

commitment  $\mathbf{K} = (\mathbf{C}, \mathcal{I})$ . A model  $M = (S, I)$ , with  $S = (D, \mathbf{R})$ , is called an intended model of  $\mathbf{L}$  according to  $\mathbf{K}$  iff

1. For all constant symbols  $c \in \mathbf{V}$  we have  $I(c) = \mathcal{I}(c)$
2. There exists a world  $w \in W$  such that, for each predicate symbol  $v \in \mathbf{V}$  there exists an intensional relation  $\rho \in \mathfrak{R}$  such that  $\mathcal{I}(v) = \rho$  and  $I(v) = \rho(w)$

The set  $\mathbf{I}_{\mathbf{K}}(\mathbf{L})$  of all models of  $\mathbf{L}$  that are compatible with  $\mathbf{K}$  is called the set of intended models of  $\mathbf{L}$  according to  $\mathbf{K}$ .

Condition 1 above just requires that the mapping of constant symbols to elements of the universe of discourse is identical. Example 2.1 does not introduce any constant symbols. Condition 2 states that there must exist a world such that every predicate symbol is mapped into an intensional relation whose value, for that world, coincides with the extensional interpretation of such symbol. This means that our intended model will be – so to speak – a description of that world. In Example 2.1, for instance, we have that, for  $w_1$ ,  $I(\text{Person}) = \{I000001, \dots, I050000, \dots\} = \text{Person}^1(w_1)$  and  $I(\text{reports-to}) = \{\dots, (I046758, I034820), (I044443, I034820), (I034820, I050000), \dots\} = \text{reports-to}^2(w_1)$ .

With the notion of intended models at hand, we can now clarify the role of an ontology, considered as a logical theory designed to account for the intended meaning of the vocabulary used by a logical language. In the following, we also provide an ontology for our running example.

**Definition 3.4 (Ontology)** Let  $\mathbf{C}$  be a conceptualization, and  $\mathbf{L}$  a logical language with vocabulary  $\mathbf{V}$  and ontological commitment  $\mathbf{K}$ . An ontology  $\mathbf{O}_{\mathbf{K}}$  for  $\mathbf{C}$  with vocabulary  $\mathbf{V}$  and ontological commitment  $\mathbf{K}$  is a logical theory consisting of a set of formulas of  $\mathbf{L}$ , designed so that the set of its models approximates as well as possible the set of intended models of  $\mathbf{L}$  according to  $\mathbf{K}$  (cf. also Fig. 2).

**Example 3.2** In the following we build an ontology  $O$  consisting of a set of logical formulae. Through  $O_1$  to  $O_6$  we specify our human resources domain with increasing precision.

*Taxonomic Information.* We start our formalization by specifying that *Researcher* and *Manager* are sub-concepts of *Person*:

$$O_1 = \{\text{Researcher}(x) \rightarrow \text{Person}(x), \text{Manager}(x) \rightarrow \text{Person}(x)\}$$

*Domains and Ranges.* We continue by adding formulae to  $O_1$  which specify the domains and ranges of the binary relations:

$$O_2 = O_1 \cup \{\text{cooperates-with}(x, y) \rightarrow \text{Person}(x) \wedge \text{Person}(y), \text{reports-to}(x, y) \rightarrow \text{Person}(x) \wedge \text{Person}(y)\}$$

*Symmetry.* *cooperates-with* can be considered a symmetric relation:

$$O_3 = O_2 \cup \{\text{cooperates-with}(x, y) \leftrightarrow \text{cooperates-with}(y, x)\}$$

*Transitivity.* Although arguable, we specify *reports-to* as a transitive relation:

$$O_4 = O_3 \cup \{\text{reports-to}(x, z) \leftarrow \text{reports-to}(x, y) \wedge \text{reports-to}(y, z)\}$$

*Disjointness* *There is no Person who is both a Researcher and a Manager:*

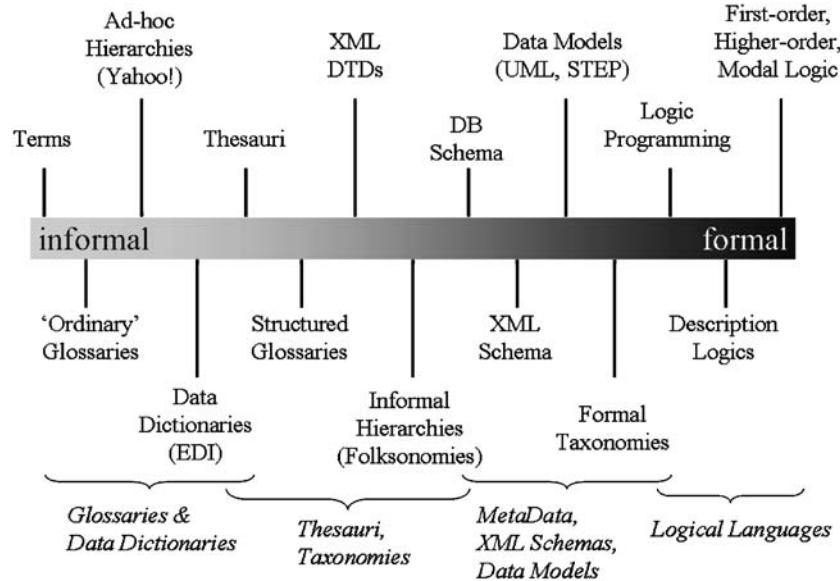
$$O_5 = O_4 \cup \{ \text{Manager}(x) \rightarrow \neg \text{Researcher}(x) \}$$

### 3.3 Choosing the Right Domain and Vocabulary

On the basis of the discussion above, we might conclude that an ideal ontology is one whose models exactly coincide (modulo isomorphisms) with the intended ones. Things are not so simple, however: even a “perfect” ontology like that may fail to exactly specify its target conceptualization, if its *vocabulary* and its *domain of discourse* are not suitably chosen. The reason for that lies in the distinction between the *logical* notion of *model* and the *ontological* notion of *possible world*. The former is basically a combination of assignments of abstract relational structures (built over the domain of discourse) to vocabulary elements; the latter is a combination of actual (observed) states of affairs of a certain system. Of course, the number of possible models depends both on the size of the vocabulary and the extension of the domain of discourse, which are chosen more or less arbitrarily, on the basis of what appears to be relevant to talk of. On the contrary, the number of world states depends on the observed variables, even those which – at a first sight – are considered as irrelevant to talk of. With reference to our example, consider the two models where the predicates of our language (whose signature is reported above) are interpreted in such a way that their extensions are those described respectively in Examples 2.1 and 2.2. Each model corresponds to a different pattern of relationships among the people in our company, but, looking at the model itself, nothing tells us what are the world states where a certain pattern of relationships holds. So, for example, it is impossible to discriminate between a conceptualization where *cooperates-with* means that two persons cooperate when they are just sharing a goal, and another where they need also do something to achieve that goal. In other words, each model, in this example, will “collapse” many different world states. The reason of this is in the very simple vocabulary we have adopted: with just two predicates, we have not enough expressiveness to discriminate between different world states. So, to really capture our conceptualization, we need to extend the vocabulary in order to be able to talk of sharing a goal or achieving a goal, and we have to introduce goals (besides persons) in our domain of discourse. In conclusion, the degree to which an ontology specifies a conceptualization depends (1) on the richness of the domain of discourse; (2) on the richness of the vocabulary chosen; (3) on the axiomatization. In turn, the axiomatization depends on language expressiveness issues as discussed in Sect. 3.4.

### 3.4 Language Expressiveness Issues

At one extreme, we have rather informal approaches for the language **L** that may allow the definitions of terms only, with little or no specification of the



**Fig. 4.** Different approaches to the language  $L$  according to [17]. Typically, logical languages are eligible for the formal, explicit specification, and, thus, ontologies

meaning of the term. At the other end of the spectrum, we have formal approaches, i.e., logical languages that allow specifying rigorously formalized logical theories. This gives rise to the continuum introduced by [17] and depicted in Fig. 4. As we move along the continuum, the amount of meaning specified and the degree of formality increases (thus reducing ambiguity); there is also increasing support for automated reasoning (cf. Chapters “Tableau-Based Reasoning” and “Resolution-Based Reasoning for Ontologies”).

It is difficult to draw a strict line of where the criterion of formal starts on this continuum. In practice, the rightmost category of logical languages is usually considered as formal. Within this rightmost category one typically encounters the trade-off between *expressiveness* and *efficiency* when choosing the language  $L$ . On the one end, we find higher-order logic, full first-order logic, or modal logic. They are very expressive, but do often not allow for sound and complete reasoning and if they do, reasoning sometimes remains untractable. At the other end, we find less stringent subsets of first-order logic, which typically feature decidable and more efficient reasoners. They can be split in two major paradigms. First, languages from the family of *description logics (DL)* (cf. chapter “Description Logics”), e.g., OWL-DL (cf. chapter “Web Ontology Language: OWL”), are strict subsets of first-order logic. The second major paradigm comes from the tradition of *logic programming (LP)* [3] with one prominent representor being F-Logic (cf. chapter “Ontologies in F-Logic”). Though logic programming often uses a syntax comparable to