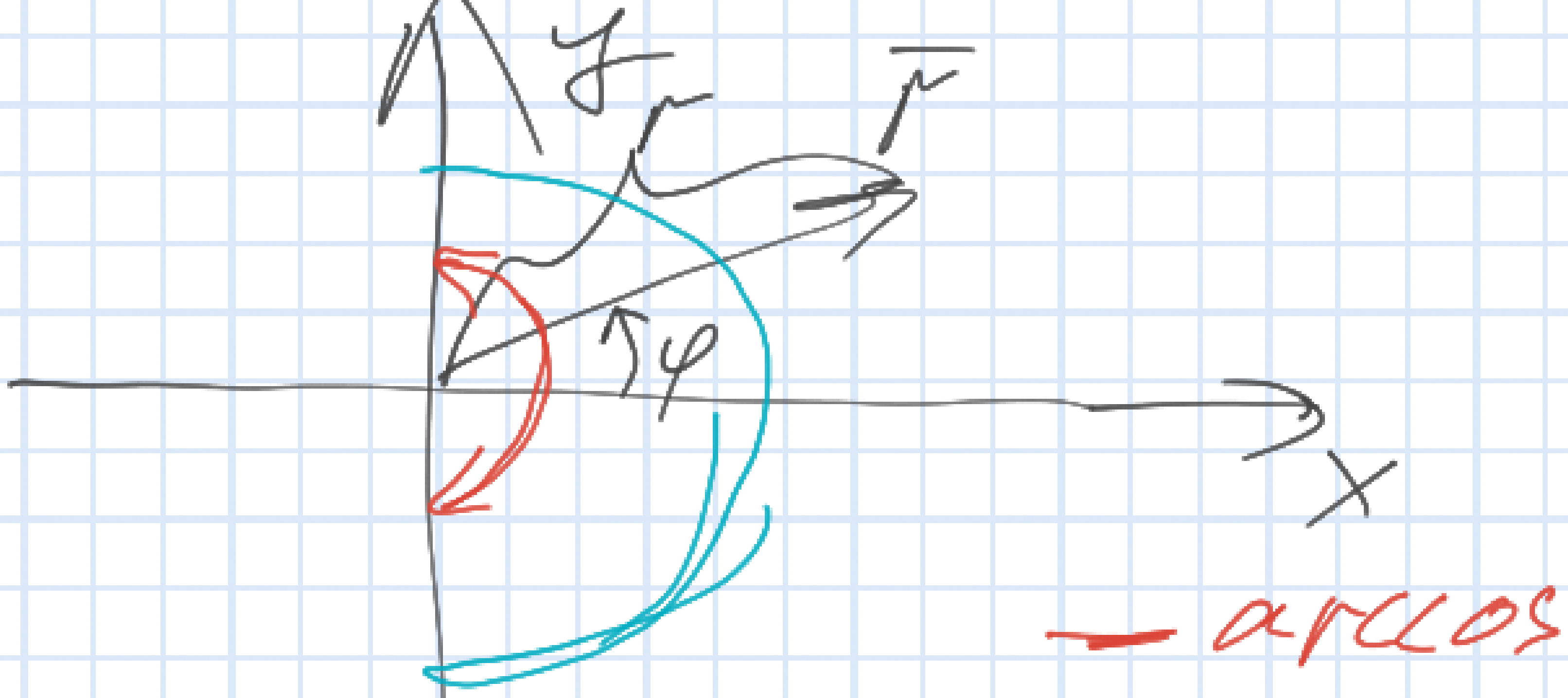


Декартовы координаты — полярные координаты

$$\bar{r} = \begin{bmatrix} x \\ y \end{bmatrix}, \begin{bmatrix} r \\ \varphi \end{bmatrix}$$

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}$$

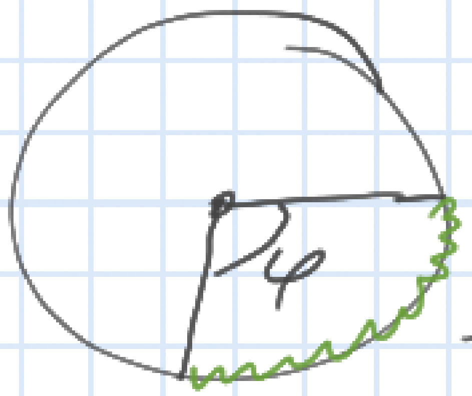
$$\begin{cases} r = \sqrt{x^2+y^2} \\ \varphi = \operatorname{atan2}(y, x) \end{cases}$$

$$\operatorname{arctg} v \in \left[\overset{-90^\circ}{-\frac{\pi}{2}}; \overset{90^\circ}{\frac{\pi}{2}}\right]$$

$$\operatorname{atan2}(y, x) \in [-\pi; \pi]$$

360° — полный оборот

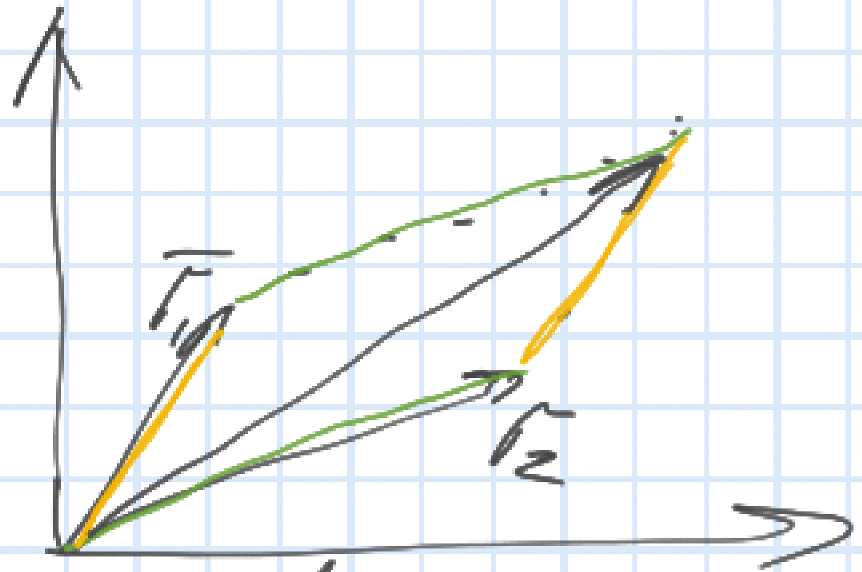
$L = 2\pi r$ $\quad$ $L_1 = 2\pi$ — полный оборот в радианах



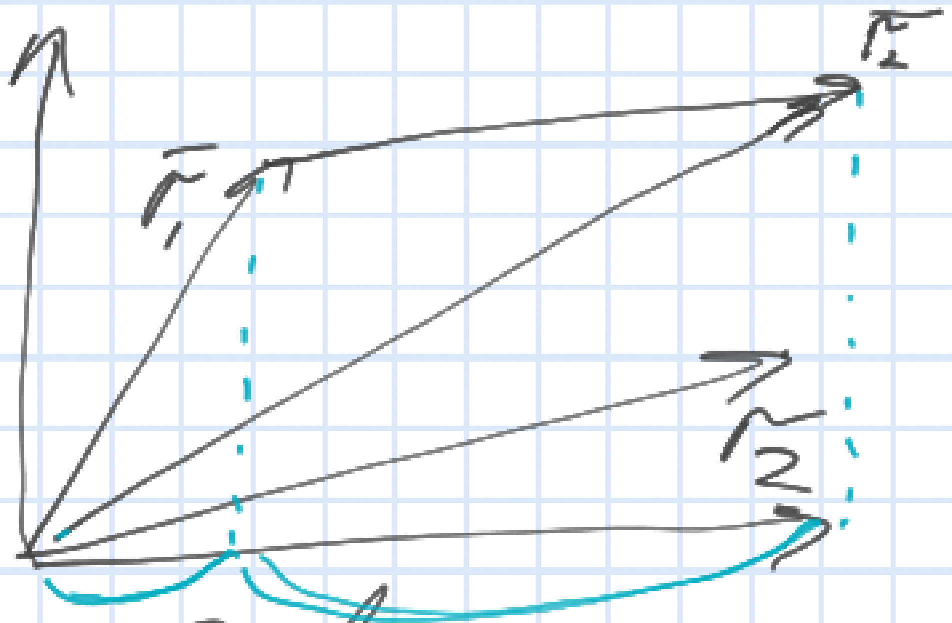
Градусы: $L_\varphi = 2\pi r \cdot \frac{\varphi}{360}$

Радианы: $L_\varphi = \cancel{2\pi} r \cdot \frac{\varphi}{\cancel{2\pi}} = r\varphi$

$$\bar{r} = \bar{r}_1 + \bar{r}_2$$



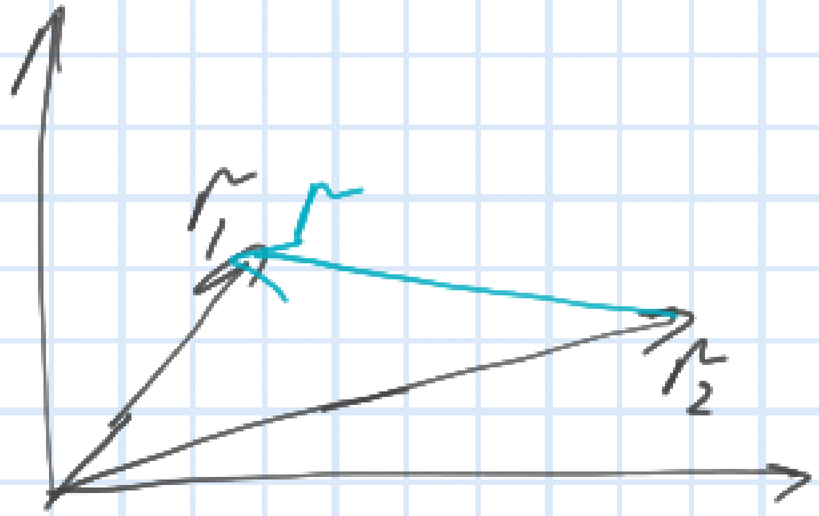
прав. паралел.

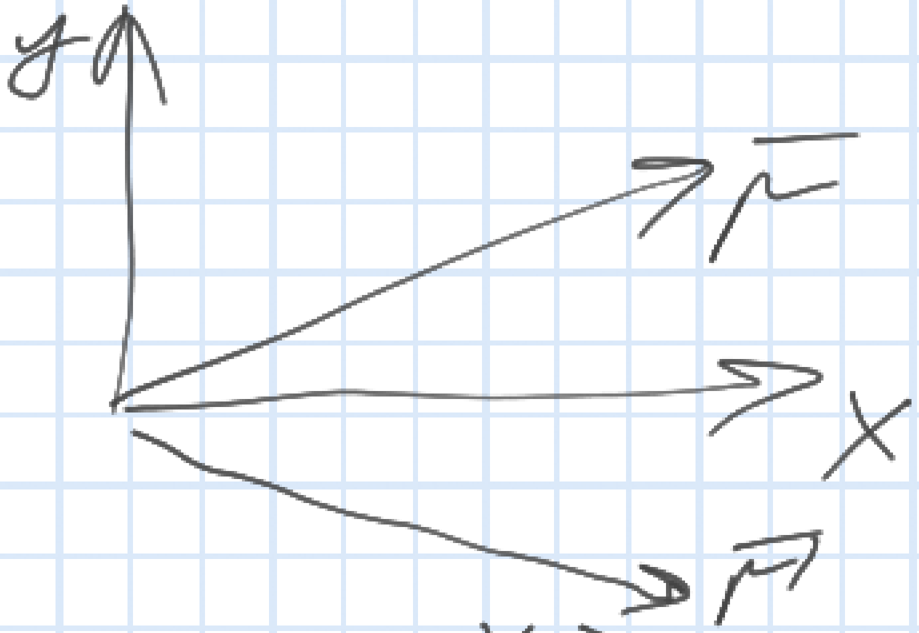


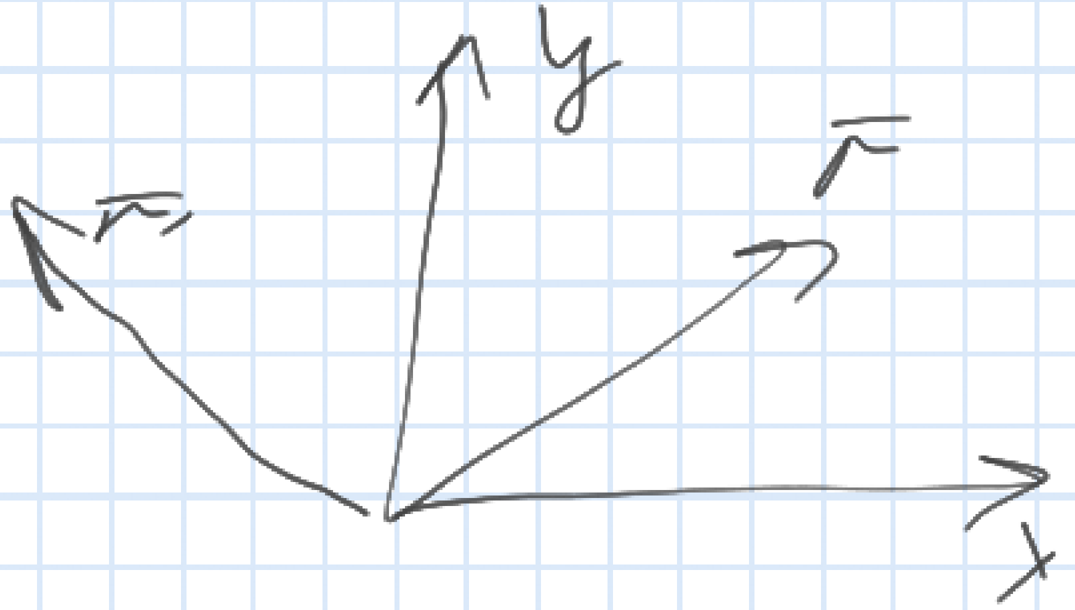
прав. треуг.

$$\bar{r} = \bar{r}_1 - \bar{r}_2$$

$$\bar{r}_1 = \bar{r} + \bar{r}_2$$

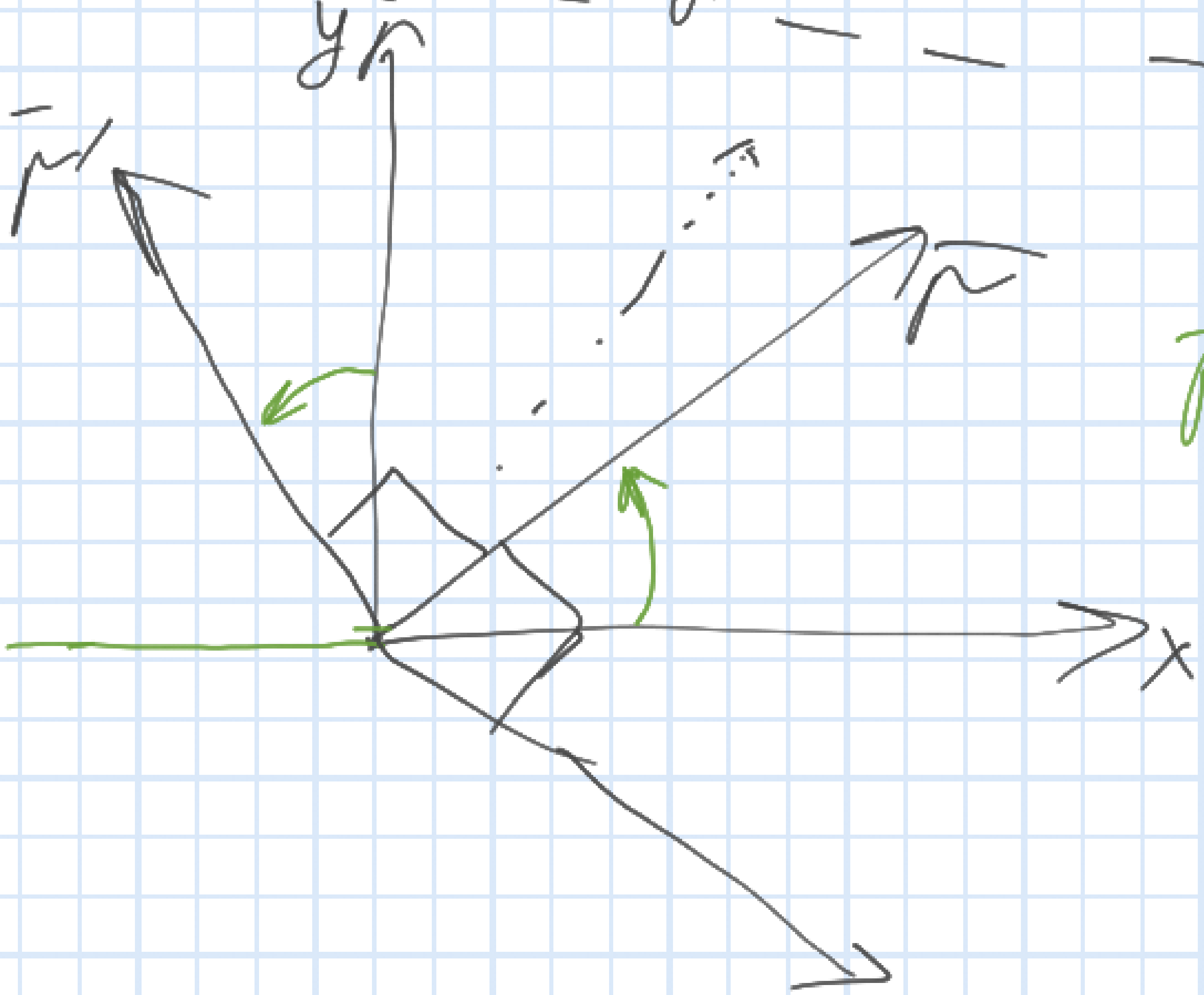






$$\bar{r}' = \begin{bmatrix} x \\ -y \end{bmatrix}$$

$$\bar{r}' = \begin{bmatrix} -x \\ y \end{bmatrix}$$



$\bar{r}' = \begin{bmatrix} -y \\ x \end{bmatrix}$ — против. часовой

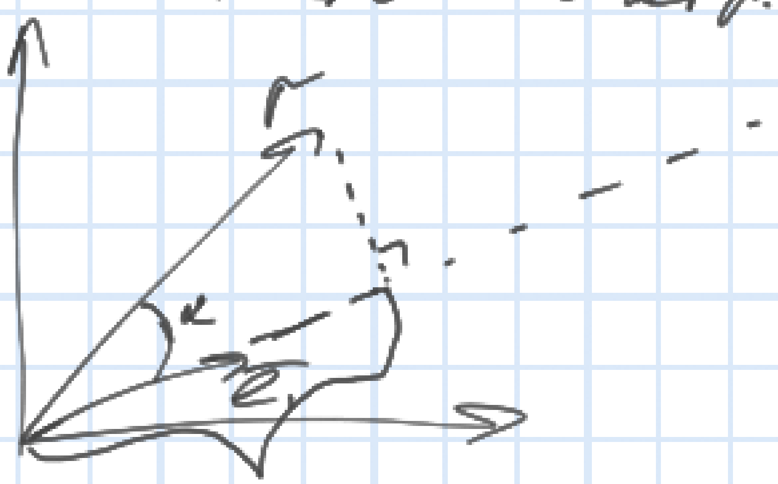
$\begin{bmatrix} y \\ -x \end{bmatrix}$ — по часовой.

Скалярное произв.

$\bar r_1, \bar r_2$ — вектора

$$(\bar r_1, \bar r_2) = x_1 x_2 + y_1 y_2 = |\bar r_1|\underbrace{|\bar r_2|\cos\widehat{\bar r_1, \bar r_2}}_{\text{пр. на } \bar r_1}$$

$\bar e_1$ — единичный вектор.



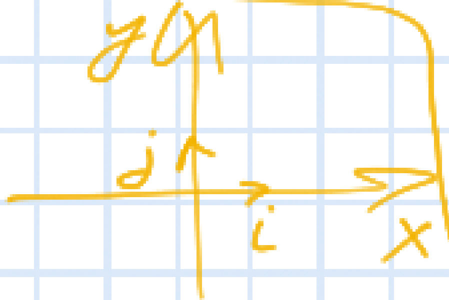
$$(\bar r, \bar e_1) = |\bar r||\bar e_1|\cos\alpha = |\bar r|\cos\alpha$$

$\bar r$ — вектор

$$\bar e_r = \frac{\bar r}{|\bar r|}$$

$$\bar r = x\bar i + y\bar j$$

i, j — базисные орты



$$(\bar r_1, \bar r_2) = (x_1\bar i + y_1\bar j, x_2\bar i + y_2\bar j) = \overset{x_1 x_2}{(x_1\bar i, x_2\bar i)} + \overset{0}{(x_1\bar i, y_2\bar j)} + \overset{0}{(y_1\bar j, x_2\bar i)} + \overset{y_1 y_2}{(y_1\bar j, y_2\bar j)} = x_1 x_2 + y_1 y_2$$

1] $(\underset{\text{число}}{\alpha}\bar r_1, \bar r_2) = \alpha(\bar r_1, \bar r_2)$

proof

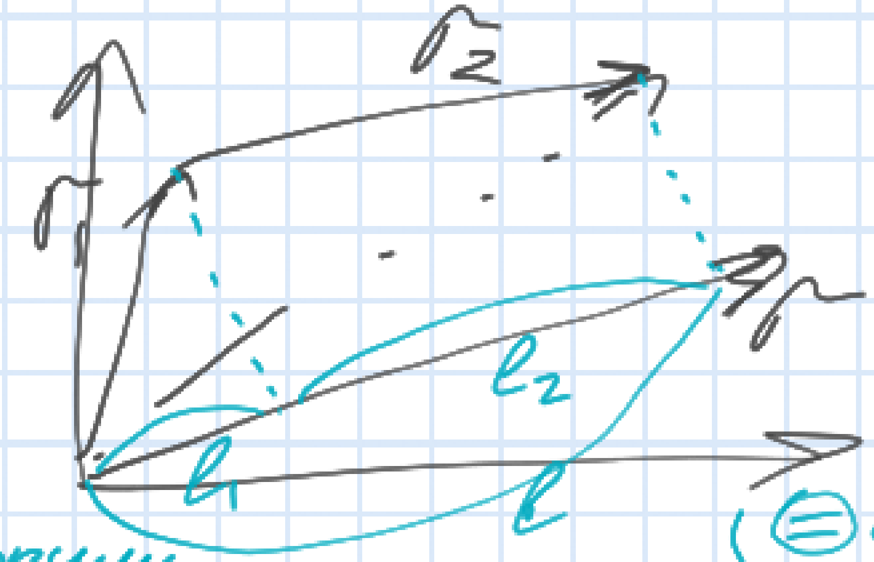
$\alpha > 0$: $|\alpha\bar r_1||\bar r_2|\cos\widehat{\alpha\bar r_1, \bar r_2} = \alpha|\bar r_1||\bar r_2|\cos\widehat{\bar r_1, \bar r_2}$

$\alpha < 0$: $|\alpha\bar r_1||\bar r_2|\cos\widehat{\alpha\bar r_1, \bar r_2} = (-\alpha)|\bar r_1||\bar r_2|(-\cos\widehat{\bar r_1, \bar r_2})$

□

2] $(\bar r_1 + \bar r_2, \bar r) = (\bar r_1, \bar r) + (\bar r_2, \bar r)$

proof



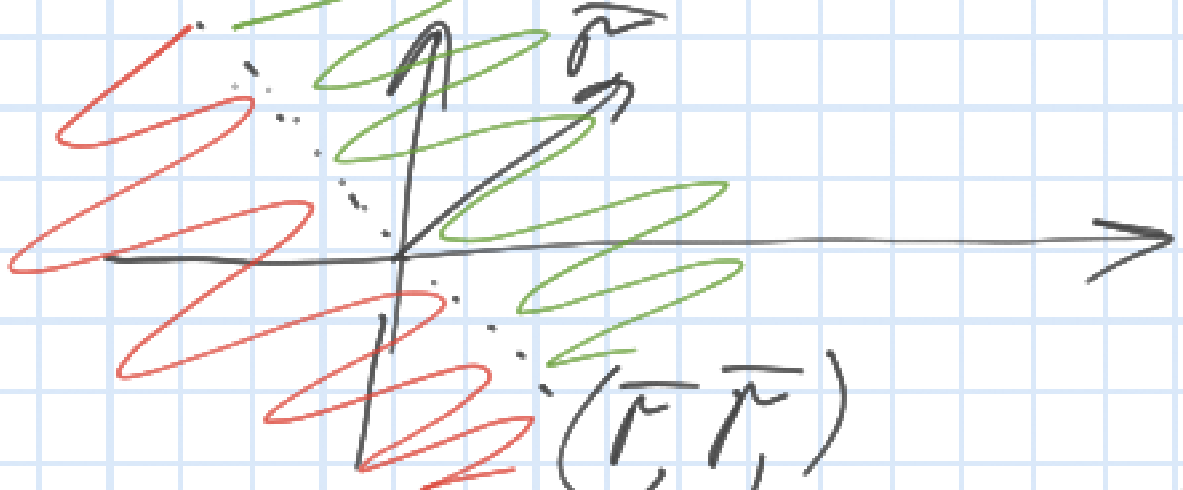
$l = l_1 + l_2$

l_1, l_2, l — проекции

$$(\bar r_1 + \bar r_2, \bar r) = l * |\bar r| = (l_1 + l_2)|\bar r| = l_1|\bar r| + l_2|\bar r| = (\bar r_1, \bar r) + (\bar r_2, \bar r)$$

□

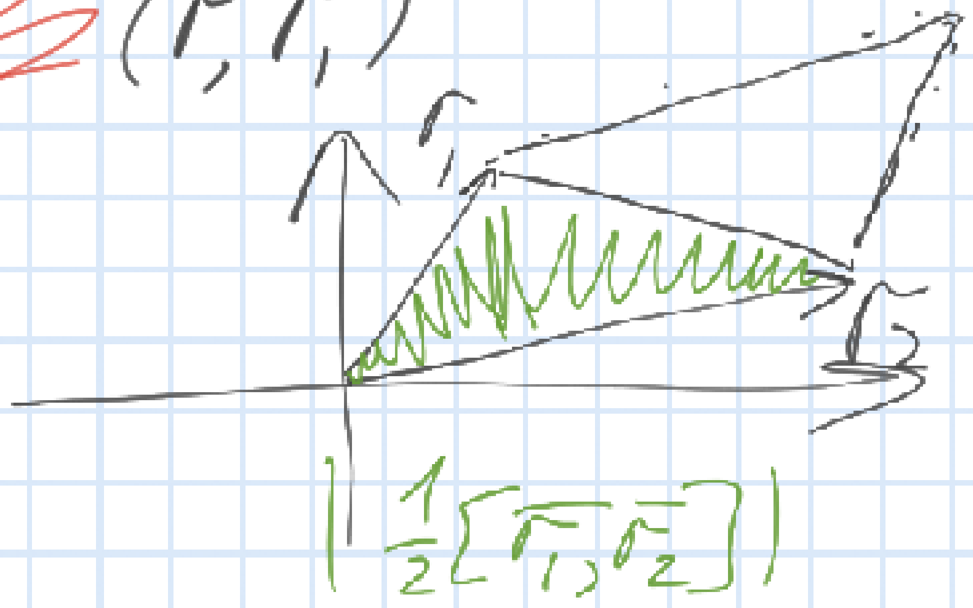
3) $(\bar r_1, \bar r_2) = (\bar r_2, \bar r_1)$



Векторное произв.

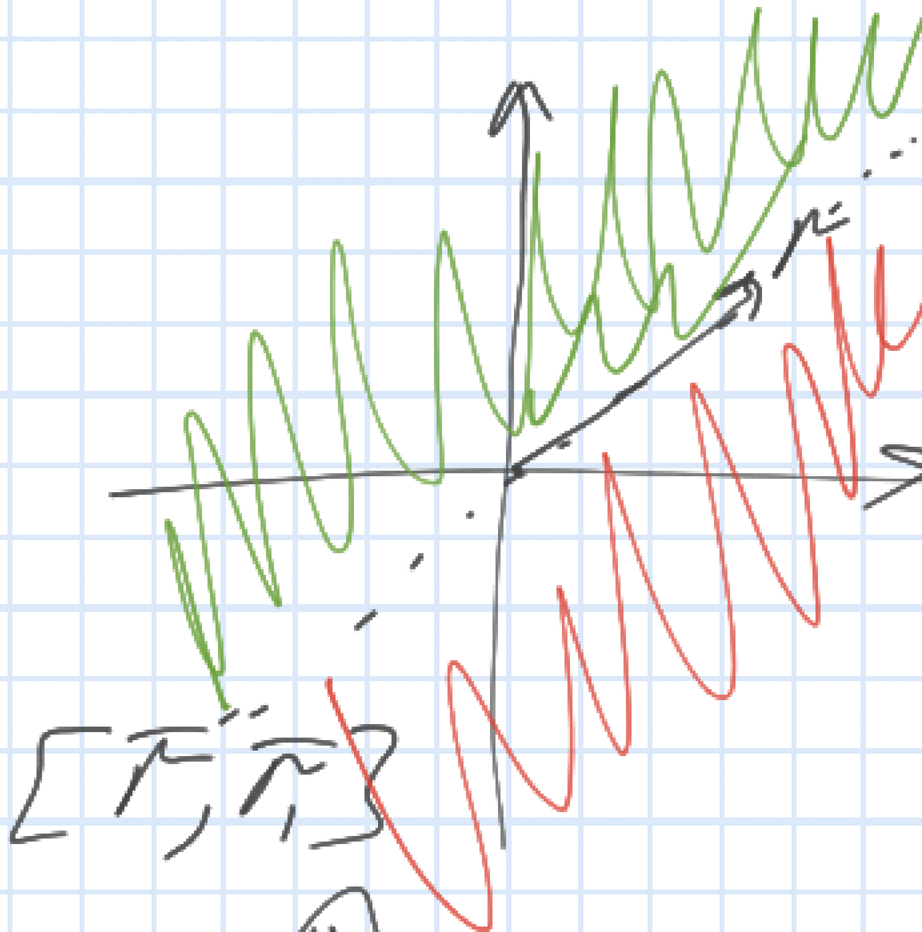
$[r_1, r_2] = x_1 y_2 - x_2 y_1 = |\bar r_1||\bar r_2| \sin \widehat{\bar r_1, \bar r_2}$

$[r_1, r_2] = \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix} = x_1 y_2 - x_2 y_1$



• — скалярное произв. (dot product)

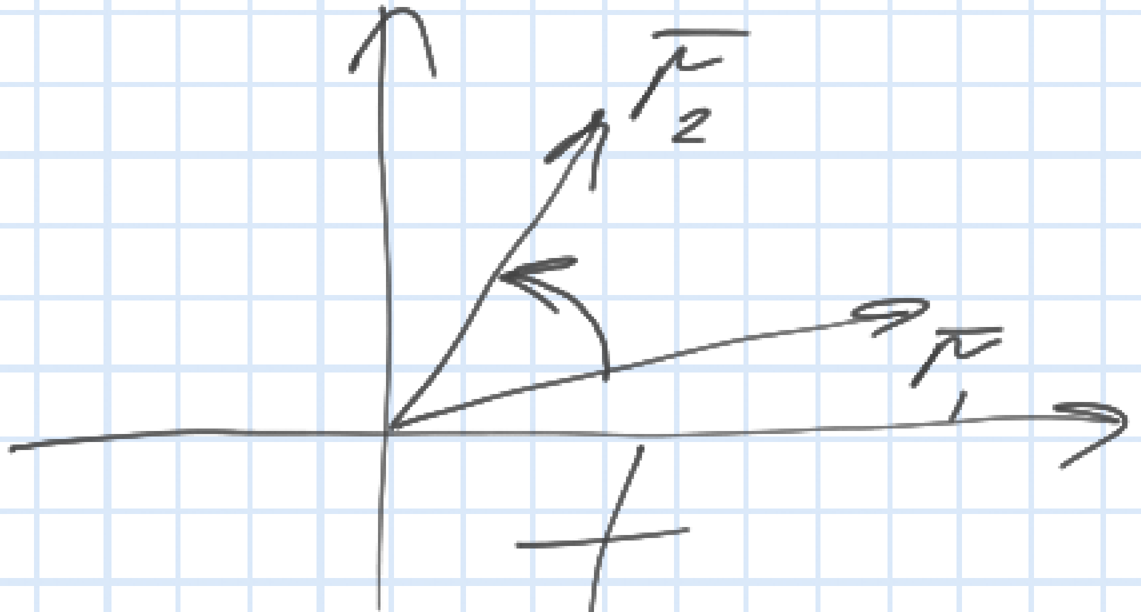
× — векторное произв. (cross product)

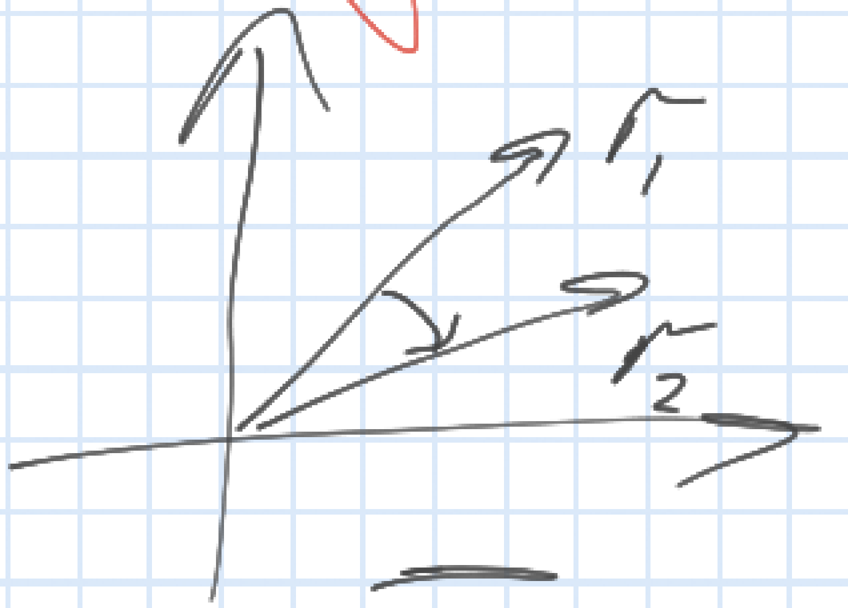


1) $[\alpha \bar r_1, \bar r_2] = \alpha [\bar r_1, \bar r_2]$

2) $[\bar r_1 + \bar r_2, \bar r] = [\bar r_1, \bar r] + [\bar r_2, \bar r]$

3) $[\bar r_1, \bar r_2] = -[\bar r_2, \bar r_1]$





Применение:

1) Площадь параллелограмма;

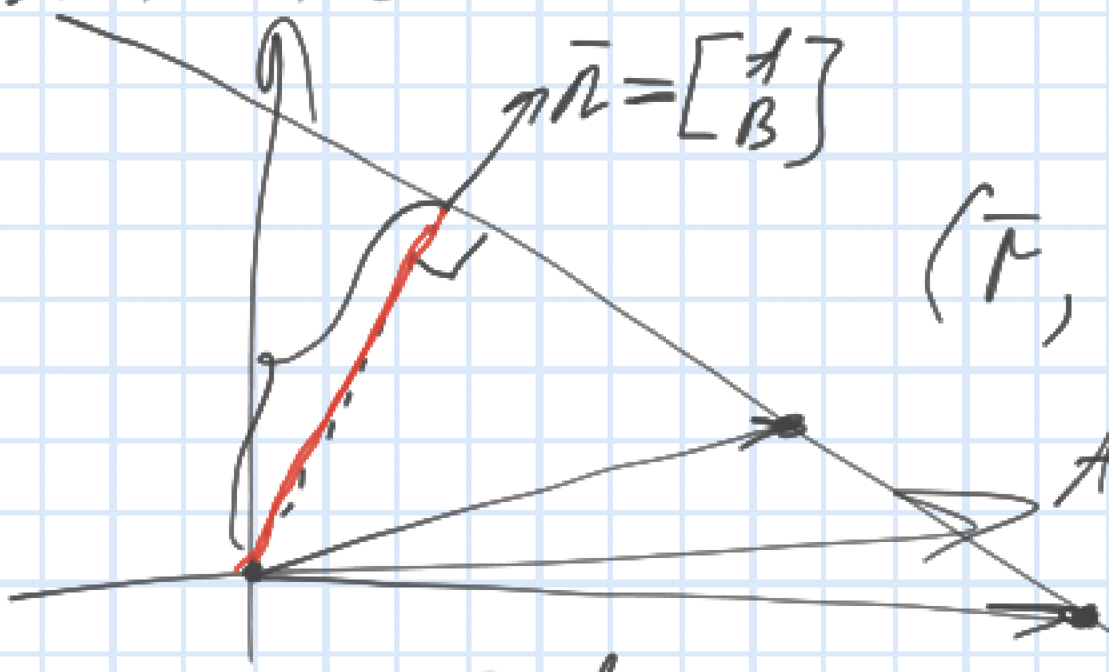
2) Проверка коллинеарности;

3) С какой стороны вектор.

$Ax + By + C = 0$ — канон. ур. пр. $\quad \underline{y = kx + b}$

$A^2 + B^2 \neq 0$

$\bar{n} = \begin{bmatrix} A \\ B \end{bmatrix}$



$(\bar{r}, \bar{n}) = -C$

$Ax + By = -C$

$Ax + By + C = 0$

$\frac{A}{|n|}x + \frac{B}{|n|}y + \frac{C}{|n|} = 0$

1] $\bar{n} = \begin{bmatrix} A \\ B \end{bmatrix}$

2] $\rho = \frac{-C}{\sqrt{A^2+B^2}}$

Точка на прям...

$\underline{Ax + By} + C = 0$

1] $x = 0$

$y = -\frac{C}{B}$

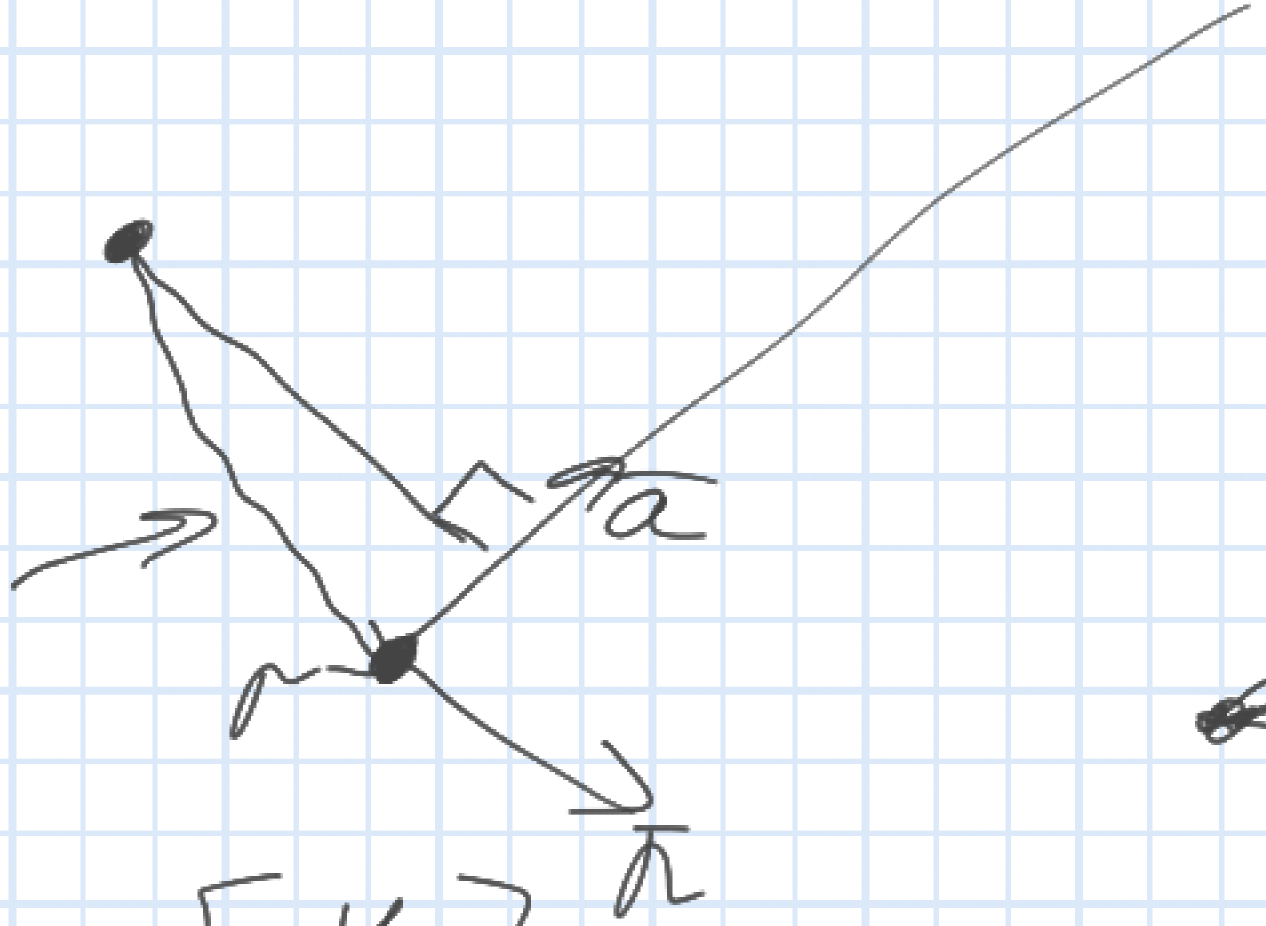
2] $x = -\frac{C}{2A}$

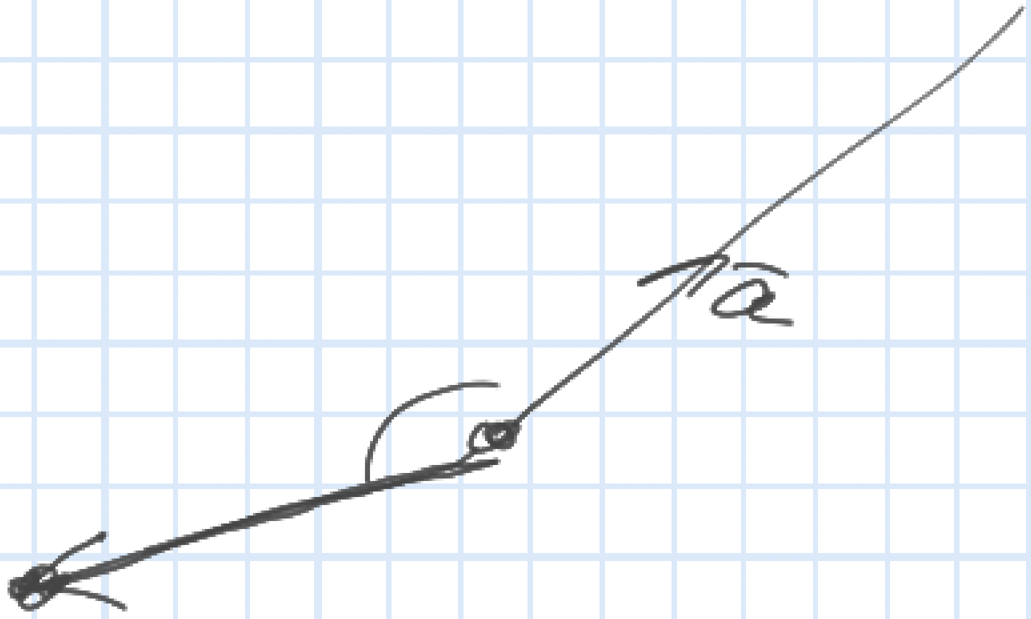
$y = -\frac{C}{2B}$

3] $x = -\frac{C}{A+B}$ $\quad -\frac{CA}{A+B} - \frac{CB}{A+B} + C = 0$

$y = -\frac{C}{A+B}$

4] $\boxed{\begin{aligned} x &= -\frac{CA}{A^2+B^2} \\ y &= -\frac{CB}{A^2+B^2} \end{aligned}}$





$$\bar{n} = \begin{bmatrix} -y_a \\ x_a \end{bmatrix}$$

$$\bar{e}_n = \frac{\bar{n}}{|\bar{n}|}$$

$$\rho = |(\bar{r}, \bar{e}_n)|$$