Chapter 8

Power in AC Circuits

Learning goals

By the end of this chapter, the students should be able to

- Explain concepts such as apparent power, active power, reactive power and power factor
- Calculate the power dissipated in circuits containing resistors, capacitors and inductors when these are used with AC signals
- Discuss the importance of the power factor in determining the efficiency of power utilisation and distribution
- Determine the power factor of a given circuit arrangement and propose appropriate additional omponents to achieve power factor correction if necessary
- Describe the measurement of power in both single phase and three-phase arrangements.

Introduction

- The instantaneous power dissipated in a component is a product of the instantaneous voltage and the instantaneous current

$p = vi$

- In a resistive circuit the voltage and current are in phase – calculation of p is straightforward
- In reactive circuits, there will normally be some phase shift between v and i, and calculating the power becomes more complicated.

Power in Resistive Components

- Suppose a voltage v = V_p sin ωt is applied across a resistance R. The resultant current i will be

$$i = \frac{v}{R} = \frac{V_P \sin \omega t}{R} = I_P \sin \omega t$$

- The result power p will be

$$p = vi = V_P \sin \omega t \times I_P \sin \omega t = V_P I_P (\sin^2 \omega t) = V_P I_P (\frac{1 - \cos 2\omega t}{2})$$

- The average value of (1 - cos 2ωt) is 1, so

$$\text{Average Power } P = \frac{1}{2} V_P I_P = \frac{V_P}{\sqrt{2}} \times \frac{I_P}{\sqrt{2}} = VI$$

where V and I are the **r.m.s. voltage and current**

Power in Resistive Components

- **Relationship between *v*, *i* and *p* in a resistor**

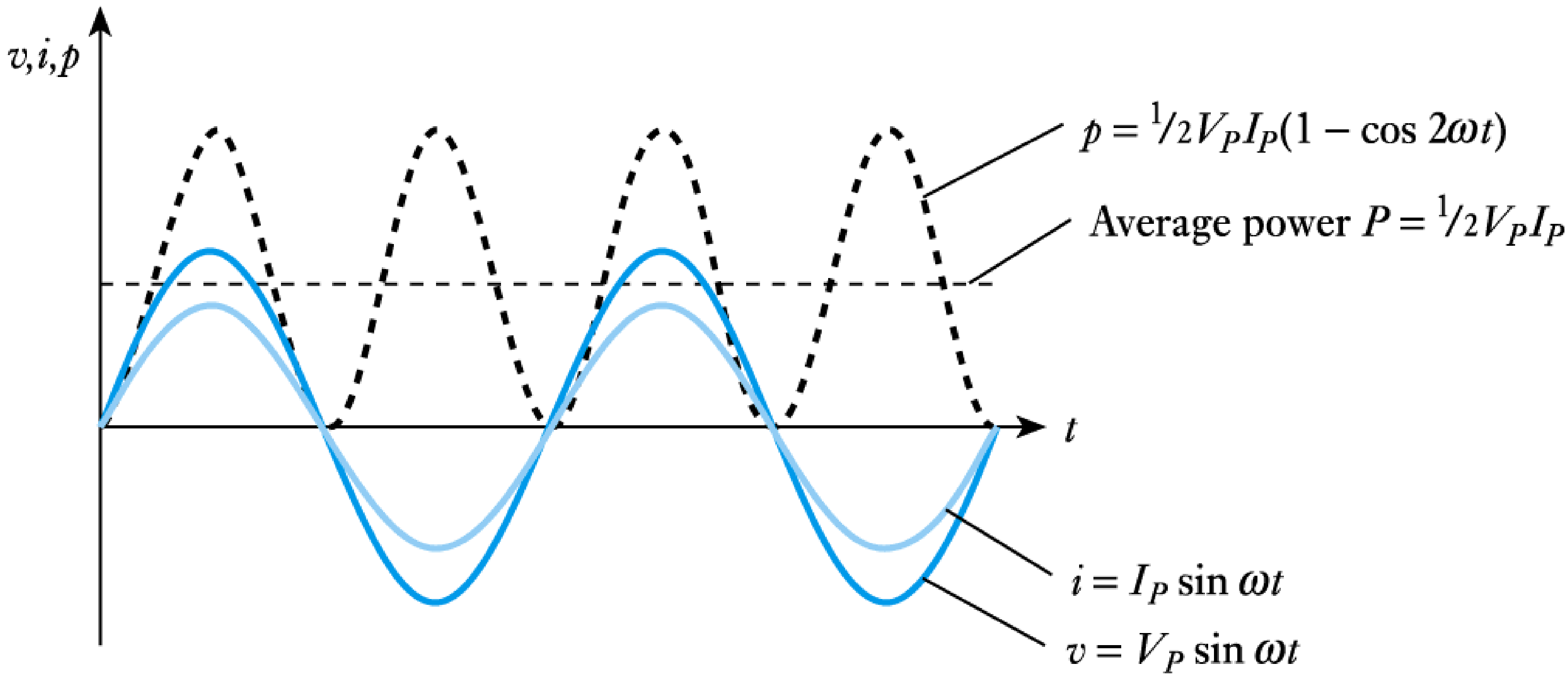

Figure 8.1 Relationship between voltage, current and power in a resistor.

Power in Capacitors

- From our discussion of capacitors we know that the current leads the voltage by 90°. Therefore, if a voltage $v = V_p \sin \omega t$ is applied across a capacitance C, the current will be given by $i = I_p \cos \omega t$
- Then

$$
\begin{aligned}
p &= vi \\
&= V_P \sin \omega t \times I_P \cos \omega t \\
&= V_P I_P (\sin \omega t \times \cos \omega t) \\
&= V_P I_P (\frac{\sin 2\omega t}{2})
\end{aligned}
$$

- The average power is zero

Power in Capacitors

Thus the capacitor stores energy for part of the cycle and returns it to the circuit again, with the average power dissipated P in the capacitor being zero.

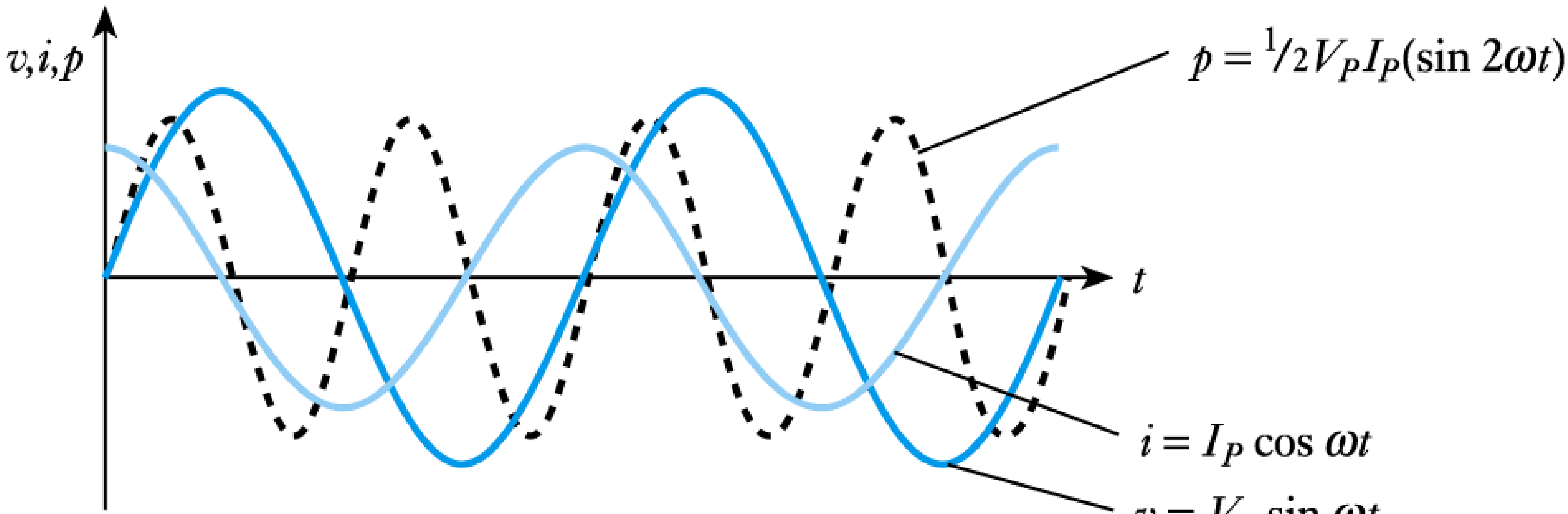

Figure 8.2: **Relationship between *v*, *i* and *p* in a capacitor**

Power in Inductors

- From our discussion of inductors we know that the current lags the voltage by 90°. Therefore, if a voltage $v = V_p \sin \omega t$ is applied across an inductance L, the current will be given by $i = -I_p \cos \omega t$
- Therefore

$$
\begin{aligned}
p &= vi \\
&= V_P \sin \omega t \times -I_P \cos \omega t \\
&= -V_P I_P (\sin \omega t \times \cos \omega t) \\
&= -V_P I_P (\frac{\sin 2\omega t}{2})
\end{aligned}
$$

- Again the average power is zero

Power in Inductors

The situation is very similar to that of the capacitor, and again the power dissipated in the inductor is zero.

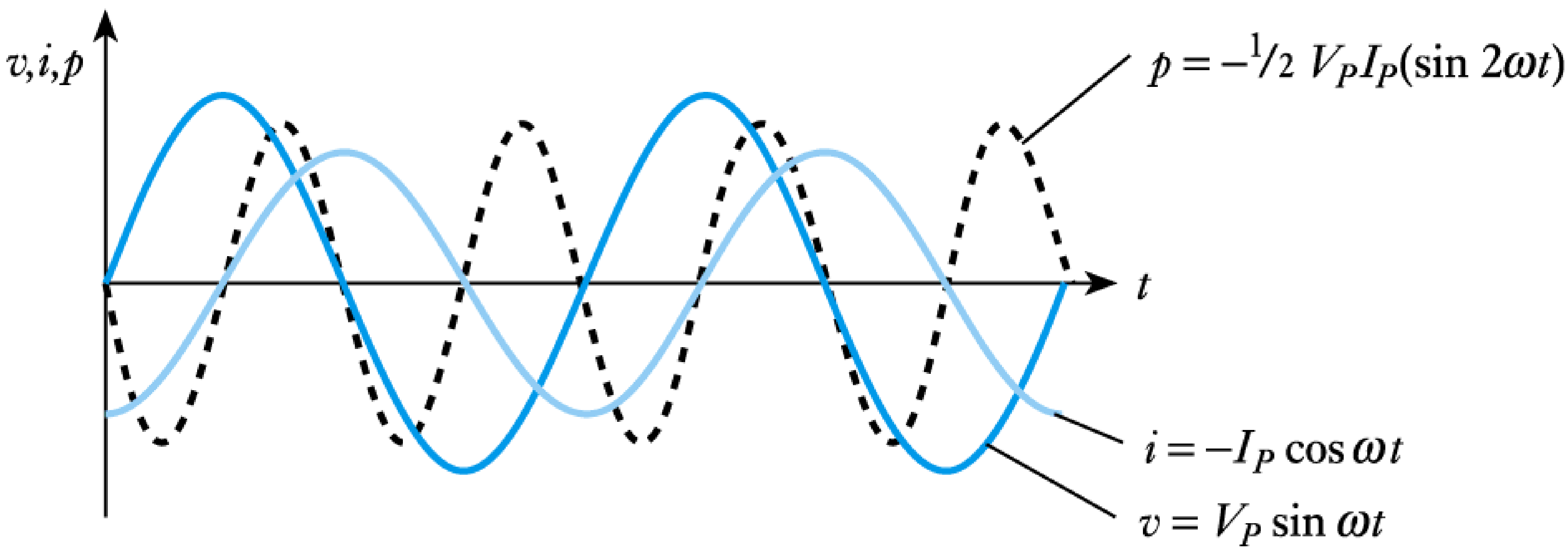

Figure 8.3: **Relationship between *v*, *i* and *p* in an inductor**

Circuit with Resistance and Reactance

- When a sinusoidal voltage $v = V_p \sin \omega t$ is applied across a circuit with resistance *and* reactance, the current will be of the general form $i = I_p \sin(\omega t - \phi)$
- Therefore, the instantaneous power, p is given by

$$\begin{aligned} p &= vi \\ &= V_P \sin \omega t \times I_P \sin(\omega t - \phi) \\ &= \frac{1}{2} V_P I_P \{\cos\phi - \cos(2\omega t - \phi)\} \end{aligned}$$

$$p = \frac{1}{2} V_P I_P \cos\phi - \frac{1}{2} V_P I_P \cos(2\omega t - \phi)$$

Circuit with Resistance and Reactance

$$p = \frac{1}{2}V_P I_P \cos\phi - \frac{1}{2}V_P I_P \cos(2\omega t - \phi)$$

- The expression for p has two components
- The second part oscillates at 2ω and has an average value of zero over a complete cycle
  - this is the power that is stored in the reactive elements and then returned to the circuit within each cycle
- The first part represents the power dissipated in resistive components. **Average power dissipation** is

$$P = \frac{1}{2}V_P I_P(\cos\phi) = \frac{V_P}{\sqrt{2}} \times \frac{I_P}{\sqrt{2}} \times (\cos\phi) = VI\cos\phi$$

Circuit with Resistance and Reactance

- The average power dissipation given by

$$P = \frac{1}{2} V_P I_P (\cos\phi) = VI\cos\phi$$

is termed the **active power** in the circuit and is measured in watts (W)

- The product of the r.m.s. voltage and current VI is termed the **apparent power**, ***S***. To avoid confusion this is given the units of volt amperes (VA)

Circuit with Resistance and Reactance

- From the above discussion it is clear that

$$P = VI\cos\phi$$
$$= S\cos\phi$$

- In other words, the active power is the apparent power times the cosine of the phase angle.
- This cosine is referred to as the **power factor**

$$\frac{\text{Active power (in watts)}}{\text{Apparent power (in volt amperes)}} = \text{Power factor}$$

$$\text{Power factor} = \frac{P}{S} = \cos\phi$$

Circuit with Resistance and Reactance

Example

The voltage across a component is measured as 50 V r.m.s. and the current through it is 5 A r.m.s. If the current leads the voltage by 30°, calculate:

(a) the apparent power;

(b) the power factor;

(c) the active power

[250 VA, 0.866, 216.5 W]

Active and Reactive Power

- When a circuit has resistive and reactive parts, the resultant power has 2 parts:
  - The first is *dissipated* in the resistive element. This is the **active power, *P***
  - The second is *stored* and *returned* by the reactive element. This is the **reactive power**, ***Q*** , which has units of **volt amperes reactive** or **var**
- While reactive power is not dissipated it does have an effect on the system
  - for example, it increases the current that must be supplied and increases losses with cables

Active and Reactive Power

- Consider an RL circuit
  - the relationship between the various forms of power can be illustrated using a power triangle

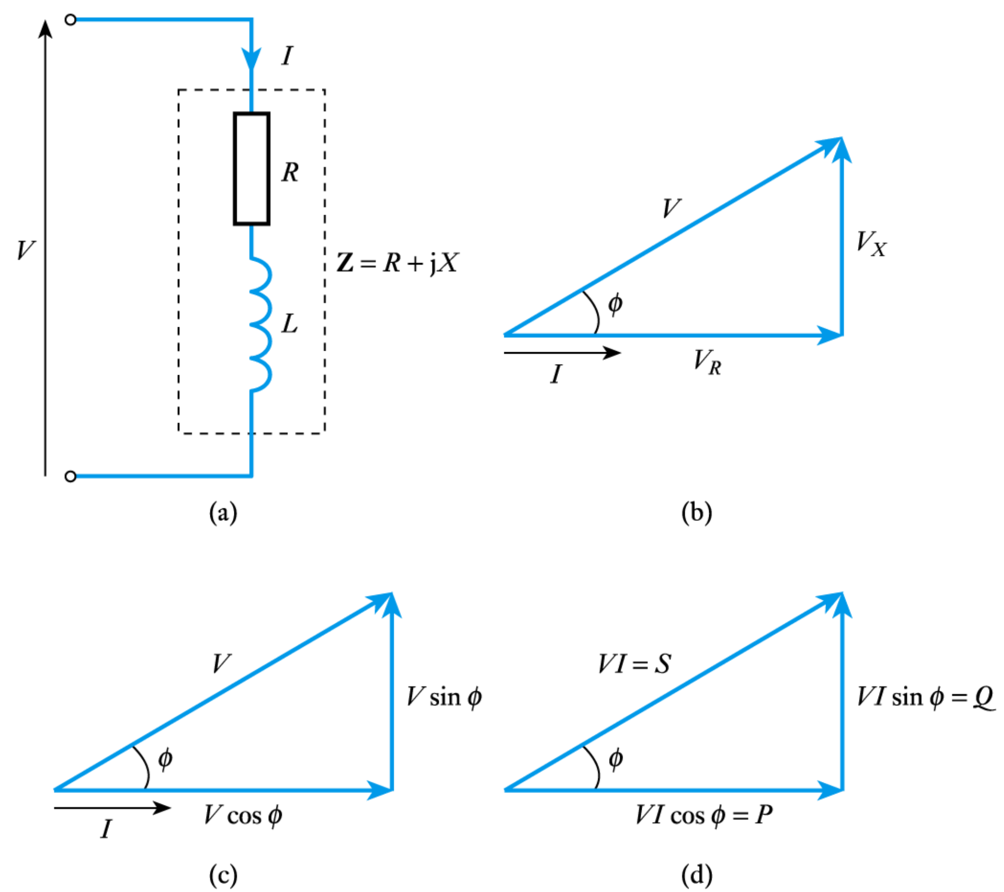

Figure 8.4

Active and Reactive Power

- Therefore

Active Power	$P = VI \cos \phi$	watts
Reactive Power	$Q = VI \sin \phi$	var
Apparent Power	$S = VI$	VA

$$S^2 = P^2 + Q^2$$

Active and Reactive Power

Example

- A 2 kVA motor operates from a 240 V supply at 50 Hz and has a power factor of 0.75. Determine the apparent power, the active power, the reactive power and the current in the motor.

Solution

The apparent power *S* of the motor is 2000 VA, since this is the rating of the motor. The power factor $(\cos \emptyset)$ is 0.75. Therefore, the active power in the motor is

- $active\ power\ P = S \cos \emptyset = 1500\ W$
- $\sin \emptyset = \sqrt{(1 - cos^2 \emptyset)} = 0.661$
- $Reactive\ power, Q = S sin\ \emptyset = 1323\ var$
- $current, I = \frac{S}{V} = 8.33\ A$

Power Factor Correction

- **Power factor** is particularly important in high-power applications
- Inductive loads have a *lagging* power factor
- Capacitive loads have a *leading* power factor
- Many high-power devices are inductive
  - a typical AC motor has a power factor of 0.9 lagging
  - the total load on the national grid is 0.8-0.9 lagging
  - this leads to major inefficiencies in power generation, and distribution.
  - power companies therefore penalise industrial users who introduce a poor power factor

Power Factor Correction

- The problem of poor power factor is tackled by adding additional components to bring the power factor back closer to unity
  - a capacitor of an appropriate size in parallel with a lagging load can 'cancel out' the inductive element
  - this is **power factor correction**
  - a capacitor can also be used in series but this is less common (since this alters the load voltage).

Power Factor Correction

Example 3

- A capacitor is to be added in parallel with the motor of Example 2 to increase its power factor to 1.0. Calculate the value of the required capacitor, and calculate the active power, the apparent power and the current after power factor correction.

Solution

- Apparent power S=2000 VA
- Active power P=1500 W
- Current, I=8.33 A
- Reactive power, Q=1323 var

The capacitor is required to cancel the *lagging* reactive power. We therefore need to add a capacitive element with a *leading* reactive power Q_C of -1323 var.

- $Q = V^2/X$. Since capacitive reactive power is negative
- $Q_C = -\frac{240^2}{X_C} = -1323\ var,\quad X_C = 43.54\ \Omega\ ,\ X_C = \frac{1}{2\pi f C},\quad C = 73\ \mu F$

Power Factor Correction

- The power factor correction does not affect the active power in the motor, and *P* is therefore unchanged at 1500 W. However, since the power factor is now 1, the apparent power is now *S* = *P* = 1500. The current is now given by

$$current, I = \frac{S}{V} = 6.25\ A$$

- Thus the apparent power is reduced from 2000 VA to 1500 VA as a result of the addition of the capacitor, while the current drops from 8.33 A to 6.25 A. The active power dissipated by the motor remains unchanged at 1500 W.
- In this example, we chose a capacitor to increase the power factor to unity, but this is not always appropriate. High-voltage capacitors suitable for this purpose are expensive, and it may be more cost effective to increase the power factor by a more modest amount, perhaps up to about 0.9.

Power Transfer

- We have already established that maximum power transfer occurs in resistive systems when the load resistance is equal to the output resistance
  - this is an example of **matching**
- When the output of an ac circuit has a reactive element maximum power transfer is achieved when the load impedance is equal to the *complex conjugate* of the output impedance
  - this is the **maximum power transfer theorem**

Power Transfer

- Thus if the output impedance $Z_o = R + jX$, maximum power transfer will occur with a load $Z_L = R - jX$

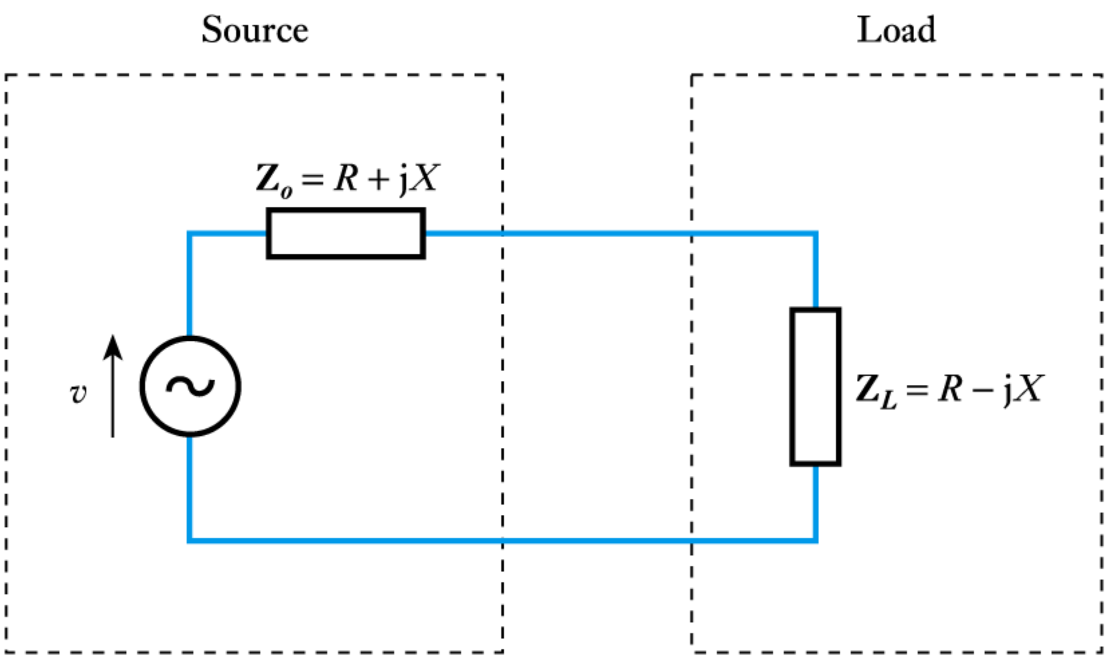

Figure 8.5

Three-Phase Systems

- So far, our discussion of AC systems has been restricted to **single-phase** arrangement
  - as in conventional domestic supplies
- In high-power industrial applications we often use **three-phase** arrangements
  - these have three supplies, differing in phase by 120 °
  - phases are labeled **red**, **yellow** and **blue** (**R**, **Y** & **B**)

Three-Phase Systems

- Relationship between the phases in a three-phase arrangement

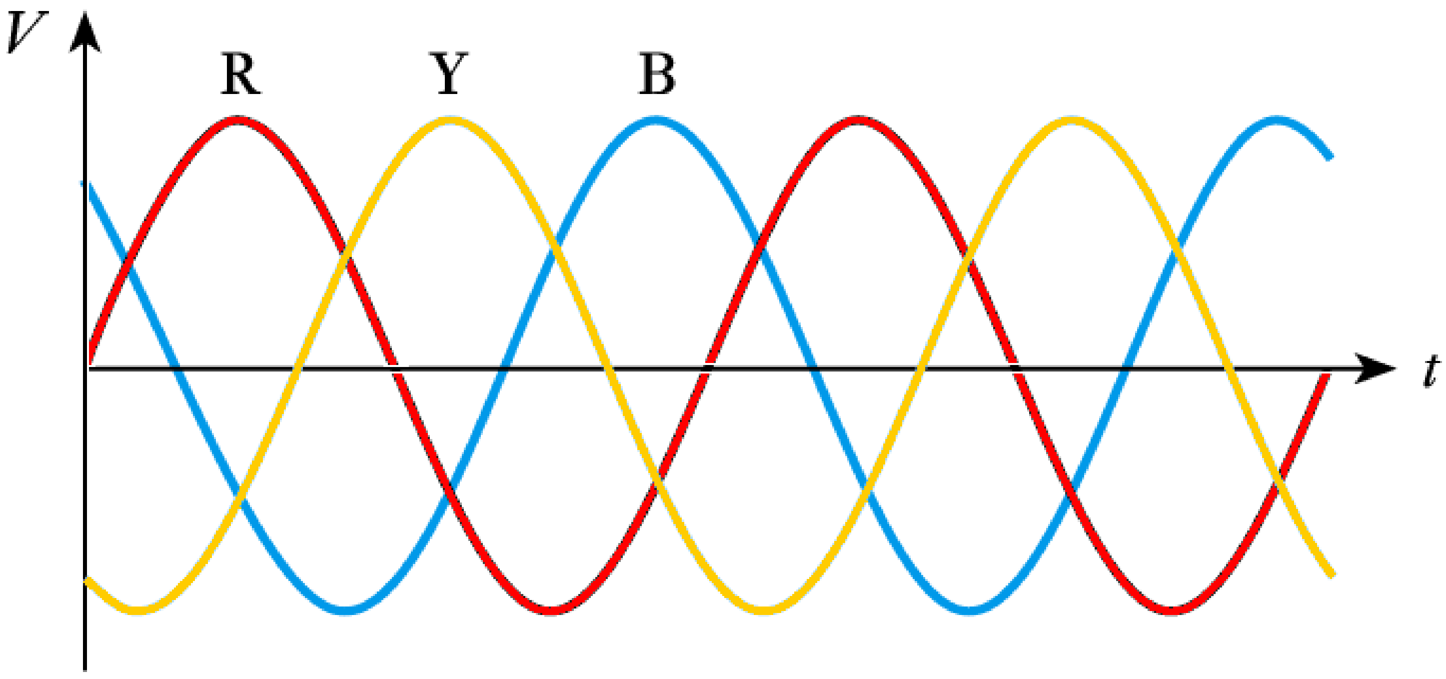

Figure 8.6: Voltage waveforms of a three-phase arrangement.

Three-Phase Systems

- Three-phase arrangements may use either 3 or 4 conductors

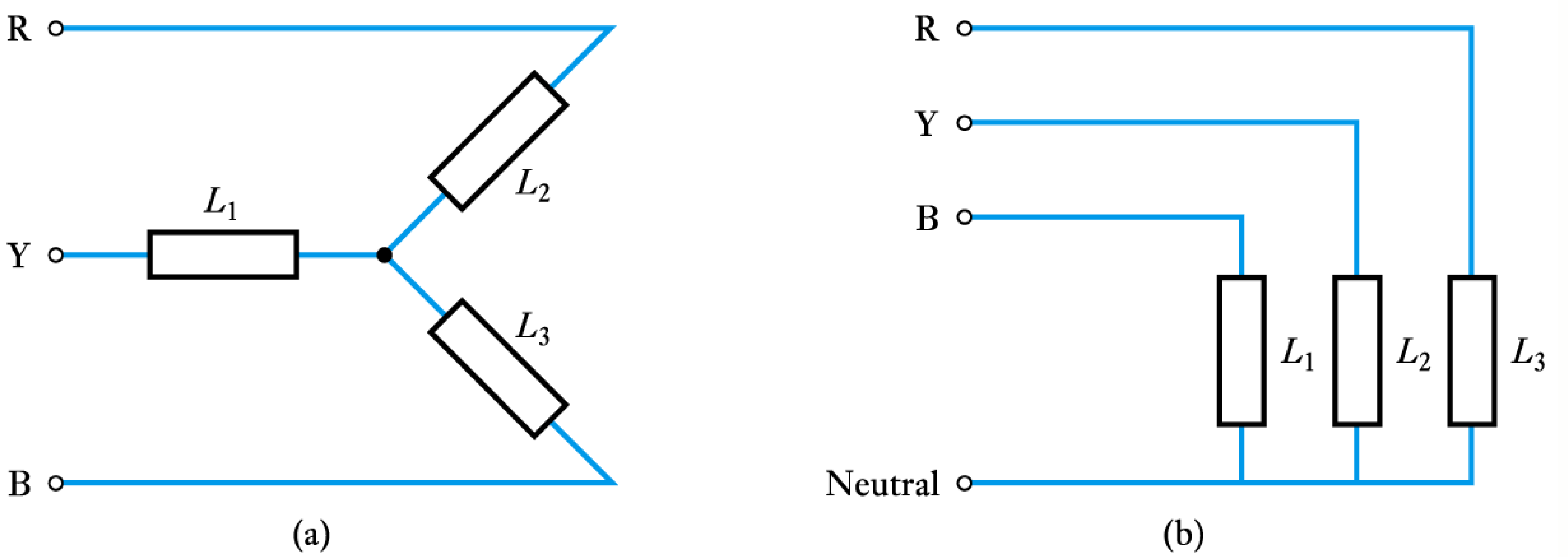

Figure 8.7: Three-phase connections.

Power Measurement

- When using AC, *power* is determined not only by the r.m.s. values of the **voltage** and **current**, but also by the **phase angle** (which determines the **power factor**)
  - consequently, you cannot determine the power from independent measurements of current and voltage
- In **single-phase systems** power is normally measured using an **electrodynamic wattmeter**
  - measures power directly using a single meter which effectively multiplies instantaneous current and voltage.

Power Measurement

- This device passes the load current through a series of low-resistance field coils and places the load voltage across a high resistance armature coil.
- The resulting deflection is directly related to the product of the instantaneous current and voltage and hence to the instantaneous power in the load.
- The device can therefore be directly calibrated in watts.

Power Measurement

- In **three-phase systems** we need to sum the power taken from the various phases
  - in three-wire arrangements we can deduce the total power from measurements using 2 wattmeter
  - in a four-wire system it may be necessary to use 3 wattmeter
  - in balanced systems (systems that take equal power from each phase) a single wattmeter can be used, its reading being multiplied by 3 to get the total power

Key Points

- In resistive circuits the average power is equal to *VI*, where *V* and *I* are r.m.s. values
- In a capacitor the current *leads* the voltage by 90° and the average power is zero
- In an inductor the current *lags* the voltage by 90° and the average power is zero
- In circuits with both resistive and reactive elements, the average power is *VI* cos ϕ
- The term cos ϕ is called the power factor
- Power factor correction is important in high-power systems
- High-power systems often use three-phase arrangements

Comments/Questions