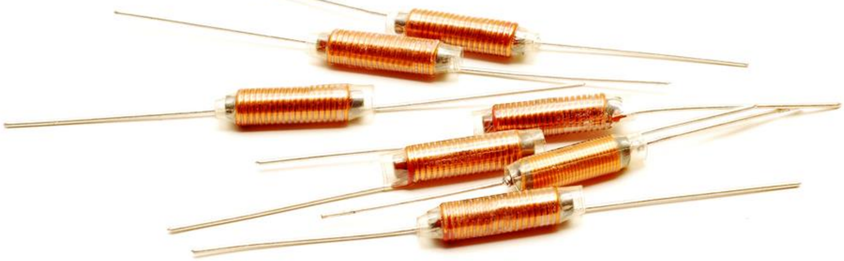
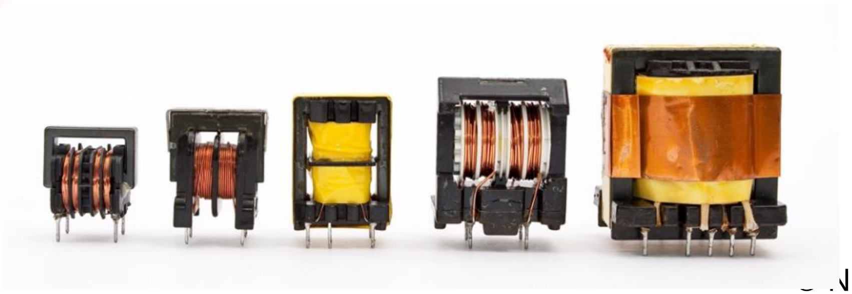
Chapter 5

Capacitance and Inductance



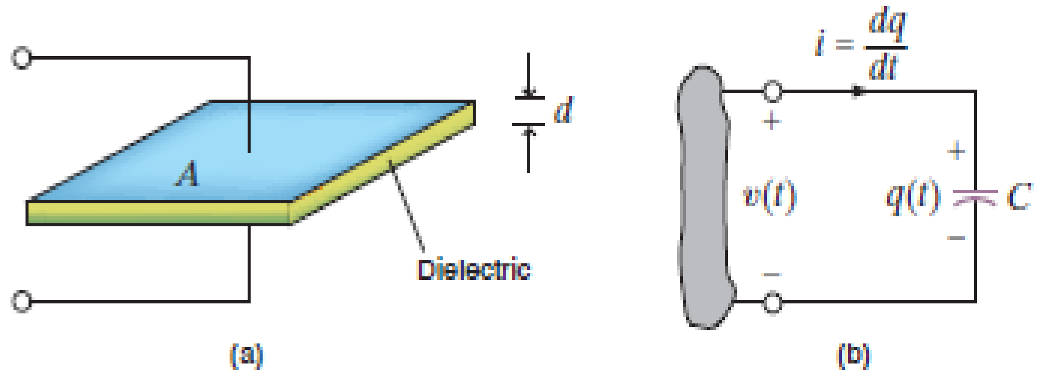


Learning goals

- By the end of this chapter, the students should be able to:
- Use circuit models for inductors and capacitors to calculate voltages, currents, and powers.
- Determine the stored energy in capacitors and inductors.
- Apply the concepts of continuity of current for an inductor and continuity of voltage for a capacitor.
- Calculate the voltages and currents for capacitors and inductors in electric circuits with dc sources.
- Determine the equivalent capacitance for capacitors in series and parallel.
- Determine the equivalent inductance for inductors in series and parallel

Capacitors

- A *capacitor* is a circuit element that consists of two conducting surfaces separated by a nonconducting, or *dielectric*, material. A simplified capacitor and its electrical symbol are shown in Fig. below.



Capacitors

- The capacitance of two parallel plates of area A, separated by distance d, is
- $C = \frac{\varepsilon_0 A}{d}$
- Where
  $\varepsilon_0 = 8.85 \times 10^{-12}\ F/m\ (Permittivity\ of\ free\ space)$
- Suppose now that a source is connected to the capacitor shown in Fig. above; then positive charges will be transferred to one plate and negative charges to the other. The charge on the capacitor is proportional to the voltage across it such that
- $Q = Cv$

Capacitors

- where C is the proportionality factor known as the capacitance of the element in farads.
- The charge differential between the plates creates an electric field that stores energy.
- Since the current is $i = \frac{dq}{dt}$, For the capacitor, $i = \frac{d}{dt}(Cv)$ whichfor a constant C is $i = C\frac{dv}{dt}$

Capacitors

- Which can be rewritten as
- $dv = \frac{1}{C} i \, dt$
- Integrating gives
- $v(t) = \frac{1}{C} \int_{-\infty}^{t} i(x) dx$
- The energy stored in the capacitor can be derived from the power that is delivered to the element. This power is given by the expression
- $p(t) = v(t)i(t) = Cv(t)\frac{dv(t)}{dt}$

Capacitors

- And hence the energy stored in the electric field is
- $w_c(t) = \int_{-\infty}^{t} Cv(x)\frac{dv(x)}{dx}dx = C\int_{-v(\infty)}^{v(t)} v(x)dv(x) = \frac{1}{2}Cv^2(x)\Big|_{v(-\infty)}^{v(t)} = \frac{1}{2}Cv^2(t)$
- The expression for the energy can be written as
- $w_c(t) = \frac{1}{2}\frac{q^2(t)}{C}$
- These equations represent the energy stored by the capacitor, which, in turn, is equal to the work done by the source to charge the capacitor.

Capacitors

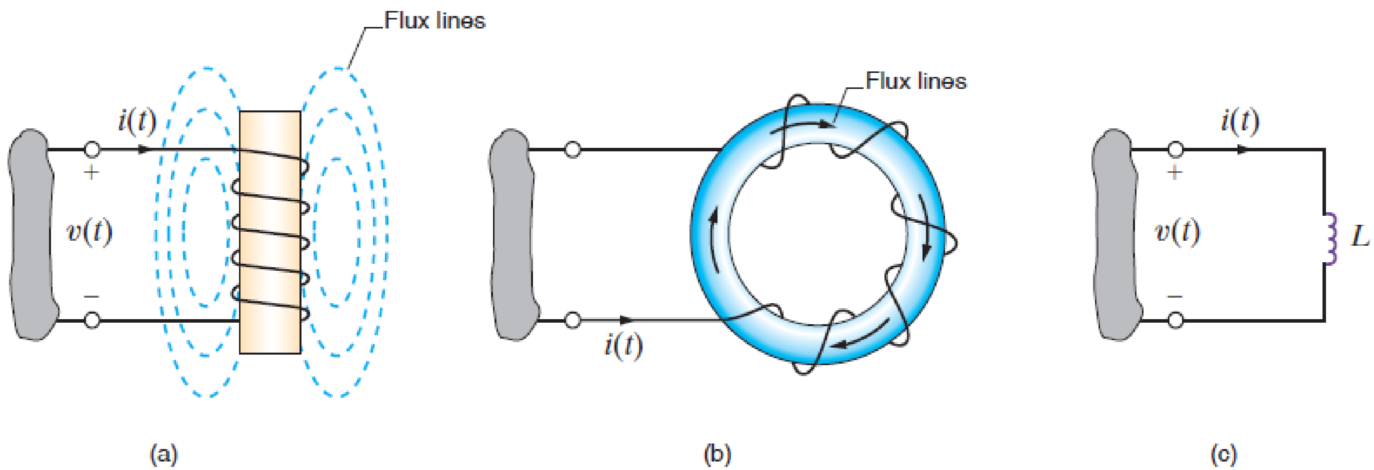
- Now let's consider the case of a dc voltage applied across a capacitor. The current flowing through the capacitor is directly proportional to the time rate of change of the voltage across the capacitor.
- A dc voltage does not vary with time, so the current flowing through the capacitor is zero.
- We can say that a capacitor is "an open circuit to dc" or "blocks dc." Capacitors are often utilized to remove or filter out an unwanted dc voltage.
- In analyzing a circuit containing dc voltage sources and capacitors, we can replace the capacitors with an open circuit and calculate voltages and currents in the circuit using our many analysis tools.

Capacitors

- **Exercise**
- If the charge accumulated on two parallel conductors charged to 12 V is 600 pC, what is the capacitance of the parallel conductors? *(50 pF)*
- A 10-μF capacitor has an accumulated charge of 500 nC. Determine the voltage across the capacitor. *(0.05 V)*

Inductors

- An *inductor* is a circuit element that consists of a conducting wire usually in the form of a coil. Two typical inductors and their electrical symbol are shown in Fig. below.



Two inductors and their electrical symbol

Inductors

- $v(t) = L\frac{di(t)}{dt}$
- The constant of proportionality *L* is called the inductance and is measured in the unit *henry*,
- The expression for the current in an inductor is
- $i(t) = \frac{1}{L}\int_{-\infty}^{t} v(x)dx$
- Which can also be written as
- $i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(x)dx$
- The power delivered to the inductor can be used to derive the energy stored in the element.

Inductors

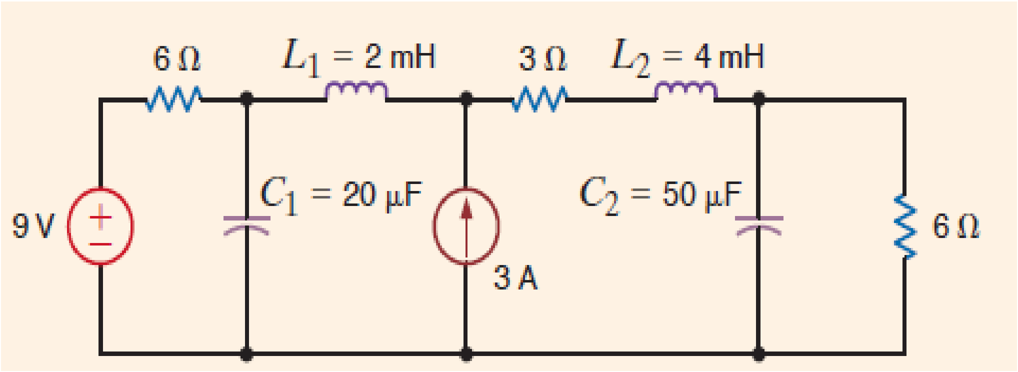
- This power is equal to
- $p(t) = v(t)i(t) = \left[L\frac{di(t)}{dt}\right]i(t)$
- Therefore, the energy stored in the magnetic field is
- $w_L(t) = \int_{-\infty}^{t}\left[L\frac{di(x)}{dx}\right]i(x)\,dx$
- From which we can obtain
- $w_L(t) = \frac{1}{2}Li^2(t)\,J$

Inductors

- Now let's consider the case of a dc current flowing through an inductor. We see that the voltage across the inductor is directly proportional to the time rate of change of the current flowing through the inductor.
- A dc current does not vary with time, so the voltage across the inductor is zero.
- We can say that an inductor is "a short circuit to dc." In analysing a circuit containing dc sources and inductors, we can replace any inductors with short circuits and calculate voltages and currents in the circuit using our many analysis tools.

Inductors

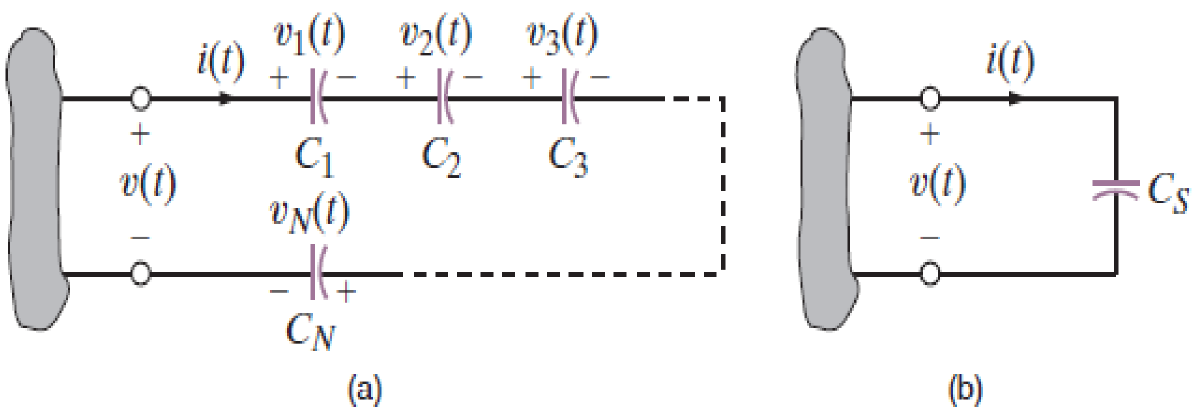
- **Example**
- Find the total energy stored in the circuit of Fig. below.



(13.46mJ)

Series Capacitors

- If a number of capacitors are connected in series, their equivalent capacitance can be calculated using KVL. Consider the circuit shown in Fig. a. For this circuit

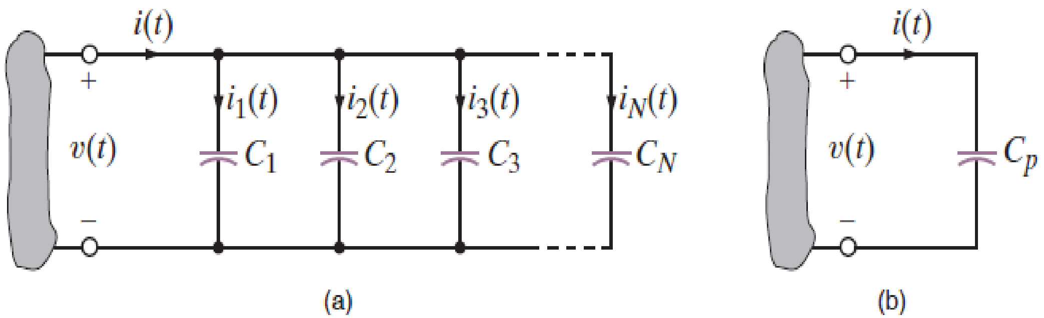


: Equivalent circuit for N series-connected capacitors.

Series Capacitors

- $v(t) = v_1(t) + v_2(t) + \cdots + v_N(t)$
- $v_i(t) = \frac{1}{C_i}\int_{t_0}^{t} i(t)dt + v_i(t_0)$
- $v(t) = \left(\sum_{i=1}^{N} \frac{1}{C_i}\right)\int_{t_0}^{t} i(t)dt + \sum_{i=1}^{N} v_i\,(t_0) = \frac{1}{C_s}\int_{t_0}^{t} i(t)dt + v(t_0)$
- Where
- $v(t_0) = \sum_{i=1}^{N} v_i\,(t_0)$
- And
- $\frac{1}{C_s} = \sum_{i=1}^{N} \frac{1}{C_i} = \frac{1}{C_1} + \frac{1}{C_1} + \cdots \frac{1}{C_N}$

Parallel Capacitors



Equivalent circuit for N capacitors connected in parallel.

We employ KCL., As can be seen from Fig. a,

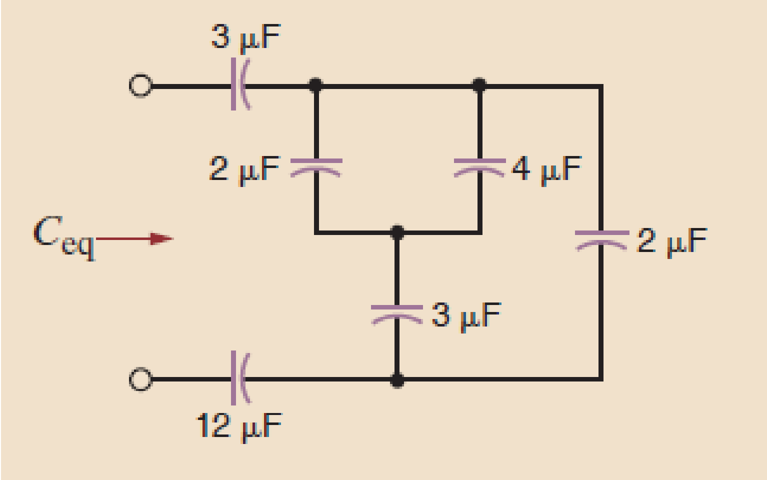
$$i(t) = i_1(t) + i_2(t) + \cdots i_N(t)$$

$$i(t) = C_1 \frac{dv(t)}{dt} + C_2 \frac{dv(t)}{dt} + \cdots C_N \frac{dv(t)}{dt} = \left(\sum_{i=1}^{N} C_i \right) \frac{dv(t)}{dt} = C_p \frac{dv(t)}{dt}$$

Where $C_p = C_1 + C_2 + \cdots C_N$

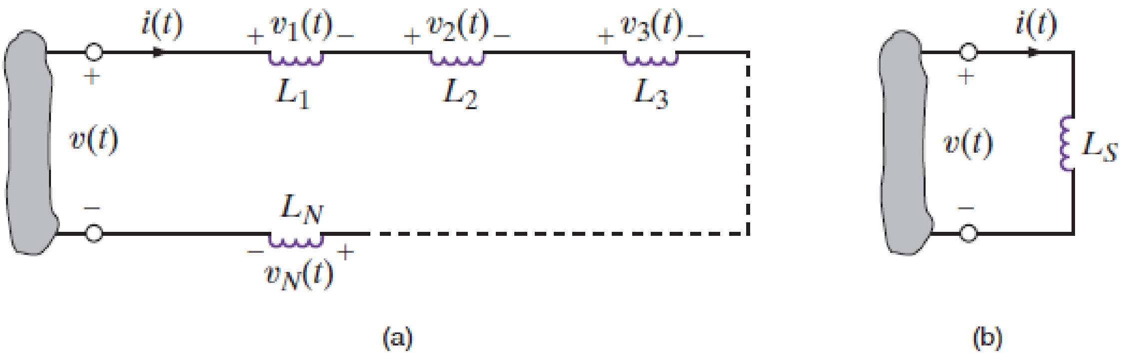
Parallel Capacitors

- **Example**
- Compute the equivalent capacitance of the network in Fig. below.



(1.5μF)

Series Inductors



Equivalent circuit for N series-connected inductors

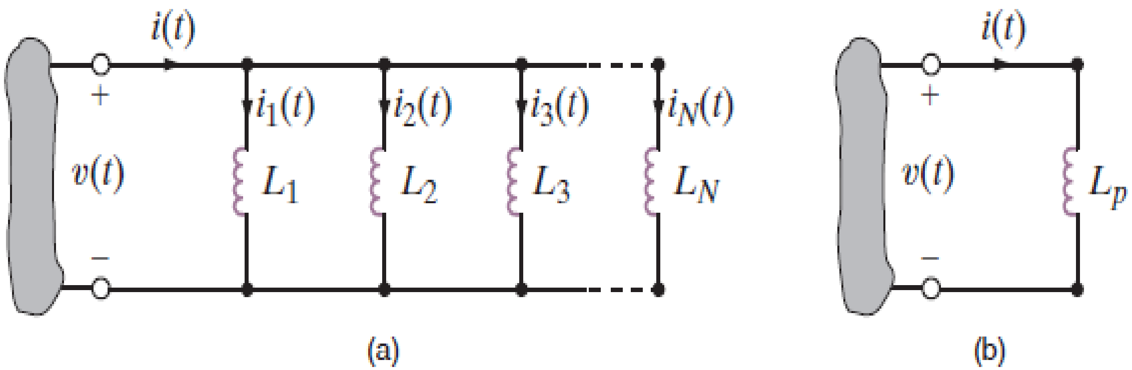
If N inductors are connected in series, the equivalent inductance of the combination can be determined as follows. Referring to Fig. a and using KVL, we see that

$$v(t) = v_1(t) + v_2(t) + \cdots v_N(t)$$

Series Inductors

- $v(t) = L_1 \frac{di(t)}{dt} + L_2 \frac{di(t)}{dt} + \cdots L_N \frac{di(t)}{dt} = \left(\sum_{i=1}^{N} L_i\right) \frac{di(t)}{dt} = L_S \frac{di(t)}{dt}$
- Where
- $L_S = \sum_{i=1}^{N} L_i = L_1 + L_2 + \cdots L_N$
- Therefore, under this condition the network in Fig. b is equivalent to that in Fig. 6.a.

Parallel Inductors



Equivalent circuits for N inductors connected in parallel.

Consider the circuit shown in Fig. a, which contains *N* parallel inductors. Using KCL, we can write

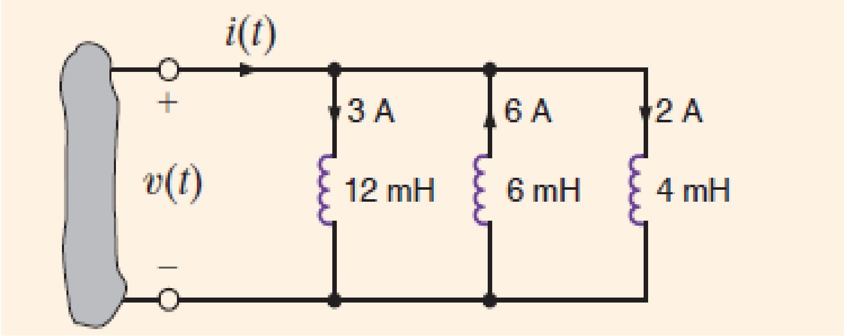
$$i(t) = i_1(t) + i_2(t) + \cdots i_N(t)$$

Parallel Inductors

- However $i_j(t) = \frac{1}{L_j}\int_{t_0}^{t} v(x)dx + i_j\,(t_0)$
- $i(t) = \left(\sum_{j=1}^{N} \frac{1}{L_j}\right)\int_{t_0}^{t} v(x)dx + \sum_{j=1}^{N} i_j(t_0) = \frac{1}{L_p}\int_{t_0}^{t} v(x)dx + i(t_0)$
- Where $\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots \frac{1}{L_N}$

Parallel Inductors

- **Examples**
- Determine the equivalent inductance and the initial current for the circuit shown in Fig. below.



(2 mH, -1 A)

Parallel Inductors

- Calculate the energy stored in the inductor in the circuit shown in Fig. below.