The Suscpetible, Infectious, Recovered Epidemic Model

HPPC, week 1

A simple mathematical description of the spread of a disease in a population is the so-called SIR model, which divides the (fixed) population of N individuals into three ”compartments” which may vary as a function of time, t:
S(t) are those susceptible but not yet infected with the disease;
I(t) is the number of infectious individuals;
R(t) are those individuals who have recovered from the disease and now have immunity to it.
The SIR model describes the change in the population of each of these compartments in terms of two parameters, β and γ. β describes how infectious the disease is: on average, in absence of immunity, every infected person infects β others. This rate is reduced by the fraction of the population that is susceptible (i.e. are not immune yet), S/N. γ is the mean recovery rate: that is, 1/γ is the mean period of time during which an infected individual can pass it on. The differential equations describing this model were first derived by Kermack and McKendrick [Proc. R. Soc. A, 115, 772 (1927)]. The inspiration from this example come from Chris Hill, Learning Scientific Programming with Python. Here are the equations:

$$\frac{dS}{dt} = 0 - \beta I \times S/N$$

$$\frac{dI}{dt} = \beta I \times S/N - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

1) Write a C++ code that integrates these equations for a disease characterised by parameters β = 0.2, γ = (10 days)$^{-1}$, in a population of N=1000 (perhaps flu in a school). The model is started with a single infected individual on day 0: I(0)=1.
2) Plot S, R, I as function of time.
3) Based on your results: how many people do you need to vaccinate to avoid an epidemic (i.e., increasing amounts of infected people)?

Also discuss the following three points in a couple of sentences each. There are no perfect answers, but there are better or worse choices, so please motivate your choices.
- how big should your time step be?
- β and γ are uncertain estimates of things that are probably not quite constant. How do you deal with this?
- how do you know your results are correct? You can *never* be sure, but provide 5 different methods to strengthen the trust in your results.