07/03/2024   Essential Statistics & Maths   Name:_____
                              Practice Test

I. Multiple Choice. For each question below, circle the best answer. (4 points each – 20 points total)

1. Consider the random experiment of rolling a fair die 6 times and recording the face value on each of the 6 rolls (a sample outcome of this random experiment is 1,5,4,3,3,6). The sample space for this random experiment consists of how many possible outcomes?

(a) 720   (b) 36   (c) 46656   (d) 12   (e) 100000   (f) None of these

2.   If Z has a standard normal distribution then the value of c (rounded to four decimal places) such that $P(Z \leq c) = .45$ is:

(a) 0.3264   (b) 0.1257   (c) 0.6736   (d) −0.1257   (e) None of these

3. If the random variable X is normally distributed with $\mu = 15$ and $\sigma = 3$, then the z-score for $X = 10.5$ is

(a) −1.5   (b) 1.5   (c) 0.9332   (d) 5.5   (e) None of these

4. Four cards are drawn <u>with</u> replacement from a well-shuffled standard deck of cards. The probability that all four cards are diamonds is:

(a) $\dfrac{13}{52}$ (b) $\left(\dfrac{13}{52}\right)^4$ (c) $\dfrac{13}{52} \cdot \dfrac{12}{51} \cdot \dfrac{11}{50} \cdot \dfrac{10}{49}$ (d) $\dfrac{13}{52} \cdot \dfrac{12}{52} \cdot \dfrac{11}{52} \cdot \dfrac{10}{52}$ (e) None of these

5. If Z has a standard normal distribution, then $P(-2.15 \leq Z) =$ (answer rounded to four decimal places):

(c) 0.9134   (a) 0.0158   (b) 0.4912   (c) 1.00   (d) 0.9842   (e) None of these

II. Probability   Two fair dice are rolled. Find the probability that the outcome is doubles (i.e. two ones, two twos, etc.) **or** that the sum of the dice is 4. You must show all work (including how you are using the laws of probability) for full credit. Hint: Try drawing a "matrix" that represents all possible outcomes. (10 points)

III. The Binomial Distribution. A multiple choice test consists of 20 questions and each question has 5 answers. Suppose a student randomly guesses the answer to every question. Please answer the following questions. (12 points total)

(a) Let the random variable X be the number of questions the student answers correctly. Then X has a binomial distribution (yes, I am telling you this!). For this binomial distribution, clearly identify the number of trials, what constitutes a "success" on each trial, and what is the probability of "success" for each trial. (4 points)

(b) Find the probability that the student gets a C or better on the test (i.e. the student answers 14 or more questions correctly). Clearly indicate how you use your calculator to solve this problem. (5 points)

(c) On average, how many questions will the student answer correctly? (3 points)

IV. **Discrete Probability Distributions and Mathematical Expectation.** A local club has decided to raise money by holding a "raffle." It will sell 100 tickets, 1 of which will win $50.00, 5 of which will win $10.00, and 10 of which will win $5.00 (the rest will not win any money). Suppose you buy a ticket for this raffle. Let the random variable
X denote the amount your ticket is worth (i.e. how much you can win from the ticket). Please answer the following: (8 points total)

| X - Amount a Ticket is Worth	$0.00	$5.00	$10.00	$50.00
$P(X)$				

(a) Please determine the probability distribution for X (place your answers in the table above). (4 points)

(b) What is the expected amount you can win if you purchase one ticket (i.e. what is the average value of a ticket)? Clearly show your work or indicate how you use your calculator to reach your answer. (4 points)

(c) **BONUS!** If the club sells the tickets for $4.00 each, all tickets are sold, and all prizes are claimed, then how much money will the club make from this raffle? (4 points)

V. **The Standard Normal Distribution.** Suppose Z is a standard normal random variable. Please answer the following. (11 points total)

(a) Find $P(Z < -1.25)$ and clearly graph this probability on the graph provided. (5 points)

Graph for part (a)
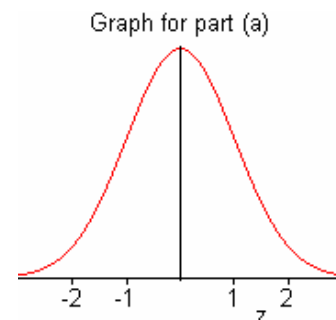
(b) Find a positive number, c, so that $P(-c < Z < c) = 0.75$. Show the numbers $-c$, c and the probability on the graph provided. (6 points)

Graph for part (b)
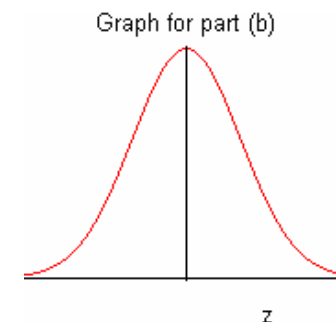
VI.   The Normal Distribution. Assume that the systolic blood pressure of adult Americans is normally distributed with a mean of 120 millimeters of mercury (mm Hg) and a standard deviation of 5.6 mm Hg. Use this information to answer the following questions. (24 points total)

(a) Draw a nice graph of the distribution of systolic blood pressure. On the graph you should show values $\pm 1, \pm 2$, and ± 3 standard deviations from the mean. Clearly label your axis. (6 points)

(b) Let X denote the systolic blood pressure of a randomly chosen American adult. Find the probability that this person's systolic blood pressure is between 110 and 115 mm Hg. Please write the probability using the random variable X and represent this probability on your graph above. You should clearly indicate any calculator functions (including inputs) you use to make this computation. (6 points)

(c) A z-score of -1.45 would correspond to what systolic blood pressure? (4 points)

(d) We will say a person has high systolic blood pressure if their blood pressure falls within the top 5% of this distribution. What is the cutoff point (i.e. the lowest blood pressure a person can have) to be considered among those with high blood pressure? Clearly show this cutoff point and the associated probability on your graph above. (8 points)