Nathan Stouffer October 9, 2018

CSCI 246 Problem 1-1

Collaborators: *Kevin Browder*

Section 3.1, Problem 29

<u>Definitions</u>

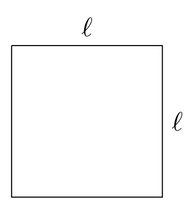
Rectangle: a plane figure with four straight sides and four right angles.

Square: a plane figure with four equal straight sides and four right angles.

(a) Original statement: $\exists x\ s.t.\ \text{Rect}(x) \wedge \text{Square}(x)$

Rewritten statement: There is a geometric figure that is both a square and a rectangle.

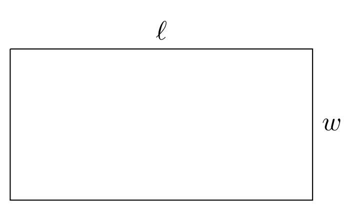
This statement is true because, by the definitions of a square and a rectangle, the following shape is a square and a rectangle with side lengths ℓ.



(b) Original statement: $\exists x\ s.t.\ \text{Rect}(x) \wedge \sim\text{Square}(x)$

Rewritten statement: There is a geometric figure that is both a rectangle and not a square.

This statement is true because, by the definitions of a square and a rectangle, the following shape is a rectangle but not a square. It has side lengths ℓ and w.



(c) Original statement: $\forall x$, $\text{Square}(x) \implies \text{Rect}(x)$

Rewritten statement: Every square is a rectangle.

This statement is true because the definition of a square satisfies the definition of a rectangle.

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CSCI 246 Problem 1-2

Collaborators: *Kevin Browder*

Section 3.2, Problem 28

Claim: All occurrences of the letter u in $Discrete\ Mathematics$ are lowercase.

Equivalent Statement: If a letter in the phrase $Discrete\ Mathematics$ is a u, then it is lowercase. Background: Truth table

capital U	Claim	Equivalent Statement
T	F	F
F	T	T

Proof: An equivalent statement to the claim is as follows: if a letter in the phrase $Discrete\ Mathematics$ is a u, then it is lowercase. This phrase is true for every letter in $Discrete\ Mathematics$.
Therefore, all occurrences of the letter u in $Discrete\ Mathematics$ are lowercase.

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CSCI 246 Problem 1-3

Collaborators: *Kevin Browder*

Section 3.2, Problem 47

Original statement: The absence of error messages during translation of a computer program is only a necessary and not a sufficient condition for reasonable [program] correctness.

Rewritten statement: The absence of error messages during translation of a computer program is required, but does not prove reasonable [program] correctness.

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CSCI 246 Problem 1-4

Collaborators: *Kevin Browder*

Section 3.4, Problem 34

1. If a writer understands human nature, then they are clever.
2. If a person cannot stir the human heart, then they are not a true poet.
3. If a person is Shakespeare, that person wrote Hamlet.
4. If a writer does not understand human nature, then that writer cannot stir the human heart.
5. If a person is not a true poet, then that person did not write Hamlet.

Nathan Stouffer October 9, 2018

CSCI 246 Problem 1-5

Collaborators: *Kevin Browder*

Question: Is Big-O notation an equivalence relation?

Claim: Big-O notation is reflexive and transitive but not symmetric. Therefore, it is not an equivalence relation.

<u>Definitions</u>
Big-O notation: For arbitrary functions $f(x)$ and $g(x)$, $f(x)$ is $O(g(x))$ if $\exists c, n_o \ s.t. \ f(x) \leq c * g(x)$

An equivalence relation must be reflexive, symmetric, and transitive. This is evaluated for Big-O notation in the following points. Let Big-O notation be represented by the relation O.

(a) Sub-claim: O is reflexive.
A relation is reflexive if $f(x) \ O \ f(x)$.
Proof: If $\exists c, n_o \ s.t. \ f(x) \leq c * f(x) \implies f(x)$ is $O(f(x))$.
Let $c = 1$ and $n_o = 1$, then the statement reads: $\forall x > 1$, $f(x) \leq 1 * f(x) \implies f(x)$ is $O(f(x))$. The left side of the relation must be true because $f(x) = f(x)$.
Therefore, O is reflexive.

(b) Sub-claim: O is not symmetric.
A relation is symmetric if $f(x) \ O \ g(x)$ and $g(x) \ O \ f(x)$.
Proof: We will find functions f and g that make O not symmetric.
Let $f(x) = x$ and $g(x) = x^2$.
Then f is $O(g)$ by letting $c = 1$ and $n_o = 2$. Then the statement would read $\forall x > 2$, $x \leq 1 * x^2 \implies x$ is $O(x^2)$. The left side of the relation must be true because x^2 grows faster than x and letting $x = 1$ means that $f(2) = 2 \leq g(2) = 2^2 = 4$.
However, $g(x) \neq O(f(x))$ because $g(x) = x * f(x)$. But, if $g = O(f)$, then $g \leq c * f$. Regardless of what c is chosen, x will always be larger at some point.
Therefore, O is not symmetric.

(c) Sub-claim: O is transitive.
A relation is transitive if $f \ O \ g$ and $g \ O \ h$, then $f \ O \ h$.
Proof: Given that $f \leq g$ and $g \leq h$, it follows that $f \leq h$.
Therefore, O is transitive.

Therefore, Big-O notation is not an equivalence relation. Q.E.D.

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CSCI 246 Problem 1-6	
Collaborators: *Kevin Browder*	

Definitions

Big-O notation: $f = O(g)$ if $\exists c, n_o$ $s.t.$ $\forall x > n_o$, $f(x) \leq c * g(x)$.

(A) Statement: The function $f(x) = 2x^2 = O(4x)$.

Claim: $2x^2 \neq O(4x)$.

Proof. For $2x^2 = O(4x)$, $2x^2 \leq c * 4x \iff \frac{x}{2} \leq c$.
There is no c for which this statement can be true for $x > n_o$.
Therefore, $2x^2 \neq O(4x)$. □

(B) Statement: The function $g(x) = 3x = \Omega(x)$.

Background: If $f = O(g)$, then $g = \Omega(f)$.

Claim: $3x = \Omega(x)$.

Proof. If $x = O(3x)$, then $3x = \Omega(x)$.
$x = O(3x)$ if $\exists c, n_o$ $s.t.$ $x \leq c * 3x$
Then $x \leq c * 3x \iff 1 \leq c * 3 \iff c \geq \frac{1}{3}$.
Let $c = \frac{1}{3}$ and $n_o = 1$, then $x = O(3x)$ because $x \leq \frac{1}{3} * 3x = x$ for $x > n_o$
Therefore, $3x = \Omega(x)$. □

(C) Statement: The function $h(x) = x^2 + log(x) = O(x^2)$.

Claim: $h(x) = x^2 + \log(x) = O(x^2)$.

Proof. We must find a c such that the following is true: $x^2 + \log(x) \leq c * x^2$.
$x^2 + \log(x) \leq x^2 + x^2 = 2x^2$.
Let $c = 2$ and $n_o = 1$, then the statement reads $x^2 + \log(x) \leq 2 * x^2$.
Therefore, $x^2 + \log(x) = O(x^2)$. □

(D) Statement: The function $k(x) = 5x^2 + x = \Theta(x)$.

Background: $f = \Theta(g)$ if $\exists c, n_o > 0$ $s.t.$ $\frac{1}{c} * g \leq f \leq c * g$.

Claim: $5x^2 + x \neq \Theta(x)$

Proof. $5x^2 + x = \Theta(x)$ if $5x^2 + x \leq x$
$5x^2 + x \leq 5x^2 + x * x = 5x^2 + x^2 = 6x^2 \nleq c * x$
Therefore, $5x^2 + x \neq \Theta(x)$. □

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CSCI 246 Problem 1-7

Collaborators: *Kevin Browder*

(A) Utah and New Mexico can have the same color despite sharing a vertex because they do not share a border.

(B) Michigan does not satisfy the conditions for the four color theorem because it is a divided country.

(C) We omit Alaska and Hawaii when constructing a four color map of the US because they do not share borders with any other states.

(D) The following statement is false: Four colors are necessary to color all maps.

Nathan Stouffer October 9, 2018

CSCI 246 Problem 1-8

Collaborators: *none*

Charles Babbage and Computer Science

References to online resources are provided as footnotes.

Charles Babbage was a British mathematician in the 1800s who is famous for inventing a calculating machine and is credited as the father of the digital computer. According to the Encyclopedia Britannica, Babbage gave "is credited with having conceived the first automatic digital computer." [1] Babbage's ideas are the foundation of the computing machines we use today.

[1] https://www.britannica.com/biography/Charles-Babbage