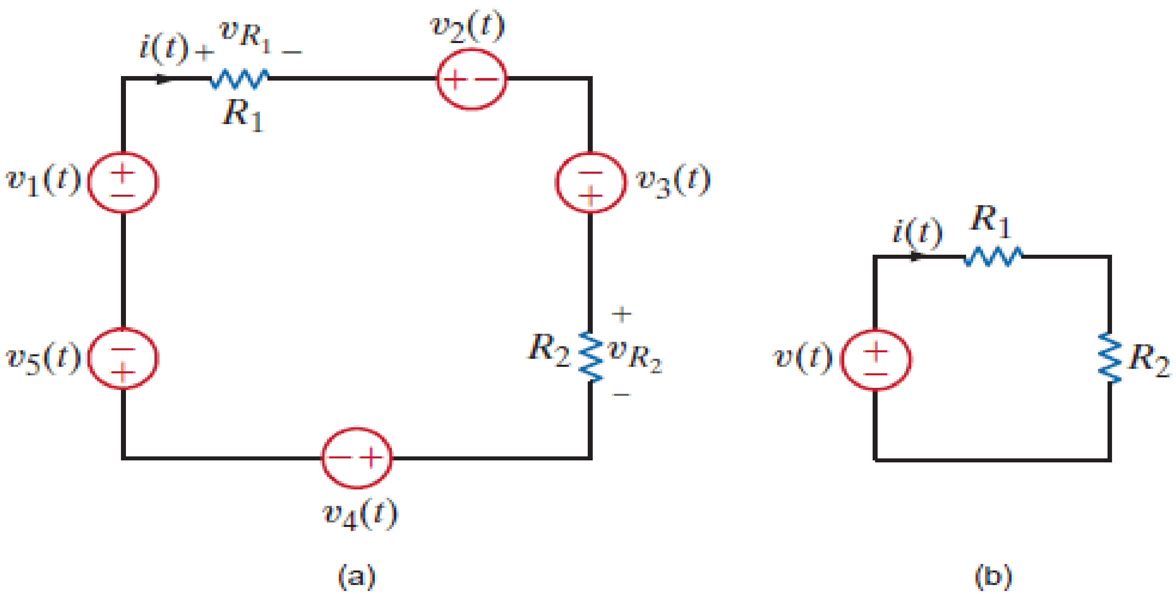
Knowing the current, we can now apply Ohm's law to determine the voltage across each resistor:

$$v_{R1} = R_1 \left[\frac{v(t)}{R_1 + R_2}\right]$$

$$v_{R2} = R_2 \left[\frac{v(t)}{R_1 + R_2}\right]$$

Voltage Division

- **MULTIPLE-SOURCE/RESISTOR NETWORKS**



Equivalent circuits with multiple sources

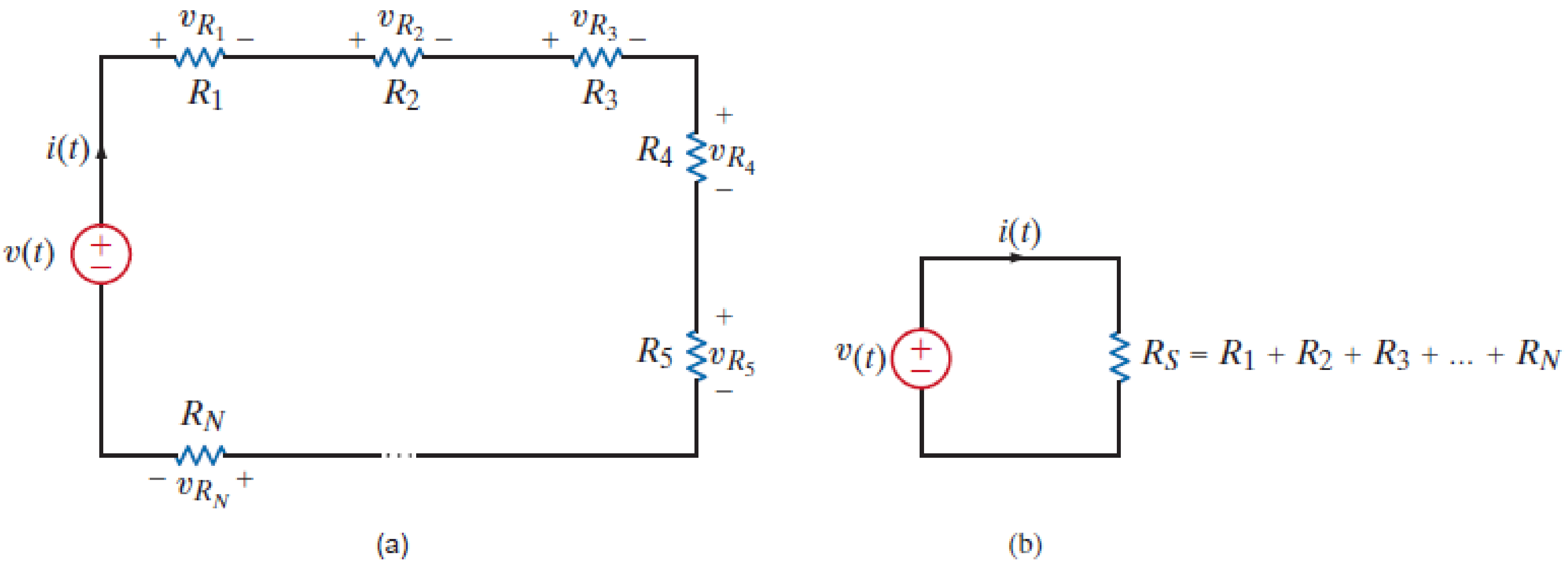
Voltage Division

- **MULTIPLE-SOURCE/RESISTOR NETWORKS**
- KVL for the foregone circuit is:
- $+v_{R1} + v_2(t) - v_3(t) + v_{R2} + v_4(t) + v_5(t) - v_1(t) = 0$
- Or by using Ohm's Law
- $(R_1 + R_2)i(t) = v_1(t) - v_2(t) + v_3(t) - v_4(t) - v_5(t)$
- Which can be rewritten as
- $(R_1 + R_2)i(t) = v(t)$
- Where $v(t) = v_1(t) + v_3(t) - [v_2(t) + v_4(t) + v_5(t)]$

Voltage Division

- so that under the preceding definitions, Fig. a is equivalent to Fig. b. In other words, the sum of several voltage sources in series can be replaced by one source whose value is the algebraic sum of the individual sources.
- This analysis can, of course, be generalized to a circuit with N series sources.

Voltage Division



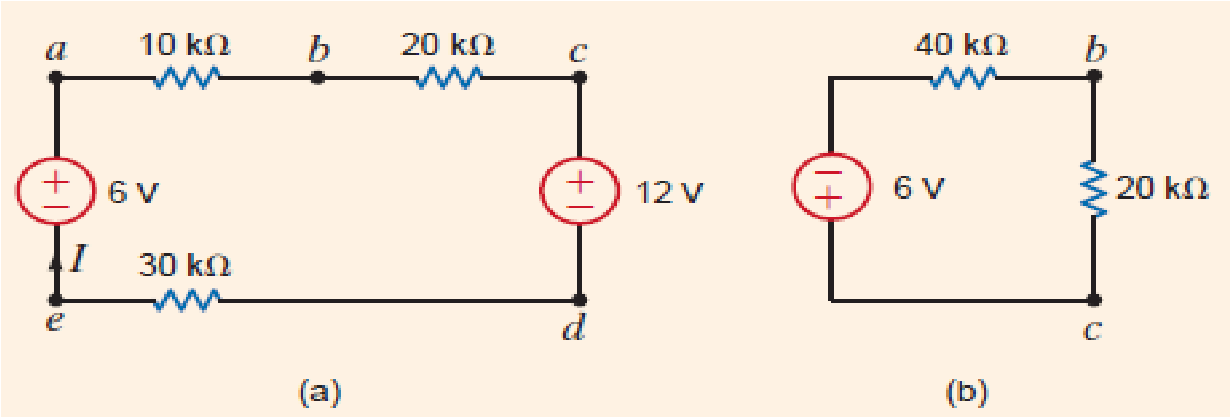
Now consider the circuit with N resistors in series, as shown in Fig. a. Applying Kirchhoff's voltage law to this circuit yields…..

Voltage Division

- $v(t) = v_{R1} + v_{R2} + \cdots v_{RN} = R_1 i(t) + R_2 i(t) + \cdots R_N i(t) +$
- And therefore
- $v(t) = R_s i(t)$
- $R_s = R_1 + R_2 + \cdots R_N$
- And hence $i(t) = \frac{v(t)}{R_s}$
- And for any resistor i,
- $v_{Ri} = \frac{R_i}{R_s} v(t)$

Voltage Division

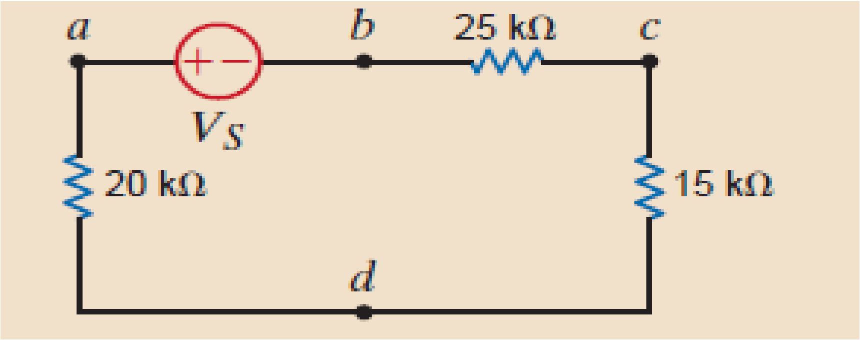
- **Example**
- Given the circuit in Fig. a, let us find I, V_{bd}, and the power absorbed by the 30-k Ω resistor. Finally, let us use voltage division to find Vbc .



(-0.1 mA, 0.3 mW, -2 V)

Voltage Division

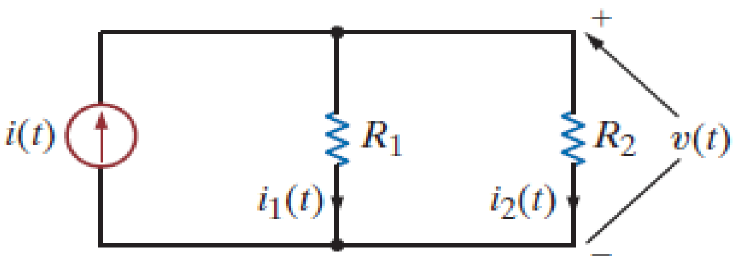
- In the network in Fig. below, if V_{ad} is 3 V, find V_S .



(9V)

CURRENT DIVISION

- **Single-Node-Pair Circuits**



Simple parallel circuits

Where $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$

$$R_p = \frac{R_1 R_2}{R_1 + R_2}$$

$$v(t) = R_p i(t) = \frac{R_1 R_2}{R_1 + R_2} i(t)$$

Therefore $i_1(t) = \frac{v(t)}{R_1} = \frac{R_2}{R_1+R_2} i(t)$

And $i_2(t) = \frac{v(t)}{R_2} = \frac{R_1}{R_1+R_2} i(t)$

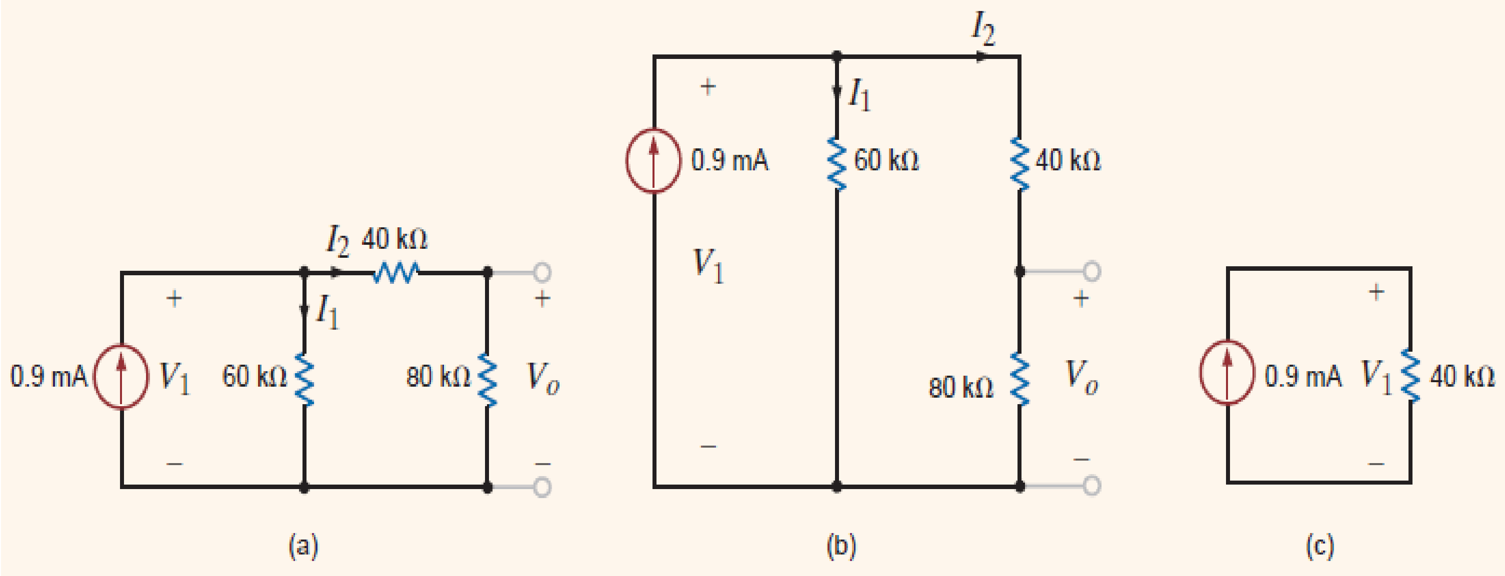
These equations are mathematical statements of current division.

$$i(t) = i_1(t) + i_2(t) \qquad KCL$$

$$i(t) = \frac{v(t)}{R_1} + \frac{v(t)}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v(t) = \frac{v(t)}{R_p}$$

Current Division

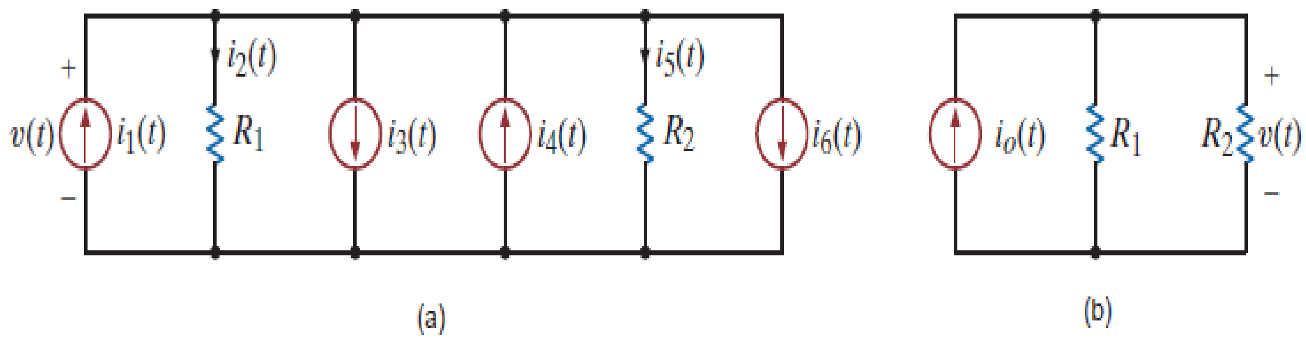
- **Example**
- Given the network in Fig. a, let us find I_1, I_2 and V_0.



(0.6 mA, 0.3 mA, 24 V)

Current Division

- **Multiple-Source/Resistor Networks**



$$i_1(t) - i_2(t) - i_3(t) + i_4(t) - i_5(t) - i_6(t) = 0$$

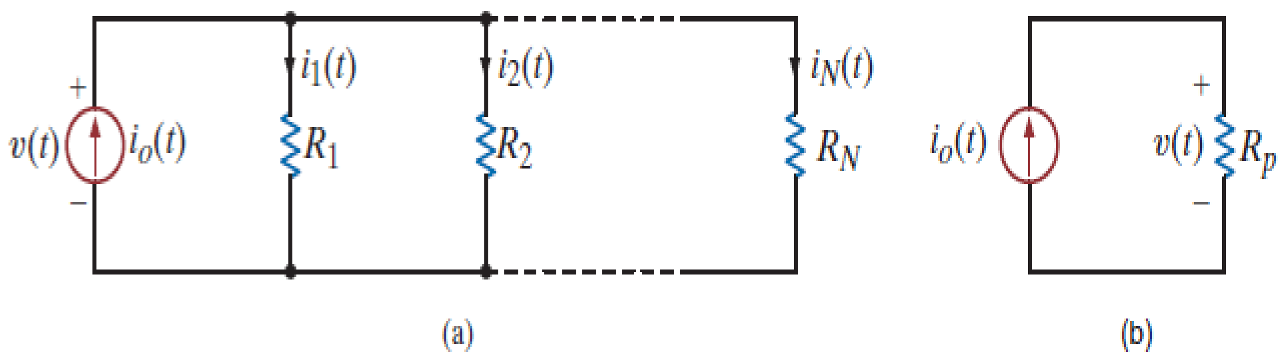
Or $i_1(t) - i_3(t) + i_4(t) - i_6(t) = i_2(t) + i_5(t)$

Current Division

- The terms on the left side of the equation all represent sources that can be combined algebraically into a single source; that is,
- $i_0(t) = i_1(t) - i_3(t) + i_4(t) - i_6(t)$
- which effectively reduces the circuit in Fig. a to that in Fig. b.
- We could, of course, generalize this analysis to a circuit with N current sources.

Current Division

- Using Ohm's law, we can express the currents on the right side of the equation in terms of the voltage and individual resistances so that the KCL equation reduces to
- $i_0(t) = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v(t)$

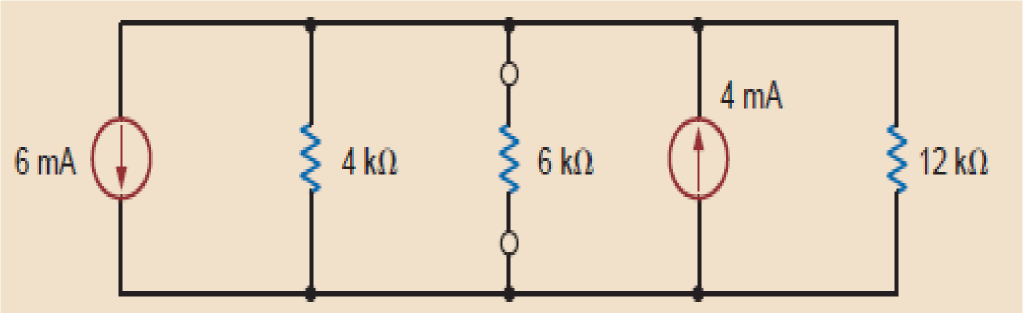


Current Division

- $i_0(t) = i_1(t) + i_2(t) + \cdots i_N(t) = \left(\frac{1}{R_1} + \frac{1}{R_2} + .. + \frac{1}{R_N}\right) v(t)$
- $i_0(t) = v(t)/R_p$
- Where $\frac{1}{R_p} = \sum_{i=1}^{N} \frac{1}{R_i}$

Current Division

- **Example**
- Find the power absorbed by the 6-kΩ resistor in the network in Fig. below



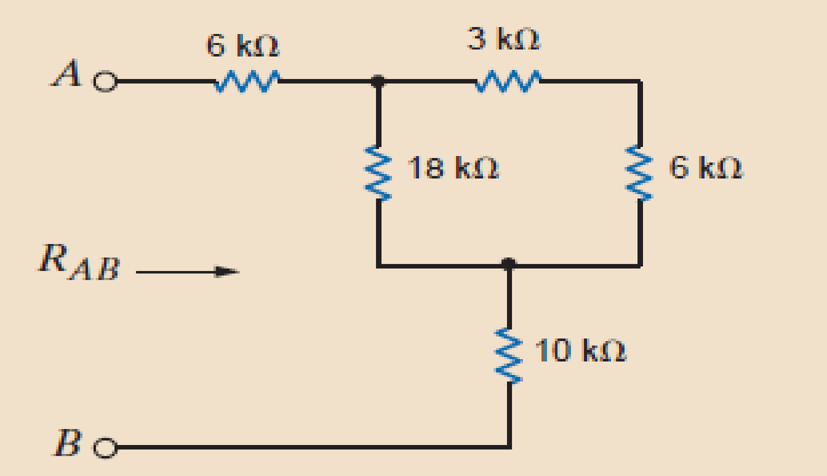
(2.67 mW)

Current Division

- **Series and Parallel Resistor Combinations**
- The equivalent resistance of N resistors in series is
- $R_S = R_1 + R_2 + \cdots R_N$
- and the equivalent resistance of *N* resistors in parallel is found from
- $\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} \ldots \frac{1}{R_N}$

Current Division

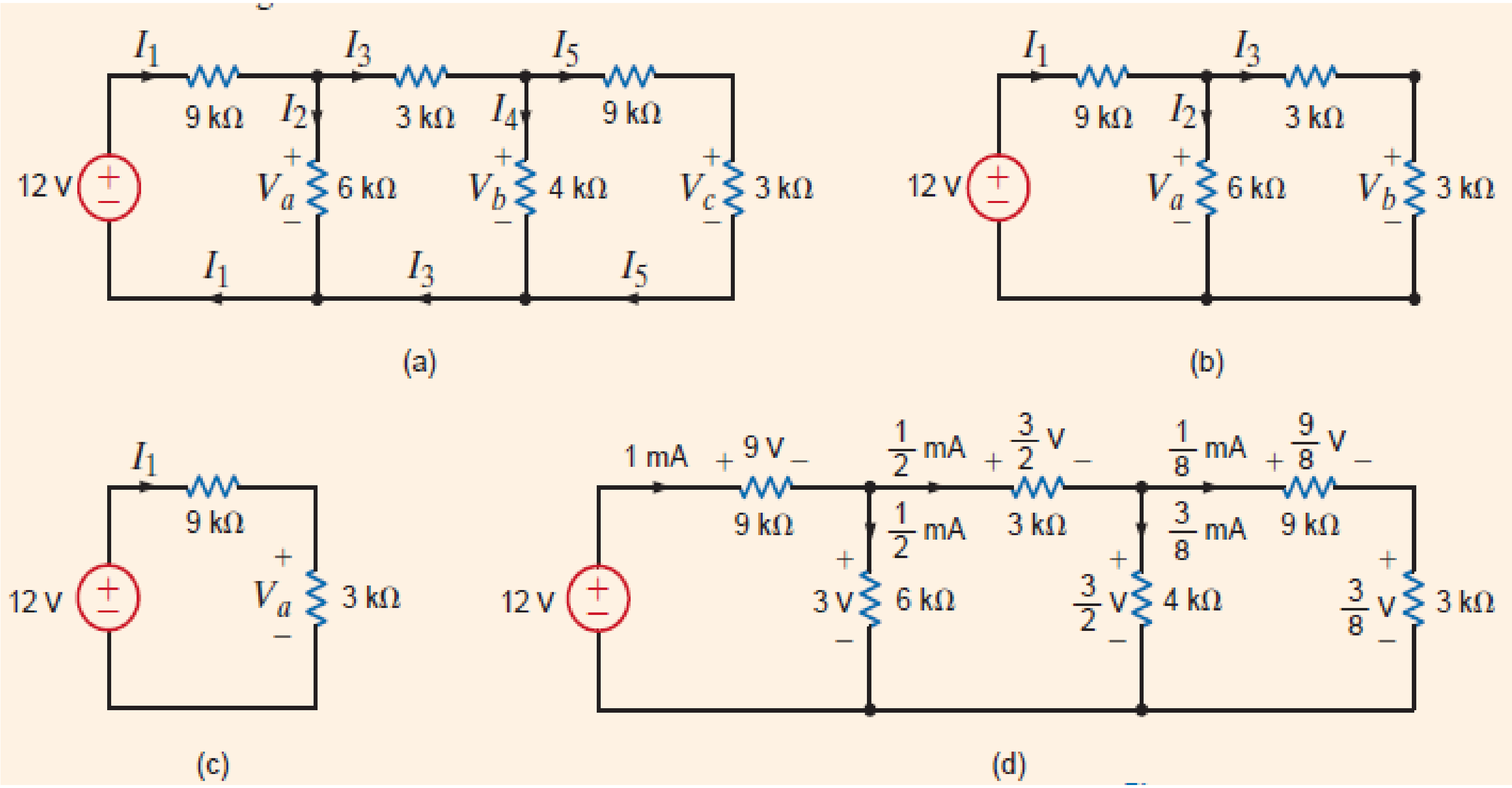
- **Examples**
- Find the equivalent resistance at the terminals *A*-*B* in the network in Fig. below



(22 kΩ)

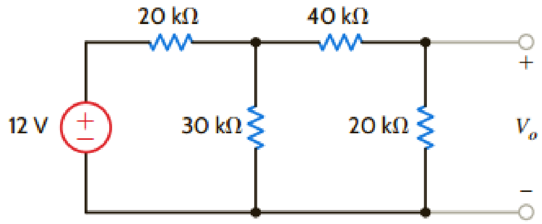
Current Division

- Find all the currents and voltages labelled in the ladder network in figure below (a)



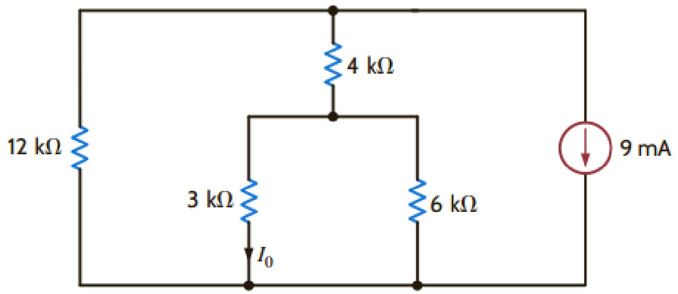
Exercise

- Find V_0 in the circuit below



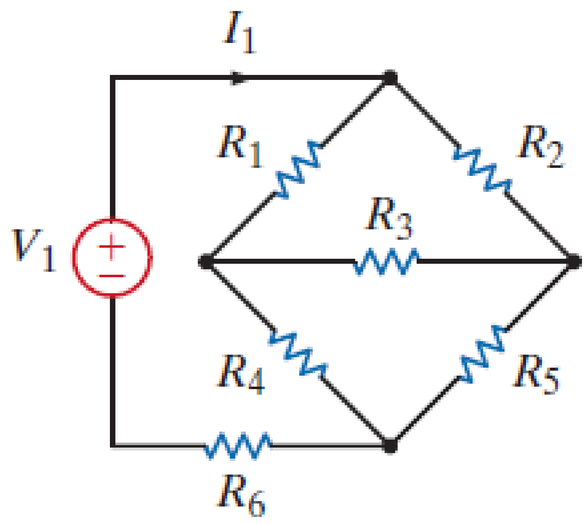
(2V)

Find I_0 in the network below

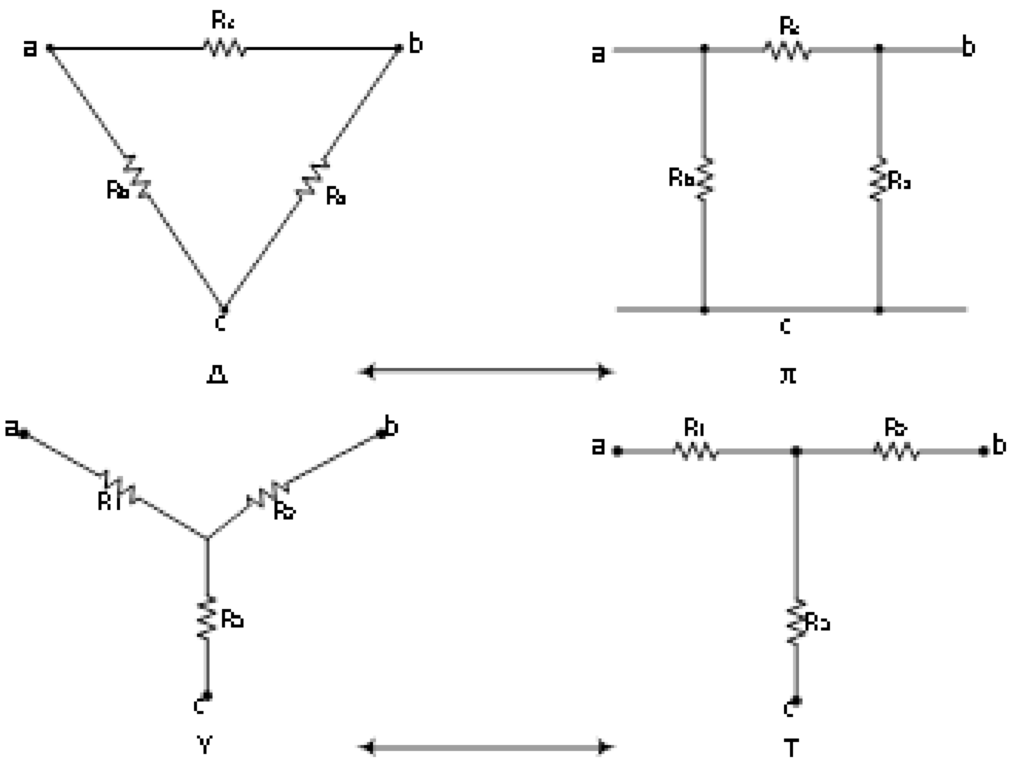


(-4 mA)

Wye ↔ Delta Transformations



Wye ↔ Delta Transformations



Wye ↔ Delta Transformations

- $R_{ab} = R_1 + R_2 = \frac{R_c(R_a+R_b)}{R_a+R_b+R_c}$
- $R_{bc} = R_2 + R_3 = \frac{R_a(R_b+R_c)}{R_a+R_b+R_c}$
- $R_{ca} = R_1 + R_3 = \frac{R_b(R_a+R_c)}{R_a+R_b+R_c}$
- Solving this set of equations simultaneously

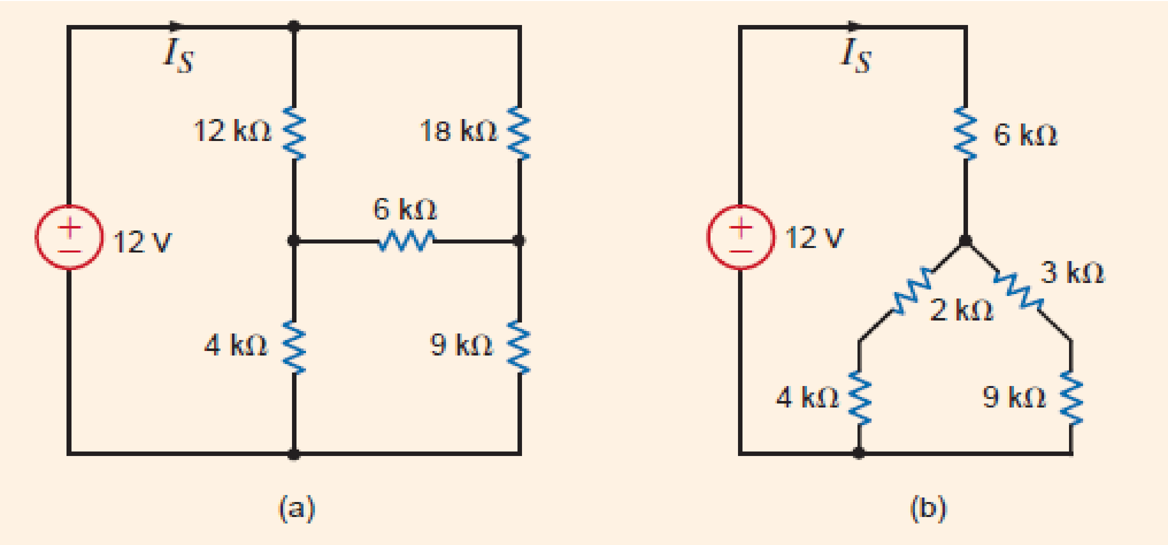
$$R_1 = \frac{R_c R_b}{R_a+R_b+R_c}, \quad R_2 = \frac{R_c R_a}{R_a+R_b+R_c} \quad , R_3 = \frac{R_a R_b}{R_a+R_b+R_c}$$

Wye ↔ Delta Transformations

- Similarly, if we solve, we obtain:
- $R_a = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1}$, $R_b = \frac{R_1R_2+R_2R_3+R_3R_1}{R_2}$, $R_c = \frac{R_1R_2+R_2R_3+R_3R_1}{R_3}$
- For the balanced case
- $R_Y = \frac{1}{3}R_\Delta$, $R_\Delta = 3R_Y$

Wye ↔ Delta Transformations

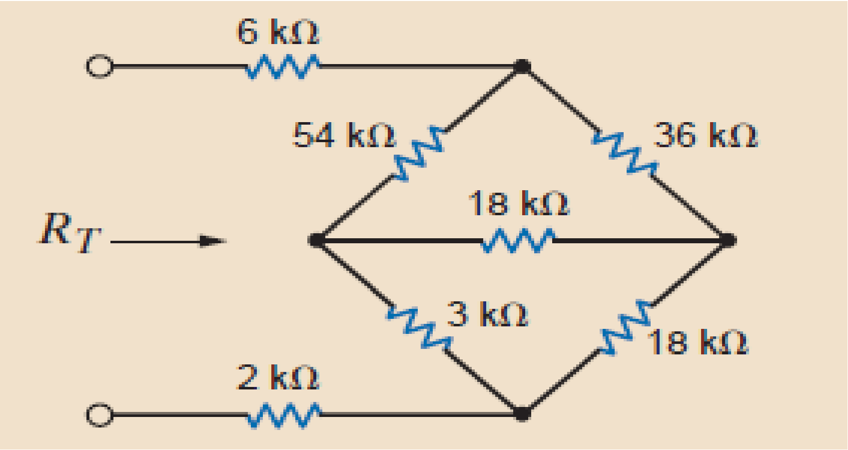
- **Example**
- Given the network in Fig. below (a), let us find the source current I_S .



$$I_S = \frac{12}{6k + 4k} = 1.2mA$$

Wye ↔ Delta Transformations

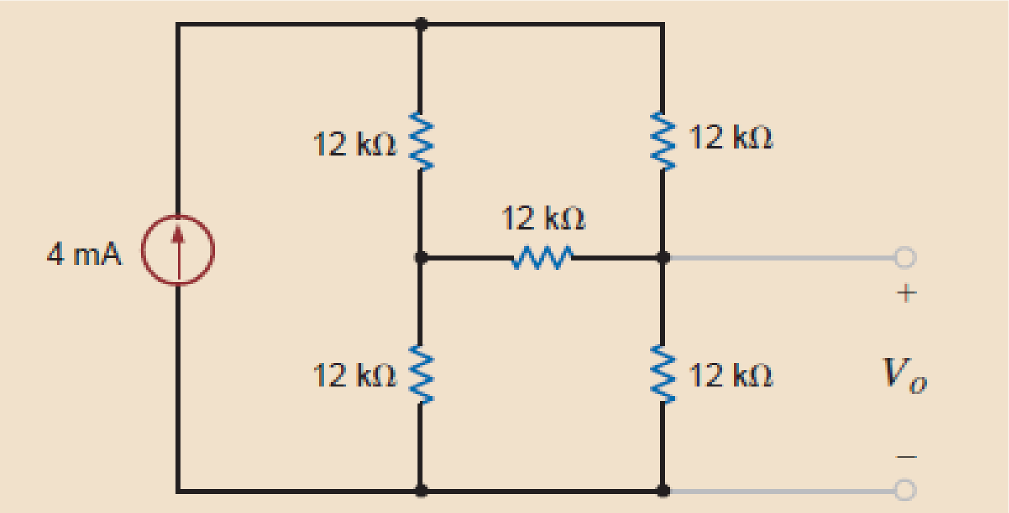
- **Exercise**
- Determine the total resistance R_T in the circuit in below



(34kΩ)

Wye ↔ Delta Transformations

- Determine V_0 in the circuit below



(24 V)