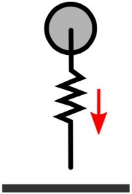
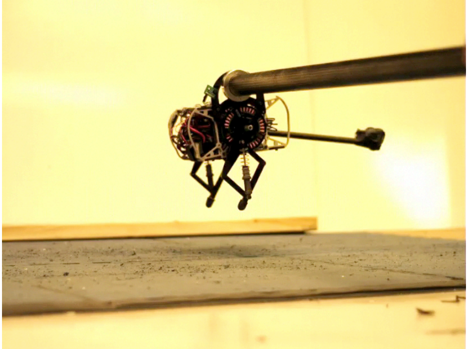
Controlled 1DOF vertical hopper (VH)



- From De and Koditschek [2015b],



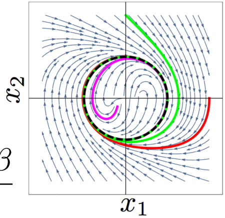
- Phase-energy EoM (De and Koditschek [2015a])

$$\dot{a} = \cos(\psi)u, \qquad \dot{\psi} = \omega - \frac{1}{\omega a}\sin(\psi)u$$

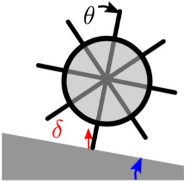
- Oscillatory energization, natural damping

$$u = (k - a\beta)\cos(\psi)$$

- On average (De and Koditschek [2015a]) $\dfrac{da}{d\psi} \overset{\text{avg}}{=} \dfrac{k - a\beta}{2}$



Controlled 1DOF Active Rimless Wheel

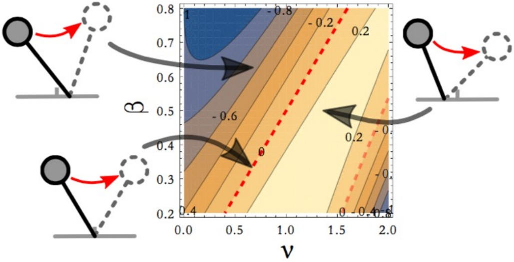


- Rimless wheel (McGeer [1990])
- Add liftoff impulse
- *Simplistic view:*

$$F(a_{\mathrm{LO}}) = a_{\mathrm{LO}} + \gamma$$
$$R(a_{\mathrm{TD}}) = \delta a_{\mathrm{TD}},\ 0 < \delta < 1$$
$$P(a) = \delta(a + \gamma)$$

- Equivalent to stepping (Raibert [1986])

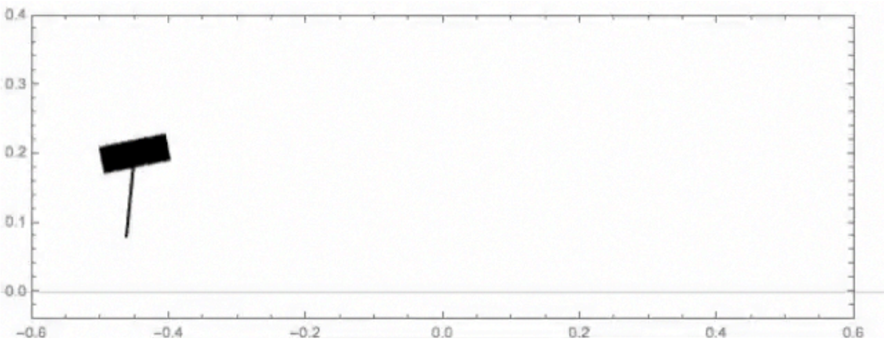
From De and Koditschek [2015a],



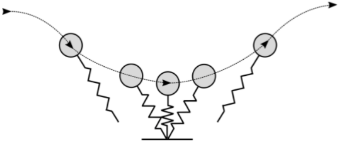
Bhounsule et al. [2012]

SLIP as a composition

Clearly works in practice



SLIP as a composition—verification



- Does SLIP anchor a VH?

- i.e. the dynamics are "the same" on an invariant (attracting) submanifold—Full and Koditschek [1999]

- Radial stance dynamics: $m\ddot{r} = -\nabla\varphi(r) - g\cos\theta - r\dot{\theta}^2$

- Say $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} := \begin{bmatrix} r \\ \dot{r} \end{bmatrix}$

- Then

$$f(x) = \begin{bmatrix} x_2 \\ -\nabla\varphi(x_1) - g\cos\theta - x_1\dot{\theta}^2 \end{bmatrix} \qquad f_{\mathrm{VH}}(x) = \begin{bmatrix} x_2 \\ -\nabla\varphi(x_1) - g \end{bmatrix}$$

- Clearly $f\big|_{\theta=0,\dot{\theta}=0} = f_{\mathrm{VH}}$
  …but not otherwise

- New research: $\bar{f} \sim \bar{f}_{\mathrm{VH}}$ —De and Koditschek [2015a]

Summary

- SLIP can be decomposed
- A vertical hopper can be controlled by active damping
- A rimless wheel can be controlled with active liftoff impulse
- SLIP as a composition empirically works
- But exact anchoring cannot be shown in general