Nathan Stouffer September 18, 2018

CSCI 246 Problem 1-1

Collaborators: *Kevin Browder*

Section 8.1, Problem 20

Let $A = \{-1, 1, 2, 4\}$ and $B = \{1, 2\}$. For all $(x, y) \in A \times B$:

$$x \ R \ y \iff |x| = |y| \text{ and}$$
$$x \ S \ y \iff x - y \text{ is even}$$

The sets $A \times B$, R, S, $R \cup S$, and $R \cap S$ are defined as:

$A \times B : \{(-1, 1), (-1, 2), (1, 1), (1, 2), (2, 1), (2, 2), (4, 1), (4, 2)\}$
$R : \{(-1, 1), (1, 1), (2, 2)\}$
$S : \{(-1, 1), (1, 1), (2, 2), (4, 2)\}$
$R \cup S : \{(-1, 1), (1, 1), (2, 2), (4, 2)\}$
$R \cap S : \{(-1, 1), (1, 1), (2, 2)\}$

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Section 8.2, Problem 21

Let $x = \{a, b, c\}$. The power set of x, $P(x)$, is $P(x) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$. For all sets $A, B \in P(x)$, the relation L is defined as: $A \ L \ B \iff n_a < n_b$ where n_k represents the number of elements in a set K.

Claim: The relation L is transitive but not reflexive and not symmetric.

1. L is reflexive if and only if $\forall n_a \in P(x)$: $n_a \ L \ n_a$. $n_a < n_a$ is never true because n_a is an integer and an integer has only one value. Therefore, it cannot be less than itself and L is not reflexive.
2. L is symmetric if and only if $\forall n_a, n_b \in P(x)$: $n_a \ L \ n_b$ and $n_b \ L \ n_a$. If $n_a \ L \ n_b$ and $n_b \ L \ n_a$ is true, then $n_a < n_b$ and $n_b < n_a$. Because n_a and n_b are both integers, only one of $n_a < n_b$ and $n_b < n_a$ can be true. Therefore, L is not reflexive.
3. L is transitive if and only if $\forall n_a, n_b, n_c \in P(x)$: if $n_a \ L \ n_b$ and $n_b \ L \ n_c$, then $n_a \ L \ n_c$. Using the definition of L, $n_a \ L \ n_b \iff n_a < n_b$ and $n_b \ L \ n_c \iff n_b < n_c$, which then means that $n_a < n_b < n_c$ and it follows that $n_a < n_c \iff n_a \ L \ n_c$. Therefore, L is transitive.

Therefore, L is transitive but not reflexive and not symmetric.

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Section 8.3, Problem 23

For all $m, n \in \mathbb{Z}$, $m\ R\ n \iff 4 \mid (m^2 - n^2)$

(a) Claim: The relation R is an equivalence relation if and only if it is reflexive, symmetric, and transitive.

1. R is reflexive if and only if $\forall m$: $m\ R\ m$. $m\ R\ m \iff 4 \mid (m^2 - m^2) \implies 4 \mid 0 \implies 0$. Therefore, R is reflexive.
2. R is symmetric if and only if $\forall m, n$: $m\ R\ n$ and $n\ R\ m$. $m\ R\ n \iff 4 \mid (m^2 - n^2) \implies 4 \mid -1 * (n^2 - m^2) \implies 4 \mid (n^2 - m^2) \iff n\ R\ m$. Therefore, R is symmetric.
3. R is transitive if and only if $\forall m, n, p \in \mathbb{Z}$: $m\ R\ n$ and $n\ R\ p$, then $m\ R\ p$. $(m\ R\ n) + (n\ R\ p) \iff (4 \mid (m^2 - n^2)) + (4 \mid (n^2 - p^2)) \implies \dfrac{m^2 - n^2}{4} + \dfrac{n^2 - p^2}{4} \implies \dfrac{m^2 - \cancel{n^2} + \cancel{n^2} - p^2}{4} \implies \dfrac{m^2 - p^2}{4} \implies 4 \mid (m^2 - p^2) \iff m\ R\ p$. Therefore, R is transitive.

(b) $m\ R\ n$ has distinct equivalence classes. If $a, b \in \mathbb{Z}$ and $c \in \{0, 1, 2, 3\}$, then a can be distinctly written as $4b + c$: $4b$, $4b + 1$, $4b + 2$, or $4b + 3$.

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CSCI 246 Problem 1-4

Collaborators: *Kevin Browder*

Section 2.1, Problem 20

Claim: $p \vee q \neq p \wedge q$

p q	$p \vee q$	$p \wedge q$
T T	T	T
T F	T	F
F T	T	F
F F	F	F

The truth tables for $p \vee q$ and $p \wedge q$ are different, therefore $p \vee q \neq p \wedge q$.

Section 2.1, Problem 22

Claim: $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

p q r	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T T T	T	T
T T F	T	T
T F T	T	T
F T T	F	F
T F F	F	F
F T F	F	F
F F T	F	F
F F F	F	F

The truth tables for $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ are the same, therefore $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$.

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CSCI 246 Problem 1-5

Collaborators: *Kevin Browder*

Section 2.3, Problem 40

Assumptions:

1. Sharky was killed by one of his henchmen (Socko, Fats, Left, and Muscles)
2. Only one of the following statements is true:
   (a) Socko: Lefty killed Sharky
   (b) Fats: Muscles didn't kill Sharky
   (c) Lefty: Muscles was shooting craps with Socko when Sharky was knocked off
   (d) Muscles: Lefty didn't kill Sharky

Notice:

1. Either Socko or Muscles must be telling the truth because their statements contradict each other.
2. Fats and Lefty are lying because there is only one truthful statement.
3. Fats lied when he said that Muscles didn't kill Sharky.

Therefore, Muscles killed Sharky.

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CSCI 246 Problem 1-6

Collaborators: *Kevin Browder*

Section 4.5, Problem 29

Claim: For $a, b, c \in \mathbb{Z}$, if $a \mid b$ and $a \nmid c$, then $a \nmid (b + c)$.

(a) Proof by Contraposition:

$$\begin{aligned} a \mid b \wedge a \nmid c &\implies a \nmid (b+c) \\ \neg(a \nmid (b+c)) &\implies \neg(a \mid b \wedge a \nmid c) \\ a \mid (b+c) &\implies a \nmid b \vee a \mid c \\ a \mid (b+c) \wedge a \mid b &\implies a \mid c \end{aligned}$$

$a \mid c$ is false because the Claim states that $a \nmid c$, therefore, if $a \mid b$ and $a \nmid c$, then $a \nmid (b + c)$.

(b) Proof by Contradiction: Assume the following statement is true:

If $a \mid b$ and $b \nmid c$, then $a \mid (b + c)$

Manipulating $a \mid (b + c)$: $a \mid (b + c) \implies \dfrac{b+c}{a} \implies \dfrac{b}{a} + \dfrac{c}{a}$. From the original statement, we know that $\dfrac{b}{a}$ is an integer and $\dfrac{c}{a}$ is a non-integer, meaning the sum of $d = \dfrac{b}{a} + \dfrac{c}{a}$ is a non-integer. If d is a non-integer, then $a \mid (b + c)$ is a false statement. Therefore, if $a \mid b$ and $a \nmid c$, then $a \nmid (b + c)$.

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CSCI 246 Problem 1-7

Collaborators: *none*

Carl Frederich Gauss and Computer Science

References to online resources are provided as footnotes.

Carl Frederich Gauss was a German mathematician in the 1800s who is famous for proving the Fundamental Theorem of Algebra and other number theory proofs. According to the Encyclopedia Britannica, Gauss gave "the first account of modular arithmetic." [1] Gauss' modular arithmetic is essential to solving many problems in computer science, even simple ones such as keeping track of time.

[1] `https://www.britannica.com/biography/Carl-Friedrich-Gauss`