Chapter 3

Nodal and Loop Analysis

Learning goals

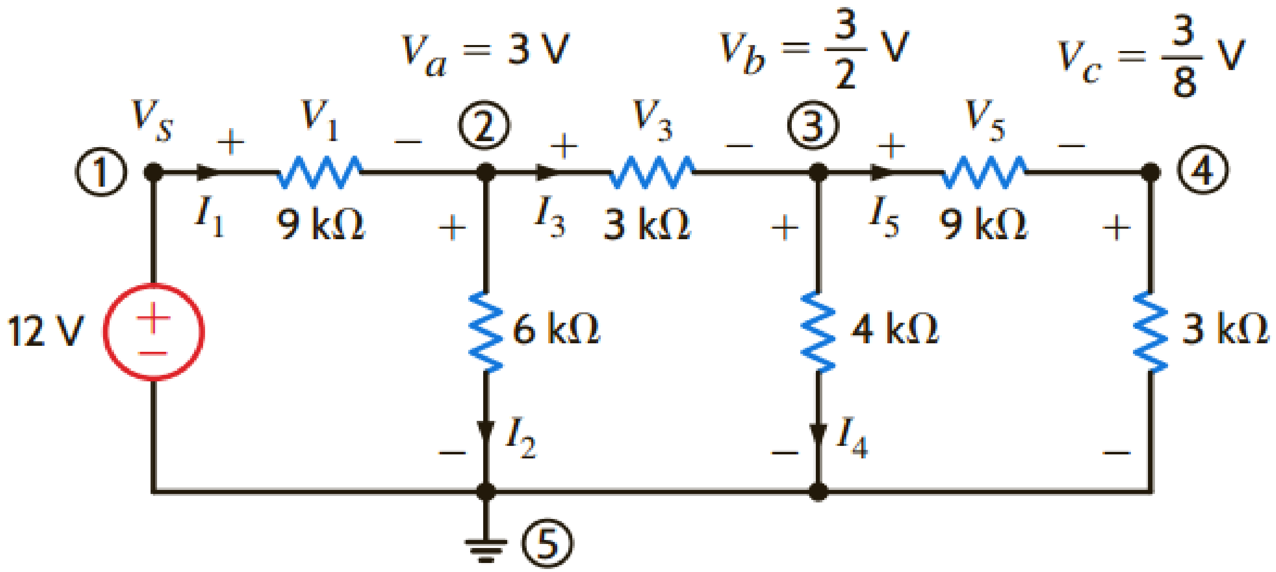
- By the end of this chapter, the students should be able to:
- Calculate the branch currents and node voltages in circuits containing multiple nodes using KCL and Ohm's law in nodal analysis.
- Calculate the mesh currents and voltage drops and rises in circuits containing multiple loops using KVL and Ohm's law in loop analysis.
- Identify the most appropriate analysis technique that should be utilized to solve a particular problem

Nodal Analysis

- In a nodal analysis, the variables in the circuit are selected to be the node voltages. The node voltages are defined with respect to a common point in the circuit. One node is selected as the reference node, and all other node voltages are defined with respect to that node.
- Quite often this node is the one to which the largest number of branches are connected. It is commonly called *ground* because it is said to be at ground-zero potential, and it sometimes represents the chassis or ground line in a practical circuit.

Nodal Analysis

- Find all the currents and voltages labelled in the ladder network .



Nodal Analysis

- The reference node is shown with the ground symbol. All the voltages are measured with reference to this node.
- Once all these voltages are known, we can calculate the branch currents, or power absorbed or supplied by each element.
- $V_1 = V_s - V_a = 12 - 3 = 9\,V$
- This is the application of KVL around the left most loop
- $-V_s + V_1 + V_a = 0$
- In a similar manner
- $V_3 = V_a - V_b$

Nodal Analysis

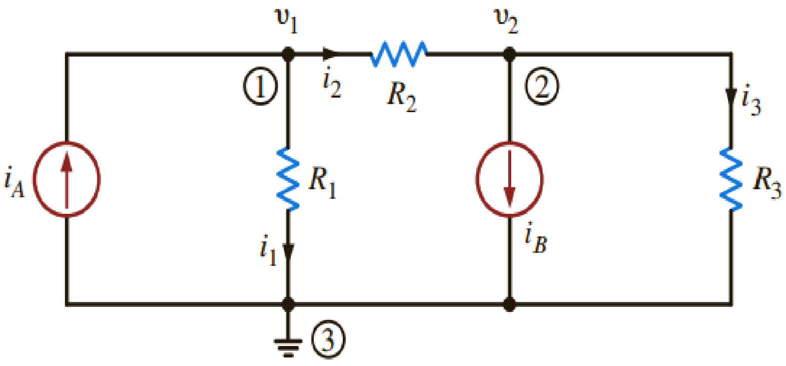
- And the currents in the resistors are
- $I_1 = \frac{V_1}{9k} = \frac{V_s - V_a}{9k}$, $I_3 = \frac{V_3}{3k} = \frac{V_a - V_b}{3k}$, $I_5 = \frac{V_5}{9k} = \frac{V_b - V_c}{9k}$,
- $I_2 = \frac{V_a - 0}{6k}$ and $I_4 = \frac{V_b - 0}{4k}$ (since the reference node 5 is at zero potential)
- Thus, as a general rule, if we know the node voltages in a circuit, we can calculate the current through any resistive element using Ohm's law; that is,
- $i = \frac{v_m - v_N}{R}$

Nodal Analysis

- **Calculating the node voltages**
- We employ KCL equations in such a way that the variables contained in these equations are the unknown node voltages of the network.
- As we have indicated, one of the nodes in an *N*-node circuit is selected as the reference node, and the voltages at all the remaining nonreference nodes are measured with respect to this reference node. Using network topology, it can be shown that exactly N-1 linearly independent KCL equations are required to determine the N-1 unknown node voltages.

Nodal Analysis

- **Circuits Containing only Independent Current Sources**



Applying KCL at node 1

$-i_A + i_1 + i_2 = 0$

$-i_A + G_1(v_1 - 0) + G_2(v_1 - v_2) = 0$

Or $G_1 v_1 + G_2(v_1 - v_2) = i_A$

KCL at node 2 yields

$-i_2 + i_B + i_3 = 0$

$-G_2(v_1 - v_2) + i_B + G_3(v_2 - 0) = 0$

Which can be expressed as

$-G_2 v_1 + (G_2 + G_3) v_2 = -i_B$

Nodal Analysis

- **Circuits Containing only Independent Current Sources**
- The above equations can be solved in many ways:
- Suppose $i_A = 1mA, R_1 = 12k\Omega, R_2 = 6k\Omega, i_B = 4mA\ and\ R_3 = 6k\Omega$. We can determine the node voltages and currents using:

Nodal Analysis

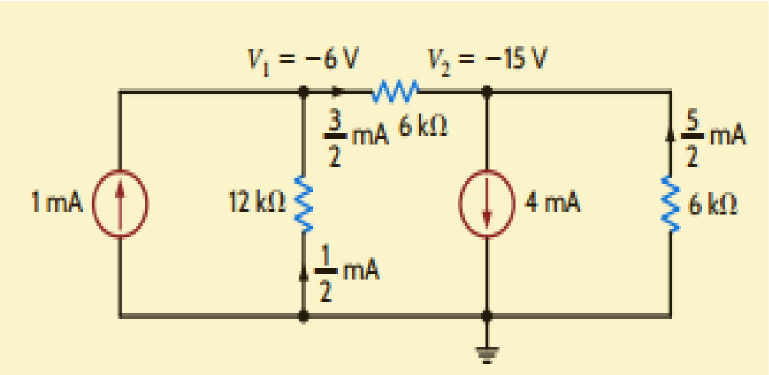
- **Circuits Containing only Independent Current Sources**
- **A: Gaussian Elimination**
- Using the given parameters
- $\frac{V_1}{4k} - \frac{V_2}{6k} = 1 \times 10^{-3}$
- $-\frac{V_1}{6k} + \frac{V_2}{3k} = -4 \times 10^{-3}$
- Solve the first equation for V_1 and then substitute in the second equation to get V_2.
- $(V_1 = -6V, V_2 = -15\ V)$

Nodal Analysis

- **Circuits Containing only Independent Current Sources**
- **B: Using Matrix Analysis**
- The general form is GV=I. Therefore $V = G^{-1}I$
- $Inverse\ of\ G, \quad G^{-1} = \frac{1}{det} \times Adjoint$
- $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 18k^2 \begin{bmatrix} \frac{1}{3k} & \frac{1}{6k} \\ \frac{1}{6k} & \frac{1}{4k} \end{bmatrix} \begin{bmatrix} 1 \times 10^{-3} \\ -4 \times 10^{-3} \end{bmatrix} = \begin{bmatrix} -6 \\ -15 \end{bmatrix}$
- We can now determine all the node currents using Ohm's law,
- $I_1 = \frac{V_1}{R_1} = -\frac{1}{2}mA, \; I_2 = \frac{V_1 - V_2}{6k} = \frac{3}{2}mA \quad and\ I_3 = \frac{V_2}{6k} = -\frac{5}{2}mA$

Nodal Analysis

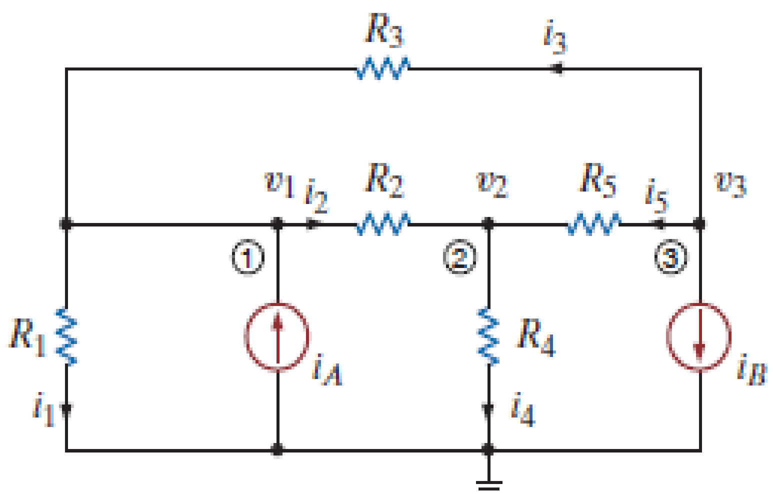
- **Circuits Containing only Independent Current Sources**
- **C: Using MATLAB- Check Text books and Appendix**



Note that KCL is satisfied at every node.

Nodal Analysis

- **Circuits Containing only Independent Current Sources**
- **Example 2:** *A four node circuit*



Nodal Analysis

- **Circuits Containing only Independent Current Sources**
- Applying KCL yields
- At node 1
- $i_1 - i_A + i_2 - i_3 = 0$
- $v_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - v_2\frac{1}{R_2} - v_3\frac{1}{R_3} = i_A$
- At node 2
- $-i_2 + i_4 - i_5 = 0$
- $-v_1\frac{1}{R_2} + v_2\left(\frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5}\right) - v_3\frac{1}{R_5} = 0$

Nodal Analysis

- **Circuits Containing only Independent Current Sources**
- At node 3
- $i_3 + i_5 + i_B = 0$
- $-v_1 \frac{1}{R_3} - v_2 \frac{1}{R_5} + v_3 \left(\frac{1}{R_3} + \frac{1}{R_5} \right) = -i_B$

Nodal Analysis

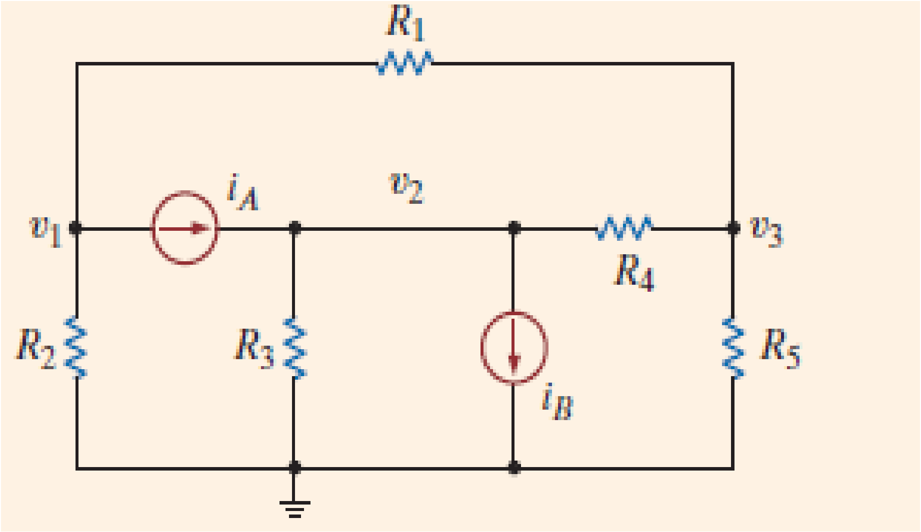
- **Circuits Containing only Independent Current Sources**
- Above equations can also be solved in many ways
- The equations can be written in matrix form
- $$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_2} & -\frac{1}{R_3} \\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5} & -\frac{1}{R_5} \\ -\frac{1}{R_3} & -\frac{1}{R_5} & \frac{1}{R_3}+\frac{1}{R_5} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} i_A \\ 0 \\ -i_B \end{bmatrix}$$

Nodal Analysis

- **Circuits Containing only Independent Current Sources**
- The G matrix for each network is a symmetrical matrix. The node equations for networks containing only resistors and independent current sources can always be written in this symmetrical form. The coefficient of v_1 is the sum of all the conductances connected to node 1 and the coefficient of v_2 is the negative of the conductances connected between node 1 and node 2.
- The RHS of the equation is the sum of the currents entering node 1 through current sources.

Nodal Analysis

- **Circuits Containing only Independent Current Sources**
- Example
- Determine the node voltages for $R_1 = R_2 = 2k\Omega, R_3 = R_4 = 4k\Omega, R_5 = 1k\Omega, i_A = 4mA, i_B = 2mA$



Nodal Analysis

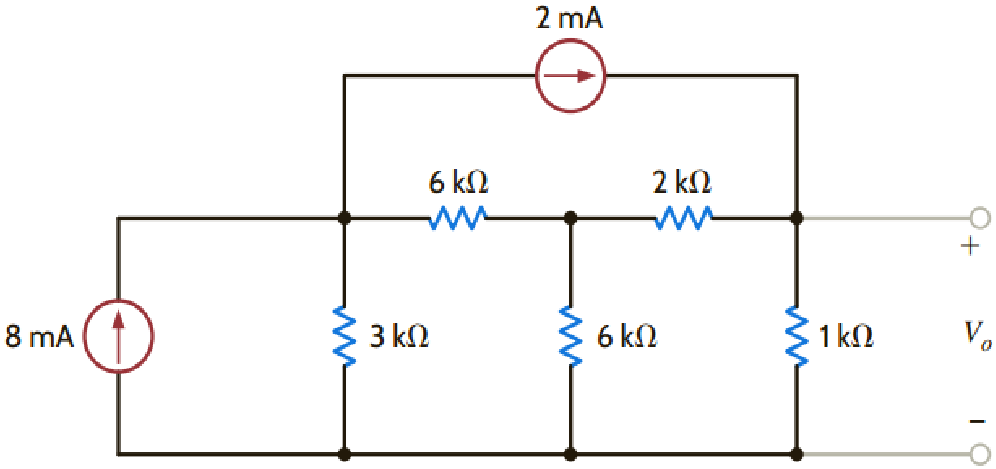
- **Circuits Containing only Independent Current Sources**
- **Solution**
- The equations are
- $v_1\left(\frac{1}{R_1}+\frac{1}{R_2}\right)-v_2(0)-v_3\left(\frac{1}{R_1}\right)=-i_A$
- $-v_1(0)+v_2\left(\frac{1}{R_3}+\frac{1}{R_4}\right)-v_3\left(\frac{1}{R_4}\right)=i_A-i_B$
- $-v_1\left(\frac{1}{R_1}\right)-v_2\left(\frac{1}{R_4}\right)+v_3\left(\frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_5}\right)=0$

Nodal Analysis

- **Circuits Containing only Independent Current Sources**
- Writing the above equations in matrix form and substituting yields
- $$\begin{bmatrix} 0.001 & 0 & -0.0005 \\ 0 & 0.0005 & -0.00025 \\ -0.0005 & -0.00025 & 0.00175 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -0.004 \\ 0.002 \\ 0 \end{bmatrix}$$
- $$Solving, \quad v_1 = -4.3636\,V, v_2 = 1.6364\,V \quad and \quad v_3 = -0.7273\,V$$

Nodal Analysis

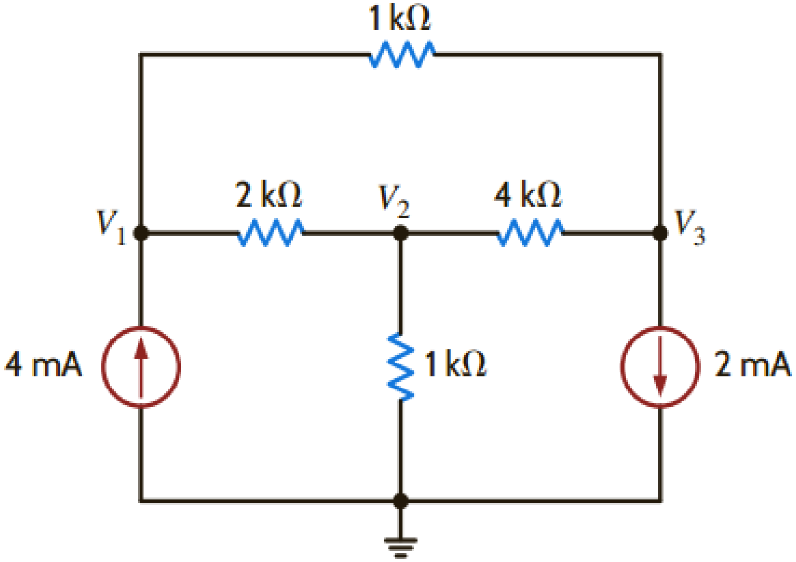
- **Circuits Containing only Independent Current Sources**



(0.952 V)

Nodal Analysis

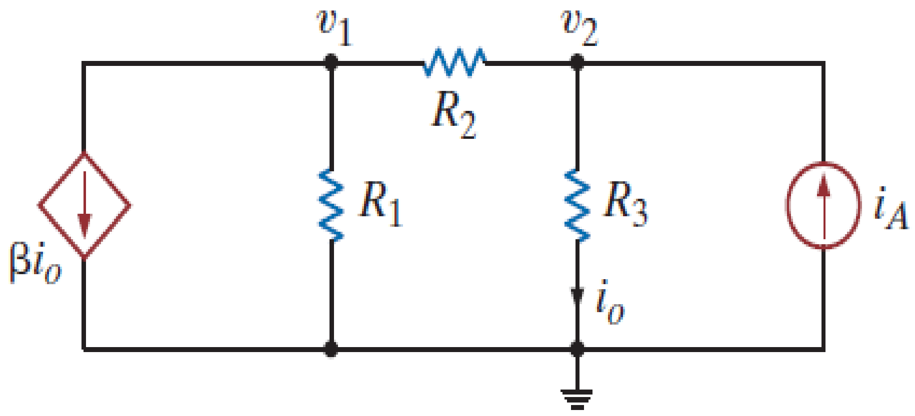
- **Circuits Containing only Independent Current Sources**
- **Exercise**
- Find the node voltages



(5.4286, 2.000, 1.1429 V)

Nodal Analysis

- **Circuits Containing Dependent Current Sources**
- The presence of a dependent source may destroy the symmetrical form of the nodal equations that define the circuit.
- **Example**

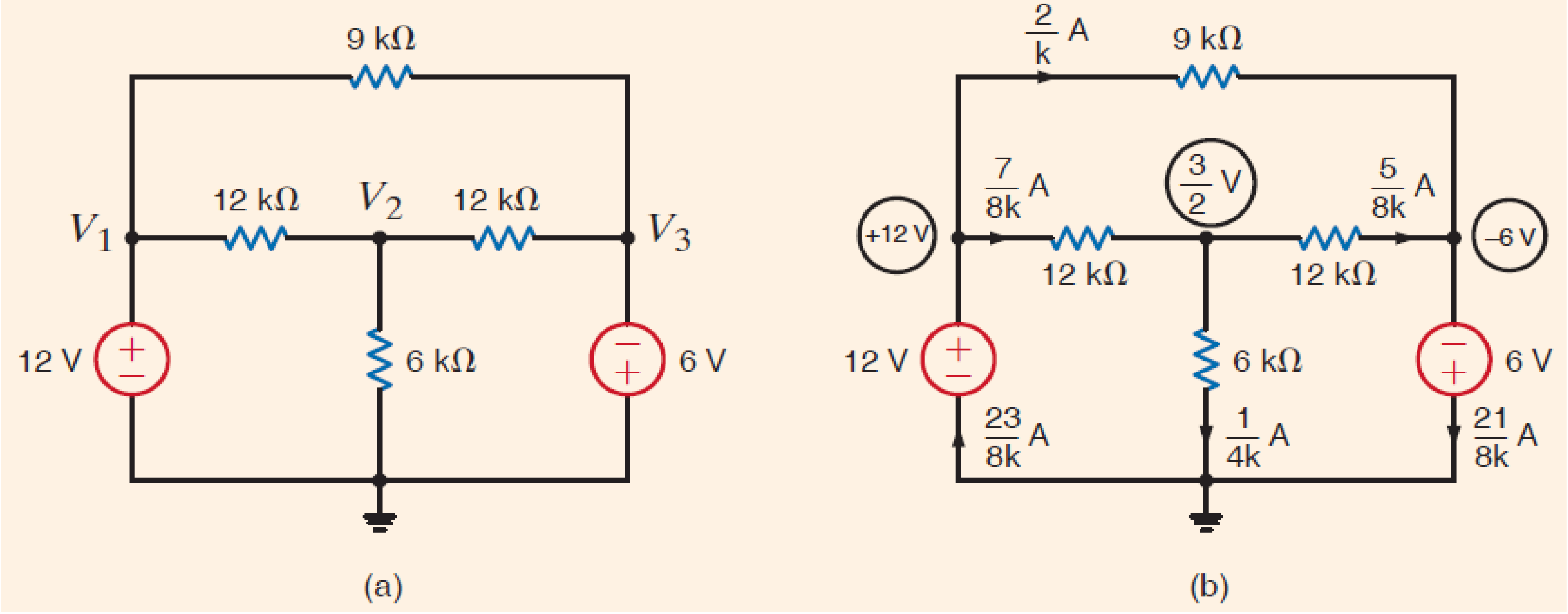


Nodal Analysis

- **Circuits Containing Dependent Current Sources**
- The KCL equations for the nonreference nodes are:
- $\beta i_0 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} = 0 \quad and \quad \frac{v_2 - v_1}{R_2} + i_0 - i_A = 0 \; where \; i_0 = \frac{v_2}{R_2}$
- Simplifying
- $(G_1 + G_2)v_1 - (G_2 - \beta G_3)v_2 = 0$
- $-G_2 v_1 + (G_2 + G_3)v_2 = i_A$
- If $\beta = 2 \quad R_1 = 12\ k\Omega\ R_2 = 6\ k\Omega, \quad R_3 = 3\ k\Omega \quad and \quad i_A = 2mA.$
- We obtain $V_1 = -\frac{24}{5}V \;\; and \; V_2 = \frac{12}{5}\ V \;\; I_0 = \frac{4}{5}mA, I_2 = \frac{6}{5}mA$

Nodal Analysis

- **Circuits Containing Dependent Voltage Sources**
- Consider the circuit shown below. Let us determine all node voltages and branch currents.

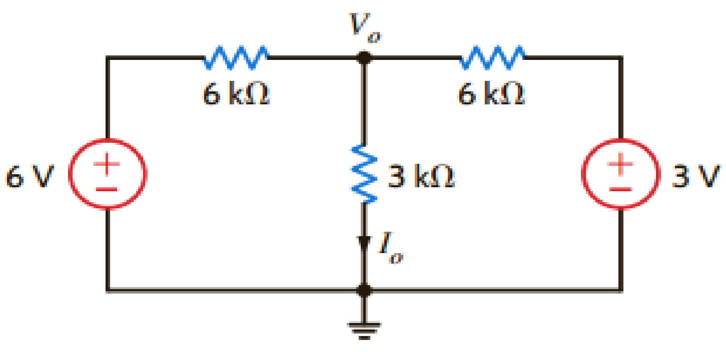


Nodal Analysis

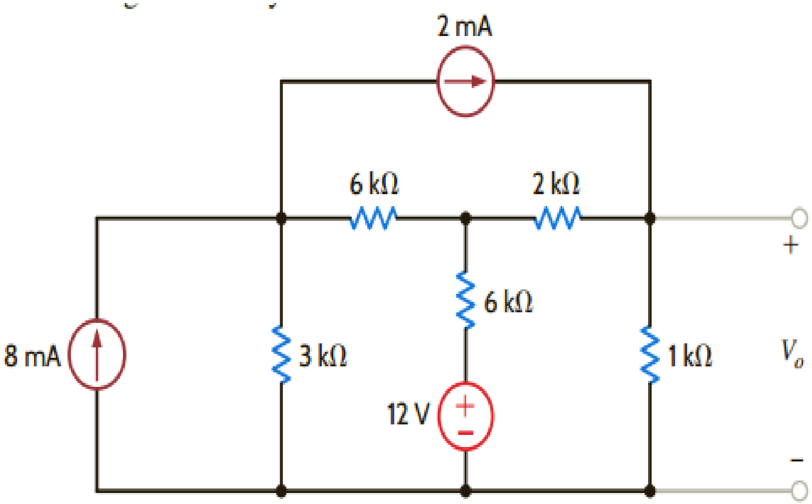
- **Circuits Containing Dependent Voltage Sources**
- Ohm's law can be used to find the branch currents. The answers are shown in figure b.
- $\frac{V_2 - V_1}{12k} + \frac{V_2 - 0}{6k} + \frac{V_2 - V_3}{12k} = 0$ $\quad or \quad$ $\frac{V_2 - 12}{12k} + \frac{V_2}{6k} + \frac{V_2 - (-6)}{12k} = 0$. $\quad V_2 = \frac{3}{2} V$

Nodal Analysis

- **Circuits Containing Dependent Current Sources and dependent voltage sources**
- **Exercise**
- Use nodal analysis to find the current I_0 and V_0 in the networks below.



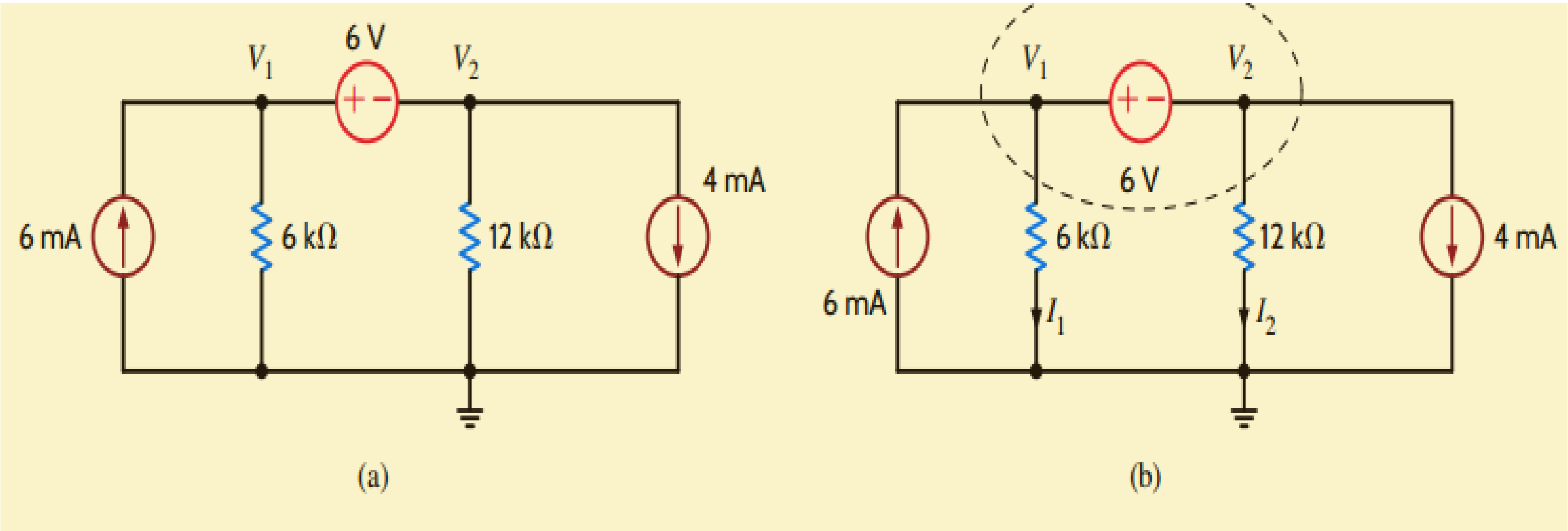
(¾ mA)



(1.89 V)

Nodal Analysis

- **Circuits Containing Dependent Current Sources**
- Find the two currents in figure a

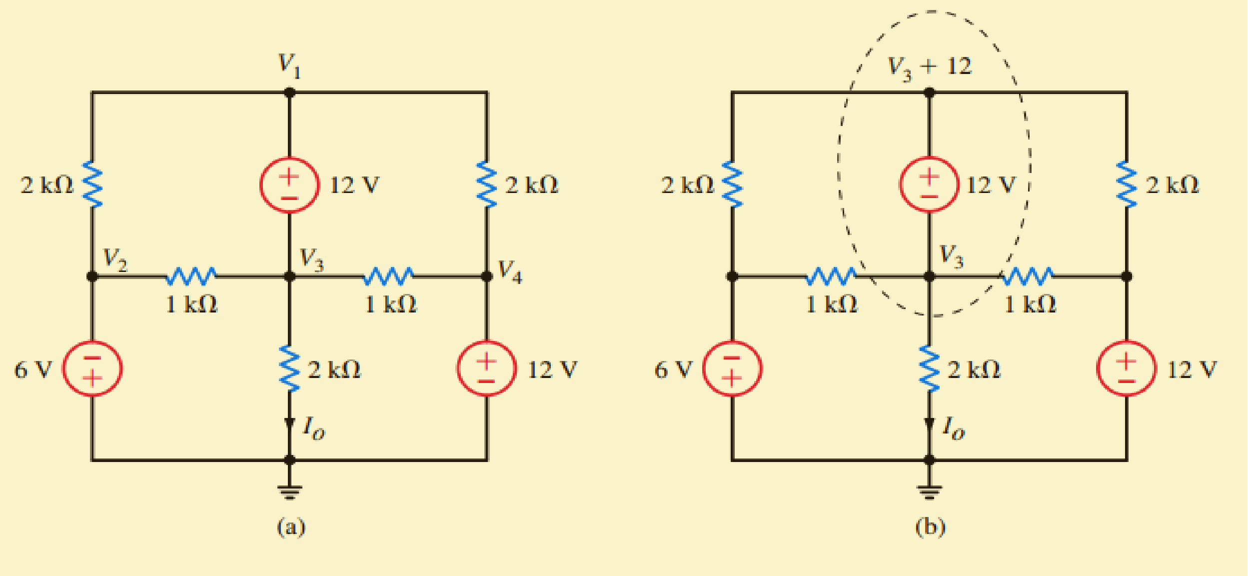


Nodal Analysis

- **Circuits Containing Dependent Current Sources**
- $V_1 - V_2 = 6$
- KCL for the super node is
- $-6m + \frac{V_1}{6k} + \frac{V_2}{12k} + 4m = 0$
- Solving the equations gives
- $V_1 = 10\,V \quad and \;\; V_2 = 4\,V \quad hence\; I_1 = \frac{5}{3} mA \quad and\; I_2 = \frac{1}{3}\, mA$

Nodal Analysis

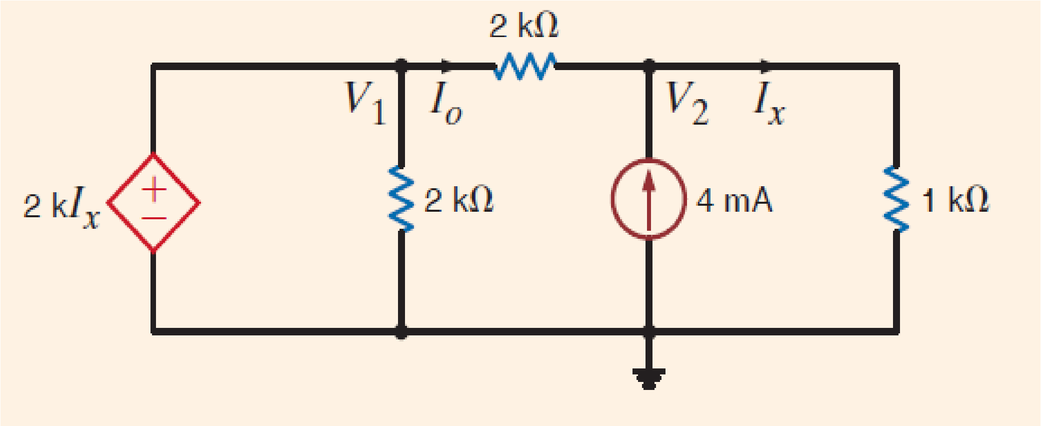
- **Circuits Containing Dependent Current Sources**
- Let us determine I_0 the current in the network in Fig.a.



(-3/7 mA)

Nodal Analysis

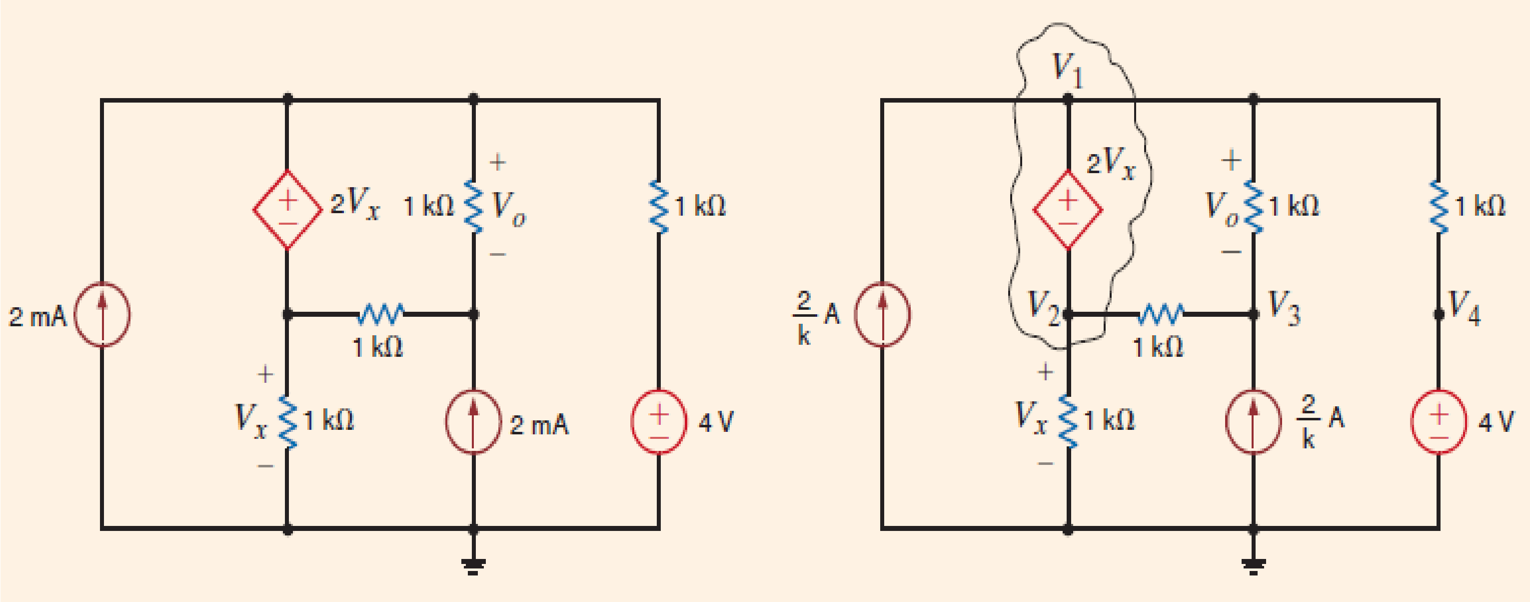
- **Circuits Containing Dependent Voltage Sources**
- Find I_o in the network



(4 mA)

Nodal Analysis

- **Circuits Containing Dependent Voltage Sources**
- Let us find V_0 in the network in Fig. a.



(a) (b)

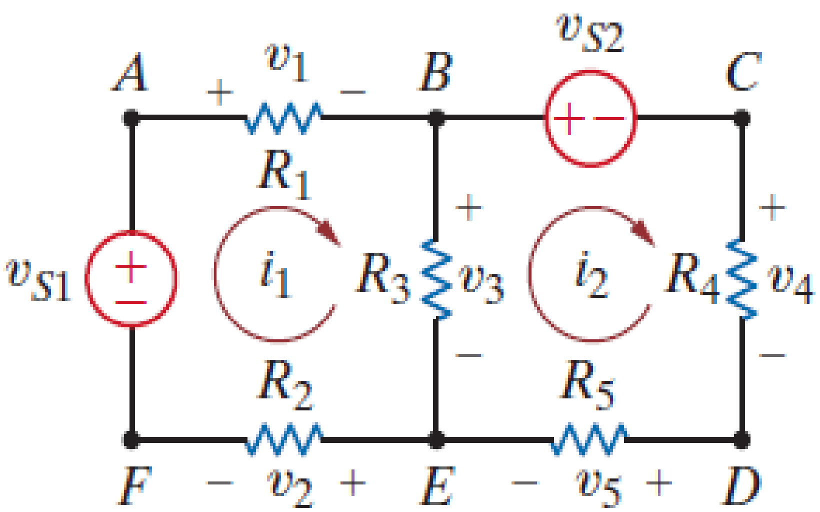
(1V)

Loop Analysis

- **Introduction**
- A loop analysis uses KVL to determine a set of loop currents in the circuit.
- Once these loop currents are known, Ohm's law can be used to calculate any voltages in the network. Via network topology we can show that, in general, there are exactly *B-N+1* linearly independent KVL equations for any network, where *B* is the number of branches in the circuit and *N* is the number of nodes.

Loop Analysis

- **Circuits Containing only Independent Voltage Sources**



Loop Analysis

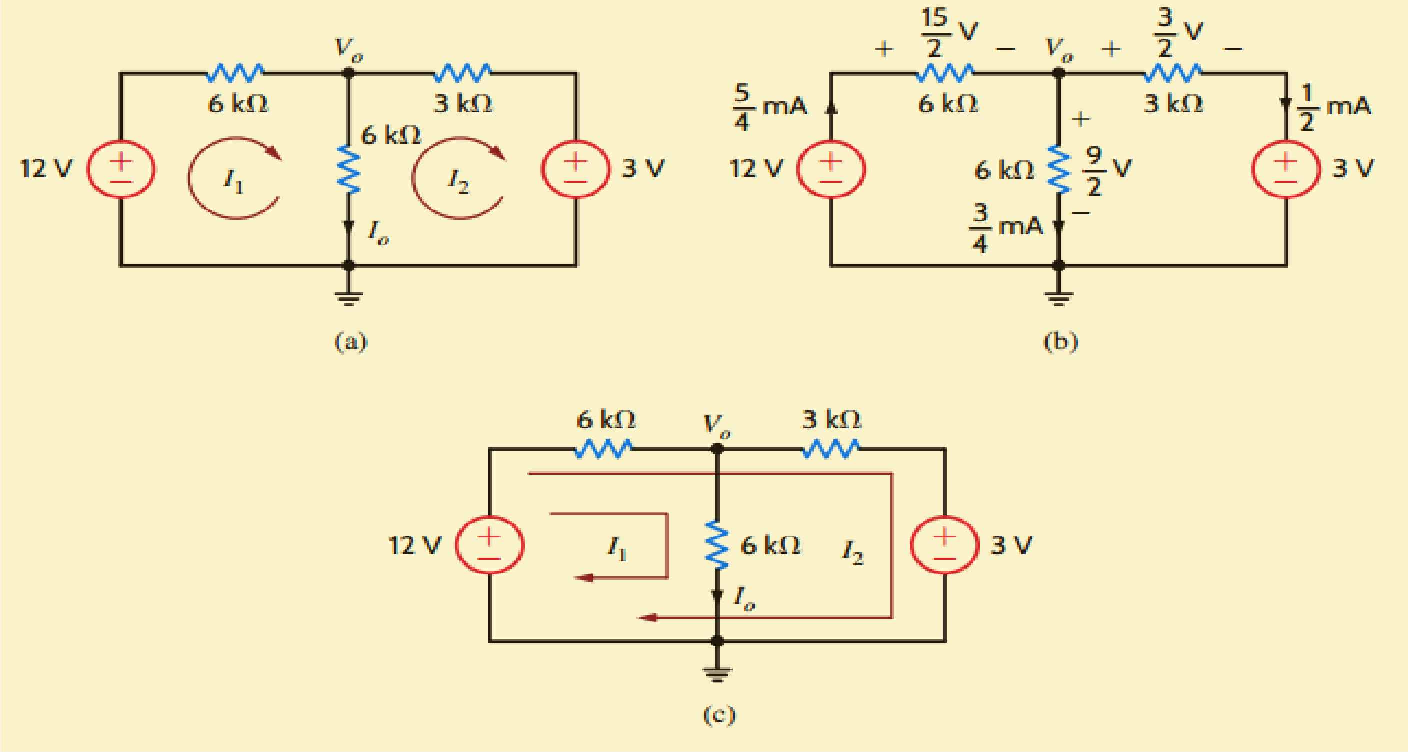
- **Circuits Containing only Independent Voltage Sources**
- This circuit has seven branches and six nodes. Thus the number of KVL equations=2.
- $+v_1 + v_3 + v_2 - v_{s1} = 0$ (Loop 1)
- $+v_{s2} + v_4 + v_5 - v_3 = 0$ (Loop 2)
- Where $v_1 = iR_1, v_2 = i_1R_2, \;\; v_3 = (i_1 - i_2)R_3, v_4 = i_2R_4 \; and \; v_5 = i_2R_5$
- Substituting these in the loop equations yields
- $i_1(R_1 + R_2 + R_3) - i_2R_3 = v_{s1}$
- $-i_1R_3 + i_2(R_3 + R_4 + R_5) = -v_{s2}$

Loop Analysis

- **Circuits Containing only Independent Voltage Sources**
- Or in matrix form
- $$\begin{bmatrix} R_1 + R_2 + R_3 & -R_3 \\ -R_3 & R_3 + R_4 + R_5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_{s1} \\ -v_{s2} \end{bmatrix}$$
- *A mesh is a special kind of loop that does not contain any loops within it.*

Loop Analysis

- **Circuits Containing only Independent Voltage Sources**



Loop Analysis

- **Circuits Containing only Independent Voltage Sources**
- Consider the circuit in figure a, calculate I_0
- $-12 + 6kI_1 + 6k(I_1 - I_2) = 0 \qquad KVL\ for\ 1st\ Mesh$
- $6k(I_2 - I_1) + 3kI_2 + 3 = 0 \qquad KVL\ for\ 2nd\ Mesh$
- *where* $I_0 = I_1 - I_2$
- *solving gives* $I_1 = 4/5mA,\ I_2 = 1/2\ m\ A, I_0 = 3/4mA$
- All the voltages and currents in the network are shown in Fig. b.

Loop Analysis

- **Circuits Containing only Independent Voltage Sources**
- Since we want to calculate the current I_0 we could use loop analysis, as shown in Fig. c.
- $I_1 = I_0$, The two loop equations in this case are.
- $-12 + 6k(I_1 + I_2) + 6kI_1 = 0$
- $-12 + 6k(I_1 + I_2) + 3kI_2 + 3 = 0$
- *Solving the above equations gives*
- $I_1 = \frac{3}{4}mA\,,\quad I_2 = \frac{1}{2}mA$

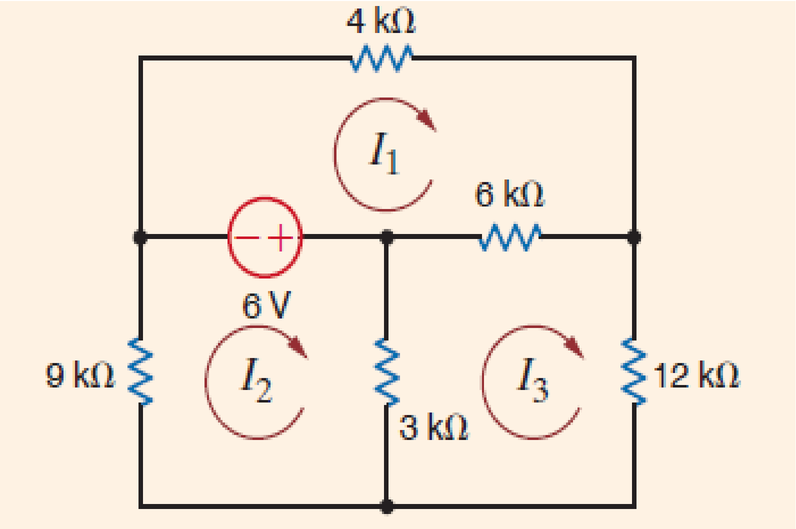
Exercise: Find I_0 using nodal analysis and compare the results.

Loop Analysis

- **Circuits Containing only Independent Voltage Sources**
- Note the symmetry and write mesh equations by inspection.
- In the first eqn, the coefficient of I_1 is the sum of the resistances through which mesh current 1 flows, and the coefficient of I_2 is the negative of the sum of the resistances common to mesh current 1 and mesh current 2. The RHS of the equation is the algebraic sum of the voltage sources in mesh 1.
- The sign of the voltage source is positive if it aids the assumed direction of the current flow and negative if it opposes the assumed flow. The first equation is KVL for mesh 1.

Loop Analysis

- **Circuits Containing only Independent Voltage Sources**
- Write the mesh equations by inspection for the network

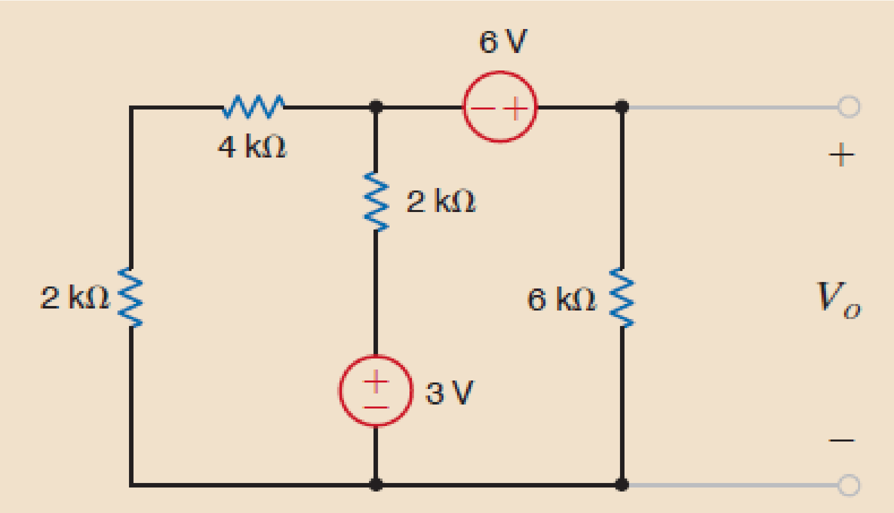


Loop Analysis

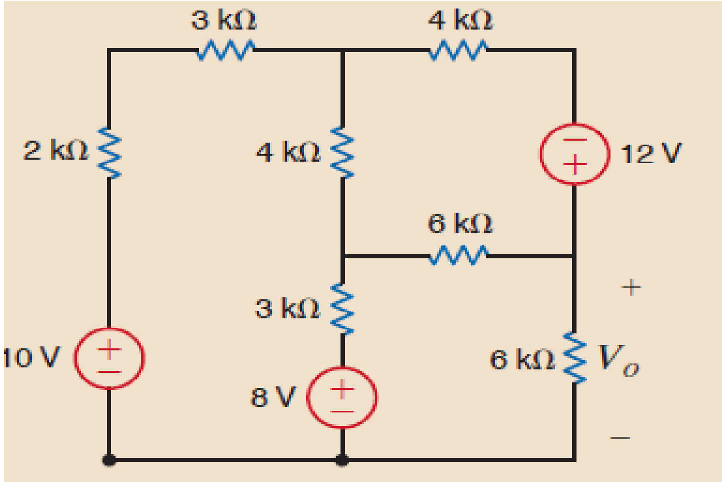
- **Circuits Containing only Independent Voltage Sources**
- $(4k + 6k)I_1 - (0)I_2 - (6k)I_3 = -6$
- $-(0)I_1 + (9k + 3k)I_2 - (3k)I_3 = 6$
- $-(6k)I_1 - (3k)I_2 + (3k + 6k + 12k)I_3 = 0$
- And in matrix form
- $$\begin{bmatrix} 10k & 0 & -6k \\ 0 & 12k & -3k \\ -6k & -3k & 21k \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 6 \\ 0 \end{bmatrix}$$
- The general form of the matrix form is: **RI=V**
- And the solution is **I=R^{-1}V**
- Solving gives $i_1 = -0.6757\ mA, i_2 = 0.4685mA, i_3 = -0.1261mA$

Loop Analysis

- **Circuits Containing only Independent Voltage Sources**
- Use mesh equations to find V_0 in the circuits



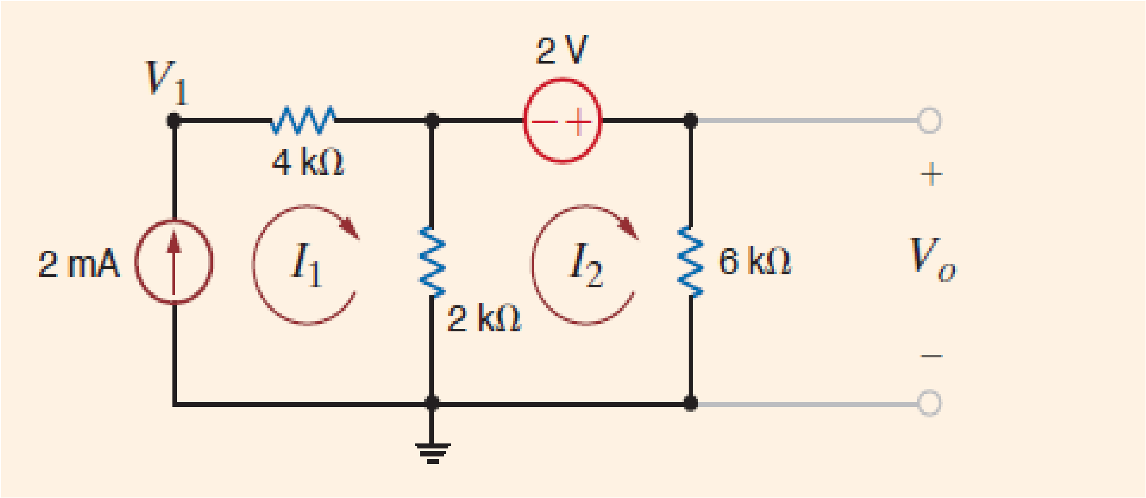
(33/5)



(8.96V)

Loop Analysis

- **Circuits Containing Independent Current Sources**
- Find V_0 and V_1 in figure below

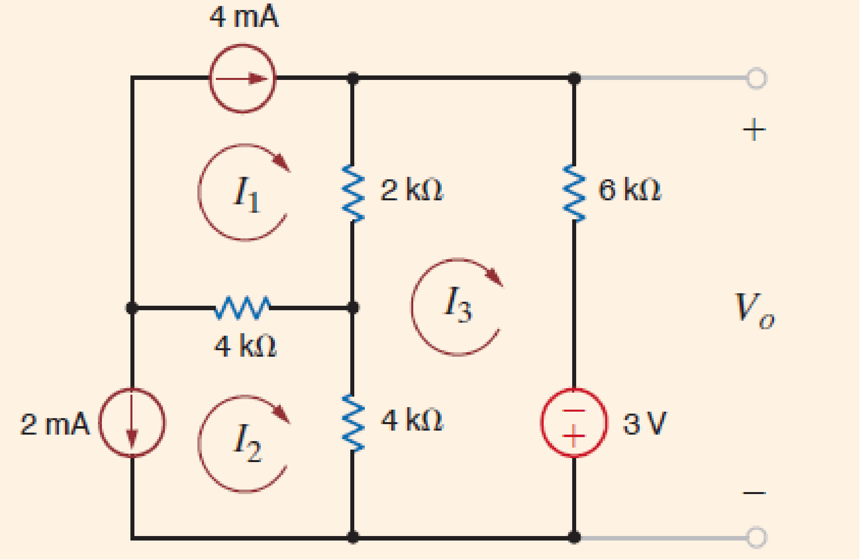


Loop Analysis

- **Circuits Containing Independent Current Sources**
- The current I_1 goes directly through the current source and, therefore I_1, is constrained to be 2 mA. Hence, only the current I_2 is unknown. KVL for the rightmost mesh is.
- $2k(I_2 - I_1) - 2 + 6kI_2 = 0$
- $I_1 = 2\,mA$
- $I_2 = \frac{3}{4}mA$ and thus $V_0 = 6kI_2 = \frac{9}{2}V$
- To obtain V_1 we apply KVL around any closed path. If we use the outer loop, the KVL equation is
- $-V_1 + 4kI_1 - 2 + 6kI_2 = 0$
- $V_1 = \frac{21}{2}V$

Loop Analysis

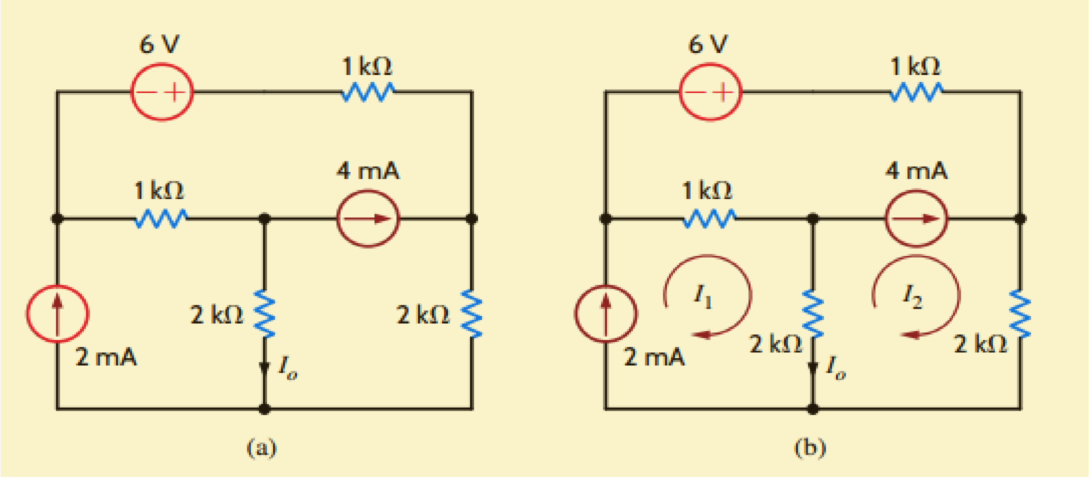
- **Circuits Containing Independent Current Sources**
- Find V_0 and V_1 in figure below



(-3/2V)

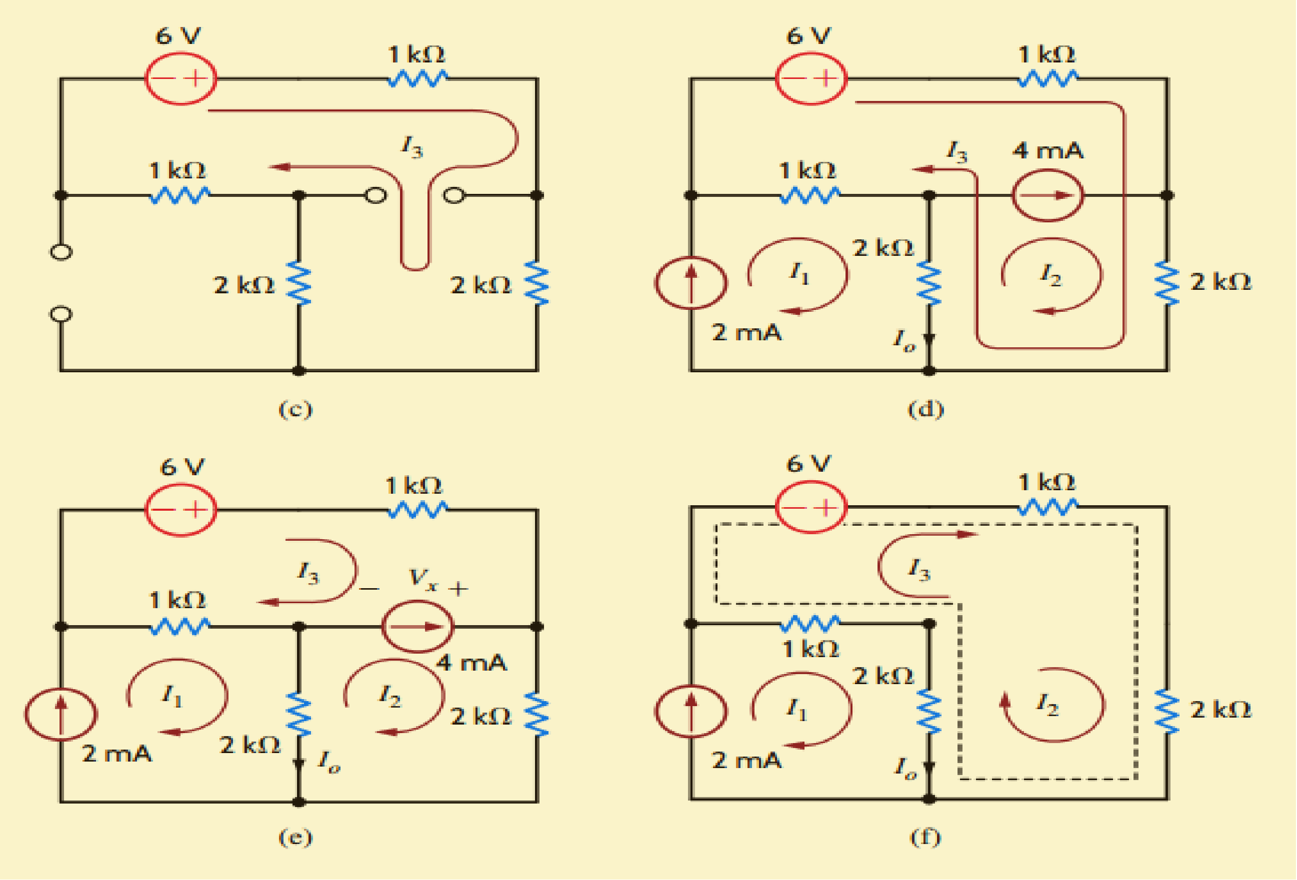
Loop Analysis

- **Circuits Containing Independent Current Sources**
- Find I_0 in figure below



Loop Analysis

- **Circuits Containing Independent Current Sources**



Loop Analysis

- **Circuits Containing Independent Current Sources**
- From Fig. b, two of the three linearly independent equations are
- $I_1 = 2 \times 10^{-3}$
- $I_2 = 4 \times 10^{-3}$
- The remaining loop current must pass through the circuit elements not covered by the two previous equations and cannot, of course, pass through the current sources. The path for this remaining loop current can be obtained by open-circuiting the current sources, as shown in Fig. c.

Loop Analysis

- **Circuits Containing Independent Current Sources**
- When all currents are labelled on the original circuit, the KVL equation for this last loop, as shown in Fig. d, is
- $-6 + 1kI_3 + 2k(I_2 + I_3) + 2k(I_3 + I_2 - I_1) + 1k(I_3 - I_1) = 0$
- Solving the equation yields $I_3 = -2/3mA$
- And thus $I_0 = I_1 - I_2 - I_3 = -4/3mA$

Loop Analysis

- **Circuits Containing Independent Current Sources**
- Next consider the supermesh technique. In this case the three mesh currents are specified as shown in Fig. e, and since the voltage across the 4-mA current source is unknown, it is assumed to be v_x . The mesh currents constrained by the current sources are
- $I_1 = 2 \times 10^{-3}$
- $I_2 - I_3 = 4 \times 10^{-3}$
- The KVL equations for meshes 2 and 3, respectively, are
- $2kI_2 + 2k(I_2 - I_1) - V_x = 0$
- $-6 + 1kI_3 + V_x + 1k(I_3 - I_1) = 0$

Loop Analysis

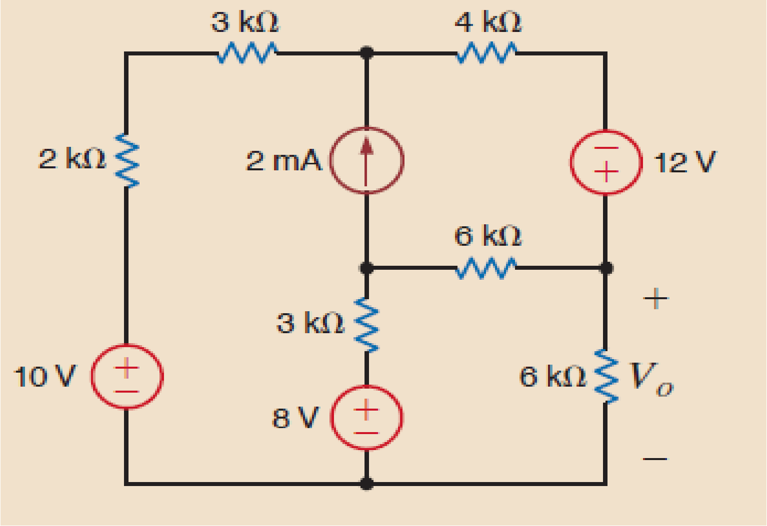
- **Circuits Containing Independent Current Sources**
- Adding the last equations yields
- $-6 + 1kI_3 + 2kI_2 + 2k(I_2 - I_1) + 1k(I_3 - I_1) = 0$
- Note that the unknown voltage V_x has been eliminated. The two constraint equations, together with this latter equation, yield the desired result.
- The purpose of the supermesh approach is to avoid introducing the unknown voltage V_x. The supermesh is created by mentally removing the 4-mA current source, as shown in Fig. f.

Loop Analysis

- **Circuits Containing Independent Current Sources**
- Then writing the KVL equation around the dotted path, which defines the supermesh, using the original mesh currents as shown in Fig. e, yields
- $-6 + 1kI_3 + 2kI_2 + 2k(I_2 - I_1) + 1k(I_3 - I_1) = 0$
- Note that this supermesh equation is the same as that obtained earlier by introducing the voltage V_x .

Loop Analysis

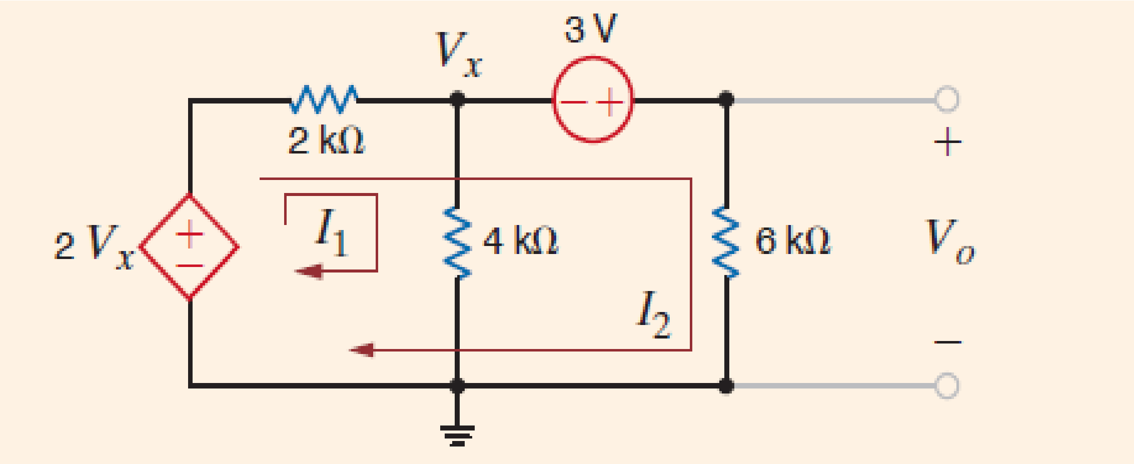
- **Circuits Containing Independent Current Sources**
- Find V_0 in the figure below using loop analysis



(9.71)

Loop Analysis

- **Circuits Containing Dependent Sources**
- Let us find V_0 in the circuit in Fig. below, which contains a voltage-controlled voltage source.

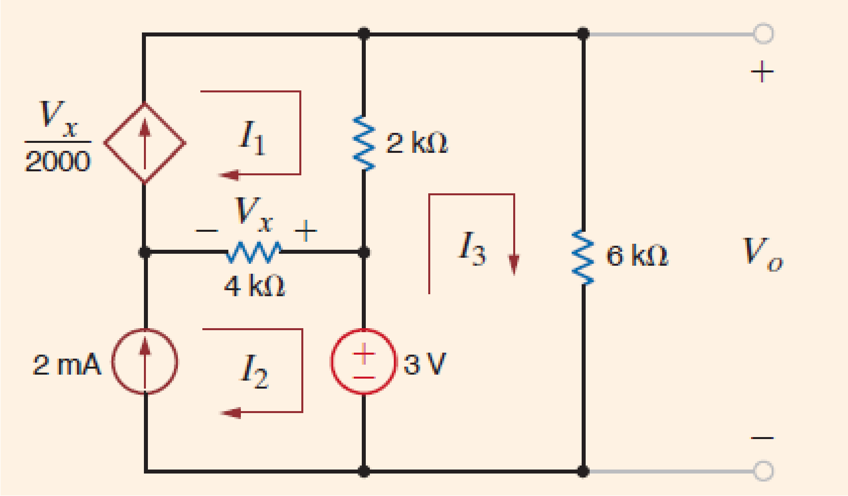


Loop Analysis

- **Circuits Containing Dependent Sources**
- $-2V_x + 2k(I_1 + I_2) + 4kI_1 = 0$
- $-2V_x + 2k(I_1 + I_2) - 3 + 6kI_2 = 0$
- Where $V_x = 4kI_1$
- These equations can be combined to produce
- $-2kI_1 + 2kI_2 = 0$
- $-6kI_1 + 8kI_2 = 3$
- These equations can be put in RI=V form and solved for I giving
- $i_1 = 1.5\ mA, i_2 = 1.5mA$
- And therefore $V_0 = 6kI_2 = 9V$
- **Exercise:** Compare this result using nodal analysis technique.

Loop Analysis

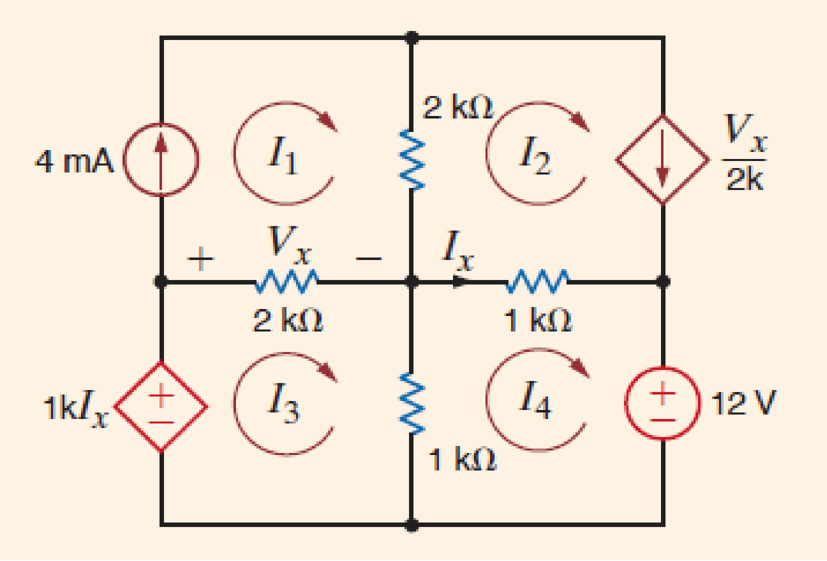
- **Circuits Containing Dependent Sources**
- Find V_0 in the circuit in Fig. below which contains a voltage-controlled current source.



(8.25V)

Loop Analysis

- **Circuits Containing Dependent Sources**
- The network in Fig. below contains both a current-controlled voltage source and a voltage controlled current source. Let us use MATLAB to determine the loop currents.

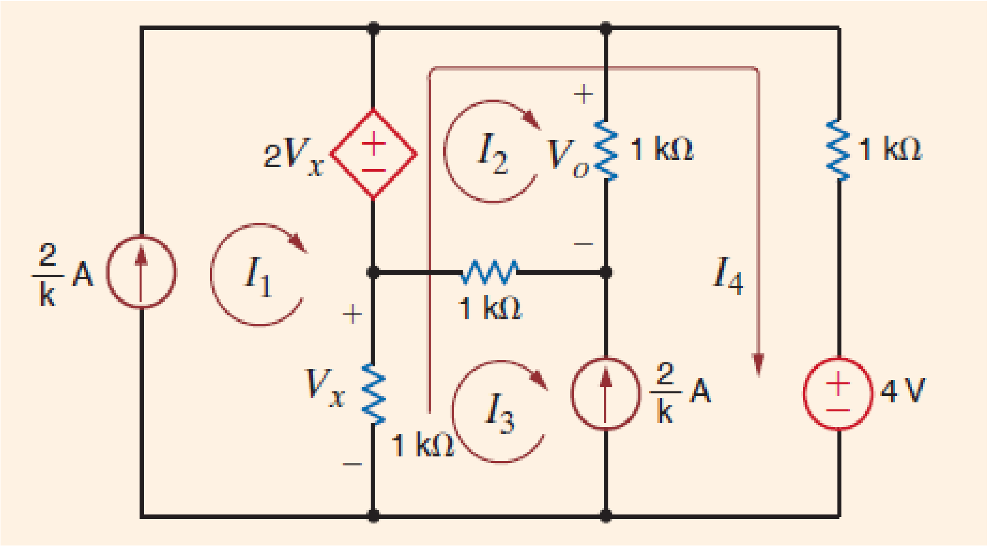


Loop Analysis

- **Circuits Containing Dependent Sources**
- The equations for the loop currents shown in the figure are:
- $I_1 = \frac{4}{k}$, $\quad I_2 = \frac{V_x}{k}$
- $-1kI_x + 2k(I_3 - I_1) + 1k(I_3 - I_4) = 0$
- $1k(I_4 - I_3) + 1k(I_4 - I_2) + 12 = 0$
- Where $V_x = 2k(I_3 - I_1)$ and $I_x = I_4 - I_2$
- Combining the equations and putting them in matrix form, gives
- $i_1 = 4mA, i_2 = 6\,mA, i_3 = -2mA, i_4 = -1mA$

Loop Analysis

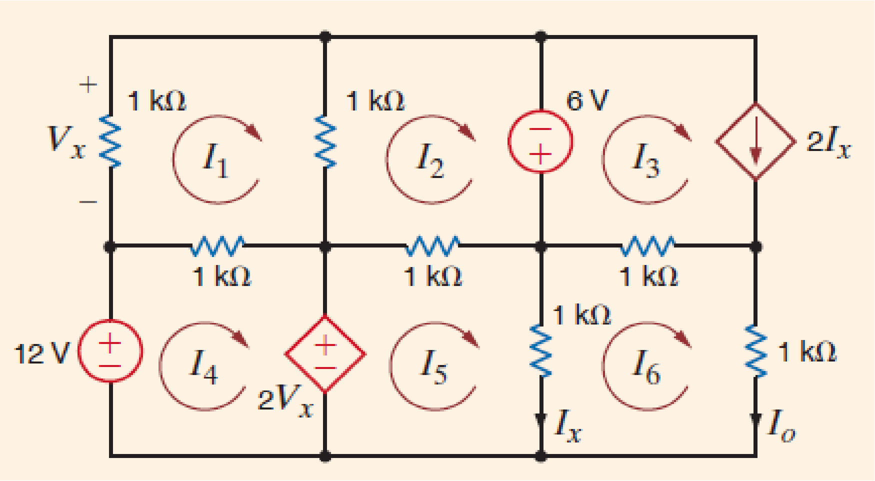
- **Circuits Containing Dependent Sources**
- Find V_0 in the circuit below



(1V)

Loop Analysis

- **Circuits Containing Dependent Sources**
- Find the mesh currents



(50, -12, -64, 162, -80, -48 mA)

Nodal and Loop Analysis

- **APPENDIX**
- **Determinant and Adjoint of a 2x2 matrix**
- $A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$
- $Adjoint\ A = \begin{bmatrix} b_2 & -b_1 \\ -a_2 & a_1 \end{bmatrix}, \quad \det A = a_1 b_2 - b_1 a_2,$
- **Cramer's rule to solve simultaneous equations**
- $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$
- $\det A = \left| \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{matrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{matrix} \right|$

Nodal and Loop Analysis

- $\det A = a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3 - (a_3b_2c_1 + b_3c_2a_1 + c_3a_2b_1)$
- Given a set of simultaneous equations
- $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$
- $D_x = \begin{bmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{bmatrix}, \quad D_y = \begin{bmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{bmatrix}, \quad D_z = \begin{bmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{bmatrix}$
- $x = \frac{\det x}{\det D}, y = \frac{\det y}{\det D}, \quad z = \frac{\det z}{\det D}$
- **Using MATLAB to solve simultaneous equations**
- $A = [a_1\ b_1\ c_1;\ a_2\ b_2\ c_2\ ;\ a_3\ b_3 c_3]$
- $b = [d_1; d_2; d_3]$
- $V = A\backslash b$ Will display the result