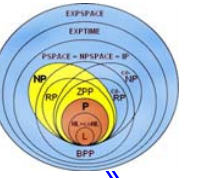
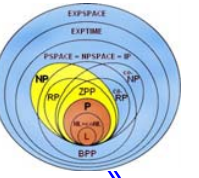
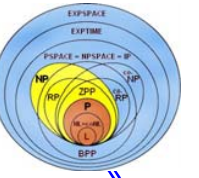


Session 7

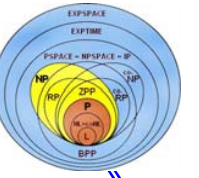
- Context-Free Grammars and Context-Free Languages
- Parse Tree for Context-Free Grammars



Context-Free Grammars and Context-Free Languages



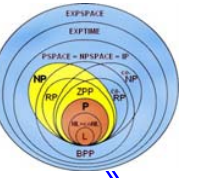
- Context-Free Grammars (CFG's) was first proposed as a description method for natural languages by Chomsky in 1956. A similar idea was used to describe computer languages: `Fortran` and `Algol` since 1960's. Sometimes CFG's are referred to as "Backus-Naur form grammars".
- Context-Free Languages (CFL's) played a central role in compilers technology. Today CFL's are increasingly important for XML and their DTD (Document-Type Definition).



Formal Definition of CFG's

A context-free grammar is a quadruple $G = (V, T, P, S)$.

- V is a finite set of *variables*.
- T is a finite set of *terminals*.
- P is a finite set of *productions* of the form $A \to \alpha$, where A is a variable and $\alpha \in (V \cup T)^*$.
- S is a designated variable called the *start symbol*.



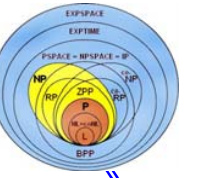
Example Consider grammar $G_{pal} = (\{S\}, \{0, 1\}, P, S)$, where

$$P = \{S \to \epsilon, S \to 0, S \to 1, S \to 0S0, S \to 1S1\}.$$

G_{pal} is a context-free grammar. Later we will prove that it describes the language of palindromes L_{pal} on alphabet $\{0, 1\}$. It is easy to verify L_{pal} is not regular language.

Sometimes we group productions with the same head, e.g.

$$P = \{S \to \epsilon|0|1|0S0|1S1\}.$$



Example Regular expressions over $\{0, 1\}$ can be defined by the grammar

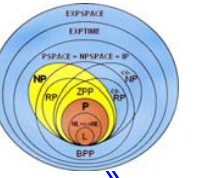
$$G_{regex} = (\{E\}, \{0, 1, (,), ., +, *, \emptyset\}, P, E)$$

where

$$P = \{E \to \epsilon, E \to 0, E \to 1, E \to \emptyset, E \to E.E, E \to E + E, E \to E^*, E \to (E)\}$$

or

$$P = \{E \to \epsilon|0|1|\emptyset|E.E|E + E|E^*|(E)\}.$$



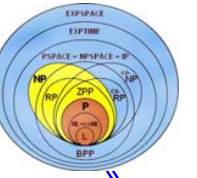
Example Consider a CFG that represent (a simplification of) expressions in a typical programming language. Operators are + and ×, and arguments are identifiers, i.e. strings in

$$L((\mathbf{a}+\mathbf{b})(\mathbf{a}+\mathbf{b}+\mathbf{0}+\mathbf{1})^*).$$

The expressions are defined by the grammar $G = (\{E, I\}, T, P, E)$ where $T = \{+, \times, (,), a, b, 0, 1\}$ and P is the following set of productions:

$$1.\, E \to I,\ 2.\, E \to E + E,\ 3.\, E \to E \times E,\ 4.\, E \to (E),$$

$$5.\, I \to a,\ 6.\, I \to b,\ 7.\, I \to Ia,\ 8.\, I \to Ib,\ 9.\, I \to I0,\ 10.\, I \to I1.$$

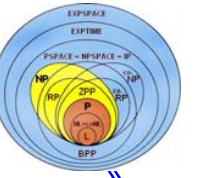


Derivations Using Grammars

We apply the productions of a CFG to infer that certain strings are in the language of a certain variable. There are two approaches to this inference.

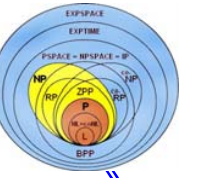
- Recursive inference, using productions from body to head.
- Derivations, using productions from head to body.

It is often more natural to think of grammar as used in derivations.



Example Let us consider some of the recursive inferences we can make using the grammar for simple expressions.

	String	Language	Production	String(s) used
(i)	a	I	$I \to a$	–
(ii)	b	I	$I \to b$	–
(iii)	$b0$	I	$I \to I0$	(ii)
(iv)	$b00$	I	$I \to I0$	(iii)
(v)	a	E	$E \to I$	(i)
(vi)	$b00$	E	$E \to I$	(iv)
(vii)	$a + b00$	E	$E \to E + E$	$(v), (vi)$
$(viii)$	$(a + b00)$	E	$E \to (E)$	(vii)
(ix)	$a \times (a + b00)$	E	$E \to E \times E$	$(v), (viii)$



Now we develop the notation for describing the derivations.

Let $G = (V, T, P, S)$ be a CFG, $A \in V$, $\{\alpha, \beta\} \subset (V \cup T)^*$, and $A \to \gamma \in P$. Then we write

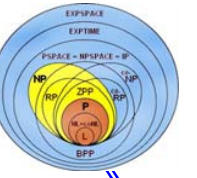
$$\alpha A \beta \underset{G}{\Rightarrow} \alpha \gamma \beta$$

or, if G is understood

$$\alpha A \beta \Rightarrow \alpha \gamma \beta$$

and say that $\alpha A \beta$ derives $\alpha \gamma \beta$.

Specially, we have $A \Rightarrow \gamma$.



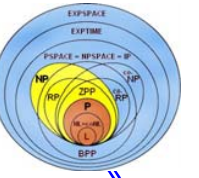
We may extend the $\Rightarrow$ relationship to present zero, one, or many derivation steps.

In other words, we define $\overset{*}{\Rightarrow}$ to be the reflexive and transitive closure of $\Rightarrow$, as follows:

Basis step: Let $\alpha \in (V \cup T)^*$. Then $\alpha \overset{*}{\Rightarrow} \alpha$.

Inductive step: If $\alpha \overset{*}{\Rightarrow} \beta$, and $\beta \Rightarrow \gamma$, then $\alpha \overset{*}{\Rightarrow} \gamma$.

☞ Use induction we can prove that if $\alpha \overset{*}{\Rightarrow} \beta$, and $\beta \overset{*}{\Rightarrow} \gamma$, then $\alpha \overset{*}{\Rightarrow} \gamma$.



Example Derivation of $a \times (a + b00)$ from E in the productions:

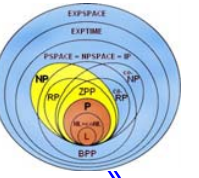
$$E \to I|E + E|E \times E|(E), \quad I \to a|b|Ia|Ib|I0|I1.$$

Here is one such derivation:

$$E \Rightarrow E \times E \Rightarrow I \times E \Rightarrow a \times E \Rightarrow a \times (E) \Rightarrow a \times (E + E) \Rightarrow a \times (I + E) \Rightarrow$$

$$a \times (a + E) \Rightarrow a \times (a + I) \Rightarrow a \times (a + I0) \Rightarrow a \times (a + I00) \Rightarrow a \times (a + b00)$$

Finally, $E \overset{*}{\Rightarrow} a \times (a + b00)$. The two viewpoints – recursive inference and derivation – are equivalent.



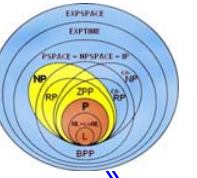
☞ At each step we might have several rules to choose from, e.g.

$$I \times E \Rightarrow a \times E \Rightarrow a \times (E), \text{ versus } I \times E \Rightarrow I \times (E) \Rightarrow a \times (E).$$

☞ Not all choices lead to successful derivation of a particular string, for instance

$$E \Rightarrow E + E$$

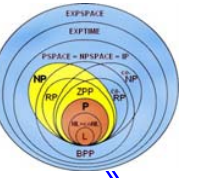
won't lead to a derivation of $a \times (a + b00)$.



Leftmost and Rightmost Derivations

In order to restrict the number of choices we have in deriving a string, it is often useful to require

- Leftmost derivation $\underset{lm}{\Rightarrow}$: Always replace the leftmost variable by one of its rule-bodies.
- Rightmost derivation $\underset{rm}{\Rightarrow}$: Always replace the rightmost variable by one of its rule-bodies.



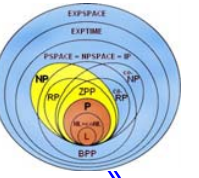
Example Derivation of $a \times (a + b00)$ from E in the productions:

$$E \to I|E + E|E \times E|(E), \quad I \to a|b|Ia|Ib|I0|I1.$$

Here is the leftmost derivation $E \overset{*}{\underset{lm}{\Rightarrow}} a \times (a + b00)$.

$$E \Rightarrow E \times E \Rightarrow I \times E \Rightarrow a \times E \Rightarrow a \times (E) \Rightarrow a \times (E + E) \Rightarrow a \times (I + E) \Rightarrow$$

$$a \times (a + E) \Rightarrow a \times (a + I) \Rightarrow a \times (a + I0) \Rightarrow a \times (a + I00) \Rightarrow a \times (a + b00)$$



We can also derive $a \times (a + b00)$ from E in the productions:

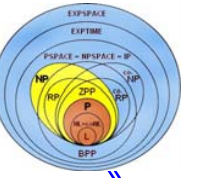
$$E \to I|E + E|E \times E|(E), \quad I \to a|b|Ia|Ib|I0|I1$$

by the rightmost derivation.

$$E \Rightarrow E{\times}E \Rightarrow E{\times}(E) \Rightarrow E{\times}(E{+}E) \Rightarrow E{\times}(E{+}I) \Rightarrow E{\times}(E{+}I0) \Rightarrow E{\times}(E{+}I00) \Rightarrow$$

$$E \times (E + b00) \Rightarrow E \times (I + b00) \Rightarrow E \times (a + b00) \Rightarrow I \times (a + b00) \Rightarrow a \times (a + b00)$$

That is $E \underset{rm}{\overset{*}{\Rightarrow}} a \times (a + b00)$.



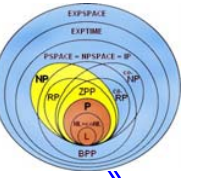
The Language of a CFG

If $G(V, T, P, S)$ is a CFG, then the language of G is

$$L(G) = \{w \in T^* \mid S \overset{*}{\underset{G}{\Rightarrow}} w\}$$

That is, the set of strings over T^* derivable from the start symbol. We call $L(G)$ a context-free language.

In general, we call $L = \{w \in T^* \mid A \overset{*}{\underset{G}{\Rightarrow}} w\}$ the language of variable A if $A \in V$.

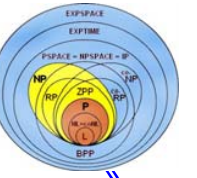


Example By the definition, we know $L(G_{pal})$ is a context-free language. Now we will show

$$L(G_{pal}) = \{w \in \{0,1\}^* \mid w = w^R\}.$$

Proof ($\supseteq$-direction) Suppose $w = w^R$. We show by induction on $|w|$ that $w \in L(G_{pal})$.

Basis step: $|w| = 0$, or $|w| = 1$. Then w is ϵ, 0, or 1. Since $S \to \epsilon$, $S \to 0$ and $S \to 1$ are productions, we conclude that $S \overset{*}{\Rightarrow} w$ in all base case.



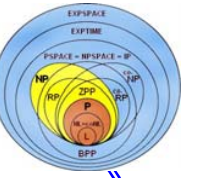
Inductive step: Suppose $|w| \geq 2$. Since $w = w^R$. we have $w = 0x0$, or $w = 1x1$, and $x = x^R$.

If $w = 0x0$ we know from the induction hypothesis that $S \stackrel{*}{\Rightarrow} x$. Then

$$S \Rightarrow 0S0 \stackrel{*}{\Rightarrow} 0x0 = w.$$

Thus $w \in L(G_{pal})$. The case for $w = 1x1$ is similar.

($\subseteq$-direction) We assume that $w \in L(G_{pal})$ and must show that $w = w^R$. Since $w \in L(G_{pal})$, we have $S \stackrel{*}{\Rightarrow} w$. We do an induction of the length of $\stackrel{*}{\Rightarrow}$.



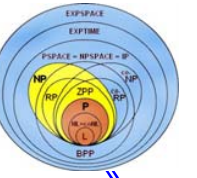
Basis step: The derivation $S \overset{*}{\Rightarrow} w$ is done in one step. Then w must be ϵ, 0, or 1, all palindromes.

Inductive step: Let $n \geq 1$, and suppose the derivation takes $n + 1$ step. Then we must have

$$w = 0x0 \overset{*}{\Leftarrow} 0S0 \Leftarrow S, \quad \text{or} \quad w = 1x1 \overset{*}{\Leftarrow} 1S1 \Leftarrow S$$

where the second derivation is done in n steps.

By the induction hypothesis x is a palindrome, so is w. The induction proof is complete. ◄



Sentential Forms

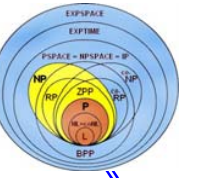
Let $G = (V, T, P, S)$ be a CFG, and $\alpha \in (V \cup T)^*$. If

$$S \overset{*}{\Rightarrow} \alpha$$

we say that α is a sentential form.

If $S \underset{lm}{\Rightarrow} \alpha$ we say that α is a left-sentential form, and if $S \underset{rm}{\Rightarrow} \alpha$ we say that α is a right-sentential form.

☞ $L(G)$ is those sentential forms that are in T^*.



Example Take G as $G = (\{E, I\}, T, P, E)$ where $T = \{+, \times, (,), a, b, 0, 1\}$ and P is

$$E \to I|E + E|E \times E|(E), \quad I \to a|b|Ia|Ib|I0|I1.$$

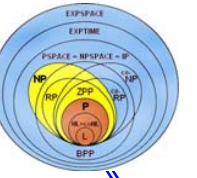
Then $E \times (I + E)$ is a sentential form since

$$E \Rightarrow E \times E \Rightarrow E \times (E) \Rightarrow E \times (E + E) \Rightarrow E \times (I + E).$$

This derivation is neither leftmost, nor rightmost.

However, $\alpha \times E$ is a left-sentential for, since

$$E \underset{lm}{\Rightarrow} E \times E \underset{lm}{\Rightarrow} I \times (E) \underset{lm}{\Rightarrow} \alpha \times E.$$



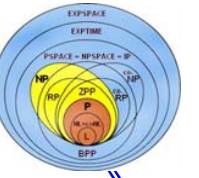
Applications of CFG's

Many aspects of a programming language have a structure that may be described by regular expressions, but not alone.

Example Typical languages use parentheses in a nested and balanced fashion. A grammar $G_{bal} = (\{B\}, \{(,)\}, P, B)$ generates all and only the strings of balanced parentheses, where P consists of the productions:

$$B \to BB|(B)|\epsilon$$

Note that $L(G_{bal})$ is not a regular language.

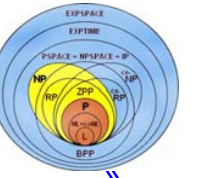


Example Consider `if` and `else` appear in a C programm. An if-clause can appear unbalanced by any else-clause, or it may be balanced by a matching else-clause.

A grammar generating the possible sequences of `if` and `else` (represented by i and e, respectively) is: $G_{ie} = (\{S\}, \{i, e\}, P, S)$ where P consists of the productions:

$$S \to \epsilon | SS | iS | iSe$$

For instance, $ieieie, iiie \in L(G_{ie})$.

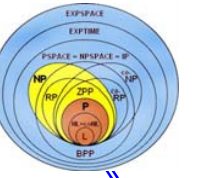


We could test for membership in $L = L(G_{ie})$ by repeatedly doing the following, starting with a string w. The string w changes during repetition.

1. If the current string begins with e, fail; w is not in L.
2. If the string currently has no e's, succeed; w is in L.
3. Otherwise, delete the first e and the i immediately to its left. Then repeat these three steps on the new string.

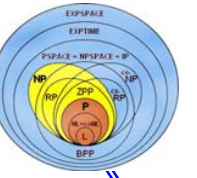
It is easy to test $ieeii \notin L$, but $iieie \in L$.


BREAK FOR 15 MINUTES

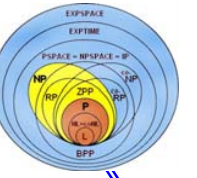


Parse Tree

for Context-Free Grammars



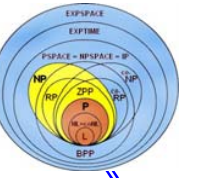
- If $w \in L(G)$, for some CFG G, then w has a parse tree, which tells us the (syntactic) structure of w. Note that w could be a program, a SQL-query, An XML-document, etc.
- Parse tree are an alternative representation to derivation and recursive inferences.
- There can be several parse trees for the same string. Ideally there should be only one parse tree for each string, i.e. the language should be unambiguous. Unfortunately, we cannot always remove the ambiguity.



Constructing Parse Trees

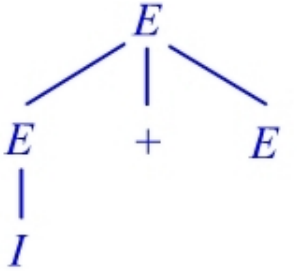
Let $G = (V, T, P, S)$ be a CFG. A tree is a parse tree for G, if:

- Each interior nodes is labeled by a variable in V.
- Each leaf is labeled by a symbol in $V \cup T \cup \{\epsilon\}$. Any ϵ-labeled leaf is the only child of its parent.
- If an interior node is labeled A, and its children (from left to right) labeled $X_1, X_2, \cdots, X_k$, then $A \to X_1 X_2 \cdots X_k \in P$.

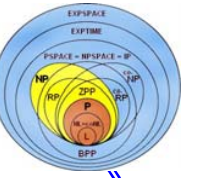


Example In the grammar $G = (\{E, I\}, T, P, E)$, where $T = \{+, \times, (,), a, b, 0, 1\}$ and $P = \{E \to I|E + E|E \times E|(E),\ I \to a|b|Ia|Ib|I0|I1\}$.

The parse tree



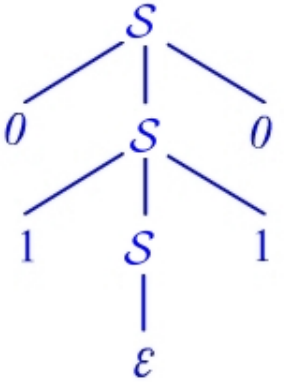
shows the derivation $E \overset{*}{\Rightarrow} I + E$.



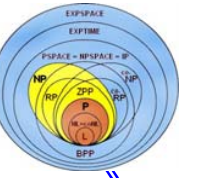
Example In the grammar $G_{pal} = (\{S\}, \{0, 1\}, P, S)$, where

$$P = \{S \to \epsilon, S \to 0, S \to 1, S \to 0S0, S \to 1S1\}.$$

The parse tree



shows the derivation $P \overset{*}{\Rightarrow} 0110$.



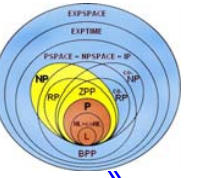
The Yield of Parse Trees

The yield of a parse tree is the string of leaves from left to right.

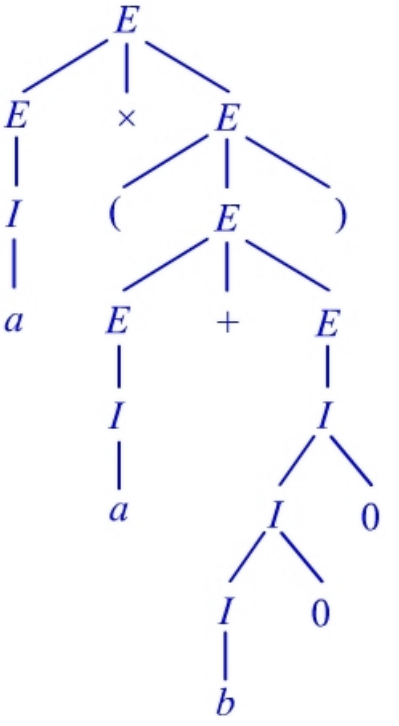
Important are those parse tree where:

- The yield is a terminal string.
- The root is labeled by the start symbol.

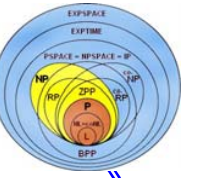
We shall see the set of yields of these important parse trees is the language of the grammar.



Example Below is an important parse tree.



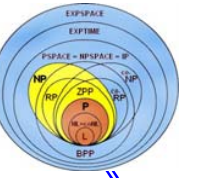
The yield is $a \times (a + b00)$.



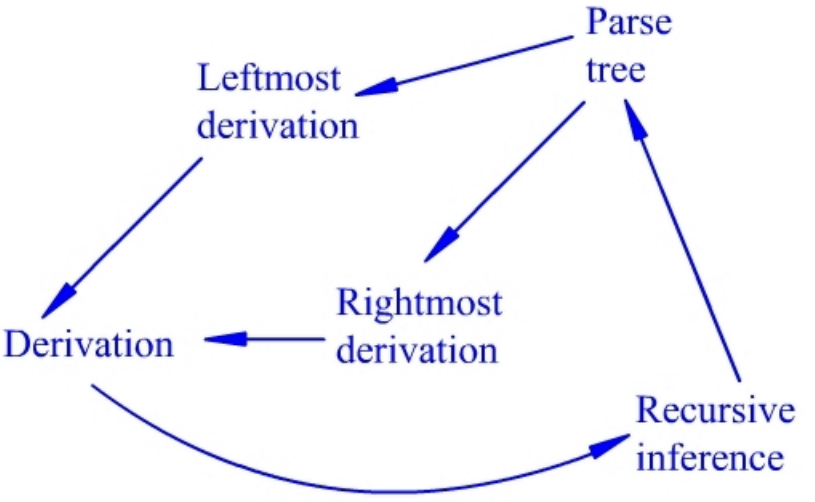
Inference, Derivations, and Parse Trees

Let $G = (V, T, P, S)$ be a CFG, and $A \in V$. We are going to show that the following are equivalent:

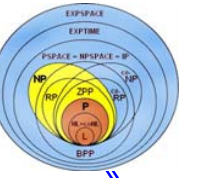
- We can determine by recursive inference that w is in the language of A.
- $A \overset{*}{\Rightarrow} w$.
- $A \underset{lm}{\Rightarrow} w$, and $A \underset{rm}{\Rightarrow} w$.
- There is a parse tree of G with root A and yield w.



To prove the equivalences, we use the following plan.



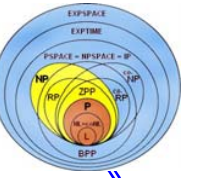
Note that two of the arcs (leftmost/rightmost derivation → derivation) are very simple and will not be proved formally, since both derivations are derivation.



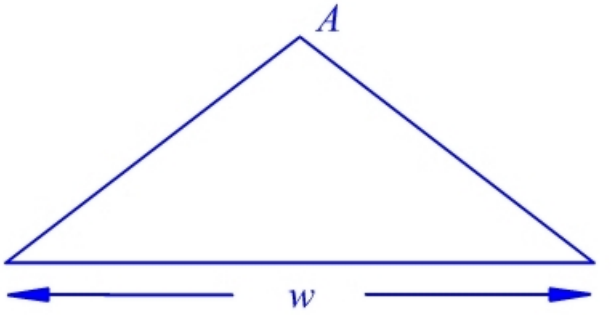
From Inferences to Trees

Theorem 5.1 *Let $G = (V, T, P, S)$ be a CFG, and suppose we can show w to be in the language of a variable A by the recursive inference procedure. Then there is a parse tree for G with root A and yield w.*

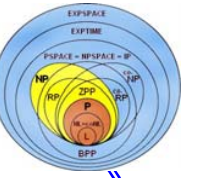
Proof We do an induction of the length of the inference.



Basis step: One step. Then we must have used a production $A \to w$. The desired parse tree is then

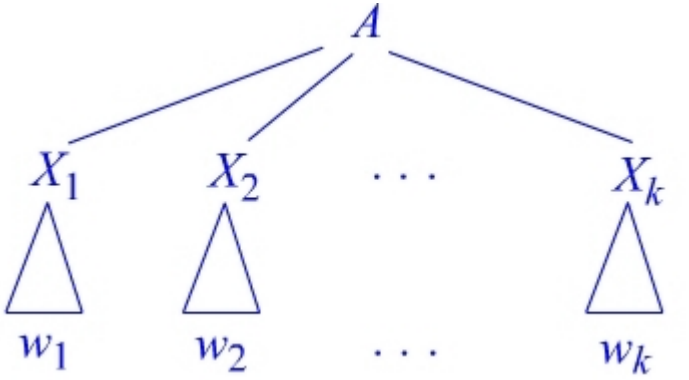


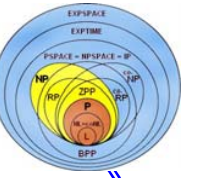
Inductive step: Suppose w is inferred in $n + 1$ steps. Consider the last step of the inference. This inference uses some production for A, say $A \to X_1X_2 \cdots X_k$, where $X_i \in V \cup T$.



We break w up as $w_1 w_2 \cdots w_k$, where $w_i = X_i$, when $X_i \in T$, and when $X_i \in V$, then w_i was previously inferred being in X_i, in at most n steps.

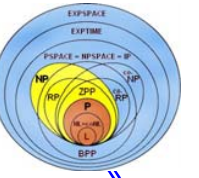
By the induction hypothesis there are parse trees i with root X_i and yield w_i. Then the following is a parse tree for G with root A and yield w:



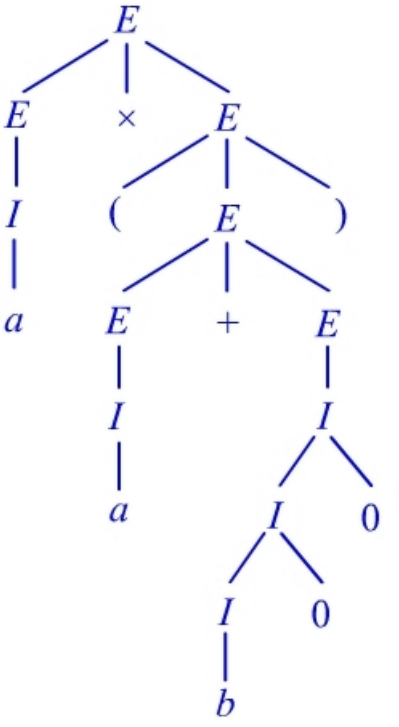


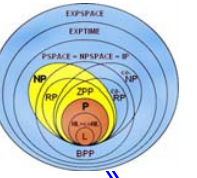
Example We have seen that $w = a \times (a + b00)$ is in the language of a variable E, and it is inferred in 9 steps. The last step of the inference is $E \to E \times E$.

	String	Language	Production	String(s) used
(i)	a	I	5	–
(ii)	b	I	6	–
(iii)	$b0$	I	9	(ii)
(iv)	$b00$	I	9	(iii)
(v)	a	E	1	(i)
(vi)	$b00$	E	1	(iv)
(vii)	$a + b00$	E	2	$(v), (vi)$
$(viii)$	$(a + b00)$	E	4	(vii)
(ix)	$a \times (a + b00)$	E	3	$(v), (viii)$



So, the parse tree for G with root E and yield $a \times (a + b00)$ is

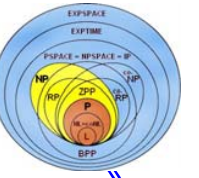




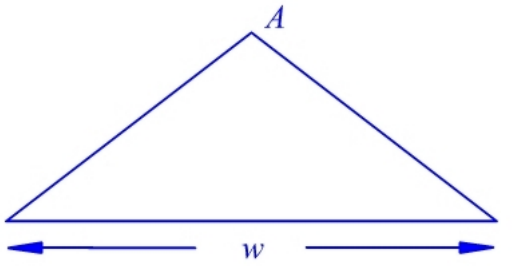
From Trees to Derivations

Theorem 5.2 *Let $G = (V, T, P, S)$ be a CFG, and suppose there is a parse tree for G with root A and yield w. Then we can construct a leftmost derivation from this parse tree: $A \overset{*}{\underset{lm}{\Rightarrow}} w$ in G.*

Proof We do an induction of the height of the parse tree.

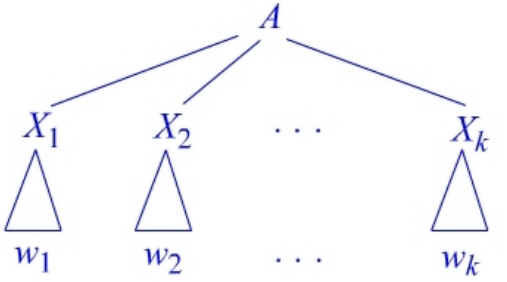


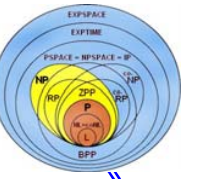
Basis step: Height is 1. The parse tree must look like



Consequently $A \to w \in P$, and $A \underset{lm}{\Rightarrow} w$.

Inductive step: Height is $n + 1$. The parse tree must look like





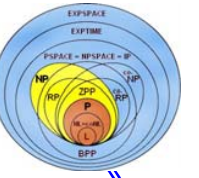
Then $w = w_1 w_2 \cdots w_k$, where

1. If $X_i \in T$, then $w_i = X_i$.
2. If $X_i \in V$, then $X_i \overset{*}{\underset{lm}{\Rightarrow}} w_i$ in G by the induction hypothesis.

Now we construct $A \overset{*}{\underset{lm}{\Rightarrow}} w$ by an (inner) induction by showing that

$$\forall\, i: \;\; A \overset{*}{\underset{lm}{\Rightarrow}} w_1 w_2 \cdots w_i X_{i+1} X_{i+2} \cdots X_k$$

When $i = k$, the result is a leftmost derivation of w from A.



Basis step: Let $i = 0$. We already know that $A \underset{lm}{\Rightarrow} X_1 X_2 \cdots X_k$.

Inductive step: Make the induction hypothesis that

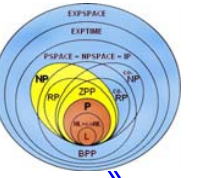
$$A \overset{*}{\underset{lm}{\Rightarrow}} w_1 w_2 \cdots w_{i-1} X_i X_{i+1} \cdots X_k$$

(Case 1): $Xi \in T$. Do nothing, since $X_i = w_i$ gives us

$$A \overset{*}{\underset{lm}{\Rightarrow}} w_1 w_2 \cdots w_{i-1} w_i X_{i+1} \cdots X_k$$

(Case 2): $X_i \in V$. By the induction hypothesis there is derivation

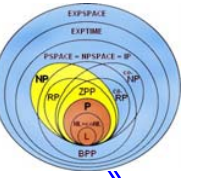
$$X_i \underset{lm}{\Rightarrow} \alpha_1 \underset{lm}{\Rightarrow} \alpha_2 \cdots \underset{lm}{\Rightarrow} w_i$$



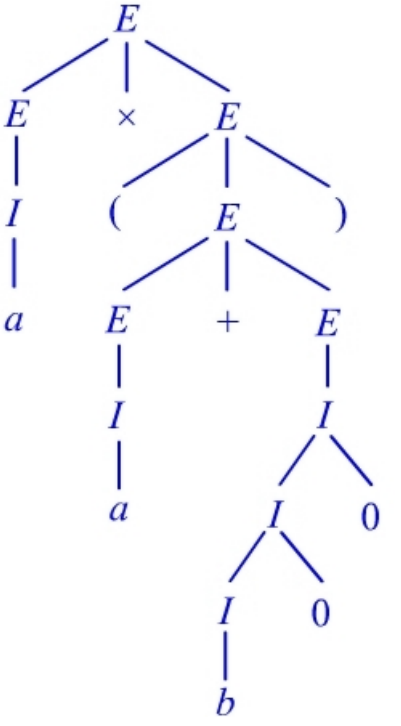
By the context-free property of derivations we can proceed with

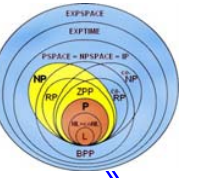
$$
\begin{aligned}
A &\overset{*}{\underset{lm}{\Rightarrow}} w_1 w_2 \cdots w_{i-1} X_i X_{i+1} \cdots X_k \\
&\underset{lm}{\Rightarrow} w_1 w_2 \cdots w_{i-1} \alpha_1 X_{i+1} \cdots X_k \\
&\underset{lm}{\Rightarrow} w_1 w_2 \cdots w_{i-1} \alpha_2 X_{i+1} \cdots X_k \quad \cdots \\
&\underset{lm}{\Rightarrow} w_1 w_2 \cdots w_{i-1} w_i X_{i+1} \cdots X_k
\end{aligned}
$$

☞ In fact, it is this property that gave rose originally to the term "context-free". There is more powerful classes of grammars, called "context-sensitive", which do not play a major role in practice today. ◄



Example Let's construct the leftmost derivation for the tree

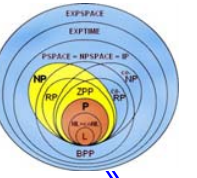




Suppose we have inductively constructed the leftmost derivation $E \underset{lm}{\Rightarrow} I \underset{lm}{\Rightarrow} a$ corresponding to the leftmost subtree. And the leftmost derivation $E \underset{lm}{\Rightarrow} (E) \underset{lm}{\Rightarrow} (E+E) \underset{lm}{\Rightarrow} (I+E) \underset{lm}{\Rightarrow} (a+E) \underset{lm}{\Rightarrow} (a+I) \underset{lm}{\Rightarrow} (a+I0) \underset{lm}{\Rightarrow} (a+I00) \underset{lm}{\Rightarrow} (a+b00)$ corresponding to the rightmost subtree.

For the derivation corresponding to the whole tree we start with $E \underset{lm}{\Rightarrow} E \times E$ and expand the first E with the first derivation and second E with the second derivation:

$$E \underset{lm}{\Rightarrow} E \times E \underset{lm}{\Rightarrow} I \times E \underset{lm}{\Rightarrow} a \times E \underset{lm}{\Rightarrow} a \times (E) \underset{lm}{\Rightarrow} a \times (E+E) \underset{lm}{\Rightarrow} a \times (I+E) \underset{lm}{\Rightarrow} a \times (a+E) \underset{lm}{\Rightarrow} a \times (a+I) \underset{lm}{\Rightarrow} a \times (a+I0) \underset{lm}{\Rightarrow} a \times (a+I00) \underset{lm}{\Rightarrow} a \times (a+b00)$$

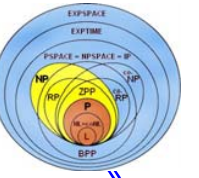


From Derivations to Inferences

Theorem 5.3 *Let $G = (V, T, P, S)$ be a CFG. Suppose $A \overset{*}{\underset{lm}{\Rightarrow}} w$, and that w is a string of terminals. Then we can infer that w is in the language of variable A.*

Proof We do an induction of the length of the derivation $A \overset{*}{\underset{G}{\Rightarrow}} w$.

Basis step: One step. If $A \underset{G}{\Rightarrow} w$ there must be a production $A \to w$ in P. Then we can infer that w is in the language of A.

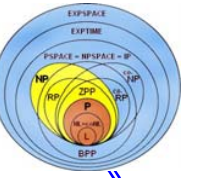


Observation: Suppose that $A \Rightarrow X_1X_2\cdots X_k \overset{*}{\Rightarrow} w$. Then $w = w_1w_2\cdots w_k$, where $X_i \overset{*}{\Rightarrow} w_i$. The factor w_i can be extracted from $A \overset{*}{\Rightarrow} w$ by looking at the expansion of X_i only.

For example, $E \Rightarrow \underbrace{E}_{X_1}\underbrace{\times}_{X_2}\underbrace{E}_{X_3}\underbrace{+}_{X_4}\underbrace{E}_{X_5} \overset{*}{\Rightarrow} a \times b + a.$

We have $E \Rightarrow E \times E + E \Rightarrow I \times E + E \Rightarrow I \times I + E \Rightarrow I \times I + I \Rightarrow a \times I + I \Rightarrow a \times b + I \Rightarrow a \times b + a.$

By looking at the expansion of $X_3 = E$ only, we can extract $E \Rightarrow I \Rightarrow b$.



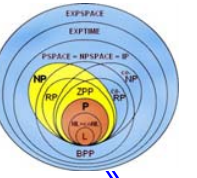
Inductive step: Suppose $A \overset{*}{\underset{G}{\Rightarrow}} w$ in $n+1$ steps. Write the derivation as

$$A \to X_1 X_2 \cdots X_k \overset{*}{\underset{G}{\Rightarrow}} w$$

Then as noted on the previous slide we can break w as $w_1 w_2 \cdots w_k$ where $X_i \overset{*}{\underset{G}{\Rightarrow}} w_i$. Furthermore, $X_i \overset{*}{\underset{G}{\Rightarrow}} w_i$ can use at most n steps.

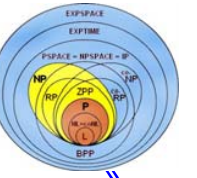
Now we have a production $A \to X_1 X_2 \cdots X_k$, and we know by the induction hypothesis that we can infer w_i to be the language of X_i.

Therefore we can infer $w_1 w_2 \cdots w_k$ to be in the language of A. ◀


EXPSPACE
EXPTIME
PSPACE = NPSPACE = IP
NP
RP
ZPP
P
NP
RP
L
BPP

Thank you!


華東師範大學
EAST CHINA NORMAL UNIVERSITY


EXPSPACE
EXPTIME
PSPACE = NPSPACE = IP
NP
RP
ZPP
P
L
BPP


華東師範大學
EAST CHINA NORMAL UNIVERSITY