Nathan Stouffer October 22, 2018

CSCI 246 Problem 1-2

Collaborators: *none*

Prove $A(n) = \sum_{i=0}^{n} F_i = F_{n+2} - 1$.

Background: $F_0 = F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ $\forall n \geq 2$.

Proof. We will prove this by mathematical induction.

Base case for $n = 2$:
$\sum_{i=0}^{n} F_i = \sum_{i=0}^{2} F_i = F_0 + F_1 + F_2 = 1 + 1 + 2 = 4$
$F_{n+2} - 1 = F_{2+2} - 1 = F_4 - 1 = (F_3 + F_2) - 1 = ((F_2 + F_1) + (F_0 + F_1)) - 1 = (F_1 + 2 * (F_0 + F_1)) - 1 = (1 + 2 * (1 + 1)) - 1 = (1 + 2 * 2) - 1 = 4$

We will now assume $A(n)$ is true and use this to prove $A(n+1)$:

$$
\begin{aligned}
\sum_{i=0}^{n+1} F_i &= F_{n+1} + \sum_{i=0}^{n} F_i \\
&= F_{n+1} + (F_{n+2} - 1) && \text{by inductive assumption} \\
&= (F_{n+1} + F_{n+2}) - 1 && \text{by associativity} \\
&= F_{n+3} - 1 && \text{by definition of Fibonacci Sequence} \\
A(n+1) &= F_{(n+1)+2} - 1
\end{aligned}
$$

Therefore, by inductive assumption, $A(n)$ is proved to be true. $\square$

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CSCI 246 Problem 1-3

Collaborators: *none*

Section 5.5, Problem 42

Prove $A(n) = \prod_{i=1}^{n} (c * a_i) = c^n \prod_{i=1}^{n} a_i$. PROOF. We will prove this by mathematical induction.

Base case for $n = 2$:
$\prod_{i=1}^{n} (c * a_i) = \prod_{i=1}^{2} (c * a_i) = (c * a_1) * (c * a_2) = c^2 * a_1 * a_2 = c^2 \prod_{i=1}^{2} a_i$

We will now assume $A(n)$ is true and use this to prove $A(n+1)$:

$$\begin{aligned}
A(n+1) &= \prod_{i=1}^{n+1} (c * a_i) \\
&= (c * a_{n+1}) \prod_{i=1}^{n} (c * a_i) \\
&= (c * a_{n+1})(c^n \prod_{i=1}^{n} a_i) && \text{by inductive assumption} \\
&= c^n * c * a_{n+1} \prod_{i=1}^{n+1} a_i && \text{by associativity} \\
A(n+1) &= c^{n+1} \prod_{i=1}^{n+1} a_i
\end{aligned}$$

Therefore, by inductive assumption, $A(n)$ is proved to be true. □

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CSCI 246 Problem 1-4

Collaborators: *none*

Section 5.5, Problem 7

$u_k = k * u_{k-1} - u_{k-2} \ \forall k \geq 3$

$u_1 = 1$
$u_2 = 1$
$u_3 = 3 * u_{3-1} - u_{3-2} = 3 * u_2 - u_1 = 3 * 1 - 1 = 2$
$u_4 = 4 * u_{4-1} - u_{4-2} = 4 * u_3 - u_2 = 4 * 2 - 1 = 7$

Section 5.5, Problem 32

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	CSCI 246 Problem 1-5	
Collaborators: *none*		

5.1 Euler's Formula: $e^{i\theta} = cos(\theta) + i * sin(\theta)$

5.2 Cube: $F_0 = 6$, $E_0 = 12$, $V_0 = 8$
Cutting a tetrahedron off each corner:
The amount of faces increases by the number of vertexes: $F_1 = F_0 + V_0 = 6 + 8 = 14$
The amount of edges increases by 3 times the number of vertexes: $E_1 = E_0 + 3V_0 = 12 + 3*8 = 12 + 24 = 36$
The amount of vertexes is 3 times the number of vertexes: $V_1 = 3V_0 = 3 * 8 = 24$

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CSCI 246 Problem 1-6

Collaborators: *none*

Leonhard Euler

References to online resources are provided as footnotes.

Leonhard Euler was a Swiss mathematician in the 1700s who is famous for discovering many theorems in number theory such as $e^{i\theta} = cos(\theta) + i * sin(\theta)$ and $F - E + V = 2$. According to the Encyclopedia Britannica, Euler "threw new light on nearly all parts of pure mathematics. " [1] Euler's ideas continue to contribute to modern theorems in mathematics and computing.

[1] `https://www.britannica.com/biography/Leonard-Euler`