# Portfolio Risk

# CAPM

**C**apital **A**sset **P**ricing **M**odel

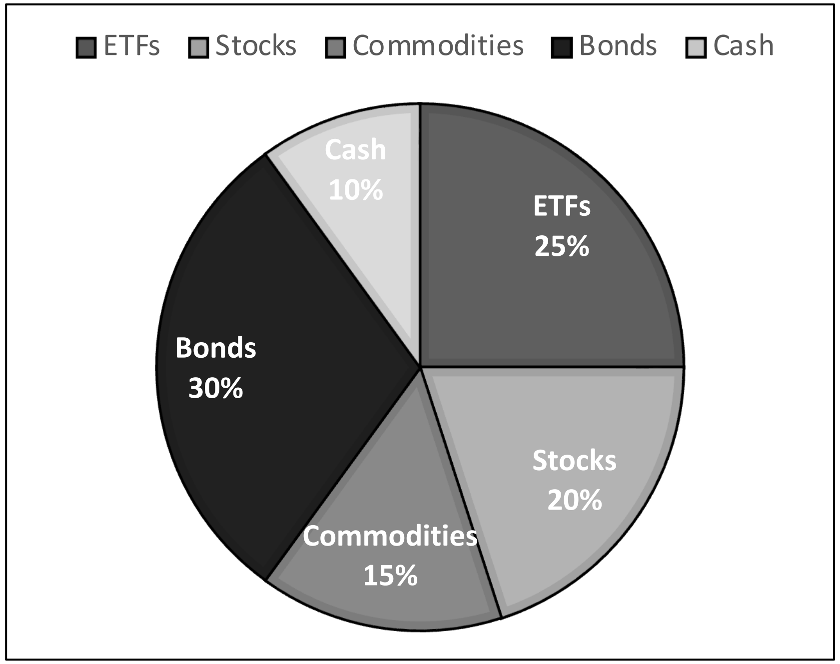
# Portfolio Overview

A basket of assets

May hold stocks, bonds, cash, commodities, derivatives, etfs

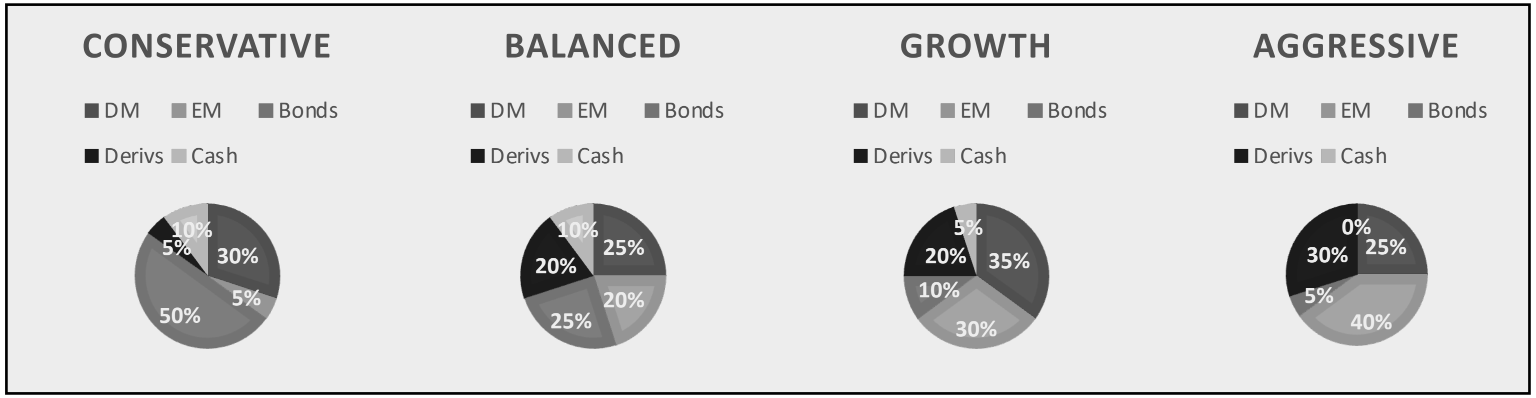
Investors aim for a return by mixing these securities in a manner that reflects
- Their appetite for risk
- Their financial goals
- Their time horizon
- …

A key concept in portfolio management is diversification
- Don’t carry all your eggs in one basket



# Portfolio Overview

Developed Markets
Emerging Markets
Derivatives
Cash

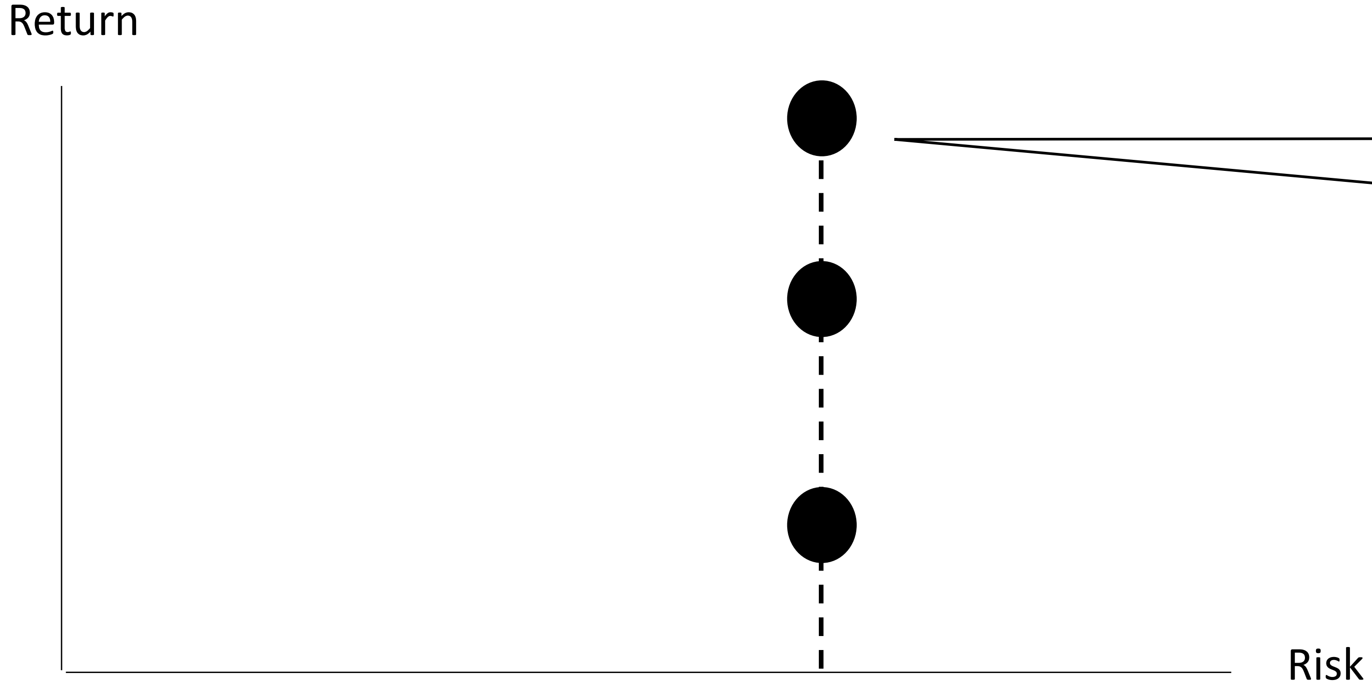



| Annual Return % | | | | |
|---|---|---|---|---|
| Average | 6.01 | 7.98 | 8.97 | 9.64 |
| Best 12 months | 31.06 | 76.57 | 109.55 | 136.07 |
| Worst 12 months | -17.67 | -40.64 | -52.92 | -60.78 |
| Best 5 years | 17.23 | 23.14 | 27.27 | 31.91 |
| Worst 5 years | 0.37 | -6.18 | -10.43 | -13.78 |

# Portfolio Overview

Assume investors are rational

Irrespective of their motivation, they want the **maximum** amount of return for the **least** amount of risk




Irrespective of their appetite for Risk
A rational investor would choose this return
i.e. the maximum return for a given amount of risk

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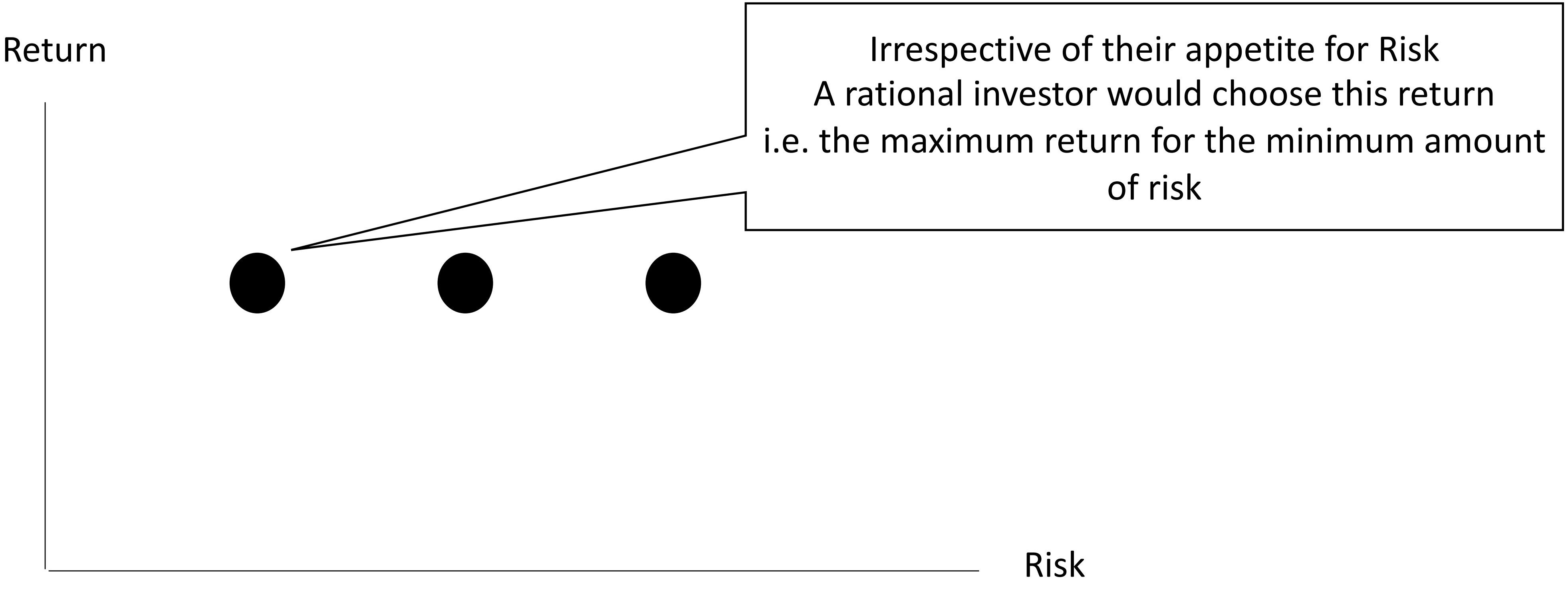

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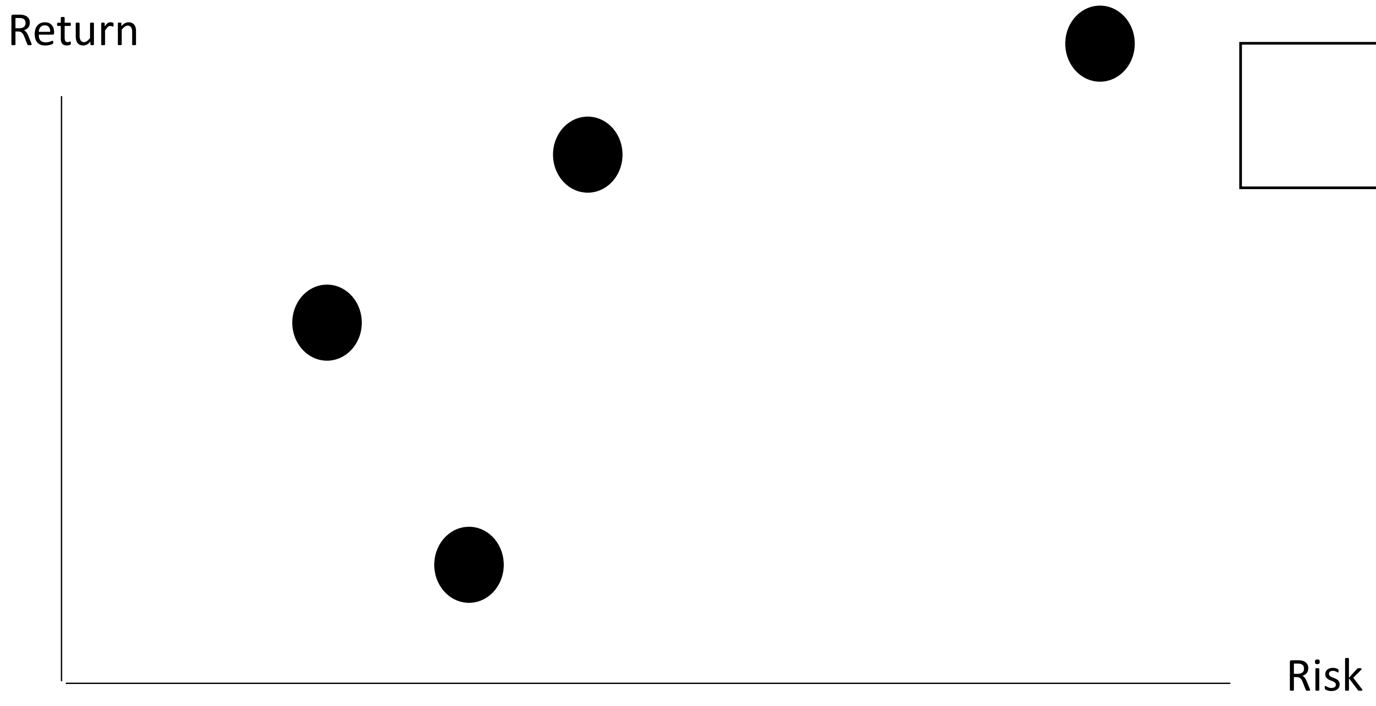



What about these returns

# Problem

Given you have **$100,000** to invest for 1 year.

Treasury bills yield 5% (TBills pay a lot less, but this is for demonstration purposes only)

One alternative is to invest in TBills for no risk where the expected return is 5%.

Another alternative is to invest in stock.

Suppose the possible outcomes from investing in stock are

| Probability | Return (%) |
|---|---|
| .05 | 50 |
| .25 | 30 |
| .4 | 10 |
| .25 | -10 |
| .05 | -30 |

May get $50K return

May also lose $30K

expected return is calculated by

$$(0.05 * 0.5) + (0.25 * 0.3) + (0.4 * 0.1) + (0.25 * -0.1) + (0.05 * -0.3)$$

$$= 0.1$$

# Quantifying Risk

Often std deviation is used

$$\sigma = \sqrt{E(R^2) - [E(R)]^2}$$

Expected Return - E(R)

= (0.05 X 0.5) + (0.25 X 0.3) + (0.4 X 0.1) + (0.25 X -0.1) + (0.05 X -0.3)

= 0.1

$[E(R)]^2 = 0.01$

$E(R^2)$

= $(0.05 \times 0.5^2) + (0.25 \times 0.3^2) + (0.4 \times 0.1^2) + (0.25 \times -0.1^2) + (0.05 \times -0.3^2)$

= 0.046

std deviation = 0.1897

# Portfolio with 2 securities

A portfolio consisting of 2 securities with returns $\boldsymbol{R_1}$ and $\boldsymbol{R_2}$

Put a proportion of money into the first $\boldsymbol{\omega_1}$

The remainder into the second $\boldsymbol{\omega_2} = \mathbf{1} - \boldsymbol{\omega_1}$

The return on the investment is $\boldsymbol{\omega_1} \mathbf{R_1} + \boldsymbol{\omega_2} \mathbf{R_2}$

The portfolio expected return $\boldsymbol{\mu_p} = \boldsymbol{\omega_1 \mu_1} + \boldsymbol{\omega_2 \mu_2}$

where

$\boldsymbol{\mu_1}$ is the expected return on the first investment

$\boldsymbol{\mu_2}$ is the expected return on the second investment

The std deviation of such a portfolio is

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2\rho w_1 w_2 \sigma_1 \sigma_2}$$

# Portfolio with 2 securities

Given the following

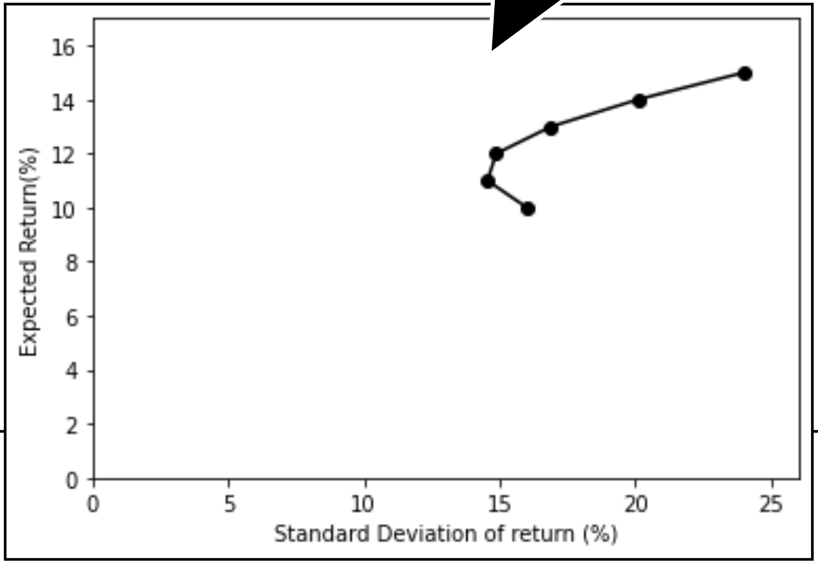
$$\mu_1 = 10\% \; \sigma_1 = 16\%$$
$$\mu_2 = 15\% \; \sigma_2 = 24\%$$
$$\rho = 20\%$$

Most investors are risk averse
They want increased expected returns while reducing std deviation of return.
They will want to move as far as possible in a **North East** direction.
Analysis such as this helps investors make more informed investing decisions.

The expected return and standard deviation for this portfolio for weights of

(0%, 100%),
(20%, 80%),
(40%, 60%),
(60%, 40%),
(80%, 20%),
(100%, 0%)

| $\omega_1$ | $\omega_2$ | $\mu_p$ | $\sigma_p$ |
|---|---|---|---|
| 0.0 | 1.0 | 15% | 24.00% |
| 0.2 | 0.8 | 14% | 20.09% |
| 0.4 | 0.6 | 13% | 16.89% |
| 0.6 | 0.4 | 12% | 14.87% |
| 0.8 | 0.2 | 11% | 14.54% |
| 1.0 | 0.0 | 10% | 16.00% |



# Efficient Frontier

An investor can
- add a third investment to the portfolio
- combine it with any combination of the first 2 produce a new risk return.

They can repeat this with a 4th, 5th investment etc.

Eventually we reach the limit of how far North West we can go.

This is known as the **efficient frontier**