Makerere University

College of Engineering, Design, Art and Technology

Department of Electrical and Computer Engineering

CMP2202: ANALYSIS AND DESIGN OF ALGORITHMS

Date: 8th June 2017 **Time: 09:00-12:00Pm**

INSTRUCTIONS

- This Examination contains Six (6) questions.
- Attempt any four (4) Questions for full Marks
- The first four questions shall be marked if more than four questions are attempted.
- All Questions carry equal marks of 25 %.
- Begin each Question on a fresh page

Question One (25 Marks)

a) Distinguish between algorithm design and analysis. Discuss the two key desired attributes of an algorithm. Explain why mathematical model formulation is an important step during algorithm design. **(7 Marks)**

b) Distinguish between space and time complexity of an algorithm .Explain why these are important considerations during design of an algorithm. Discuss why asymptotic analysis is preferred to step count in the analysis of algorithm complexity. Hence or otherwise distinguish between worst –case complexity and average-case complexity interpretations of an algorithm upper bound complexity. **(8 Marks)**

c) Discuss the following asymptotic notations:

(i) Big Theta **(4 Marks)**

(ii) Big Oh **(3 Marks)**

(iii)Big Omega **(3 Marks)**

Question Two (25 Marks)

a) Using a relevant example, distinguish between sorting and searching. Hence discuss the working principle of the insertion sort algorithm. Explain three drawbacks associated with this algorithm. **(7 Marks)**

b) With reference to recursive and iterative algorithms;

(i) Give the key distinction between recursion and iteration. State two advantages and two drawbacks of the recursive algorithms in comparison to iterative algorithms. State where each of these algorithms is suited for application justifying your answer. **(6 Marks)**

(ii) Using any method of your choice, solve the following recurrence relation stating all assumptions made if any. **(4 Marks)**

$$x(n) = x(n-1) + 5 \quad \text{for } n > 1, x(1) = 0$$

c) Write **brief notes** about the following algorithms with focus on general working principle, time complexity, advantages and draw backs.

(i) Divide and Conquer **(4 Marks)**

(ii) Binary search **(4 Marks)**

Question Three (25 Marks)

a) In the context of *leader election* algorithms;

(i) Define the problem of leader election and briefly discuss how a leader election algorithm could provide a functionality necessary to some other protocol discussed during the course. **(3 Marks)**

(ii) Discuss one algorithm of your choice, clearly stating its assumption, the algorithm's advantages and drawbacks . **(5 Marks)**

b) What is the consensus problem? Hence discuss the concept of atomic commitment as regards the consensus problem. Briefly explain the limitation of the two-phase commit protocol and explain how the three-phase commit protocol mitigates this issue. **(6 Marks)**

c) The figure Q3-c below depicts three different executions of the two phase commit (2PC) protocol on a network consisting of a coordinator and three workers.

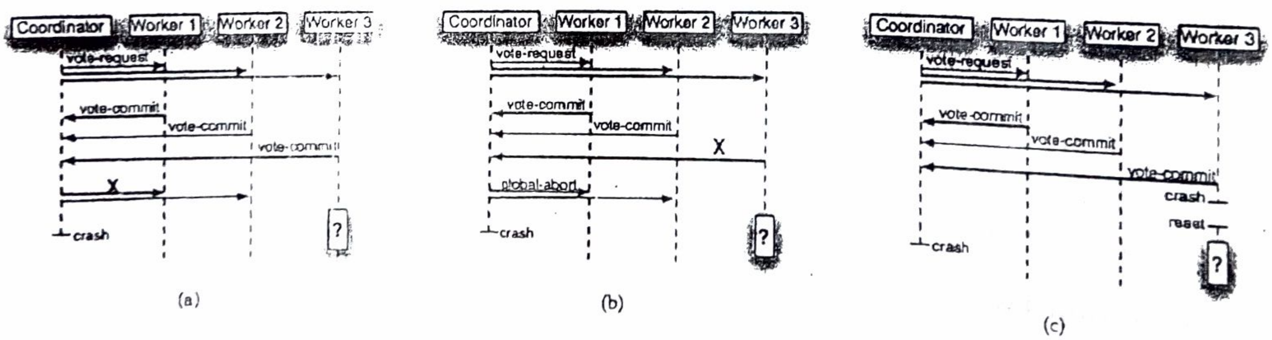

Fig Q3-c

Assuming that the coordinator always has a positive vote, answer the following:

(i) What is the meaning of the two X messages in execution 1(a) and 1(b)? For all three executions, describe the behavior of worker 3 in the box marked with a question mark. What is the overall outcome (i.e., commit or abort) of the 2PC protocol? **(7 Marks)**

(ii) In which of the three executions would it be possible for worker 3 to take a *local* decision (i.e., without contacting any other process)? Justify your answer. **(4 Marks)**

Question Four (25 Marks)

a) In the context of the consensus problem with Byzantine faults;

(i) Define agreement, validity and termination. **(6 Marks)**

(ii) Briefly discuss the consensus problem and explain the constraint that must be satisfied to have consensus. State three situations that may necessitate consensus among processes in a practical distributed system. **(5 Marks)**

b) How many non-faulty processes are required to provide n-fault tolerance in case of byzantine behavior? Hence explain why the Floodset algorithm, the solution studied for tolerating crash failures, does not work when byzantine failures are present. **(4 Marks)**

c) (i) What is the two generals problem description? Does it have a solution?, explain your answer. **(5 Marks)**

(ii) Consider a case of traitorous general sending messages to royal Lieutenants as depicted in fig Q4-c below. Show what are the messages exchanged and which is the consensus in this scenario assuming. Justify your answer. **(5 Marks)**

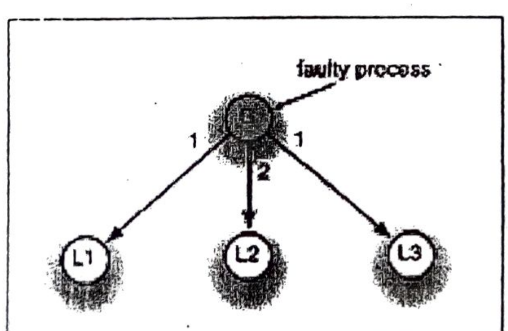

Fig Q4-c

Question Five (25 Marks)

a) Message delivery orderings are of paramount importance in distributed computing. With respect to this aspect;

(i) Provide a *precise* definition of the FIFO, Causal and Total message delivery policies. **(6 Marks)**

(ii) Describe the relationships between the above policies: are they somewhat related? If so, to what extent? If no, why? (4 Marks)

b) Apply the above definitions to the distributed executions shown in fig Q5-b indicating the message ordering adopted in each case. In case some definitions cannot be satisfied, indicate the set of messages responsible for that and the reason why they invalidate the definition.

(15 Marks)

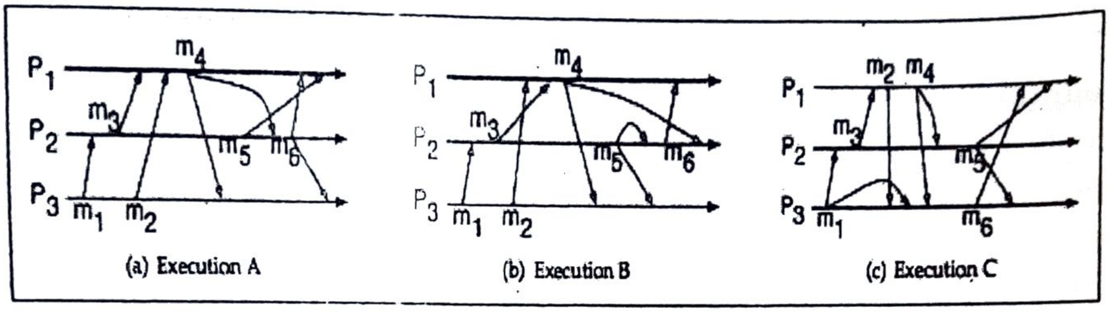

Fig Q5-b

Question Six (25 Marks)

a) Define replication and give 2 reasons why replication is important. Hence distinguish between passive and active replication stating the main drawback associated with each. (6 Marks)

b) The key challenge of any replication algorithm is to identify the type of content to propagate and how this content is to be propagated in case of an update. With reference to this context;

(i) Give three types of contents propagated and three update strategies employed in replication. (6 Marks)

(ii) Distinguish between server-initiated and client-initiated replicas. State the drawback associated with of each of these techniques and state scenario where each is suited for application. (6 Marks)

c) With reference to epidemic algorithms;

(i) Explain what is meant by an infective server, a susceptible server and a removed server (3 Marks)

(ii) Briefly discuss the Anti-entropy and Rumor mongering propagation strategies. (4 Marks)

END

MAKERERE UNIVERSITY

College of Engineering, Design, Art and Technology

School of Engineering

Department of Electrical and Computer Engineering

Bachelor of Science in Computer Engineering

Year II Semester II 2010/2011 Examinations

CMP2202: Analysis and Design of Algorithms

DATE: Saturday, 28th May 2011 TIME: 09:00 A.M. -12:00 P.M.

Attempt Only FOUR Questions

4 **Question One (25 Marks)**

a) Algorithms have found widespread use in all fields involving computation, describe the role that Design and Analysis of Algorithms has played in any three (3) of these fields (9 marks)

b) In the study of algorithms explain the different *complexity* considerations made. (5 marks)

c) The security controls for an e-commerce application require an RSA algorithm for encryption that involves computing greatest common divisors (GCD) of large numbers. Propose an efficient algorithm that can be used for GCD computation as part of the RSA algorithm (4 marks)

d) (i) Describe an efficient algorithm for multiplication of BigNums (4 marks)

(ii) For the above algorithm, perform a dry-run for the case (1235 x 5687) (3 marks)

1 **Question Two (25 marks)**

a) Give formal definitions for the following asymptotic notations for algorithm growth

(i) Little o notation (2 marks)
(ii) Theta (Θ) notation (3 marks)
(iii) Omega (Ω) notation (2 marks)

b) Peter is given the task of sorting the papers of his classmates in ascending order of performance. He decides to pick a pivot mark 68% and divides the papers in tow groups, one having marks ≥ 68 and the other marks < 68. For each group he selects another pivot and repeats the procedure of dividing, which procedure he follows until he has each group having on paper, putting these back together into one group gives a sorted group. Outline the pseudo-code for Peter's sorting algorithm. (8 marks)

c) For Peter's sorting algorithm, find the tight big Oh bound for the best case and worst case running times (10 marks)

Question Three (25 marks)

Using a suitable example in each case, explain the following algorithmic strategies.

a) Backtracking Algorithms (5 m...

b) Heuristic Algorithms (5 mar...

c) Greedy Algorithms (5 marks)

d) Divide and Conquer Algorithms (5 marks)

e) Dynamic Programming (5 marks)

Question Four (25 marks)

a) Solve the following recurrence relations, giving asymptotically tight solutions

(i) $T(n) = T(n/3) + \Theta(n)$ (4 marks)

(ii) $T(n) = 2T(\sqrt{n}) + 1$ (5 marks)

b) Using only two LIFO data structures outline the Pseudocode for the insertion and Removal Operations of a FIFO data structure. (6 marks)

c) Given a hash-table with a hashing function $h(x) = (x \bmod 5)$ and a Universe of Keys defined by the set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 11\}$

i) Illustrate the hash-table including appropriate collision avoidance (6 marks)

ii) Compute the load factor and element-access time (4 marks)

Question Five (25 marks)

a) Outline the Pseudocode for the Search, Insert and Delete operations in a linked list, in each case computing the algorithm running time (9 marks)

b) For the directed, weighted graph G with vertices V = {a, b, c, d, f} with Edges in the format *(start-vertex, end-vertex, weight)* including (a, b, 5), (b, a, 4), (b, c, 1), (d, f, 9), (b, d, 5), (c, f, 3)

(i) Give a diagrammatic representation of the Graph G (3 marks)

(ii) For graph G define both the Adjacency-List and Adjacency Matrix (8 marks)

(iii) Which of the two representation in b (ii) is most suitable and why? (5 marks)

~END~

MAKERERE UNIVERSITY

College of Engineering, Design, Art and Technology

School of Engineering

Department of Electrical and Computer Engineering

Bachelor of Science in Computer Engineering

Year II Semester II 2011/2012 Final Examination

CMP2202 Analysis and Design of Algorithms

DATE: Thursday, 9th May 2013 TIME: 09:00 A.M. -12:00 P.M.

Attempt Only FOUR Questions

Question One (25 Marks)

a) Describe what a "good" algorithm is. What are their characteristics? (4 marks)

b) Describe one algorithm that has had extensive application in the field of computer and telecommunication networks. (6 marks)

c) The security controls for an e-commerce application require an RSA algorithm for encryption that involves computing greatest common divisors (GCD) of large numbers. Propose an efficient algorithm that can be used for GCD computation as part of the RSA algorithm (7 marks)

d) (i) Describe an efficient algorithm for finding the power of a given number (x^n) (4 marks)

(ii) For the above algorithm, perform a dry-run for the case (6^5) (3 marks)

Question Two (25 marks)

a) Describe the two dimension of algorithm complexity (i.e. space and time) (5 marks)

b) Explain the difference amongst the three (3) asymptotic notations for defining algorithm running time (6 marks)

c) John is given the task of sorting the papers of his classmates in ascending order of performance. He decides to pick a pivot mark 72% and divides the papers in two groups, one having marks ≥ 72 and the other marks < 72. For each group he selects another pivot and repeats the procedure of dividing, which procedure he follows until he has each group having on paper, putting these back together into one group gives a sorted group. Outline the pseudo-code for Peter's sorting algorithm. (8 marks)

d) For Peter's algorithm, find the big O for the worst case (6 marks)

Question Three (25 marks)

a) Explain the concept of recursion (3 marks)

b) For each of the following algorithms write the pseudocode for a recursive algorithm solution:

i. Finding the smallest number in an array of integers (4 marks)
ii. Finding the nth Fibonacci number where n is an integer (4 marks)
iii. Finding the factorial of n where n is a non-negative integer (4 marks)

c) For each of the algorithms in 3(b), determine the time complexity in terms of big O (10 marks)

Question Four (25 marks)

a) Compare and contrast a Stack and a Queue (6 marks)

b) Write pseudo-code for Enqueue and Dequeue algorithms for a Queue taking into consideration both underflow and overflow and analyze their running times (9 marks)

c) Show how to implement a queue using two stacks and analyze the Θ asymptotic notation of the fundamental Queue Operations (10 marks)

Question Five (25 marks)

a) Solve the recurrence $T(n) = 2T(\sqrt{n}) + 1$ giving an asymptotically tight solution (5 marks)

b) Give an analytical comparison of the Quick sort and Merge Sort algorithms (8 marks)

c) Differentiate between direct address tables and hash tables (6 marks)

d) Using suitable example, explain collision-avoidance in hashing using chaining, explaining the significance of load factor. (6 marks)

~END~

MAKERERE UNIVERSITY

COLLEGE OF COMPUTING & INFORMATION SCIENCES

SCHOOL OF COMPUTING & INFORMATICS TECHNOLOGY

END OF SEMESTER I EXAMINATION 2012/2013

PROGRAMME: BSC

YEAR OF STUDY: SE I, CS II

COURSE NAME: Discrete Mathematics

COURSE CODE: MTH 3105

DATE: 26th November, 2012 **TIME:** 12:00 – 15:00

EXAMINATION INSTRUCTIONS

1. ATTEMPT ALL QUESTIONS IN SECTION A (40 MARKS)
2. ATTEMPT THREE (03) QUESTIONS IN SECTION B (60 MARKS)
3. DO NOT OPEN THIS EXAM UNTIL YOU ARE TOLD TO DO SO
4. ATTEMPT EACH QUESTION IN SECTION B ON A NEW PAGE
5. ALL ROUGH WORK SHOULD BE IN YOUR ANSWER BOOKLET

SECTION A [40 Marks]

Each question in this section carries 4 marks

a) A number n is a sum of two squares if $n = a^2 + b^2$ for some integers a and b. If x and y are both sum of two squares, prove that xy is also a sum of two squares.

b) Show that $(p \rightarrow q) \rightarrow (q \vee \neg p)$

c) Define $\gcd(a_1, a_2, \ldots, a_n)$ as the GCD of $a_1, a_2, \ldots, a_n$.

Show that $\gcd(a_1, a_2, \ldots, a_n) = \gcd(\gcd(a_1, a_2, \ldots, a_{n-1}), a_n)$.

d) It is known that $|x + y| \leq |x| + |y|$ for any two real numbers x and y.

Show that for any n real numbers $x_1, x_2, \ldots, x_n$,

$|x_1 + x_2 + \ldots + x_n| \leq |x_1| + |x_2| + \ldots + |x_n|$

e) Construct truth tables for

i. $\neg(p \vee q) \rightarrow r$

ii. $\neg((p \vee q) \rightarrow r)$

f) There are 7 glasses on a table, all standing upside down. You are allowed to turn over any 4 of them in one move. Is it possible to have all the glasses right-side-up? Give reasons to support your answer

g) Prove that $\sqrt{6}$ is irrational.

h) Show that $m^{13} - m$ is divisible by 13.

i) Let R be the relation on $\{1, 2, 3, 4, 5\}$ defined by mRn if and only if $m - n$ is odd. Draw a picture for this relation.

j) Write down all elements in the power set of $\{\emptyset, \{1, 2\}\}$.

SECTION B (60 Marks)

Question 1:

a. Show that a 6 x n board ($n \geq 2$) can be tiled with L-shaped tiles, without gap and overlapping. Each L-shaped tiles cover three squares. (4 Marks)

b. Prove that for any integer n, $n^{6k} - 1$ is divisible by 7 if gcd(n, 7) = 1 and k is a positive integer. (4 Marks)

c. Prove by induction

i. $1^2 + 2^2 + 3^2 + \cdots + (2n)^2 = \dfrac{n(2n+1)(4n+1)}{3}$ (6 Marks)

ii. $1 + 4 + 7 + \cdots + (3n-2) = \dfrac{n(3n-1)}{2}$ (6 Marks)

Question 2:

a. Prove that $\sqrt{10}$ is irrational (5 Marks)

b. Show that $1 + \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{3}} + \cdots + \dfrac{1}{\sqrt{n}} \leq 2\sqrt{n}$ (5 Marks)

c. Show that $2! \cdot 4! \cdot 6! \ldots (2n)! \geq ((n+1)!)^n$ (5 Marks)

d. Define the sequence a0, a1, a2, ... by the recursive formula $a_{n+1} = 2a_n - a_n^2$.
Show that $a_n = 1 - (1 - a_0)^{2^n}$. (5 Marks)

Question 3:

a. Convert the following to First Order Logic (2 Marks @)

i. Every student who has an Exam card is cool

ii. Some students took MTH3105 in Winter 2012

iii. Any person can fool some of the people all of the time, all of the people some of the time but not all of the

b. Write down the negations of the following quantified statements (2 Marks @)

i. $\exists x(Q(x) \rightarrow \forall y P(y))$ ii. $\forall V \, \exists P$, kill(P, V) iii. $\exists x \, \forall y \, [P(x) \rightarrow Q(y)]$

c. Assume that $\forall x \exists y P(x, y)$ is false and the domain of them is nonempty. Which of the following must be false? (4 Marks)

i. $\forall x \forall y P(x, y)$ ii. $\exists x \forall y P(x, y)$ iii. $\exists x \exists y P(x, y)$

d. Assume that $\exists x \forall y P(x, y)$ is true and the domain of them is nonempty. Which of the following must be true? (4 Marks)

i. $\forall x \forall y P(x, y)$ ii. $\forall x \exists y P(x, y)$ iii. $\exists x \exists y P(x, y)$

Question 4:

a. Write a truth table for $[(p \leftrightarrow q) \vee (p \rightarrow r)] \rightarrow (\neg q \wedge p)$ (4 Marks)

b. Write a logical formula for the below truth table and then simplify it. (6 Marks)

p	F	F	F	F	T	T	T	T
q	F	F	T	T	F	F	T	T
r	F	T	F	T	T	T	F	T
Output	F	T	T	F	T	T	T	F

c. State whether the following arguments are valid or not. Show your steps.

i. $p \rightarrow q, q \rightarrow r, \quad \therefore p \rightarrow r$ (5 Marks)

ii. $p \rightarrow q, q \rightarrow r, r \rightarrow p \quad \therefore p \wedge q \wedge r$ (5 Marks)

Question 5:

a. Which of the following describe equivalence relations? For those that are not equivalence relations, specify which property fails. (2 Marks @)

i. $L_1 \parallel L_2$ for straight lines in the plane if L_1 and L_2 are the same or are parallel.

ii. $L_1 \perp L_2$ for straight lines in the plane if L_1 and L_2 are perpendicular.

iii. $(m, n) \approx (k, l)$ if $m + l = n + k$; $m, n \in \mathbb{N}$.

iv. $(m, n) \approxeq (p, q)$ if $mq = np$; $(m, n), (p, q) \in \mathbb{Z} \times \mathbb{P}$.

b. Solve the followings, or give "unsolvable" if it has no solution. (4 Marks @)

i. $10x \equiv 35 \pmod{42}$

ii. $49x \equiv 98 \pmod{21}$

iii. $14x \equiv 7 \pmod{28}$

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MAKERERE UNIVERSITY

Faculty of Technology

Department of Electrical Engineering and Computer Engineering

Bachelor of Science Computer Engineering

Year II Semester II 2010/2011

CAT II: CMP2201 Discrete Mathematics and Random Processes

Date: Thursday 21st May 2011 — Time: 08 00 - 09 00 Hours

Attempt ALL Questions — Martina Alinda

Question One (25 marks)

i) What is a proof? (2 marks)

ii) What is the relevance of proofs in mathematical sciences? (2 marks)

iii) Differentiate between a proof and a lemma. (2 marks)

b) Write brief notes on any three(3) methods of formal proofs (9 marks)

c) Use the direct proof method to prove that if n is an odd integer, then n^2 is odd. (5 marks)

d) Use mathematical induction to prove the inequality $n < 2^n$ (5 marks)

$3 | n^3 - n$

Question Two (25 marks)

Define a random process. (4 marks)

A random process is given by $U(t) = A + B\cos(wt + \theta)$ [illegible] uniformly distributed random variables with distributions such that $A \sim U[-3, 3]$ [illegible] $-\frac{\pi}{2}, \frac{3\pi}{2}$. The mean/expectation of B is $E(B)$ [illegible]

$S \subseteq T$ $\quad T = \{1, 2, 3, 4\}$

$S = \{2, 4\}$?

i. The mean of the random process $X(t)$. (8 marks)

ii. The variance $\delta^2_{x(t)}$, of the random process (7 marks)

iii. Is $X(t)$ wide sense stationary? (6 marks)

$2^n + 2^n$

THE END

$x = 3k+1$

$n+1 < 2^n + 2^n = \dfrac{2^{n^2}}{2^n}$ $\quad x^2 = (3k+1)^2$

Quantifiers $\exists$ → there exists $\quad \dfrac{2^n + 2^n}{}$ $\quad x^2 = 9k^2 + 6k + 1$

Quantifiers $\forall$ - for all

$= 3(3k^2 + 2k) + 1$

$k \equiv 1 \pmod 3$ $\Rightarrow$

n some integer n.

$\exists n \; k - 1 = 3n$ $\quad 3n+1$

$\exists n \; k = 3n + 1$

Faculty of Technology

Department of Electrical Engineering and Computer Engineering

Bachelor of Science Computer Engineering

Year II Semester II 2010/2011

CAT I: CMP2201 Discrete Mathematics and Random Processes

Date Thursday 24th March 2011 Time: 08 00 -09 00 Hours

Attempt ALL Questions

Martina Alinda

Question One (20 marks)

a) Let p and q be the propositions;

p: Swimming at the Lake Victoria shore is allowed

q: Sharks have been spotted near the shore.

Express each of the following compound propositions as an English sentence

ii) $\neg p \rightarrow \neg q$ iii) $p \leftrightarrow \neg q$ (6 marks)

b) Given the conditional statement;

p: If it snows tonight, then I will stay home

Write the converse, contrapositive and inverse of p; (6 marks)

c) Construct a truth table for the compound proposition $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$ (5 marks)

d) Find the bitwise OR, bitwise AND, and bitwise XOR the following pair of bit strings;

101 1110, 010 0001 (3 marks)

Question Two (25 marks)

a) Differentiate between propositional logic and predicate logic (4 marks)

b) Determine the truth values of the following statements if the domain consists of all real numbers

i) $\exists x(x^3 = -1)$ ii) $\forall x(2x > x)$ True (4 marks)

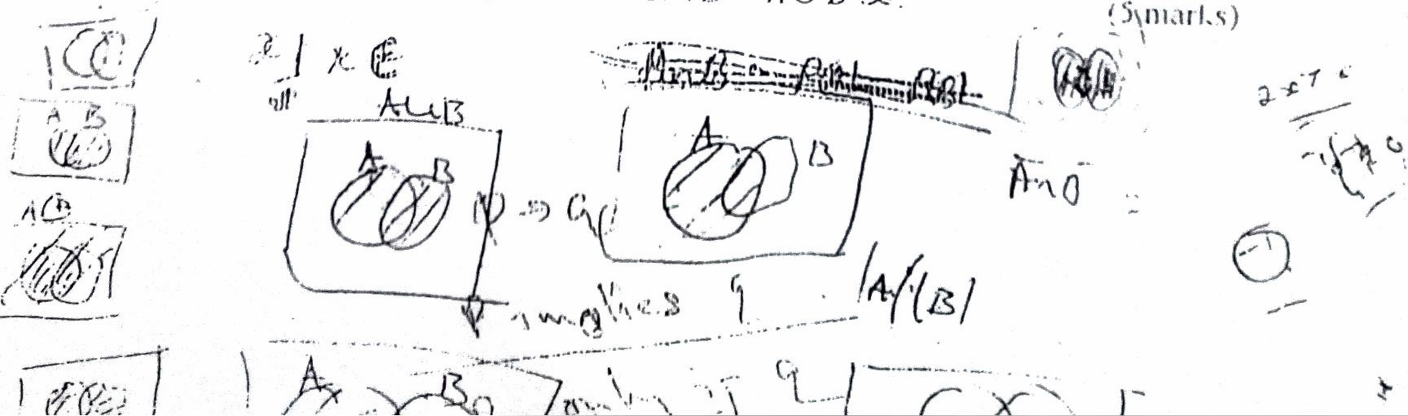
c) Let $Q(x, y, z)$ be the statement "$x + y = z$", where the domain of all variables consists of all real numbers. Write down the English translations of the following statements and state their truth values

i) $\forall x \forall y \exists z Q(x, y, z)$ (2 marks)

ii) $\exists z \forall x \forall y Q(x, y, z)$ (2 marks)

d) Given any two sets, A and B. With the aid of appropriate diagrams, arrange the following in order of increasing size $|\emptyset|$, $|A|+|B|$, $|A \oplus B|$, $|A - B|$, $|A \cup B|$, $|A|$ (8 marks)

e) Use the set builder notation to prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ (5 marks)



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MAKERERE UNIVERSITY

College of Engineering, Design, Art and Technology

Department of Electrical Engineering and Computer Engineering

Bachelor of Science in Computer Engineering

Year II, Semester II 2012/2013

Martina Alinda

CAT I: CMP2201 Discrete Mathematics and Random Processes

Date: Wednesday 3rd April 2013 — Time: 08 00 – 09 00 Hours

Attempt ALL Questions

... 25 marks)

... understanding contrast between Discrete and Continuous Mathematics.
... describe Real World Applications of Discrete Mathematics (6 marks) – Boolean Alg – Set Theory

... the following five Relations on the set A = {1, 2, 3}
... 2), (1, 3), (3, 3)} ... (1,2)
... (2, 1), (2, 2), (3, 3)} ...
... 1/2, (2, (2,3) (3, 4)
... 2, 3 ...
Empty relation
Universal relation

... whether or not each of the above relations on A is
Reflexive
Symmetric (3 marks)

... the truth value of each of the following statements where U = {1, 2, 3} is the universal set. (3 marks) There exist — for all

$\exists x\ \forall y,\ x^2 < y + 1$ {1} True (2 marks)
$\forall x\ \exists y,\ x^2 + y^2 < 12$ {1}? True (2 marks)
$\forall x\ \forall y,\ x^2 + y^2 < 12$ $p \to q = \neg q \Rightarrow \neg p$ False (2 marks)

... the Contrapositive of each statement

... Mark studies will he pass the test (1 mark)

... truth table verify that the proposition $(p \wedge q) \wedge \neg(p \vee q)$ is a contradiction. (1 mark)

(5 marks)

A discrete random variable is one takes on integer value
no. of students in class
while continuous → is variable that can take on any
real value (on an interval average volume of soda, heights of students
is how we defined $P(X=x)$, & $F(X=x$ — continuous

COLLEGE OF ENGINEERING, DESIGN, ART AND TECHNOLOGY

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

B.Sc. (COMP) SECOND YEAR – Continuous Assessment Test I – 2012/2013

CMP2202: ANALYSIS AND DESIGN OF ALGORITHMS

DATE: March 26, 2013 TIME: 8am – 9am

INSTRUCTIONS:

Attempt all the questions.

Question 1:

a) Briefly describe what you understand by an algorithm and give the characteristics of a good algorithm. **(4 Marks)**

b) Describe three practical fields in which algorithms have played an important role in Computer Engineering. **(6 Marks)**

c) Mr. Andrew has been employed by the high security level where they are dealing with only classified documents. He has been asked to design an RSA encryption algorithm of the information that involves computing the GCD (Greatest Common Divisor). Propose an algorithm that can be used for GCD computation as part of the RSA algorithm.**(5 Marks)**

d) Using simple code snippets describe the difference between the Iterative and recursion versions of getting the next number in the sequence 0,1,1,2,3,5,8,13,... and give the advantage of one over the other. **(10 marks)**

Question 2:

a) Describe the goal of analysis of algorithms and what you understand by running time analysis. **(3 Marks)**

b) Explain what you understand by the following asymptotic notations. O notation, Ω notation and Θ notation. **(6 Marks)**

c) A second year student is participating in CEDAT cards games competition and is thinking about organizing his cards in ascending order so that he can easily pick a card from his cards. Using pseudo code describe an algorithm that can help him to organize his cards in ascending order. **(8 Marks)**

~ END ~

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COLLEGE OF ENGINEERING, DESIGN, ART AND TECHNOLOGY (CEDAT)

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING
CMP 2201: DISCRETE MATHEMATICS AND RANDOM PROCESSES
B Sc COMPUTER ENGINEERING
END OF SEMESTER II EXAMINATIONS. DATE: 7TH MAY, 2013, 9-12.00

INSTRUCTIONS: Attempt all the five questions. Each question carries twenty marks.

QUESTION ONE

a) With illustrations, contrast between Discrete and Continuous Mathematics. Give and briefly describe Real World Applications of Discrete Mathematics (6 marks)

b) Represent a disjoint union of two sets on a venn diagram. (set2) **(2 marks)**

c) Consider the following five Relations on the set A = {1, 2, 3}

R = {(1, 1), (1, 2), (1, 3), (3, 3)}

S = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)}

T = {(1, 1), (1, 2), (2, 2), (2, 3)}

{} = Empty relation

A x A = Universal relation

Determine whether or not each of the above relations on A is

i. Reflexive (3 marks)

ii. Symmetric (3 marks)

d) Determine the Contrapositive of each statement.

i. If Eric is a poet, then he is poor. (1 mark)

ii. Only if Marc studies will he pass the test. (1 mark)

e) What is meant by the term Cartesian product?

If $A = \{1,2\}$, $B = \{3,4\}$, $C = \{5,6,7\}$ what is $A \times B \times C$ and determine its cardinality (set1) (4 marks)

f) Determine if $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a contradiction, tautology or neither. (5 marks)

$\neg q \wedge (\neg p \vee q)) \rightarrow \neg p$

QUESTION TWO

a) Distinguish and relate a Theorem and a Proof (4 marks)

b) Give two applications of study of Proofs in the field of Computer Engineering or Computer Scienc (2 marks)

c) Describe with illustrations any three methods of formal Proofs (9 marks)

d) If n is a non – negative integer, then $5 \mid (n^5 - n)$. Prove this statement by the method of Induction. (5 marks)

QUESTION THREE

a) Let $f : Z \rightarrow R$ be given by $f(x) = x^2$

i. What are the domain and co-domain?

ii. What's the image of -3 ?

iii. What are the pre-images of 3, 4?

iv. What is the range $f(Z)$? (5 marks)

b) Functions are like non-void Java methods. Here is an example of a java function, what is the domain, codomain and possible outcome of the function (Range). (5 marks)

```
int f(double x){
        return x<0 ? –1 :
                ( x>0 ? 1 : 0 );
    }
```

c) Which of the following are 1-to-1, onto, a bijection? If f is invertible, what is its inverse?

i. $f : Z \rightarrow R$ is given by $f(x) = x^2$

ii. $f : Z \rightarrow R$ is given by $f(x) = 2x$

iii. $f : R \rightarrow R$ is given by $f(x) = x^3$ (6 marks)

d) Given two functions $f(x) = \frac{1}{x} + 4$ and $g(x) = \frac{1}{x-4}$, find $(f.g)^{-1}(x)$. Comment on your answer. (6 marks)

e) Define the term Ceiling and floor as applied to discrete mathematics. (3 marks)

QUESTION FOUR

a) Define the following

i. Random process

ii. Brownian process

iii. Poisson process

iv. Markov process (2 marks each)

b) Explain/write short notes on the following

i. Properties of power spectral density (5 marks)

ii. Digital Modulation using PSK(Phase Shift Keying) (5 marks)

c) Explain the Gaussian random process and its relative importance in communication theory. (7 marks)

QUESTION FIVE

a) Suppose that the probability that an item produced by a certain machine will be defective is 1. Find the probability that a sample of 10 items will contain at most one defective item. Assume the quality of successive items is independent (5 marks)

b) If the average number of claims handled daily by an insurance company is 5:

i. What proportion of days has less than 3 claims? (2 marks)

Lillian   Nabunnya Bkwa...

MAKERERE UNIVERSITY

COLLEGE OF ENGINEERING, DESIGN, ART AND TECHNOLOGY

DEPARTMENT OF ELECRICAL AND COMPUTER ENGINEERING

BACHELOR OF SCIENCE IN COMPUTER ENGINEERING

YEAR II 2011/2012 SEMESTER TWO EXAMINATIONS

CMP2201: DISCRETE MATHEMATICS & RANDOM PROCESSES

May 8, 2012   Martha Atiinda   **TIME:** 0900-1200 HOURS

5.3   25=4 ?

Attempt any Four (4) Questions

QUESTION 1 (Propositional Logic and Proof Techniques)

a. Define a ***proposition*** and give one such an example. [4]

i. Determine if $\left(\neg q \wedge (p \to q)\right) \to \neg p$ is a *tautology* or a *contradiction* or *neither*. [5]

ii. ... *You can drive a car"* and q = *"You are over 18 years old"* and r = *"You have an instructor"*, translate the following sentence into a compound proposition using p, q and r.
"if you are not over 18 years old and you have a training instructor, you can drive a car" [5]

Prove by **contradiction** that the hypotenuse of a non-trivial right-angled triangle is shorter than ... its other two sides. [6]

... direct **proof** to show that the arithmetic mean $\frac{1}{2}(x+y)$ is always greater than the ... mean $\sqrt{xy}$ where $x, y \in \mathbb{Z}^{+}$.

Eg (1,1) (1,2) (1,3) (2,1) (2,2) (2,3) ... (3,3)

An ordered pair ... of elements a b where a is first element b second element (a,b)

QUESTION 2 (Relations, Sets, Functions and Series)

... is a set of 2 of ordered pairs from A to ... each ordered pair ... element ... from A & second B relation

i. Differentiate a **binary relation** from an **ordered pair**. [4]

ii. Let $A = \{2, 3, 4\}$ and $B = \{-2, 1, 0\}$. Determine whether there exists a relation R from set A to set B such that R is both transitive and symmetric. [2]

... power set — Set of all subsets. [2]

ii. ... Venn diagrams or propositional logic, prove that $\overline{\left(\overline{A} \cup \overline{B}\right)} = A \cap B$ [6]

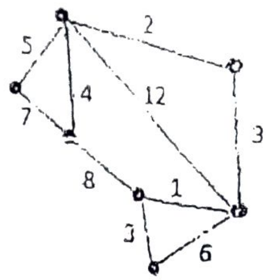
iii. Given two functions $f(x) = \frac{1}{x} + 4$ and $g(x) = \frac{1}{x-4}$, find $(f \circ g)^{-1}(x)$. Comment on your answer. [6]

Using your knowledge of Geometric Progressions, evaluate $\sum_{i=0}^{n} 2^{-n}$ where n is large [5]

... (Graphs and Trees)

Let A & B be sets, A binary relation from A to B is subset of A×B

... the edges in the graph below show the cost it will take in million... up a road network connecting different towns in Masaka district. Use Prim's algorithm ... minimum cost of constructing a road network that connects all 7 towns.



c. A graph has n vertices, and one edge from each vertex to every other vertex. Show that there are a total of $\frac{1}{2}n(n-1)$ edges in the graph.

8

QUESTION 4 (Recurrence relations and Counting)

a. Let there be an n^{th} term in a series, $f(n)$, defined in terms of the terms after it as

$$8f(n) = f(n+2) - 7f(n+1)$$

If we know that $f(0) = f(1) = 3$, express $f(n)$ in terms of n and hence write down the 7^{th} and 8^{th} terms of the series [10]

$A = \frac{1}{5}$ $A = \frac{2}{3}$ $B = \frac{7}{3}$

b.

i. How many different six lettered arrangements of the word "ARRANGEMENT" do not contain the letter R? [4]

ii. How many possible arrangements of 5 letters are there in the word NAKASONGOLA which contain an S? [6]

c. Show that, if repetitions are permitted, there are N^p possible arrangements of p objects from an ensemble of N objects. [5]

8

$C(n+r-1, r) = C(n+r-1, n-1)$ $\frac{(n+r-1)!}{r!(n-1)!}$

QUESTION 5 (Probability, Random Variables and Random Processes)

a. Define the term "**mutually exclusive**" as applied to two events in probability theory. [2]

b. Mulindwa is a Cake salesman for parties. He has his cakes baked either by Nakabuye Cakes Ltd or Ngoboka & Co. Pastries Ltd. The companies make 30% and 70% of his cakes respectively. Given that a cake is from Ngoboka & Co. Pastries Ltd, the probability that it is delicious is 0.67. Now, he has received 450 cakes delivered and found that 250 were not delicious, how many delicious cakes can we expect from Nakabuye Cakes Ltd. for every batch of 100 that they produce? [8]

c. Given the following pdf,

$$\int f(x)\,dx = 1 \qquad f(x) = \begin{cases} k\left(\frac{x}{3} - 1\right), & 0 \ldots 3 \\ 0 & \text{elsewhere} \end{cases}$$ where k is a constant.

$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

i. Find its mean [3]

ii. Show that its n^{th} moment is $3^p\left[\frac{1}{(n+1)(n+2)}\right]$ where $p = n + \log_3 2$ [5]

d. Define an **ergodic** random process and give one example of such process. [1]

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DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

BACHELOR OF SCIENCE IN COMPUTER ENGINEERING

CMP 2201: Discrete Mathematics and Random Processes

FINAL EXAMINATIONS SECOND SEMESTER 2010/2011

May 2011 Martina Alinda Time: 09:00 – 12:00 HRS

Instructions:

1. Attempt only 5(five) out of 6(six) questions
2. All questions carry 20 marks

Question 1

a) Let p and q be the propositions.

p : Swimming at the Lake Victoria shore is allowed.

q : Sharks have been spotted near the shore.

Express each of the following compound propositions as an English sentence

i) $\neg p \vee q$ ii) $\neg p \to \neg q$ iii) $p \leftrightarrow \neg q$ (6 marks)

b) Given the conditional statement;

p : If productivity increases, then wages will rise. $\neg q \to \neg p$ — Contrapositive

State the converse, contrapositive and inverse of p; $p \to \neg q$ (6 marks)

c) Show that $\neg(p \to q)$ and $p \wedge \neg q$ are equivalent:

i) Using a truth table (5 marks)

ii) Using logical equivalencies Converse $q \to p$ (3 marks)

$\neg p \to \neg q$ — inverse

Question 2

a) Distinguish between a proposition and a predicate. (4 marks)

b) Determine the truth values of the following statements if the domain consists of all real numbers

i) $\exists x(x^3 = -1)$ (2 marks)

ii) $\forall x(x^2 \geq x)$ (2 marks)

c) Let $Q(x, y, z)$ be the statement "$x + y = z$", where the domain of all variables consists of all real numbers. Write down the English translation of the following statement, and state its truth value

$\forall x \forall y \exists z Q(x, y, z)$ (3 marks)

d) Write down in English the negation of the following quantified statement:

"Every student has at least one course where the lecturer is a teaching assistant."

(3 marks)

e) Use Venn diagrams to show that $(C \cup B) - A = (C - A) \cup (B - A)$ (6 marks)

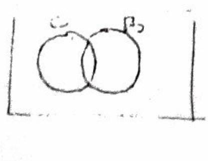
Question 3

a) Determine whether the following functions are bijections from R to R. Please explain.

i) $f(x) = x^2$ non 2. 1^2 2 because it does for does give one for one (4 marks)

ii) $f(x) = x^3 - 3$ -1 -3 onto (4 marks)

$x^2 \to$ set integer



$f(1) = f(-1) = 1$

$f(x) = f(-x)$

b) Given the set $A = \{1,2,3,4\}$, and the following relations (4,1) (1,2) (2,1)

$R_1 = \{(1,1),(1,2),(1,4),(2,1),(2,2),(3,3),(4,1),(4,4)\}$

$R_2 = \{(1,1),(1,2),(1,3),(1,4),(2,2),(2,3),(2,4),(3,3),(3,4),(4,4)\}$

For each of the two relations R_1 and R_2, explain whether it is:

i) Reflexive (4 marks)

ii) Symmetric (4 marks)

iii) Transitive (4 marks)

(Hint: Yes/No answers carry no marks)

Question 4

a) i) What is a proof?

ii) Differentiate between a theorem and a lemma. (2 marks)

b) Distinguish between proof by contradiction and proof by contraposition (2 marks)

c) Use an indirect proof method to prove that if n is an odd integer, then n^2 is odd. (6 marks)

d) Use mathematical induction to prove the inequality $\sum_{i=1}^{n} \sqrt{i} > \frac{n}{2}$ for $n \geq 2$ (4 marks) (6 marks)

Question 5

a) Define a random process. (3 marks)

b) A random process is given by $U(t) = A + B\cos(wt + \theta)$ where A and θ are uniformly distributed random variables with distributions such that $A \sim U[-3,3]$ and $\theta \sim U[-\frac{\pi}{2}, \frac{\pi}{2}]$. The mean/expectation of B is $E(B) = 0$, the variance of B is $\sigma_B^2 = 4$ and w is a constant.

Determine the

i) The mean of the random process $U(t)$ (6 marks)

ii) The variance $\sigma^2_{X(t)}$, of the random process (6 marks)

iii) Is $U(t)$ wide sense stationary? (5 marks)

Question 6

a) A company has 12 job applicants from which to select 4 programmers and 3 management trainees. How many ways can they do this if only 7 of the applicants are qualified to be programmers? Assume no one is offered more than one job and any of the applicants can be management trainees. (6 marks)

b) i) Define is a simple graph. (3 marks)

iii) How many possible edges are there for a graph G with n vertices(nodes) (4 marks)

c) Distinguish between a multigraph and a pseudograph. (4 marks)

d) Construct and label clearly the digraph represented by the following edges

$e_1 \to (1,2)$, $e_2 \to (1,2)$, $e_3 \to (2,2)$, $e_4 \to (2,3)$, $e_5 \to (2,3)$, $e_6 \to (3,3)$, $e_7 \to (3,3)$

(5 marks)

- THE END -

MAKERERE UNIVERSITY

SCHOOL OF ENGINEERING

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

B.Sc (CE), SECOND YEAR, SECOND SEMESTER 2016/17 FINAL EXAMINATIONS

CMP2201: DISCRETE MATHEMATICS AND RANDOM PROCESSES

Date: Tuesday 6th June 2017 Time: 9:00 AM – 12:00 PM

Instructions:

1. This paper consists of **FIVE (5)** questions. Attempt only **FOUR (4)** questions.
2. All questions carry 25 marks

Question 1 (25 Marks)

a) Define Cartesian product. If $A = \{1,2\}, B = \{3,4\}$ *and* $C = \{5,6,7\}$. Determine the value of |A X B X C|. **(5 Marks)**

b) If $f(x) = 3 + 2x$ and $g(x) = 5 - 4x^2$. Determine: **(5 Marks)**

i) gf(-1).

ii) fg(-1).

Is gf always equal to fg?

c) For the following relations on S = {1, 2, 3, 4} specify which of the properties reflexive, antireflexive, symmetric, antisymmetric and transitive the relations satisfy. **(10 Marks)**

i) $(m,n) \in R_1$ *if* $m + n = 5$

ii) $(m,n) \in R_2$ *if* $m - n$ *is even*

iii) $(m,n) \in R_3$ *if* $m < n$

iv) $(m,n) \in R_4$ *if* $m + n \leq 5$

Which of the relations above are equivalence relations?

d) Let $g: Z \to Z$ be defined by $g(x) = ax + b$ where Z denotes the set of integers and $a,\ b \in Z$ *with* $a \neq 0$. **(5 Marks)**

i) Prove that g is one to one

ii) What must be true about a and b if g is onto?

Question 2 (25 Marks)

a) What are the methods of mathematical proof? Distinguish between mathematical induction and strong induction. **(8 Marks)**

b) Prove the following statement. **(10 Marks)**

For any nonnegative integer n and any real number $x \neq 1$

$$1 + x + x^2 + \dots + x^n = \frac{x^{n+1} - 1}{x - 1}$$

c) Prove that $\sqrt{40}$ is irrational. **(3 Marks)**

d) Prove or disprove the following statement: The sum of two odd integers is odd. **(4 Marks)**

Question 3 (25 Marks)

a) Write the negations of the following statements: **(3 Marks)**

i) $9 - 100 \neq 91$

ii) Some birds cannot fly

iii) All students pass calculus

b) Write the i) converse ii) inverse iii) contrapositive of the following statement: "If this is Friday, then I will go to the movies. **(3 Marks)**

c) i) Let the universe of discourse for x be the set of students in your discrete class and the universe of discourse for y is the set of examples in their lecture notes. R(x, y) is the predicate "x understands y". Use quantifiers to express the following statements: **(8 Marks)**

1. There exists a student in this class who understands every example in the lecture notes
2. For every example in the lecture notes there is a student in the class who understands that example
3. Every student in this class understands at least one example in the lecture notes
4. There is an example in the notes that every student in this class understands

ii) Give the negations of the quantifiers in ci). **(3 Marks)**

d) The digits 1 – 6 are to be used to make four digit numbers? **(8 Marks)**

i) How many such numbers can be made if repetition is allowed?

ii) How many such numbers can be made if repetition is not allowed?

iii) How many of the numbers in (ii) begin with 3?

iv) How many of the numbers in (ii) contain 2?

Question 4 (25 Marks)

a) Briefly define the following terms and use diagrams where applicable: **(5 Marks)**

i) graph ii) simple graph iii) multigraph iv) tree v) spanning tree

b) Show that the given pairs of statements are logically equivalent. **(10 Marks)**

i. $\sim (p \rightarrow q)$ and $\sim q \wedge (p \vee q)$

ii. $p \leftrightarrow q$ and $(\sim p \vee q) \wedge (\sim q \vee p)$

c) Use the laws of logical proposition to prove that: **(10 Marks)**

$$(z \wedge w) \vee (\sim z \wedge w) \vee (z \wedge \sim w) \equiv z \vee w$$

Question 5 (25 Marks)

a) There are 4 defective capacitors in a box of 24 capacitors. 6 capacitors are chosen at random. What is the probability that a defective capacitor is not chosen? **(5 Marks)**

b) Transistors of the same kind are bought from three suppliers A, B and C. Let M be the event "the transistor fails before 10000 hours' operation." The probabilities that the transistors from each of the three suppliers fail that soon are:

$P(M|A) = 0.20$; $P(M|B) = 0.15$; $P(M|C) = 0.45$.

The transistors are picked at random from a batch. 30% of the transistors are from A, 45% from B and 25% from C.

i) What is the probability that a transistor chosen at random and installed fails before 10000 hours? **(5 Marks)**

ii) Given that a transistor fails before 10000 hours' operation determine the probabilities that it came from each of the three suppliers. **(5 Marks)**

c) The probability that a certain diode fails before operating 1000 hours is 0.15. There are 20 such diodes.

i) What is the probability that at most 4 diodes will fail before operating 1000 hours? **(5 Marks)**

ii) What is the probability that at least 5 diodes will fail before operating 1000 hours? **(5 Marks)**

END

MAKERERE UNIVERSITY

COLLEGE OF ENGINEERING, DESIGN AND TECHNOLOGY

DEPARTMENT OF ELECTRICAL ENGINEERING

SEMESTER TWO EXAMINATION 2015/16

CMP2204 OPERATING SYSTEM TECHNOLOGIES

DATE: 26-05 - 2016 **TIME: 2:00-5:00 pm**

INSTRUCTIONS:

1. *Attempt any **FIVE** out of Six Questions.*
2. *All questions carry equal marks*
3. *Do not write on any part of this examination question paper*

QUESTION ONE

a) Describe what you understand by the following terms
   i. Turnaround time (2 marks)
   ii. Waiting time (2 marks)

b) An operating system is a program that acts as an interface between the user and the computer hardware. Describe the following important function of an operating system, giving a brief description of the activities undertaken in each:
   i. Memory management (5 marks)
   ii. Process management (5 marks)
   iii. Device management (3 marks)
   iv. Security management (3 marks)

QUESTION TWO

a) Differetiate between the following terms CPU Bound processes and I/O Bound processes . Describe the process state diagram. Explain the transitions that are valid between the states and state an event that might cause such a transition.(12 marks)

b) Differetiate between deadlock prevention and deadlock avoidance (4 marks)

c) Assume we have a system with a page fault service time of 20ms and a memory access time of 90ms . Find the effective access time of such a system. (4 marks)

QUESTION THREE

a) Describe the two deadlock avoidance algorithms below:
   i. Resource allocation graph (4 marks)
   ii. Bankers deadlock algorithm (8 marks)

b) There are four processes which are going to share nine tape drives. Their current and maximum number of allocation numbers are shown below, Is the system in safe state or not?Why? (8 marks)

process	current	maximum
p1	3	6
p2	1	2
p3	4	9
p4	0	2

QUESTION FOUR

a) Every access to memory should go through the page table. Therefore, it must be Implemented in an efficient way. Describe the various storage places of the page table and there different implementation. (12 marks)

b) Is sharing a desirable occurrence? Why? Describe clearly the process of sharing in Paging and segmentation processes. (8 marks)

QUESTION FIVE

a) Describe what you understand by
   i. hit ratio (1 mark)
   ii. Page fault (1 mark)

b) What are the necessary steps taken in handling a page fault. (3 marks)

c) With the help of an example, use the different page replacement algorithm to evaluate the performance of different algorithms (15 marks)
   7,0,1,2,0,3,0,4,2,3,0,3,2,0,1,7,0,1

QUESTION SIX

a) Describe the process of paged-segmemtation in operating systems. Is segementation superior to paging?Discuss (12 marks)

b) Generate the memory map according to the given segment table. (2 marks)

Segment	limit	Base
1	1000	1000
2	700	2500
3	250	3700
4	1300	5000

c) Assume the generated logical address is shown below, find the corresponding physical address.
   i. < 1, 302> (2 marks)
   ii. <3, 300 > (2 marks)
   iii. <5 , 200> (2 marks)

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MAKERERE UNIVERSITY
COLLEGE OF ENGINEERING, DESIGN, ART AND TECHNOLOGY
BACHELOR OF SCIENCE IN COMPUTER ENGINEERING
SECOND YEAR 2012/2013 SEMESTER TWO
CMP 2204: OPERATING SYSTEM TECHNOLOGIES
CAT II

Date: Friday, April 26 2013

Time: 0800 – 0900 Hours

ATTEMPT ALL(7) QUESTIONS

1. Name main differences between logical and physical addresses [1]
2. Give a scenario where choosing a large file system block size might be a benefit; Give an example where it might be a hinderance. [3]
3. What file allocation strategy is most appropriate for random access files? — Indexed allocation [1]
4. Which of the three goals of computer security is the following an attack on: [3]
   a) Network snooping — Confidentiality
   b) A distributed denial of service attack → System availability
   c) Modifiying your marks in the student records database → data integrity
5. What is a covert channel? [2]
6. Compare RAID 0, RAID 1, and RAID 5 in terms of performance improvement, ability to withstand failure, and space overhead required to implement the particular RAID level. [6]
7. Which of RAID 1 or RAID 5 would you use in the following scenarios and why: [4]
   a) A transaction database that performs many thousands of updates per second.
   b) A web server that serves almost entirely static web content.

A covert channel is a noisy channel which contains lots of extraneous/irrelevant information.

Forms of the covert channels

1. modulating CPU usage
2. Paging rate modulation.
3. locking and unlocking files

Makerere University

College of Engineering, Design, Art and Technology

Department of Electrical and Computer Engineering

CMP2203: **DIGITAL LOGIC**

Date: 13th **June, 2017** **Time: 09:00 A.M -12:00 P.M**

INSTRUCTIONS

- This Examination contains Six (6) questions.
- Attempt any four (4) Questions for full Marks
- The first four questions shall be marked if more than four questions are attempted.
- All Questions carry equal marks of 25 %.

Begin each Question on a fresh page

Question One [25 Marks]

a) Give five reasons for minimization of Boolean expressions during digital circuit design. **(5 Marks)**

b) Simplify the following logic function using Boolean algebra and compare your results using K-Map. Draw the resulting logic circuit using NAND gates only. **(8 Marks)**

$$Y = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}C + \overline{A}B$$

c) Consider the following Boolean expression as stated below.

$$Y = f(A,B,C) = \overline{(A+B)}(\overline{B}+C)$$

(i) Without simplifying the expression, draw the logic circuit realization of the above expression and give the corresponding complete truth table. **(5 Marks)**

(ii) Convert the Boolean expression above to its Demorgan equivalent. Hence draw the logic circuit for the realised Demorgan equivalent Boolean equation. **(6 Marks)**

Question Two [25 Marks]

a) Explain the difference between a timer and a counter. Give a practical example where each of these finds application. **(5 Marks)**

b) Consider a counter circuit that contains 3 ~~six~~ FFs wired as shown in the arrangement in fig Q2-b below.

(i) Give the state transition diagram of the counter. **(5 Marks)**

(ii) Determine the frequency at the output of FF Q2 when the input clock frequency is 1 GHz. (3 Marks)

(iii) Determine the MOD number and counting range for this counter. Also determine the counter's state after 130 pulses assuming an initial state of 000000. (7 Marks)

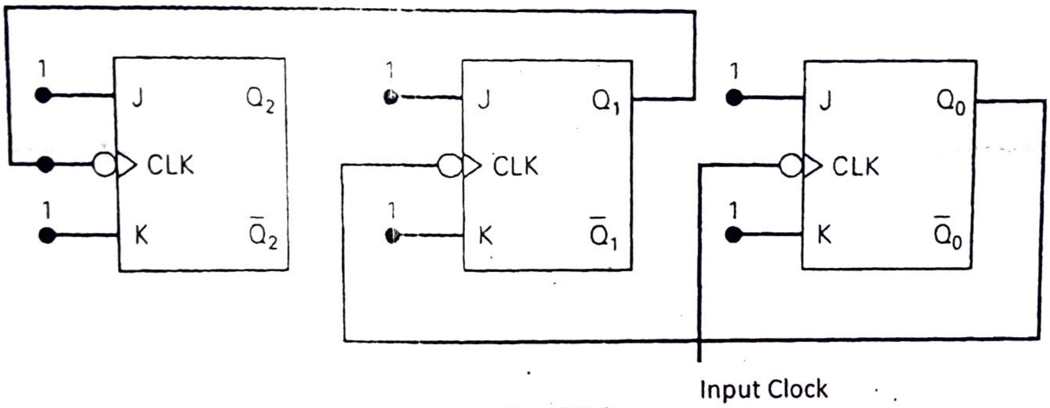

Fig Q2-b

c) Consider five seats, numbered 0 to 4, arranged in a circle and described by Boolean variables i_o *to* i_4. Boolean variable i_o is true if seat 0 is occupied and i_o is false if the seat is not occupied, likewise for i_1, i_2, i_3 *and* i_4. Write a Boolean expression that's true if at least two people are sitting next to each other and at least one seat is not occupied. Draw the logic circuit to implement this expression. (3 Marks)

Question Three [25 Marks]

a) State the function of the following components of the digital arithmetic circuit:

(i) Accumulator register. (2 Marks)

(ii) B register. (2 Marks)

(iii) Control unit. (2 Marks)

b) Draw a complete logic circuit of the 4-bit parallel adder including the associated registers. By using LOAD, CLEAR and TRANSFER signal lines, describe the sequence of operations by which the logic circuit will add binary numbers 1001_{LSB} and 0101_{LSB}. (9 Marks)

c) Write **brief** notes about the following shift registers with focus on logic circuits, operation, advantages and draw backs:

(i) Ring counters (5 Marks)

(ii) Johnson Counters (5 Marks)

Question Four [25 Marks]

a) Indicate how a Nor gate can be used to implement each of the following logic gates:

(i) Inverter (2 Marks)

(ii) AND gate (2 Marks)

(iii) OR gate (2 Marks)

b) (i) What is the difference between a latch and a flip-flop? Hence draw a block diagram of a D latch and a D flip-flop and give their corresponding truth tables. (5 Marks)

(ii) On the graph Q5-b below, inputs CLK and D are inputs to both a D latch and a D flip-flop. CLK goes into the EN or C input of the D latch. Ignoring setup and hold time requirements and assuming both outputs are initially 0 at the start of the graph, write the output of the D latch as Q_{DL} and the output of the D flip-flop as Q_{DFF} on the same graph. Do the two outputs differ, and if so, why? (4 Marks)

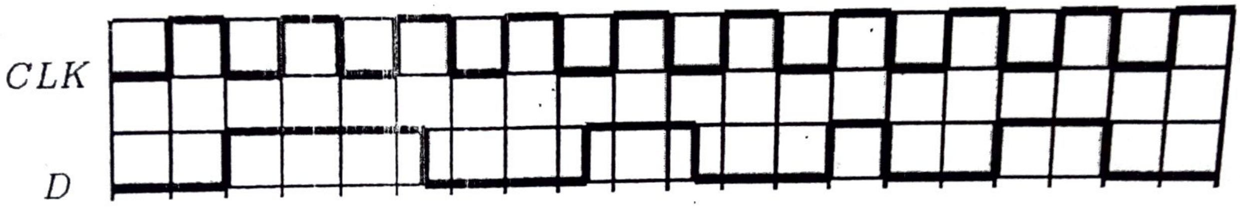

Fig Q5-b

c) (i) Draw a logic circuit used for serial transfer of a DATA word $X_2X_1X_0 = 101$ from one register X to another register Y leaving register X with a DATA word $X_2X_1X_0 = 000$ at the end of the transfer operation. Consider X_0 to be the LSB of the data word and use D Flip-flops. (6 Marks)

(ii) Modify the logic circuit in part c(i) above so that the original DATA word stored in register X is present in both registers at the end of the transfer operation. (4 Marks)

Question Five [25 Marks]

a) With reference to sequential circuit design, explain the following concepts and give their significance:

(i) State diagram. (2 Marks)

(ii) State reduction. (2 Marks)

(iii) Transition equation (2 Marks)

b) Consider a sequential circuit implemented using T-Flip-flops as shown in Fig Q5-b below. For this circuit, derive the corresponding;

(i) Excitation equations. (4 Marks)

(ii) Next state equation and state table. (6 Marks)

(iii) State diagram. (3 Marks)

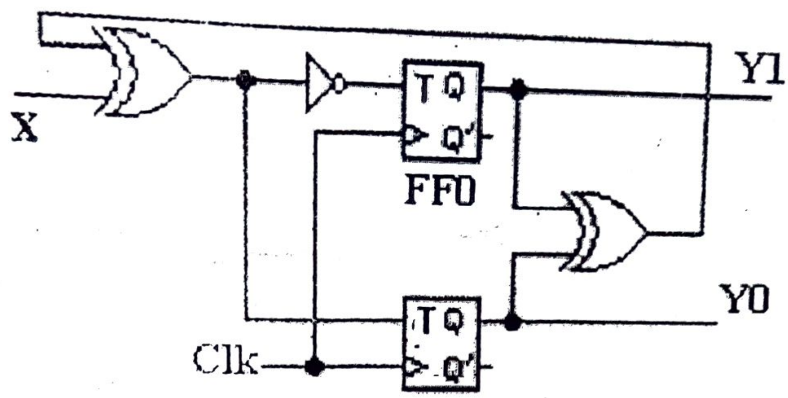

Fig Q5-b

c) Using the 74ALS163 counter shown in fig Q5-c below and logic gates, design a counter that counts in the sequence 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 3, 4,... Connect all unused inputs. The counter may cycle through several unwanted states before settling into the final count sequence. Q_D is the most significant bit (MSB) of the counter output.

(6 Marks)

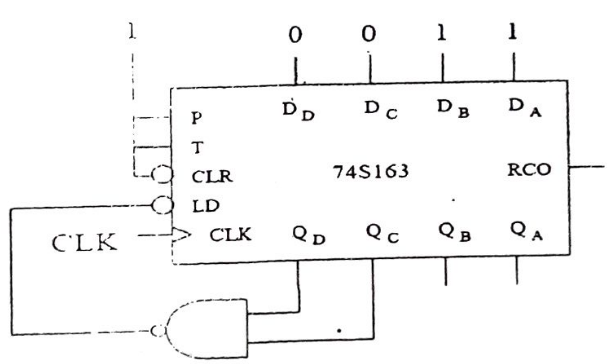

Fig Q5-c

Question Six [25 Marks]

a) Give one application of multiplexers and encoders in digital circuitry. Draw a block diagram of a 4-to-2 encoder labeling all inputs and outputs. How is this encoder different from a 4-to-1 multiplexer? **(6 Marks)**

b) Show how the 8-input multiplexers can be implemented using the components indicated below. In each case, the three select input bits should be labeled s_2, s_1, s_0 with s_0 being the least significant bit. The data inputs should be labeled 0 to 7:

(i) Two 4-input multiplexers and a 2-input multiplexer **(4 Marks)**

(ii) Four 2-input multiplexers and a 4-input multiplexer **(4 Marks)**

(iii) A decoder and logic gates. **(4 Marks)**

c) Mukasa has half adders and full adders available to use as components in his tool bag. If Mukasa wishes to add two 4-bit numbers;

(i) Draw a **block** diagram for his 4-bit adder using half adder and full adders. Show and label all inputs and outputs. **(5 Marks)**

(ii) Assume that a half adder has a maximum propagation delay of Δ, and a full adder has a maximum propagation delay of 2Δ. What is the maximum propagation delay for his 4-bit adder, from LSB to MSB output? **(3 Marks)**

END